PROBABILITY & STATISTICS

BS 1402

Contents

- Sample Spaces
- Probability
- Addition Rules of Probability
- Multiplication Rules of Probability

Basic Concepts of Probability

A **probability experiment** is a chance process that leads to well-defined results called outcomes.

An **outcome** is the result of a single trial of a probability experiment.

A **sample space** is the set of all possible outcomes of a probability experiment.

Some examples of sample spaces are:

Experiment	**Sample space**
Toss one coin	Head, tail
Roll a die	1, 2, 3, 4, 5, 6
Answer a true/false question	True, false
Toss two coins	Head-head, tail-tail, head-tail, tail-head

Example 1:

Find the sample space for rolling two dice.

Solution

Since each die can land in six different ways, and two dice are rolled, the sample space can be presented by a rectangular array, as shown in Figure 4–1. The sample space is the list of pairs of numbers in the chart.

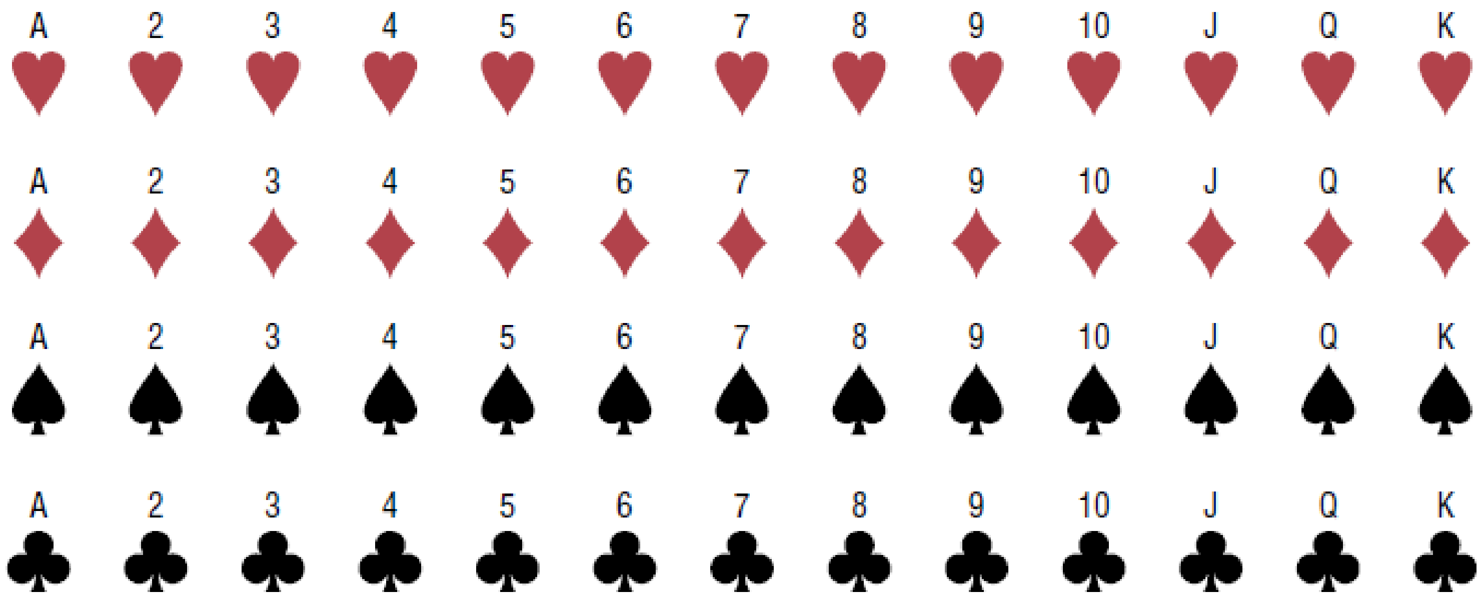
	Die 2					
Die 1	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Example 2:

Find the sample space for drawing one card from an ordinary deck of cards.

Solution

Since there are 4 suits (hearts, clubs, diamonds, and spades) and 13 cards for each suit (ace through king), there are 52 outcomes in the sample space. See Figure 4–2.



Example 3:

Find the sample space for the gender of the children if a family has three children. Use B for boy and G for girl.

Solution

There are two genders, male and female, and each child could be either gender. Hence, there are eight possibilities, as shown here.

BBB BBG BGB GBB GGG GGB GBG BGG

A **tree diagram** is a device consisting of line segments emanating from a starting point and also from the outcome point. It is used to determine all possible outcomes of a probability experiment.

