Propositional Logic

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Overview

Tautology, Contradiction and Contingency

Logical Equivalence

Derived Implications

Well formed formula

Tautology, Contradiction and Contingency

- Tautology
  A tautology is a proposition which is always true .
  Classic Example: $P \vee \neg P$
- Contradiction
  A contradiction is a proposition which is always false .
  Classic Example: $P \wedge \neg P$
- Contingency
  A contingency is a proposition which neither a tautology nor a contradiction.
  Example: $(P \vee Q) \implies \neg R$

Example 1:

Show that each of the following is a tautology by using truth tables:

1. $(p \wedge q) \implies p$
2. $\neg p \implies (p \implies q)$
3. $\neg(p \implies q) \implies p$

The compound propositions p and q are called logically equivalent if $p \iff q$ is a tautology. The notation $p \equiv q$ denotes that p and q are logically equivalent.

Example 2:

Show that $p \implies q$ and $\neg p \vee q$ are logically equivalent.

Example 3 :

Show that $\neg(\neg p)$ and p are logically equivalent.

(Double Negation Law)

De Morgan's Laws

1. $\neg(p \wedge q) \equiv \neg p \vee \neg q$
2. $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Verify using truth table.

Derived Implications

1. Contrapositive : $\neg q \implies \neg p$
2. Converse : $q \implies p$
3. Inverse : $\neg p \implies \neg q$

Note : Conditional and contrapositive have same truth value. Converse and Inverse have same truth value.

Example 4: Let p: Today is friday
q: We have a DMS class today
$p \implies q$: If today is Friday, then we have a DMS class today.
Write the contrapositive, converse and inverse.

well formed formula

A statement or proposition may consist of variables, paranthesis and connective symbols. A gramatically correct expression is called a well formed formula.

1. Every atomic statement is a well formed formula
2. if p is wff then $\neg p$ is also wff.
3. if p and q are wff, then $p \wedge q$, $p \vee q$ and $p \implies q$ are also wff.

Example 5:

Check whether the following are wff :

1. $\neg(p \wedge q)$
2. $(p \implies (p \vee q)$
3. $(p \vee q) \implies (\wedge p)$

Contribution

- Augustus De Morgan
- Augusta Ada
- Henry Maurice Sheffer

Queries?