

Tutorial Solution – 08

1. (a) f is not a function because the element b of A does not have an image in B .
- (b) f is not a function because the element c of A does not have a unique image in B .
- (c) f is a function because every element of A has a unique image in B .
- (d) f is a function because every element in A has a unique image in B .

2. We have to prove this function is both injective and surjective.

If $f(x_1) = f(x_2)$, then $2x_1 - 3 = 2x_2 - 3$ and it implies that $x_1 = x_2$.

Hence, f is injective.

Here, $2x - 3 = y$

So, $x = (y + 3)/2$ which belongs to R and $f(x) = y$.

Hence, f is surjective.

Since f is both surjective and injective, we can say f is bijective.

3. The function f is invertible because it is a one-to-one correspondence. The inverse function f^{-1} reverses the correspondence given by f , so $f^{-1}(1) = c$, $f^{-1}(2) = a$, and $f^{-1}(3) = b$.

4.(c) From the definition of the sum and product of functions, it follows that $(f_1 + f_2)(x) = f_1(x) + f_2(x) = x^2 + (x - x^2) = x$ and $(f_1 f_2)(x) = x^2 (x - x^2) = x^3 - x^4$. When f is a function from a set A to a set B , the image of a subset of A can also be defined.

5. (a) The composition of f and g is given by $f(g(x))$ which is equal to $2(3x + 4) + 1$.

6. Thus, the correct option is B.

7. a) The conversion formula $r = (\pi/180) \cdot d$ stipulates r as a function of d . The opposite conversion formula $d = (180/\pi) \cdot r$ specifies the same relationship between r and d , only this time from the other direction, so it defines a different function. In both cases, the domain and co-domain can be taken to be the set R of real numbers.

b) Since a definite amount of money is either owed to or by each person with a social security number, P is a function from the set of social security numbers to the set of positive and negative numbers given to two decimal places. P is a function even though there is no set formula for calculating $P(x)$; in fact, while a complicated equation might be devised, it would be of no use whatsoever as a formula. It only summarizes a discrete set of data; it would not cover any new cases that arise. On the other hand, if x indicated taxable income instead of social security numbers, a usable formula could be found, though it would not be a single equation. Currently, the tax percentage changes as income increases; there are different levels of taxation (formulas) depending on income level.

8. No domain is given; we will assume it is the largest set of real numbers for which the formula makes sense, namely, $R - \{-3\}$. To prove that f is one-to-one, we will show that the pre-images of two equal images are themselves equal. So suppose $x_1 / x_1 + 3 = x_2 / x_2 + 3$. Cross-

multiplying and simplifying yields $x_1 = x_2$. Thus, f is one-to-one. f is obviously onto its range by definition. To determine the range, we must see what y -values result from real x -values different from 3.

Suppose $y = x/(x + 3)$.

All such y are real numbers, but not all real numbers can be put into this form. To see which ones can, we will solve this equation for x . This will show explicitly which x -values in the domain, if any, can generate a given y -value. Cross-multiplying, $xy + 3y = x$, so $x(1 - y) = 3y$. Hence $x = 3y/(1 - y)$, which is defined for all y except 1. So, $y = 1$ has no pre-image x . However, given any real number y is not equal to 1, these x -values will produce it: $(3y/(1 - y))/(3y/(1 - y) + 3) + 3 = 3y/(3y + (3 - 3y)) = 3y/3 = y$. Thus, the only y -value that must be excluded from R is 1. This would give $1 = x/(x + 3)$, leading to $3 = 0$, which is false. Therefore, the range of f is $R - \{1\}$. Note that here $y = 1$ is the equation of the asymptote for the function f .

9. Correct answer: (C)

$$f(x) = 0.75(x-1) + 5$$

Explanation:

Notice that the question describes a linear equation because there is a constant rate of change (the cost per topping). This means we can use slope intercept form to describe the scenario.

Recall that slope intercept form is

$$y = mx + b$$

The value of b is the y -value when $x=0$. In this case $x=0$ means there are zero additional toppings and the question tells us that this pizza costs \$5 so:

$$5 = m(0) + b$$

$$5 = b$$

m is the slope, or rate of change as x increases. In this case each additional topping costs 75 cents. Notice that the units have shifted from dollars to cents so we must write this as 0.75

$$m = 0.75$$

Notice that the first topping is included in the \$5 cost. The 75 cent charge only applies to additional toppings. So one less than the number of toppings is equivalent to:

$$x - 1$$

Putting all these steps together we get: $0.75(x-1) + 5$

10. It is a function because there is only one output (profit) for every input (year). Functions don't have to be constant or follow a specific pattern.