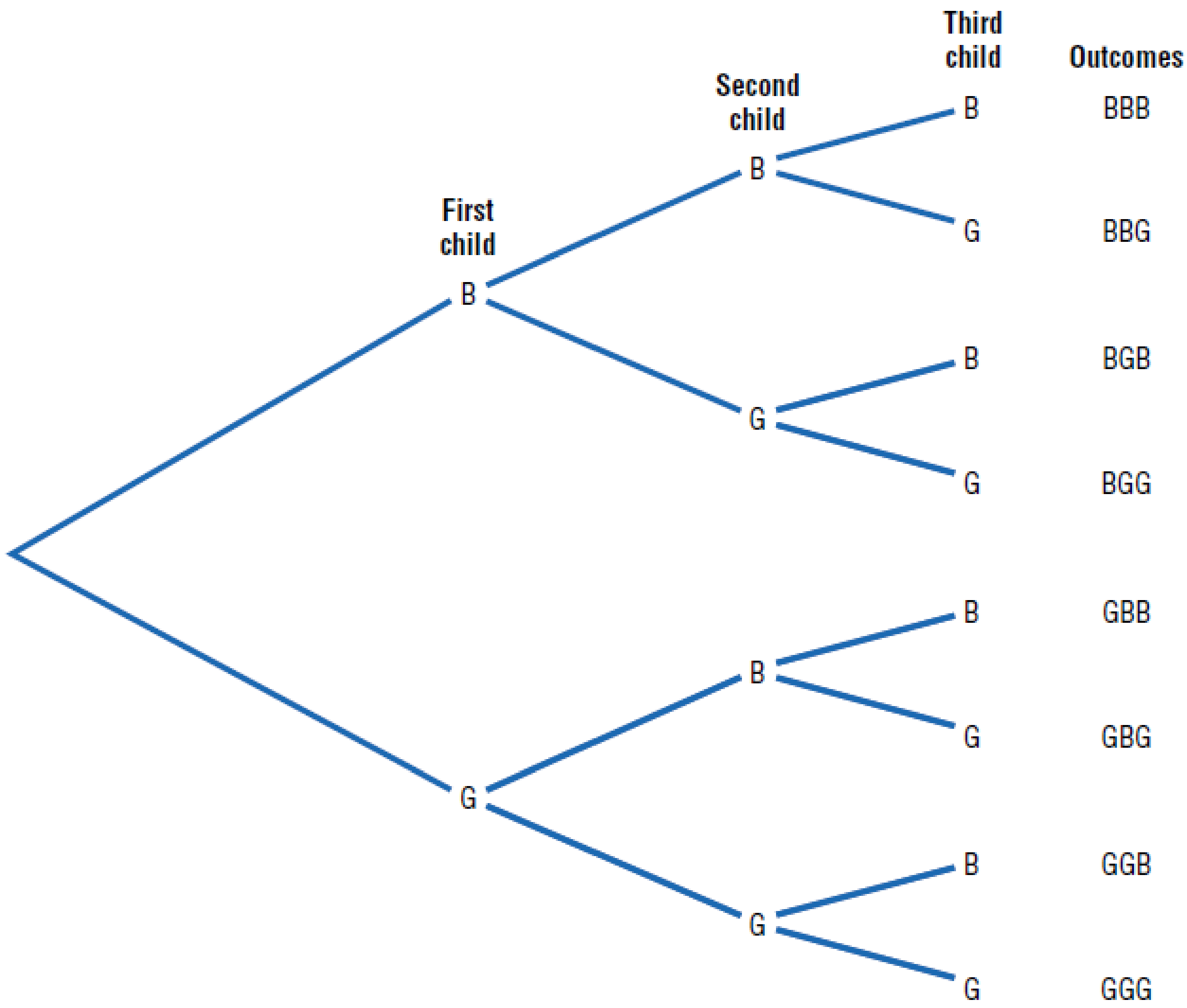
Example 4:

Use a tree diagram to find the sample space for the gender of three children in a family, as in Example 3

Solution

Since there are two possibilities (boy or girl) for the first child, draw two branches from a starting point and label one B and the other G. Then if the first child is a boy, there are two possibilities for the second child (boy or girl), so draw two branches from B and label one B and the other G. Do the same if the first child is a girl. Follow the same procedure for the third child. The completed tree diagram is shown in Figure 4–3. To find the outcomes for the sample space, trace through all the possible branches, beginning at the starting point for each one.

Example 4: (cont.)



An **event** consists of a set of outcomes of a probability experiment.

An event can be one outcome or more than one outcome. For example, if a die is rolled and a 6 shows, this result is called an *outcome*, since it is a result of a single trial. An event with one outcome is called a **simple event.** The event of getting an odd number when a die is rolled is called a **compound event,** since it consists of three outcomes or three simple events. In general, a compound event consists of two or more outcomes or simple events.

Equally likely events are events that have the same probability of occurring.

The probability of any event E is

$$\frac{\text{Number of outcomes in } E}{\text{Total number of outcomes in the sample space}}$$

This probability is denoted by

$$P(E) = \frac{n(E)}{n(S)}$$

Probabilities can be expressed as fractions, decimals, or—where appropriate—percentages. If you ask, "What is the probability of getting a head when a coin is tossed?" typical responses can be any of the following three.

