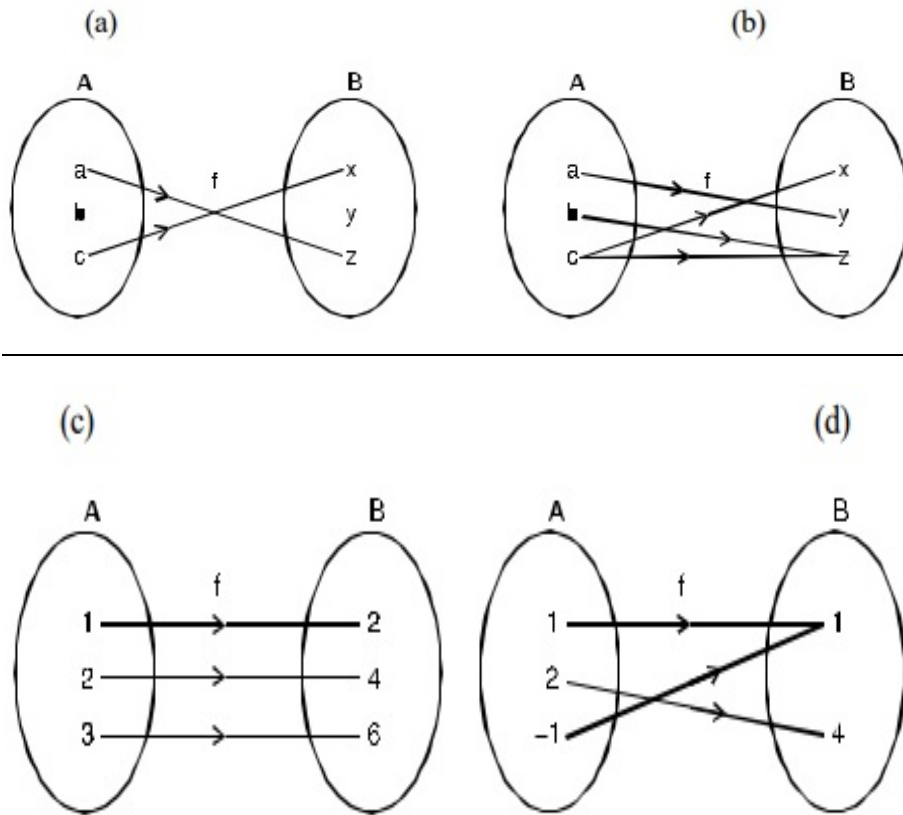


## Tutorial 8

1. State whether each of the following relations represent a function or not:



2. "A function  $f: A \rightarrow B$  is bijective or one-to-one correspondent if and only if  $f$  is both injective and surjective." Prove that a function  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 2x - 3$  is a bijective function.
3. Let  $f$  be the function from  $\{a, b, c\}$  to  $\{1, 2, 3\}$  such that  $f(a) = 2$ ,  $f(b) = 3$ , and  $f(c) = 1$ . Is  $f$  invertible, and if it is, what is its inverse?
4. Let  $f_1$  and  $f_2$  be functions from  $\mathbb{R}$  to  $\mathbb{R}$  such that  $f_1(x) = x^2$  and  $f_2(x) = x - x^2$ . What are the functions  $f_1 + f_2$  and  $f_1 f_2$ ?
  - (a)  $2x$  and  $x^2 - x^3$
  - (b)  $x^{2+2x}$  and  $x^2(x - x^2)$
  - (c)  $x$  and  $x^3 - x^4$
  - (d) None of the above.
5. Let  $f$  and  $g$  be the function from the set of integers to itself, defined by  $f(x) = 2x + 1$  and  $g(x) = 3x + 4$ . Then the composition of  $f$  and  $g$  is \_\_\_\_\_
  - a)  $6x + 9$
  - b)  $6x + 7$
  - c)  $6x + 6$
  - d)  $6x + 8$

6. Consider the set of all functions  $f: \{0,1, \dots, 2014\} \rightarrow \{0,1, \dots, 2014\}$  such that  $f(f(i)) = i$ , for all  $0 \leq i \leq 2014$ . Consider the following statements:

P. for each such function it must be case that for every  $i$ ,  $f(i)=i$ .

Q. For each such function it must be case that for some  $i$ ,  $f(i)=i$

R. Each such function must be onto.

Which one of the following is CORRECT?

- (A) P, Q and R are true
- (B) Only Q and R are true
- (C) Only P and Q are true
- (D) Only R is true.

7. Explain why the following define functions.

a) The formula for converting degree measure into radian measure is given by  $r = (\pi/180) \cdot d$ .

b) Let  $P(x)$  denote the refund/income tax payment calculated on a tax form for a given year that is owed to/by the person whose social security number is  $x$ .

8. Show that  $y = f(x) = x / (x + 3)$  is one-to-one onto its range and determine the range.

9. At Joe's pizzeria a pizza costs \$5 with the first topping, and then an additional 75 cents for each additional topping.

If  $x$  represents the number of toppings on a pizza, what function represents the cost of a pizza with at least one topping?

- (A)  $f(x)=5+75x$
- (B)  $f(x)=5x+0.75$
- (C)  $f(x)=0.75(x-1)+5$
- (D)  $f(x)=0.75+5(x+1)$

10. Martin starts a company and he wants to know the financial condition of his company. He asks his assistant to construct a table with the number of years that he operates the company and profit in million dollars as variables. Will the ordered pairs from the assistant's data represent a function or a relation?

- (A) Relation, because the profit of the company is neither constant nor predictable.
- (B) Function, because the company can only have one profit at a given year.