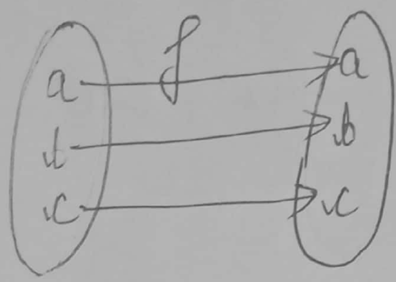
Identity Function →

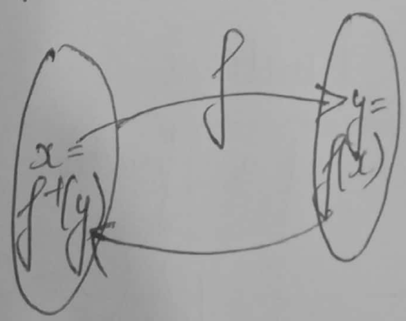
Let X be a non empty set.

$I_x : X \to X$ is called an Identity Function if $I_x(x) = x, \forall x \in X$



f) Invertible Function →

$f : X \to Y$ will be invertible if its inverse, f^{-1} is a function from Y to X such that $\forall y \in Y$, f^{-1} assigns a unique value of X.



(a) Inverse can be defined only for functions which are bijection.

(b) Not every function may have its inverse possible.

eg1 Check if $f(x) = x^2$ $f: Z \to Z$ an Inverse function.

$$y = x^2$$

$$x = \pm\sqrt{y}$$

Not an Inverse function

eg2 Check if $f(x) = (2x+3)$ is an Inverse function.

$$y = 2x+3$$

$$y-3 = 2x$$

$$x = \frac{(y-3)}{2}$$

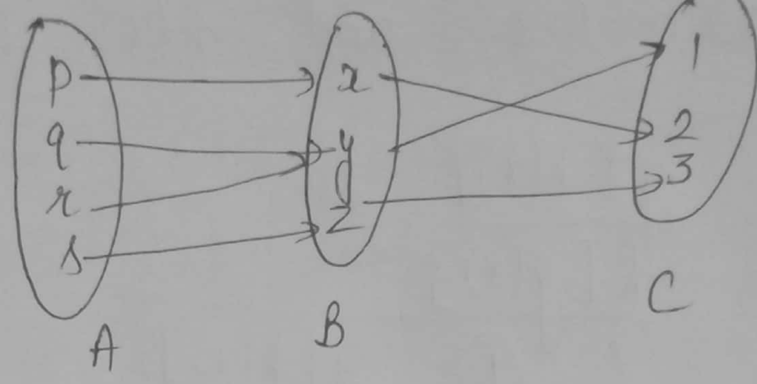
$$\therefore f^{-1}(x) = \frac{y-3}{2}$$ Yes an Inverse function

(III) Composition of functions

(function of a function)

$F: A \to B$ $\qquad$ $g: B \to C$

1st Codomain = Domain 2nd



$$B = f(A)$$
$$C = g(B)$$
$$gof = g[f(A)]$$

Let $f : X \to Y$ and $g : Y \to Z$,

$$\therefore \; gof = g(f(x)), \; \forall x \in X$$

eg1 Let $f : R \to R$ and $g : R \to R$ be two functions defined as $f(x) = x^2$ and $g(x) = 3x+1$. Find $gof(x)$ and $fog(x)$.

$$gof(x) = g(f(x)) = g(x^2) = 3x^2 + 1$$
$$fog(x) = f(g(x)) = f(3x+1) = (3x+1)^2$$

Thus, $gof(x) \neq fog(x)$

eg2

If $g(x) = 1-x$ and $h(x) = \frac{x}{x-1}$,

then $\frac{g[h(x)]}{h[g(x)]}$ is

a) $\frac{h(x)}{g(x)}$

b) $-1/x$

c) $\frac{g(x)}{h(x)}$

d) $\frac{x}{(1-x)^2}$

① $g[h(x)] =$

$$g\left(\frac{x}{x-1}\right) = 1 - \frac{x}{x-1}$$

$$= \frac{x-1-x}{x-1}$$

$$= \frac{-1}{x-1}$$

② $h(1-x) = \frac{1-x}{1-x-1} = \frac{1-x}{-x}$

$$= \frac{x-1}{x}$$

③ $\frac{g[h(x)]}{h[g(x)]} = \frac{-1}{x-1} \times \frac{x}{x-1}$

$$= \frac{-x}{(x-1)^2}$$

$$\frac{h(x)}{g(x)} = \frac{x}{(x-1)(1-x)} = \frac{-x}{(x-1)^2} = a$$

IV)

Sum and Product of Functions

Let f_1 and f_2 be functions from a set X to R. Then f_1+f_2 and f_1f_2 are also functions from X to R.

$$(f_1+f_2)(x) = f_1(x) + f_2(x)$$
$$(f_1f_2)(x) = f_1(x) * f_2(x)$$

eg1 Let f_1 and f_2 be functions from a set R to R such that $f_1(x) = x+2$ and $f_2(x) = x-3$. Find f_1+f_2 and f_1f_2

$$(f_1+f_2)(x) = f_1(x) + f_2(x)$$
$$= x+2+x-3$$
$$= 2x-1$$

$$(f_1f_2)(x) = f_1(x) \times f_2(x)$$
$$= (x+2)(x-3)$$
$$= x^2-3x+2x-6$$
$$= x^2-x-6$$

eg 2 # Let f_1 and f_2 be functions from a set R to R such that $f_1(x) = x^2$ and $f_2(x) = x+1$

find $(f_1+f_2)(2)$ and $(f_1 f_2)(1)$

$\downarrow$ 7 Ans

$\downarrow$ 2 Ans