"One-half."

"Point five."

"Fifty percent."[1]

These answers are all equivalent. In most cases, the answers to examples and exercises given in this chapter are expressed as fractions or decimals, but percentages are used where appropriate.

Example 5:

Find the probability of getting a red ace when a card is drawn at random from an ordinary deck of cards.

Solution

Since there are 52 cards and there are 2 red aces, namely, the ace of hearts and the ace of diamonds, $P(\text{red ace}) = \frac{2}{52} = \frac{1}{26}$.

Example 6:

If a family has three children, find the probability that two of the three children are girls.

Solution

The sample space for the gender of the children for a family that has three children has eight outcomes, that is, BBB, BBG, BGB, GBB, GGG, GGB, GBG, and BGG. (See Examples 4–3 and 4–4.) Since there are three ways to have two girls, namely, GGB, GBG, and BGG, $P(\text{two girls}) = \frac{3}{8}$.

In probability theory, it is important to understand the meaning of the words *and* and *or.* For example, if you were asked to find the probability of getting a queen *and* a heart when you were drawing a single card from a deck, you would be looking for the queen of hearts. Here the word *and* means "at the same time." The word *or* has two meanings. For example, if you were asked to find the probability of selecting a queen *or* a heart when one card is selected from a deck, you would be looking for one of the 4 queens or one of the 13 hearts. In this case, the queen of hearts would be included in both cases and counted twice. So there would be 4 +13 - 1 = 16 possibilities.
On the other hand, if you were asked to find the probability of getting a queen *or* a king, you would be looking for one of the 4 queens or one of the 4 kings. In this case, there would be 4 + 4 = 8 possibilities. In the first case, both events can occur at the same time; we say that this is an example of the *inclusive or.* In the second case, both events cannot occur at the same time, and we say that this is an example of the *exclusive or.*

Example 7:

A card is drawn from an ordinary deck. Find these probabilities.

a. Of getting a jack

b. Of getting the 6 of clubs (i.e., a 6 and a club)

c. Of getting a 3 or a diamond

d. Of getting a 3 or a 6

Example 7 (cont.):

Solution

a. Refer to the sample space in Figure 4–2. There are 4 jacks so there are 4 outcomes in event E and 52 possible outcomes in the sample space. Hence,

$$P(\text{jack}) = \tfrac{4}{52} = \tfrac{1}{13}$$

b. Since there is only one 6 of clubs in event E, the probability of getting a 6 of clubs is

$$P(6 \text{ of clubs}) = \tfrac{1}{52}$$

c. There are four 3s and 13 diamonds, but the 3 of diamonds is counted twice in this listing. Hence, there are 16 possibilities of drawing a 3 or a diamond, so

$$P(3 \text{ or diamond}) = \tfrac{16}{52} = \tfrac{4}{13}$$

This is an example of the inclusive or.

d. Since there are four 3s and four 6s,

$$P(3 \text{ or } 6) = \tfrac{8}{52} = \tfrac{2}{13}$$

This is an example of the exclusive or.

Probability Rules

There are four basic probability rules. These rules are helpful in solving probability problems, in understanding the nature of probability, and in deciding if your answers to the problems are correct.

Probability Rule 1

The probability of any event E is a number (either a fraction or decimal) between and including 0 and 1. This is denoted by $0 \leq P(E) \leq 1$.

Rule 1 states that probabilities cannot be negative or greater than 1.

Probability Rule 2

If an event E cannot occur (i.e., the event contains no members in the sample space), its probability is 0.

Probability Rule 3

If an event E is certain, then the probability of E is 1.

Probability Rule 4

The sum of the probabilities of all the outcomes in the sample space is 1.

Example 8:

When a single die is rolled, find the probability of getting a 9.

Solution

Since the sample space is 1, 2, 3, 4, 5, and 6, it is impossible to get a 9. Hence, the probability is $P(9) = \frac{0}{6} = 0$.

Example 9:

When a single die is rolled, what is the probability of getting a number less than 7?

Solution

Since all outcomes—1, 2, 3, 4, 5, and 6—are less than 7, the probability is

$$P(\text{number less than 7}) = \frac{6}{6} = 1$$

The event of getting a number less than 7 is certain.

In other words, probability values range from 0 to 1. When the probability of an event is close to 0, its occurrence is highly unlikely. When the probability of an event is near 0.5, there is about a 50-50 chance that the event will occur; and when the probability of an event is close to 1, the event is highly likely to occur.

For example, in the roll of a fair die, each outcome in the sample space has a probability of $\frac{1}{6}$. Hence, the sum of the probabilities of the outcomes is as shown.

