(I) Predicates

1. Mohan is a student.
2. Shrikant is a student.
3. Shefali is a student.

→ The part "is a student" of the sentence is common in all these sentences.

→ So, it can be written as 'x is a student'.

where <u>is a student</u> is called a predicate.

and

a set X of students from where x can take its values.

Here, the set X is called the <u>universe of discourse</u> for x.

→ A predicate is denoted as $P(x)$, $Q(x)$, $R(x)$ .... etc

Eg:- $P(x)$: x is a student.

$Q(x)$: x is an animal.

→ If we assign a particular value to x, then the predicate is converted into a proposition.

Eg:- $P(x)$: x is less than five.

The universe of discourse for x is the set of real numbers.

Thus, $P(2)$ is a proposition whose truth value is <u>true</u>.

→ Two variables are also possible :-

Eg:- $P(x, y)$: x is greater than y

if x is 6 and y is 3 the '6 is greater than 3'

becomes a proposition whose truth value is <u>true</u>.

II <u>Quantifiers</u>

- Rakesh is brilliant and Mohan is brilliant and Alka is brilliant.
- The above sentence can be written as :-

  " All the students of the set A are brilliant ".

  where set A = { Rakesh, Mohan, Alka }
- In the predicate form the given sentence can be stated as:

  $\boxed{P(x) : x \text{ is brilliant}}$ and set A becomes the Universe of discourse

But how to represent '<u>all the students</u>' ?

- To tackle this we need some representation of the phrase " <u>for all</u> ".
- This leads to the use of quantifiers.

- There are two types of quantifiers:

Quantifiers

(a) Universal ($\forall$)   (b) Existential ($\exists$)

(a) Example of Universal Quantifier

(i) Let $\boxed{P(x): x \text{ is even numbers}}$ and the universe of discourse for x is the set $\{1,2,3,4\}$.

Find the truth value of $\forall(x)\, P(x)$.

Ans. $\forall x\, P(x)$ is FALSE.

(b) Example of Existential Quantifier

(i) Let $\boxed{P(x): x \text{ is even number}}$ and the universe of discourse for x is the set $\{1,2,3,4\}$. Find the truth value of $\exists x\, P(x)$.

Ans. $\exists x\, P(x)$ is TRUE.

(III) Free and Bound Variables

→ A variable in a predicate is bound by a quantifier.

→ A variable is free if it is not bounded.

Eg:- $\forall x \ P(x, y)$ and $\exists x \ P(x, y)$

In both cases, x is a bound variable and y is a free variable.

(IV) Negation of Quantifiers

→ Every politician is clever

$P(x)$: x is clever.

UoD : set of politicians

$\therefore$ $\boxed{\forall x \ P(x)}$.

Here, the negation of the statement would be "It is not the case that every politician is clever".

OR

"There is a politician who is not clever".

Symbolically, it can be represented as:

$$\boxed{\exists x \sim P(x)}$$

→ Laws of Equivalence

$$\sim(\forall x \ P(x)) \iff \exists x \sim P(x)$$
$$\sim(\exists x \ P(x)) \iff \forall x \sim P(x)$$

→ Negation of Quantified Statements with more than One Variable :

- Quantified statements with more than one variable may be obtained by successively applying the rule for negating a statement with only one quantifier.
- Thus, each $\forall$ is changed to $\exists$ and each $\exists$ is changed to $\forall$ as the negation symbol passes through the statement from left to right.

Eg:- ① $\sim[\forall x \exists y \ P(x,y)] \equiv \exists x [\sim \exists y \ P(x,y)]$
$\equiv \exists x \forall y [\sim P(x,y)]$.

② $\sim[\exists x \exists y \sim P(x,y) \wedge \forall x \forall y \ Q(x,y)]$
$\equiv \forall x \forall y \ P(x,y) \vee \exists x \exists y \sim Q(x,y)$

Q) Write the negation of the following statements:

(a) All states in India are highly populated.

(b) Some states in India are highly populated.

→ Removing Quantifiers from Predicates

- Let $P(x)$ be the predicate on x and the universe of discourse of x is the set $\{x_1, x_2, x_3, \dots, x_n\}$

Then, (a) $\forall x \; P(x) \iff P(x_1) \wedge P(x_2) \wedge \dots \wedge P(x_n)$

(b) $\exists x \; P(x) \iff P(x_1) \vee P(x_2) \vee \dots \vee P(x_n)$

Eg:- Let us assume that $P(x)$ and $Q(x)$ are two predicates on x where $x \in \{1, 2, 3\}$. Remove the quantifiers from the following:

(a) $\exists x \; P(x) = P(1) \vee P(2) \vee P(3)$

(b) $\forall x \; P(x) = P(1) \wedge P(2) \wedge P(3)$

(c) $\exists x \; P(x) \wedge \forall x \; Q(x)$

→ Nested Quantifiers

$\forall x \exists y \; P(x,y)$: This proposition is same as $\forall x \; Q(x)$

where, $\boxed{Q(x) \text{ is } \exists y \; P(x,y)}$.

Let the UoD for the variable x and y be the set of positive integers and let $P(x,y): x^2 = y$.

Translate $\forall x \exists y \; P(x, y)$ into an english sentence.

"The square of every positive integer is a positive integer".

(V) Fallacies

- A fallacy is an error in reasoning that results in an invalid argument.
- Two types of fallacies are:
  1. The fallacy of affirming the consequent (or affirming the converse).
  2. The fallacy of denying the hypothesis (or assuming the inverse).

Eg:- (1.) If Siddharth solved this problem correctly, then he obtained the answer 5.

Siddharth obtained the answer 5.

Therefore, Siddharth solved the problem correctly.

Note :- The argument is of the form $p \to q$ and q then p. and is invalid because $[(p \to q) \wedge q] \to p$ is not a tautology.

This is $\boxed{\text{Fallacy of affirming the consequent}}$

(2) Test the validity of the following argument:

If two sides of a triangle are equal, then the opposite angles are equal.

Two sides of a triangle are not equal.

Therefore, the opposite angles are not equal.