Assignment 3

1. Solve the system of equations, if possible

$$\begin{cases} x+2y+z-3w=1 \\ 2x+4y+3z+w=3 \\ 3x+6y+4z-2w=5, \end{cases} \qquad \begin{cases} x+2y+z-3w=1 \\ 2x+4y+3z+w=3 \\ 3x+6y+4z-2w=4, \end{cases}$$

2. Determine the conditions for which the system

$$\begin{cases} x+y+z=1 \\ x+2y-z=b \\ 5x+7y+az=b^2, \end{cases}$$

admits of (i) only one solution (ii) no solution (iii) many solution.

3. Examine the consistency and solve if possible

$$\begin{cases} 2x+4y+3z+w=15 \\ 3x+7y+2w=16 \\ 5x+3y+2z+3w=21, \end{cases} \qquad \begin{cases} x+y+z=1 \\ 2x+y+2z=2 \\ 3x+2y+3z=5, \end{cases}$$

4. Find all real values of c for which the rank of the matrix
$\begin{pmatrix} 1+c & 2 & 3 & 4 \\ 1 & 2+c & 3 & 4 \\ 1 & 2 & 3+c & 4 \\ 1 & 2 & 3 & 4+c \end{pmatrix}$ **is less than 4.**

5. Examine if the set S is a Subspace of R^3

(i) $S = \{(x, y, z) : x + y - z = 0,\ 2x - y + z = 0\}$

(ii) $S = \{(x, y, z) : x^2 + y^2 = z^2\}$

(iii) $S = \{(x, y, z) : x + y + z = 0\}$

(iv) $S = \{(x, y, z) : x + y + z = 1\}$

6. Express $(-1, 2, 4)$ as linear combination of $(-1, 2, 0), (0, -1, 1)$ and $(3, -4, 2)$ in the vector space R^3 over real field.

7. Show that $(1, 2, 5)$ in R^3 can not be expressed as linear combina tion of $(1, 2, 3), (2, 4, 6), (1, 3, 4)$ and $(-3, 1, -2)$.

8. Find whether the set $\{(2, 4, 0), (0, 1, 0), (2, 6, 2)\}$ are linearly independent in real vector space R^3.

9. Find whether the set $\{(1, 0, 0), (0, 1, 0), (8, -1, 0)\}$ are linearly independent in real vector space R^3.

10. For what value of of k the set $\{(1, 1, 2), (k, 1, 1), (1, 2, 1)\}$ are linearly independent in real vector space R^3.

11. For what value of of k the set $\{(1, -1, 2), (0, k, 3), (-1, 2, 3)\}$ are linearly dependent in real vector space R^3.