Outcome	1		2		3		4		5		6
Probability	$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$
Sum	$\frac{1}{6}$	$+$	$\frac{1}{6}$	$+$	$\frac{1}{6}$	$+$	$\frac{1}{6}$	$+$	$\frac{1}{6}$	$+$	$\frac{1}{6} = \frac{6}{6} = 1$

Complementary Events

The **complement of an event** *E* is the set of outcomes in the sample space that are not included in the outcomes of event *E*. The complement of *E* is denoted by $\overline{E}$ (read "*E* bar").

When a die is rolled, for instance, the sample space consists of the outcomes 1, 2, 3, 4, 5, and 6. The event *E* of getting odd numbers consists of the outcomes 1, 3, and 5. The event of not getting an odd number is called the *complement* of event *E*, and it consists of the outcomes 2, 4, and 6.

Example 10:

Find the complement of each event.

a. Rolling a die and getting a 4

b. Selecting a letter of the alphabet and getting a vowel

c. Selecting a month and getting a month that begins with a J

d. Selecting a day of the week and getting a weekday

Solution

a. Getting a 1, 2, 3, 5, or 6

b. Getting a consonant (assume *y* is a consonant)

c. Getting February, March, April, May, August, September, October, November, or December

d. Getting Saturday or Sunday

Rule for Complementary Events

$$P(\overline{E}) = 1 - P(E) \quad \text{or} \quad P(E) = 1 - P(\overline{E}) \quad \text{or} \quad P(E) + P(\overline{E}) = 1$$

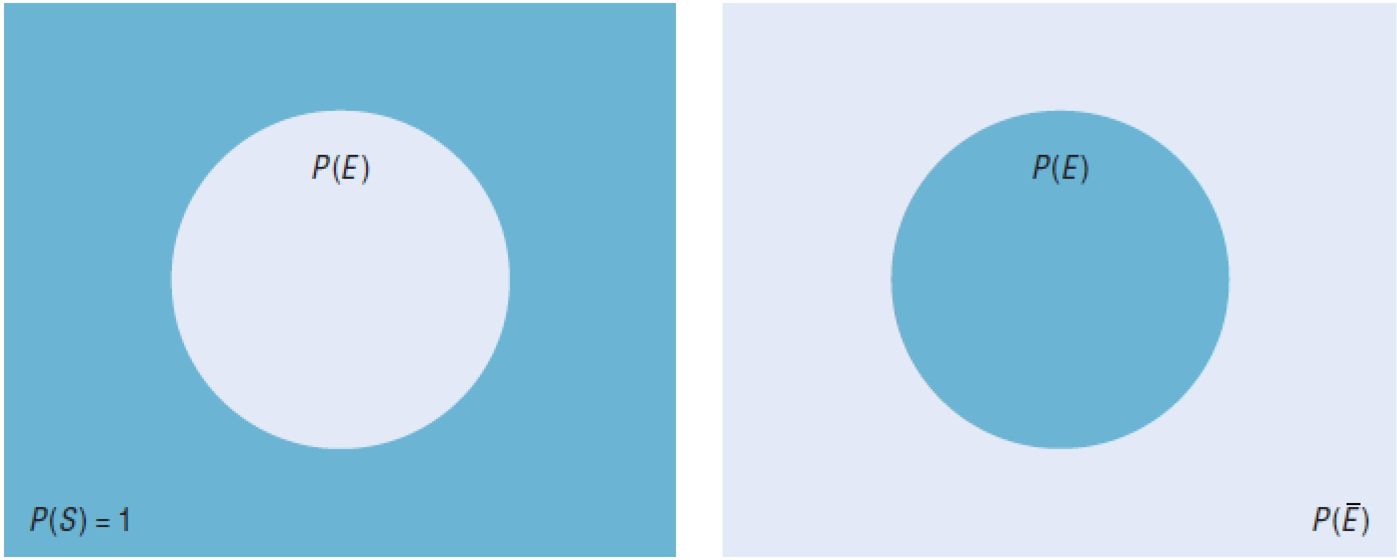
Example 11:

If the probability that a person lives in an industrialized country of the world is $\frac{1}{5}$, find the probability that a person does not live in an industrialized country.

Solution

$$\begin{aligned} P(\text{not living in an industrialized country}) &= 1 - P(\text{living in an industrialized country}) \\ &= 1 - \tfrac{1}{5} = \tfrac{4}{5} \end{aligned}$$

Probabilities can be represented pictorially by **Venn diagrams.** Figure 4–4(a) shows the probability of a simple event E. The area inside the circle represents the probability of event E, that is, $P(E)$. The area inside the rectangle represents the probability of all the events in the sample space $P(S)$.



(a) Simple probability **(b)** $P(\bar{E}) = 1 - P(E)$

The Venn diagram that represents the probability of the complement of an event $P(\overline{E})$ is shown in Figure 4–4(b). In this case, $P(\overline{E}) = 1 - P(E)$, which is the area inside the rectangle but outside the circle representing $P(E)$. Recall that $P(S) = 1$ and $P(E) = 1 - P(\overline{E})$. The reasoning is that $P(E)$ is represented by the area of the circle and $P(\overline{E})$ is the probability of the events that are outside the circle.

