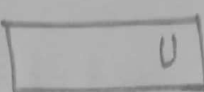
Graphical Representation of Sets

1. *) **Venn Diagrams** :- It is a schematic or diagrammatic representation of Sets.

Symbols Used →

1) 

→ Set

2) 

→ Universal Set

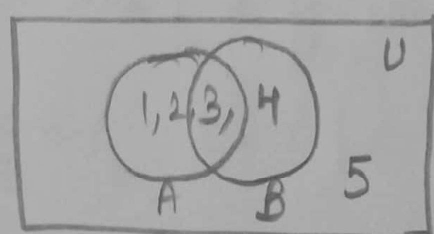
e.g

$U = \{1, 2, 3, 4, 5\}$

$A = \{1, 2, 3\}$

$B = \{3, 4\}$

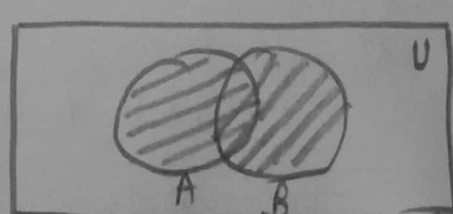
Venn Diagram



2. *) **Operations on Sets**

1) Union ($\cup$) $A \cup B$, set of all elements of set A, as well as B

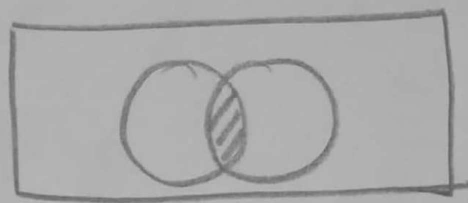
$A \cup B = \{x \mid x \in A \vee x \in B\}$



$A \cup B = \{1, 2, 3, 4\}$

2) **Intersection ($\cap$)** $A \cap B$, set of elements which belong to both A and B (common elements)

$$A \cap B = \{ x \mid x \in A \wedge x \in B \}$$



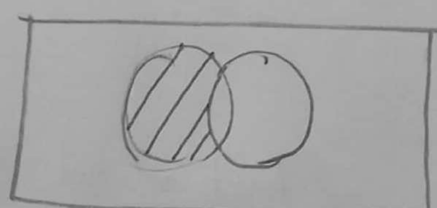
$A \cap B = \{3\}$

If $A \cap B = \phi$, then A and B are called Disjoint Sets.

3) **Set Difference ($A - B$)**

Set with elements of A, that are not in B.

$$A - B = \{ x \mid x \in A \wedge x \notin B \}$$

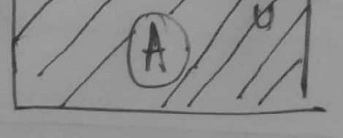


$A - B = \{1, 2\}$

4) **Complement (A^c)**

A^c, set with elements that are not in A

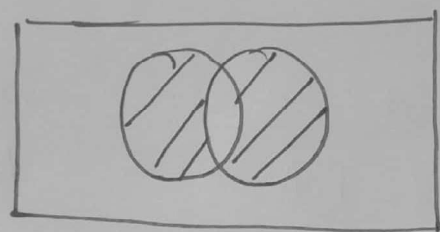
$$A^c = U - A$$
$$= \{ x \mid x \in U \wedge x \notin A \}$$



5) Symmetric Difference ($A \Delta B$)

$A \Delta B$, set with elements of A or B, but not in both.

$$A \Delta B = \{ x \mid (x \in A \vee x \in B) \wedge x \notin (A \cap B) \}$$



$$A \Delta B = \{1, 2, 4\}$$

6) Cartesian Product ($A \times B$)

$$A \times B = C \{ (x, y) \}$$

↳ pair of 2 elements wrapped in a bracket

↳ called ordered pair because they follow an order

$$(2,3) \neq (3,2)$$

→ Eg $A = \{1, 2, 3\}$ $B = \{a, b\}$

$$A \times B = \{ (1,a), (2,a), (3,a), (1,b), (2,b), (3,b) \}$$

$A \times B$, set of all ordered pairs such that first member of ordered pair is from set A and the second is from set B.

$$A \times B = \{ (x, y) \mid x \in A \wedge y \in B \}$$

Cardinality of $A \times B = n(A) \times n(B)$

$= |A| \times |B|$

(3) *) Set Identities

	Union	Intersection
(1) Commutative	$A \cup B = B \cup A$	$A \cap B = B \cap A$
(2) Associative	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
(3) Distributive	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
(4) Absorption	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
(5) De Morgan's Law	$(A \cup B)^c = A^c \cap B^c$	$(A \cap B)^c = A^c \cup B^c$

Symmetric Difference

(1) Commutative $\quad A \Delta B = B \Delta A$

(2) Associative $\quad (A \Delta B) \Delta C = A \Delta (B \Delta C)$

Set Difference

1. Commutative $A-B \neq B-A$
2. Associative $(A-B)-C \neq A-(B-C)$

Cartesian Product

1. Commutative $A \times B \neq B \times A$
2. Associative $(A \times B) \times C \neq A \times (B \times C)$

4. *) Formulas for Cardinality

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$
3. $n(A-B) = n(A) - n(A \cap B)$

Q In a class of 200 Students, 125 have taken programming language, 85 took data Structure, 65 have taken computer organization, 50 took both programming and data Structure, 35 took data structure and computer organization, 30 took programming and computer organization, 15 took all three. How many have not taken any course?

Solⁿ → $U = 200$

→ $n(P) = 125$ $n(C) = 65$

$n(D) = 85$

→ $n(P \cap D) = 50$ $n(P \cap C) = 30$

$n(D \cap C) = 35$

→ $n(P \cap D \cap C) = 15$

$n(P \cup D \cup C) = 125 + 85 + 65 - 50 - 35 - 30 + 15$

$= 175$

$n(P \cup D \cup C)^c = U - n(P \cup D \cup C)$

$= 200 - 175$

$= \underline{\underline{25}}$

(5) *) Partition

Let X be a set and $S = \{ A_i \mid A_i \subseteq X, i \in N \}$ be the set of subsets of X.

S is said to be a partition of X, if the elements of S hold the following properties →

1) The union of all A_i's is the set X
   i.e $\bigcup_i A_i = X$
2) All A_i's are disjoint
   i.e if $A_i, A_j \in S$ then,
   $A_i \cap A_j = \phi$

Eg

Let $X = \{1, 2, 3, 4, 5, 6, 7\}$

a) $S = \{ \{1,2,3\}, \{3,4,5\}, \{6,7\} \}$

All sets are not disjoint

$\{1,2,3\} \cap \{3,4,5\} = \{3\} \neq \phi$

$\therefore$ S is not a partition

b) $S = \{ \{1,2\}, \{3,4\}, \{5,6,7\} \}$

Yes, a Partition

c) $S = \{ \{1,2\}, \{3\}, \{5,6,7\} \}$

Not a Partition