Ordinary Differential Equations

(Lecture-1)

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Books

Reference Books

- S. L. Ross, “Differential Equations”, 3rd Edition, Wiley India, 1984.
- Erwin Kreyszig, “Advanced Engineering Mathematics”, Tenth Edition, Wiley India, 2016.

Learning Outcome of the Lecture

We learn

- Differential Equation
  - Definition
  - Examples
- Classification of DE's
  - Type: ODE/PDE
  - Order
  - Linear/Nonlinear

Differential Equation

Definition

An equation involving derivatives of one or more dependent variables with respect to one or more independent variables is called a differential equation.

Differential equations occur naturally in real life problems encountered in science and engineering.

Examples:

- Radioactive decay
- Motion of Projectile, rocket, satellite, or planet
- Current in the electric Circuit
- Heat conduction in a rod
- Vibration of a wire
- Motion of simple pendulum

Types of Differential Equations

A DE is classified based on its type as Ordinary Differential Equation (ODE) or Partial Differential Equation (PDE).

Definition

Let $y(x)$ denotes a function in the variable x. An ordinary differential equation (ODE) is an equation containing one or more derivatives of an unknown function y.
In general, a differential equation involving ordinary derivatives of one or more dependent variables with respect to a single independent variable is called an ordinary differential equation.

Definition

A differential equation involving partial derivatives of one or more dependent variables with respect to more than one independent variable is called a partial differential equation.

Examples

(i) $\dfrac{d^2y}{dx^2} + xy\left(\dfrac{dy}{dx}\right)^2 = 0,$ Ordinary Differential Equation

(ii) $\dfrac{d^4x}{dt^4} + 5\dfrac{d^2x}{dt^2} + 3x = \sin t,$ Ordinary Differential Equation

(iii) $\dfrac{\partial v}{\partial s} + \dfrac{\partial v}{\partial t} = v,$ Partial Differential Equation

(iv) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0.$ Partial Differential Equation

Order of Differential Equation

Definition

The order of the highest ordered derivative involved in a differential equation is called the order of the differential equation.

Examples:

$$(i) \quad \frac{d^2y}{dx^2} + xy\left(\frac{dy}{dx}\right)^2 = 0, \qquad \text{DE of second order}$$

$$(ii) \quad \frac{d^4x}{dt^4} + 5\frac{d^2x}{dt^2} + 3x = \sin t, \qquad \text{DE of fourth order}$$

$$(iii) \quad \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} = v, \qquad \text{DE of first order}$$

$$(iv) \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0. \qquad \text{DE of second order}$$

Linear Differential Equations

Definition

Linear ODE: A linear DE of order n, in the dependent variable y and the independent variable x, is an equation that is in, or can be expressed in, the form

$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)y = b(x),$$

where $a_0(x)$ is not identically zero.

Check list: If the dependent variable is y, derivatives occur upto first degree only, no products of y and/or its derivatives are there.

Examples:

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0,$$

$$\frac{d^4y}{dx^4} + x^2\frac{d^3y}{dx^3} + x^3\frac{dy}{dx} = xe^x.$$

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Nonlinear Differential Equation

Definition

A nonlinear ordinary differential equation is an ordinary differential equation that is not linear.

Examples:

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y^2 = 0,$$

$$\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 + 6y = 0,$$

$$y\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0,$$

$$\frac{d^2y}{dx^2} + 5y\frac{dy}{dx} + 6y = 0.$$