Example 12:

Suppose, for example, that a researcher for the American Automobile Association (AAA) asked 50 people who plan to travel over the Thanksgiving holiday how they will get to their destination. The results can be categorized in a frequency distribution as shown.

Method	**Frequency**
Drive	41
Fly	6
Train or bus	3
	50

Now probabilities can be computed for various categories. For example, the probability of selecting a person who is driving is $\frac{41}{50}$, since 41 out of the 50 people said that they were driving.

Given a frequency distribution, the probability of an event being in a given class is

$$P(E) = \frac{\text{frequency for the class}}{\text{total frequencies in the distribution}} = \frac{f}{n}$$

Example 12 (cont.):

In the travel survey just described, find the probability that a person will travel by airplane over the Thanksgiving holiday.

Solution

$$P(E) = \frac{f}{n} = \frac{6}{50} = \frac{3}{25}$$

Example 13:

In a sample of 50 people, 21 had type O blood, 22 had type A blood, 5 had type B blood, and 2 had type AB blood. Set up a frequency distribution and find the following probabilities.

a. A person has type O blood.

b. A person has type A or type B blood.

c. A person has neither type A nor type O blood.

d. A person does not have type AB blood.

Solution

Type	Frequency
A	22
B	5
AB	2
O	21
Total	50

Example 13 (cont.):

a. $P(\text{O}) = \frac{f}{n} = \frac{21}{50}$

b. $P(\text{A or B}) = \frac{22}{50} + \frac{5}{50} = \frac{27}{50}$

(Add the frequencies of the two classes.)

c. $P(\text{neither A nor O}) = \frac{5}{50} + \frac{2}{50} = \frac{7}{50}$

(Neither A nor O means that a person has either type B or type AB blood.)

d. $P(\text{not AB}) = 1 - P(\text{AB}) = 1 - \frac{2}{50} = \frac{48}{50} = \frac{24}{25}$

(Find the probability of not AB by subtracting the probability of type AB from 1.)

Addition Rules for Probability

Two events are **mutually exclusive events** if they cannot occur at the same time (i.e., they have no outcomes in common).

For example, the events of getting a 4 and getting a 6 when a single card is drawn from a deck are mutually exclusive events, since a single card cannot be both a 4 and a 6. On the other hand, the events of getting a 4 and getting a heart on a single draw are not mutually exclusive, since you can select the 4 of hearts when drawing a single card from an ordinary deck.

Example 14:

Determine which events are mutually exclusive and which are not, when a single die is rolled.

a. Getting an odd number and getting an even number

b. Getting a 3 and getting an odd number

c. Getting an odd number and getting a number less than 4

d. Getting a number greater than 4 and getting a number less than 4

Solution

a. The events are mutually exclusive, since the first event can be 1, 3, or 5 and the second event can be 2, 4, or 6.

b. The events are not mutually exclusive, since the first event is a 3 and the second can be 1, 3, or 5. Hence, 3 is contained in both events.

c. The events are not mutually exclusive, since the first event can be 1, 3, or 5 and the second can be 1, 2, or 3. Hence, 1 and 3 are contained in both events.

d. The events are mutually exclusive, since the first event can be 5 or 6 and the second event can be 1, 2, or 3.

Example 15:

Determine which events are mutually exclusive and which are not, when a single card is drawn from a deck.

a. Getting a 7 and getting a jack

b. Getting a club and getting a king

c. Getting a face card and getting an ace

d. Getting a face card and getting a spade

Solution

Only the events in parts *a* and *c* are mutually exclusive.

The probability of two or more events can be determined by the *addition rules.* The first addition rule is used when the events are mutually exclusive.

Addition Rule 1

When two events A and B are mutually exclusive, the probability that A or B will occur is

$$P(A \text{ or } B) = P(A) + P(B)$$

Example 16:

A box contains 3 glazed doughnuts, 4 jelly doughnuts, and 5 chocolate doughnuts. If a person selects a doughnut at random, find the probability that it is either a glazed doughnut or a chocolate doughnut.

Solution

Since the box contains 3 glazed doughnuts, 5 chocolate doughnuts, and a total of 12 doughnuts, $P(\text{glazed or chocolate}) = P(\text{glazed}) + P(\text{chocolate}) = \frac{3}{12} + \frac{5}{12} = \frac{8}{12} = \frac{2}{3}$. The events are mutually exclusive.

Example 17:

At a political rally, there are 20 Republicans, 13 Democrats, and 6 Independents. If a person is selected at random, find the probability that he or she is either a Democrat or an Independent.

Solution

$$\begin{aligned} P(\text{Democrat or Independent}) &= P(\text{Democrat}) + P(\text{Independent}) \\ &= \tfrac{13}{39} + \tfrac{6}{39} = \tfrac{19}{39} \end{aligned}$$

Example 18:

A day of the week is selected at random. Find the probability that it is a weekend day.

