Maths Question Paper

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Subject - Maths | Duration - 1 hr
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Section A

Q1) Show that the following limits exist and find them:

$$(a)\lim_{n\to\infty}\frac{n!}{n^n} \qquad (b)\lim_{n\to\infty}(\frac{n}{n^2+1}+\frac{n}{n^2+2}+...+\frac{n}{n^2+n})$$

Q2) Prove that the following sequences are convergent by showing that they are mono- tone and bounded. Also find their limits

$$(a)a_1=\sqrt{2}, a_{n+1}=\sqrt{a_n+1}), \forall \geq 1$$

Q3) Find the radius and the interval of convergence of the following power series.

$$(a)\sum_{n=1}^{+\infty}(-1)^{n+1}\frac{n^2}{n^4+1} \qquad (b)\sum_{n=1}^{+\infty}\frac{(-1)^n}{1+\sqrt{n}}$$

Q4) Find the volumes of the solids generated by revolving the regions bounded by the lines and curves about the y- axis.
The region is enclosed by :-

$$x=2sin(2y), 0\geq y\geq \pi/2, x=0. \qquad and \qquad x=\sqrt{cos(\frac{\pi x}{4})}$$

Section B

Q1) Evaluate the following improper integrals :-

$$(a)\int_{-1}^{\infty}\frac{dx}{\sqrt{x^2+5x+6}}$$

$$(b) \int_0^{\infty} \frac{(xsin(x) + x^3)^2}{\sqrt{x}}$$

Q2) Prove the following reduction formulae and state the values of n for which they are valid. Note that m,n are nonnegative integers.

$(a) If U_n = \int_0^{\pi} \theta cos(\theta)^n$ then prove that $U_n = \frac{-1}{n^2} + \frac{n}{n-1} U_{n-2}$

(b)$If I_n = \int_{\frac{\pi}{4}}^{\pi} cot(x)^n \, dx$ then prove that $I_n = \frac{1}{n-1} - I_{n-2}$ Hence evaluate I_6

Q3)If A $= \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 0 \\ 12 & -1 & 0 \end{bmatrix}$ and $A^{-1} = \frac{A^2+cA+d}{6}$ then the values of c and d are respectively is -

(A) -6, -11

(B) 6,11

(C) -11, 11

(D) None

-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x-x