Tutorial Solution - 01

Q1) The truth table for the two given statements are shown below:-

P	Q	$\sim P$	$\sim Q$	$P \to Q$	$(\sim P) \vee (\sim Q)$	$(\sim P \vee \sim Q) \to Q$
T	T	F	F	T	F	T
T	F	F	T	F	T	F
F	T	T	F	T	T	T
F	F	T	T	T	T	F

From the table it follows that $P \to Q$ is <u>false</u>, then the value of $(\sim P \vee \sim Q) \to Q$ is <u>false</u>.

Q2) Let P: Food is Good, Q: Food is cheap.

Then, the statement "Good food is not cheap' is written as:

$$P \to \sim Q$$

and the statement 'Cheap food is not good' is written as:

$$Q \to \sim P$$

The truth table for the Statements are given below:-

P	Q	$\sim P$	$\sim Q$	$P \to \sim Q$	$Q \to \sim P$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

Q3) (a) $Q \wedge \sim P$

(b) $\sim Q \wedge \sim P$

(c) $P \vee Q$

(d) $\sim (P \vee Q)$

(e) $\sim (\sim Q \vee \sim P)$

Q4) The number of possible combinations are $2^4 = 16$ Ans

Q5) (I) $A \wedge B \wedge C \wedge (D \vee E) \wedge F \wedge G$.

(II) No, your GPA in the major is too low.

Q6) (a) Let us consider the truth table:-

P	Q	$\sim P$	$\sim Q$	$(P \vee Q)$	$\sim(P \vee Q)$	$(\sim P) \vee (\sim Q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

The statement is false as visible from the last two columns of the truth table.

Q.6) (b.) let us consider the truth table:

P	Q	R	$P \vee Q \vee R$	$P \vee (Q \vee R)$	$(P \vee Q) \vee R$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	F	F	F

The statement is True as visible from the last three columns of the truth table.

Q7) (a) $P \rightarrow [(\neg P) \rightarrow Q]$

P	Q	$\neg P$	$(\neg P) \rightarrow Q$	$P \rightarrow [(\neg P) \rightarrow Q]$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

Yes, it is a tautology as visible from the last column of the truth table that for all the combination of the inputs, $P \rightarrow [(\neg P) \rightarrow Q]$ is True.

Q7) (b) $(P \wedge Q) \vee Q$

P	Q	$P \wedge Q$	$(P \wedge Q) \vee Q$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	F

No, it is not a tautology as $(P \wedge Q) \vee Q$ is not true for all combinations of the input.

Q8) (a) Let P : 'a man has discovered something he will die for'

Q: 'he is fit to live'.

Original	$(\neg P) \rightarrow (\neg Q)$	'If a man hasn't discovered something he will die for, then he isn't fit to live.
Contrapositive	$Q \rightarrow P$	'If a man is fit to live, then he has discovered something he will die for.
Converse	$(\neg Q) \rightarrow (\neg P)$	If a man isn't fit to live, then he hasn't discovered something he will die for.
Inverse	$P \rightarrow Q$	If a man has discovered something he will die for then he is fit to live.

Q8) (b) If the original is true, we know that contrapositive is also true. We also know that the converse and inverse have the same truth value.

Q9) (a) $(P \wedge Q)$ and $(\neg P \vee \neg Q)$ are they logically equivalent.

P	Q	$(P \wedge Q)$	$(\neg P \vee \neg Q)$
T	T	T	F
T	F	F	T
F	F	F	T
F	F	F	T

Since the last two column of the truth table shows that $(P \wedge Q)$ and $(\neg P \vee \neg Q)$ have different truth values for the different combinations of truth values, therefore they are not logically equivalent.

Q9) (b) $(P \to Q) \vee P$ and $(P \vee \neg Q) \wedge Q$

P	Q	$(P \to Q)$	$(P \to Q) \vee P$	$(P \vee \neg Q)$	$(P \vee \neg Q) \wedge Q$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	T	F	F
F	F	T	T	T	F

They are also not logically equivalent.

Q10) Let P: I work hard.

Q: I get a job.

So, the statement :-

I work hard or I get a job and I do not work hard and I do not get a job.

Symbolically, it can be represented as:-

$$(P \vee Q) \wedge (\sim P \wedge \sim Q)$$

P	Q	$(P \vee Q)$	$(\sim P \wedge \sim Q)$	$(P \vee Q) \wedge (\sim P \wedge \sim Q)$
T	T	T	F	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	F

Since, $(P \vee Q) \wedge (\sim P \wedge \sim Q)$ is False for all combinations, Therefore, the above statement is a contradiction.

11. We have four variables p, q, r, s. Three of them are true and the other is false. However, we don't know which is the false variable. $\boxed{\neg \equiv \sim}$

Case 1 p is false. Then the other variables are true.

$\therefore \neg p \wedge q \wedge r \wedge s$

Case 2 q is false, & the others are true.

$p \wedge \neg q \wedge r \wedge s$

Case 3 r is false, other three are true

$p \wedge q \wedge \neg r \wedge s$

Case 4 s is false, other three are true

$p \wedge q \wedge r \wedge \neg s$

Conclusion One of the four cases has to be true. This means we need to take conjunction of the propositions in each case.

Result $(\neg p \wedge q \wedge r \wedge s) \vee (p \wedge \neg q \wedge r \wedge s) \vee (p \wedge q \wedge \neg r \wedge s) \vee (p \wedge q \wedge r \wedge \neg s)$

12.

$\neg r$	p	q	r	$p \vee q$	$(p \vee q) \wedge (\neg r)$	$(p \vee q) \wedge (\neg r) \leftrightarrow q$
1	0	0	0	0	0	1
0	0	0	1	0	0	1
1	0	1	0	1	1	1
0	0	1	1	1	0	0
1	1	0	0	1	1	0
0	1	0	1	1	0	1
1	1	1	0	1	1	1
0	1	1	1	1	0	0

13.

p	q	p∨q	¬p	¬q	¬p∨¬q	(p∨q)∧(¬p∨¬q)
0	0	0	1	1	1	0