Tutorial 11

1. How many bit strings of length eight either start with a 1 bit or end with the two bits 00?

2. a) How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen?
b) How many must be selected to guarantee that at least three hearts are selected?

3. How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on a scale from 0 to 100 points?

4. Suppose that a computer science laboratory has 15 workstations and 10 servers. A cable can be used to directly connect a workstation to a server. For each server, only one direct connection to that server can be active at any time. We want to guarantee that at any time any set of 10 or fewer workstations can simultaneously access different servers via direct connections. Although we could do this by connecting every workstation directly to every server (using 150 connections), what is the minimum number of direct connections needed to achieve this goal?

5. What is the least number of area codes needed to guarantee that the 25 million phones in a state can be assigned distinct 1 O-digit telephone numbers? (Assume that telephone numbers are of the form NXX-NXX-XXXX, where the first three digits form the area code, N represents a digit from 2 to 9 inclusive, and X represents any digit.)

6. How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?

7. Suppose that a saleswoman has to visit eight different cities. She must begin her trip in a specified city, but she can visit the other seven cities in any order she wishes. How many possible orders can the saleswoman use when visiting these cities?

8. How many permutations of the letters ABCDEFGH contain the string ABC?

9. A group of 30 people have been trained as astronauts to go on the first mission to Mars. How many ways are there to select a crew of six people to go on this mission (assuming that all crew members have the same job)?

10. Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department?

11. A car has 8 seats (4 in front row and 4 in back row). In how many ways can 5 people be seated in the car so that 2 people sit on the front seats and rest three on the back row?

12. Determine the number of solutions possible for the equation $x_1 + x_2 + x_3 + x_4 = 30$ if $x_1 \geq 2, x_2 \geq 4, x_3 \geq 5, and\ x_4 \geq 6$ and all are integers.