# Tutorial – 7

1) Prove with the help of an example that every asymmetric relation is also an antisymmetric relation i.e. $(Asymmetric\ Relation \subseteq Antisymmetric\ Relation)$.

2) Let $R$ be a relation on a set of positive integers defined as $xRy$ if and only if $x|y$ that is $x$ divides $y$. Determine whether the relation $R$ is transitive or not.

3) Determine whether the following are true or false:
"If a relation is"
(a) Symmetric and Transitive ⇒ Reflexive
(b) Irreflexive and Symmetric ⇒ Transitive

4) The binary relation $R = \{(1,1),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4)\}$on a set $A = \{1,2,3,4\}$ is
(a) Reflexive, symmetric and transitive
(b) Neither reflexive, nor irreflexive but transitive
(c) Irreflexive, symmetric and transitive
(d) Irreflexive and antisymmetric

5) Compute the maximum number of relations possible from a set.

6) If a relation $R$ on a set of integers $Z$ is define as

$$R = \{(x,y)|x \in Z, y \in Z, (x-y)\ is\ divisible\ by\ 6\ \}$$

Then prove that $R$ is an equivalence relation.

7) Let $R$ be a relation on $A$. Prove that "$R$ is antisymmetric if and only if $R \cap R^{-1} \subseteq I_A$".

8) Let $R$ be the relation from $S = \{1,2,3,4\}$ to $T = \{a,b,c\}$ represented as

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(a) Show that $RoR^{-1}$ is a symmetric relation on $S$.
(b) Show that $R^{-1}oR$ is a symmetric relation on $T$.
(c) Are the relation $RoR^{-1}$ and $R^{-1}oR$ equivalent relation?

9) A relation $R$ defined on $A = \{1,2,3,4,5,6\}$ as

$$R = \{(1,2), (1,6), (2,3), (3,3), (3,4), (4,1), (4,3), (4,5), (6,4)\}$$

Draw the graph of $R$ and $R^2$ and hence find $M_{R^2}$.

10) Consider the relation $R$ defined on $A$ as follows:
$R = \{(a,b),(b,c),(d,c),(d,a),(a,d),(d,d)\}$ on $A = \{a,b,c,d\}$
Find the reflexive, transitive and symmetric closure of $R$.

11) Let a set $X$ contain $n$ elements. How many reflexive relations will there be on $X$?

12) Let A be a set of books.
   a) Let R1 be a binary relation on A such that (a,b) is in R1 if book A costs more and contains fewer pages than book B. In general, is R1 Reflexive? Symmetric? Antisymmetric? Transitive?
   b) Let R2 be a binary relation on A such that (a,b) is in R2 if book A costs more or contains fewer pages than book B. In general, is R2 Reflexive? Symmetric? Antisymmetric? Transitive?

13) Let P be the set of all people. Let R be a binary relation on P such that (a,b) is in R if a is a brother of b. Is R reflexive? Symmetric? Antisymmetric? Transitive? An equivalence relation?A partial order relation?