Solution

$$P(\text{Saturday or Sunday}) = P(\text{Saturday}) + P(\text{Sunday}) = \tfrac{1}{7} + \tfrac{1}{7} = \tfrac{2}{7}$$

When events are not mutually exclusive, addition rule 2 can be used to find the probability of the events.

Addition Rule 2

If A and B are *not* mutually exclusive, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example 19:

In a hospital unit there are 8 nurses and 5 physicians; 7 nurses and 3 physicians are females. If a staff person is selected, find the probability that the subject is a nurse or a male.

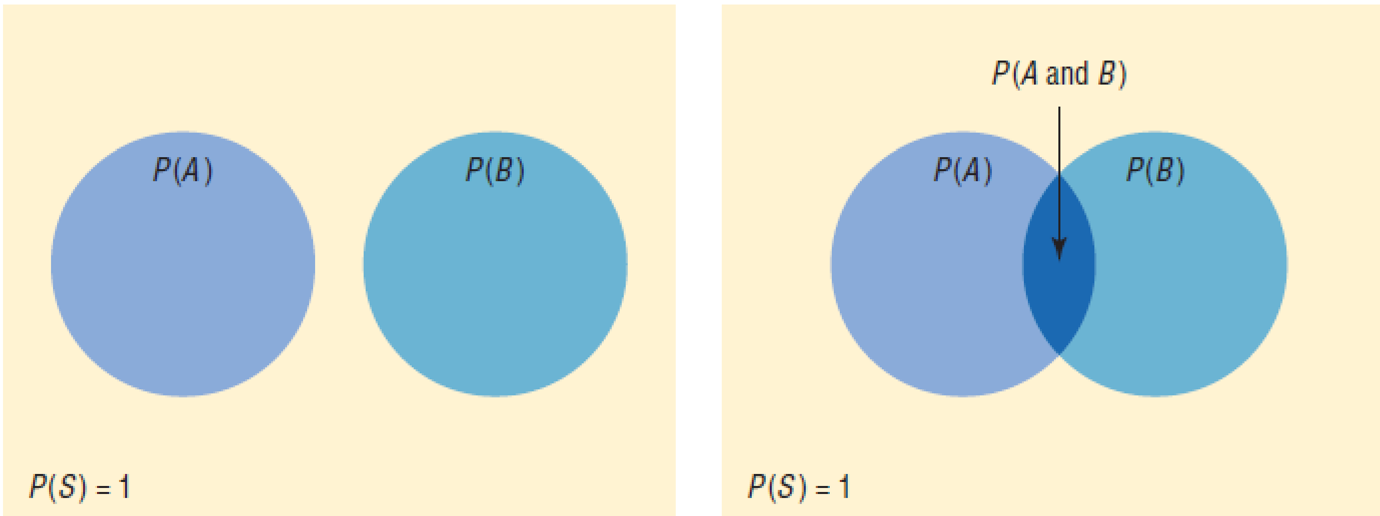
Solution

The sample space is shown here.

Staff	Females	Males	Total
Nurses	7	1	8
Physicians	3	2	5
Total	10	3	13

The probability is

$$P(\text{nurse or male}) = P(\text{nurse}) + P(\text{male}) - P(\text{male nurse})$$
$$= \frac{8}{13} + \frac{3}{13} - \frac{1}{13} = \frac{10}{13}$$



(a) Mutually exclusive events
$P(A \text{ or } B) = P(A) + P(B)$

(b) Nonmutually exclusive events
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Example 20:

On New Year's Eve, the probability of a person driving while intoxicated is 0.32, the probability of a person having a driving accident is 0.09, and the probability of a person having a driving accident while intoxicated is 0.06. What is the probability of a person driving while intoxicated or having a driving accident?

Solution

$$
\begin{aligned}
P(\text{intoxicated or accident}) &= P(\text{intoxicated}) + P(\text{accident}) \\
&\quad - P(\text{intoxicated and accident}) \\
&= 0.32 + 0.09 - 0.06 = 0.35
\end{aligned}
$$

Independent Events

Two events A and B are **independent events** if the fact that A occurs does not affect the probability of B occurring.

Here are other examples of independent events:

Rolling a die and getting a 6, and then rolling a second die and getting a 3.

Drawing a card from a deck and getting a queen, replacing it, and drawing a second card and getting a queen.

To find the probability of two independent events that occur in sequence, you must find the probability of each event occurring separately and then multiply the answers. For example, if a coin is tossed twice, the probability of getting two heads is $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$. This result can be verified by looking at the sample space HH, HT, TH, TT. Then $P(\text{HH}) = \frac{1}{4}$.

Multiplication Rules

Multiplication Rule 1

When two events are independent, the probability of both occurring is

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Example 21:

A coin is flipped and a die is rolled. Find the probability of getting a head on the coin and a 4 on the die.

