Counting Techniques

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Overview

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Inclusion-Exclusion Principle

Suppose two tasks A and B can occur in n_1 and n_2 ways, where some of the n_1 and n_2 ways may be same. In this case, just $n_1 + n_2$ cannot be done as it may count the same ways twice. Therefore, in such a case, inclusion-exclusion principle is applied.
E.g. If we have two sets A and B, then the number of elements in the set $A \cup B$ is given by
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Example

In how many ways can we select an ace or a heart from a pack of cards?

Solution: There are 4 aces and 13 cards of heart in a pack of cards and 1 card is common to both.

If E_1 is the event of getting an ace.

E_2 is the event of getting an heart, then

$n(E_1) = 4, n(E_2 = 13)$ and $n(E_1 \cap E_2) = 1$

$=4+13-1$

$=16$

Example

There are 70 students in a class. The class teacher decided to organize two competitions – singing and dancing. Every student has to participate in at least one competition.

The students who participate in the dance competition, also get a chance to perform during the annual function.

The student who participated in both the activities get an additional 10 points in general proficiency, 50 students participate in the singing competition and only 30 students do not get additional points in their general proficiency. Find the number of students who get the chance to perform during the annual function, but do not get additional points.

Solution

Q1) Let S denote the set of students who participate in the singing competition and D denote the set of students who participate in the dancing competition.

Given that $|S \cup D| = 70$ and $|S| = 50$

$|D|$ = No. of students who get a chance to perform during the annual function.

$|S \cap D|$ = No. of students who get 10 additional points in general proficiency.

Given that $|(S \cap D)'| = 30$, $|S \cap D| = 70 - 30 = 40$

We know that $|S \cup D| = |S| + |D| - |S \cap D|$.

Hence, $|D| = 70 - 50 + 40 = 60$

Thus, the number of students who get the chance to perform during the annual function but do not get additional point is

$|D| - |S \cap D| = 60 - 40 = 20$ Ans.

Permutation

Any arrangement of n objects in a given order is called a permutation of the object (taken all at a time). The placement of any $r \leq n$ of these objects is called an r-permutation. The number of r permutations of a set with n distinct elements is denoted by P(n,r) or n_{P_r}.
e.g. Consider a set of letters a,b,c
A) abc,acb,bac,bca,cab,cba : permutations of 3 objects taken all at a time.
B) ab,ba,ac,ca,bc,cb: permutations of any 2 of the 3 objects.
$P(n,r) = \frac{n!}{(n-r)!}$
When r=n, we have $P(n,n) = \frac{n!}{(n-n)!}$=n!/0!=n!

1. If all the n objects are arranged in a row, that is the permutation of n objects taken all at a time, then total permutations will be n!
2. If all the n objects are arranged in a circle, then the total number of permutations shall be (n-1)! (also called circular permutation). In this case, two extreme places in a line will coincide.

1. In how many ways can five students arrange themselves in a row?

Solution

The permutation of n objects taken all at a time is given by $n!$

Thus total number of ways is $5! = 120$

2. In how many ways can 5 students arrange themselves in a circle?

Solution

The permutation of n objects in a circle is (n-1)!. Thus the total number of ways is $4! = 24$

3. How many different 3 digit numbers can be formed by using the digits 1,2,3,4 and 5 when repetition is not allowed?

Solution

Here n=5 and we have to find 3-permutations from a set of 5 elements.

Thus the total number of ways is 5_{P_3}=5!/(5-3)!=5!/2!=60

4. In how many ways can 6 students arrange themselves in a row, if two particular students always sit together?

Solution

Since two students always sit together, assuming that 2 students form 1 unit,we have to arrange 5 units in a row, this can be done in 5! ways.

Moreover, for each of these 5! ways, the two students can be arranged in 2! ways. Thus the total number of ways $=5! \times 2! = 240$

5. In how many ways can 7 students arrange themselves in a row if two particular students always take the corner seats.

Solution

Let the two students taking the corner seat be A and B. Then the positions of the 7 students can be represented as follows:

A——B or B——A

The 5 positions between A and B can be arranged in 5! ways.

Since the corner positions can be arranged in 1.1=1 way, each representation can be done in 5! ways.

Thus the total n umber of ways for the given arrangement of students is 5!+5!=240