Mathematical Logic

→ Logical reasoning is the essence of mathematics and is therefore, an important starting point for study of discrete mathematics.

→ Logic among other things, have provided the theoretical basis for many areas of computer science such as digital logic design, automata theory and computability, artificial intelligence, etc.

→ Mathematical Logic can have the following categories:-

(a) Proposition Calculus

(b) Predicate Calculus.

(a) Propositional logic deals with statements with values true and false and is concerned with analysis of propositions

→ Def:- A proposition or statement is a declarative sentence that is either true or false but not both.

Eg:- Three plus three equals six → True

Three plus three equals seven → False

Here, both are propositions or statements

Similarly, $x + y > 1$ is not a proposition because for some values of x and y the sentence is true whereas for others it is false.

Eg:- $x = 1$, $y = 2$ → sentence is true.

$x = -3$, $y = 1$ → sentence is false.

→ The truth or falsity of a statement is called its Truth Value.

→ Since only two possible truth values are possible, the logic is also sometimes referred to as two-valued logic.

→ Questions, exclamations and commands are not propositions.

Eg:- (1) The sun rises in the west.

(2) $2 + 4 = 6$

(3) $(5, 6) \subset (7, 6, 5)$

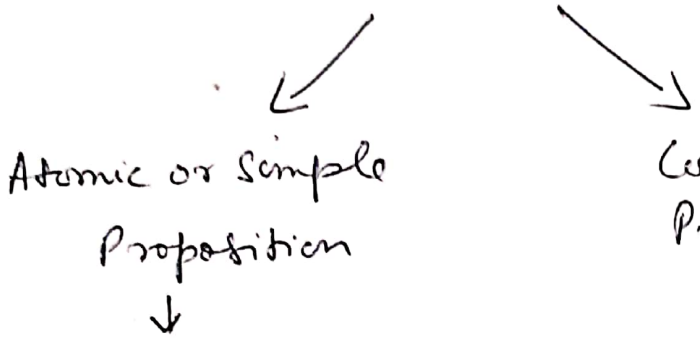
(4) Do you speak Hindi?

(5) $4 - x = 8$

(6) Close the door

(7) What a hot day!

→ Propositions can be of two types:-



→ The words or symbols used to form compound propositions are called connectives.

→ There are five basic connectives which are as follows:

(a)	Negation	not	$\sim, \neg$	$\sim p$
(b)	Conjunction	and	$\wedge$	$p \wedge q$
(c)	Disjunction	or	$\vee$	$p \vee q$
(d)	Conditional	if.... then	$\rightarrow$	$p \rightarrow q$
(e)	Bi-Conditional	if and only if	$\leftrightarrow$	$p \leftrightarrow q$

(a) Negation :-

p : Paris is in France.

$\neg p$: Paris is not in France.

(b) Conjunction :-

p : Ram is healthy q : He has blue eyes.

$p \wedge q$: Ram is healthy and he has blue eyes.

(c) Disjunction :-

p : $5 < 5$ q : $5 < 6$

$p \vee q$: $5 < 5 \vee 5 < 6$

(d) Conditional :-

$p \rightarrow q$ can be interpreted as $\boxed{\neg p \vee q}$.

Let p : I have 1L petrol in my bike.

q : I travel a distance of 35 Kms.

$p \rightarrow q$: If I have 1L petrol in my bike, then I travel a distance of 35 kms.

(e) Biconditional :-

$p \leftrightarrow q$ can be interpreted as $[(p \rightarrow q) \wedge (q \rightarrow p)]$

Let p: I participate in all the activities of the class.

q: I get grade A.

$p \leftrightarrow q$: I participate in all the activities of the class if and only if I get grade A.

Q1) Write sentences which describes each of the following statements.

(a) $\sim p$ (b) $p \wedge q$ (c) $p \vee q$

(d) $p \vee \sim q$

where p: It is cold.

q: It is raining.

→ Rules of Precedence

① $\sim$
② $\wedge$
③ $\vee$
④ $\rightarrow$
⑤ $\leftrightarrow$