Solution

$$P(\text{head and } 4) = P(\text{head}) \cdot P(4) = \tfrac{1}{2} \cdot \tfrac{1}{6} = \tfrac{1}{12}$$

Note that the sample space for the coin is H, T; and for the die it is 1, 2, 3, 4, 5, 6.

The sample space for above example is:

H1 H2 H3 H4 H5 H6 T1 T2 T3 T4 T5 T6

The solution is $\frac{1}{12}$, since there is only one way to get the head-4 outcome.

Example 22:

An urn contains 3 red balls, 2 blue balls, and 5 white balls. A ball is selected and its color noted. Then it is replaced. A second ball is selected and its color noted. Find the probability of each of these.

a. Selecting 2 blue balls

b. Selecting 1 blue ball and then 1 white ball

c. Selecting 1 red ball and then 1 blue ball

Solution

a. $P(\text{blue and blue}) = P(\text{blue}) \cdot P(\text{blue}) = \frac{2}{10} \cdot \frac{2}{10} = \frac{4}{100} = \frac{1}{25}$

b. $P(\text{blue and white}) = P(\text{blue}) \cdot P(\text{white}) = \frac{2}{10} \cdot \frac{5}{10} = \frac{10}{100} = \frac{1}{10}$

c. $P(\text{red and blue}) = P(\text{red}) \cdot P(\text{blue}) = \frac{3}{10} \cdot \frac{2}{10} = \frac{6}{100} = \frac{3}{50}$

Multiplication rule 1 can be extended to three or more independent events by using the formula

$$P(A \text{ and } B \text{ and } C \text{ and } \ldots \text{ and } K) = P(A) \cdot P(B) \cdot P(C) \cdots P(K)$$

Example 23:

A Harris poll found that 46% of Americans say they suffer great stress at least once a week. If three people are selected at random, find the probability that all three will say that they suffer great stress at least once a week.

Solution

Let S denote stress. Then

$$\begin{aligned} P(S \text{ and } S \text{ and } S) &= P(S) \cdot P(S) \cdot P(S) \\ &= (0.46)(0.46)(0.46) \approx 0.097 \end{aligned}$$

Dependent Events

When the outcome or occurrence of the first event affects the outcome or occurrence of the second event in such a way that the probability is changed, the events are said to be **dependent events.**

Here are some examples of dependent events:

Drawing a card from a deck, not replacing it, and then drawing a second card.

Selecting a ball from an urn, not replacing it, and then selecting a second ball.

Conditional Probability

To find probabilities when events are dependent, use the multiplication rule with a modification in notation. For the problem just discussed, the probability of getting an ace on the first draw is $\frac{4}{52}$, and the probability of getting a king on the second draw is $\frac{4}{51}$. By the multiplication rule, the probability of both events occurring is

$$\frac{4}{52} \cdot \frac{4}{51} = \frac{16}{2652} = \frac{4}{663}$$

The event of getting a king on the second draw *given* that an ace was drawn the first time is called a *conditional probability.*

Conditional Probability

Conditional probability of an event B in relationship to an event A is the probability that event B occurs after event A has already occurred. The notation for conditional probability is $P(B|A)$. This notation does not mean that B is divided by A; rather, it means the probability that event B occurs given that event A has already occurred.

Example 24:

At a university in western Pennsylvania, there were 5 burglaries reported in 2003, 16 in 2004, and 32 in 2005. If a researcher wishes to select at random two burglaries to further investigate, find the probability that both will have occurred in 2004.

Solution

In this case, the events are dependent since the researcher wishes to investigate two distinct cases. Hence the first case is selected and not replaced.

$$P(C_1 \text{ and } C_2) = P(C_1) \cdot P(C_2|C_1) = \tfrac{16}{53} \cdot \tfrac{15}{52} = \tfrac{60}{689}$$

Example 25:

Three cards are drawn from an ordinary deck and not replaced. Find the probability of these events.

a. Getting 3 jacks

b. Getting an ace, a king, and a queen in order

c. Getting a club, a spade, and a heart in order

d. Getting 3 clubs

Example 25 (cont.):

Solution

$$a.\ P(3\text{ jacks}) = \frac{4}{52}\cdot\frac{3}{51}\cdot\frac{2}{50} = \frac{24}{132{,}600} = \frac{1}{5525}$$

$$b.\ P(\text{ace and king and queen}) = \frac{4}{52}\cdot\frac{4}{51}\cdot\frac{4}{50} = \frac{64}{132{,}600} = \frac{8}{16{,}575}$$

$$c.\ P(\text{club and spade and heart}) = \frac{13}{52}\cdot\frac{13}{51}\cdot\frac{13}{50} = \frac{2197}{132{,}600} = \frac{169}{10{,}200}$$

$$d.\ P(3\text{ clubs}) = \frac{13}{52}\cdot\frac{12}{51}\cdot\frac{11}{50} = \frac{1716}{132{,}600} = \frac{11}{850}$$

Formula for Conditional Probability

The probability that the second event B occurs given that the first event A has occurred can be found by dividing the probability that both events occurred by the probability that the first event has occurred. The formula is

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Example 26:

A box contains black chips and white chips. A person selects two chips without replacement. If the probability of selecting a black chip *and* a white chip is $\frac{15}{56}$, and the probability of selecting a black chip on the first draw is $\frac{3}{8}$, find the probability of selecting the white chip on the second draw, *given* that the first chip selected was a black chip.

Solution

Let

$$B = \text{selecting a black chip} \qquad W = \text{selecting a white chip}$$

Then

$$P(W|B) = \frac{P(B \text{ and } W)}{P(B)} = \frac{15/56}{3/8}$$

$$= \frac{15}{56} \div \frac{3}{8} = \frac{15}{56} \cdot \frac{8}{3} = \frac{\overset{5}{\cancel{15}}}{\underset{7}{\cancel{56}}} \cdot \frac{\overset{1}{\cancel{8}}}{\underset{1}{\cancel{3}}} = \frac{5}{7}$$

Hence, the probability of selecting a white chip on the second draw given that the first chip selected was black is $\frac{5}{7}$.

Example 27:

A recent survey asked 100 people if they thought women in the armed forces should be permitted to participate in combat. The results of the survey are shown.

Gender	Yes	No	Total
Male	32	18	50
Female	8	42	50
Total	40	60	100

Find these probabilities.

a. The respondent answered yes, given that the respondent was a female.

b. The respondent was a male, given that the respondent answered no.

Example 27 (cont.):

Solution

Let

M = respondent was a male $\qquad$ Y = respondent answered yes

F = respondent was a female $\qquad$ N = respondent answered no

a. The problem is to find $P(Y|F)$. The rule states

$$P(Y|F) = \frac{P(F \text{ and } Y)}{P(F)}$$

The probability $P(F \text{ and } Y)$ is the number of females who responded yes, divided by the total number of respondents:

$$P(F \text{ and } Y) = \frac{8}{100}$$

Example 27 (cont.):

The probability $P(F)$ is the probability of selecting a female:

$$P(F) = \frac{50}{100}$$

Then

$$P(Y|F) = \frac{P(F \text{ and } Y)}{P(F)} = \frac{8/100}{50/100}$$

$$= \frac{8}{100} \div \frac{50}{100} = \frac{\overset{4}{\cancel{8}}}{\underset{1}{\cancel{100}}} \cdot \frac{\overset{1}{\cancel{100}}}{\underset{25}{\cancel{50}}} = \frac{4}{25}$$

b. The problem is to find $P(M|N)$.

$$P(M|N) = \frac{P(N \text{ and } M)}{P(N)} = \frac{18/100}{60/100}$$

$$= \frac{18}{100} \div \frac{60}{100} = \frac{\overset{3}{\cancel{18}}}{\underset{1}{\cancel{100}}} \cdot \frac{\overset{1}{\cancel{100}}}{\underset{10}{\cancel{60}}} = \frac{3}{10}$$

Example 28:

A game is played by drawing 4 cards from an ordinary deck and replacing each card after it is drawn. Find the probability that at least 1 ace is drawn.

Solution

It is much easier to find the probability that no aces are drawn (i.e., losing) and then subtract that value from 1 than to find the solution directly, because that would involve finding the probability of getting 1 ace, 2 aces, 3 aces, and 4 aces and then adding the results.

Let E = at least 1 ace is drawn and $\overline{E}$ = no aces drawn. Then

$$\begin{aligned} P(\overline{E}) &= \frac{48}{52} \cdot \frac{48}{52} \cdot \frac{48}{52} \cdot \frac{48}{52} \\ &= \frac{12}{13} \cdot \frac{12}{13} \cdot \frac{12}{13} \cdot \frac{12}{13} = \frac{20{,}736}{28{,}561} \end{aligned}$$

Hence,

$$P(E) = 1 - P(\overline{E})$$

$$P(\text{winning}) = 1 - P(\text{losing}) = 1 - \frac{20{,}736}{28{,}561} = \frac{7825}{28{,}561} \approx 0.27$$

Example 29:

A coin is tossed 5 times. Find the probability of getting at least 1 tail.

Solution

It is easier to find the probability of the complement of the event, which is "all heads," and then subtract the probability from 1 to get the probability of at least 1 tail.

$$P(E) = 1 - P(\overline{E})$$

$$P(\text{at least 1 tail}) = 1 - P(\text{all heads})$$

$$P(\text{all heads}) = \left(\tfrac{1}{2}\right)^5 = \tfrac{1}{32}$$

Hence,

$$P(\text{at least 1 tail}) = 1 - \tfrac{1}{32} = \tfrac{31}{32}$$

Thank You.