

\pm binary
 ± 25 0.1
=16

Sign Number and Addition, Subtraction



How To Represent Signed Numbers

- Plus and minus sign used for decimal numbers: 25 (or +25), -16, etc.
- For computers, desirable to represent everything as *bits*.
- Three types of signed binary number representations:
 - signed magnitude,
 - 1's complement,
 - 2's complement.
- In each case: left-most bit indicates sign: positive (0) or negative (1).



Signed Magnitude

Consider ***signed magnitude***:

8 bit Signed mag.
 $\overbrace{1\ 0000\ 111}^{\text{sign mag}} = -7$
↑
sign mag

$\overbrace{0\ 0001\ 100}_2 = 12_{10}$
↑ ↑
Sign bit Magnitude

$\overbrace{1\ 0001\ 100}_2 = -12_{10}$
↑ ↑
Sign bit Magnitude

0 → + Sign
1 → - Sign



One's Complement Representation

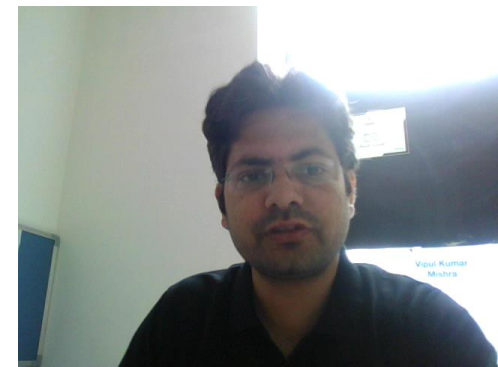
- The one's complement of a binary number involves inverting all bits.
- 1's comp of 00110011 is **11001100**
- 1's comp of 10101010 is **01010101**
- To find negative of 1's complement number take the 1's complement.

+ve
Same
-ve

15 → +15 → 01111
-15 → 10000

→ 00001100₂ = 12₁₀
Sign bit Magnitude

11110011₂ = -12₁₀
Sign bit Magnitude



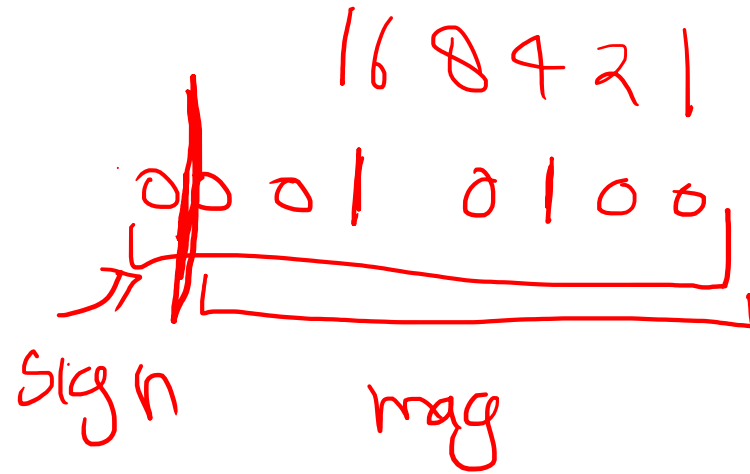
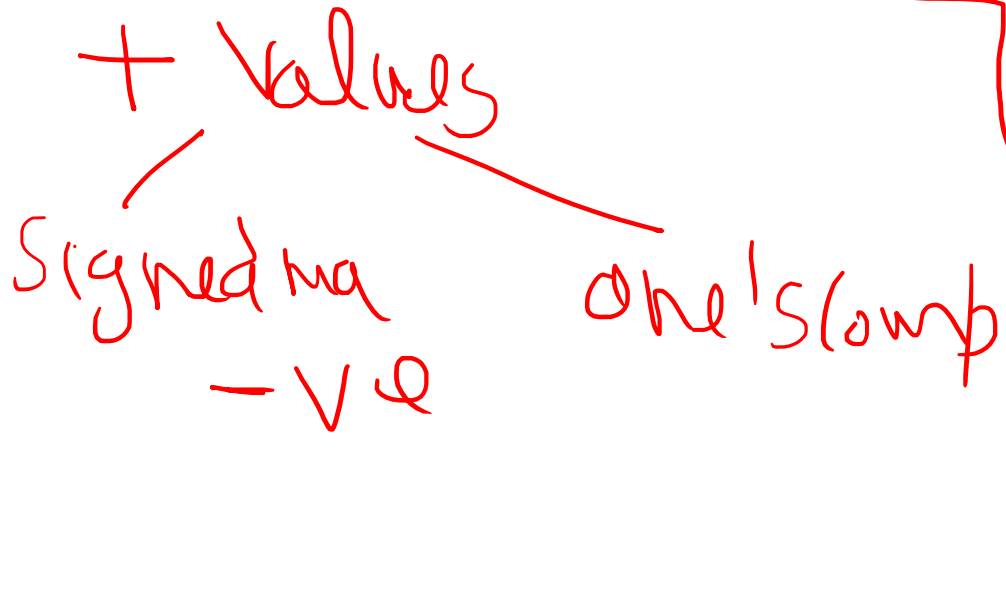
One's complement

8 bit \rightarrow

+20

\downarrow

-20



+15 \rightarrow

-15

00010100

\downarrow is complement

11101011

↑
Sign



Two's Complement Representation

- The two's complement of a binary number involves inverting all bits and adding 1.
- 2's comp of 00110011 is **11001101**
- 2's comp of 10101010 is **01010110**
- To find negative of 2's complement number take the 2's complement.

$$\begin{array}{c} \text{Sign bit} \nearrow \text{00001100}_2 = 12_{10} \nwarrow \text{Magnitude} \end{array}$$

$$\begin{array}{c} \text{Sign bit} \nearrow \text{11110100}_2 = -12_{10} \nwarrow \text{Magnitude} \end{array}$$

$$\begin{array}{l} -12 \\ 00001100 \\ \downarrow \text{1's com} \\ (11110011) \text{ 1's com} \\ + 1 \\ \hline (11110100) \\ \rightarrow -12 \text{ in } 2^{\text{nd}} \text{ comp} \end{array}$$



Two's Complement Representation

+17

Signed mag \rightarrow $\left\{ \begin{array}{c} \overbrace{00010001}^{\text{mag}} \\ \underbrace{\uparrow}_{\text{sign}} \end{array} \right\}$

16 8 4 2 1

1's comp \rightarrow $\left\{ \begin{array}{c} 00010001 \\ 00010001 \end{array} \right\}$

2's comp \rightarrow $\left\{ \begin{array}{c} 00010001 \\ 00010001 \end{array} \right\}$

-17

Signed ho \rightarrow $\left\{ \begin{array}{c} \overbrace{10010001}^{\text{mag}} \\ \underbrace{\downarrow}_{\text{sign}} \end{array} \right\}$ ✓

1's complement \rightarrow 00010001 ✓

\downarrow
 $(11101110) \rightarrow -17$

2's complement \rightarrow

11101110 ✓

$(11101111) \rightarrow -17$



Binary Addition

- Binary addition is very simple.
- This is best shown in an example of adding two binary numbers...

$$\begin{array}{r} \begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ + & & 1 & 0 & 1 & 1 & 1 \end{array} & \leftarrow \text{carries} \\ \hline 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{array}$$



Binary Subtraction

- We can also perform subtraction (with borrows in place of carries).
- Let's subtract $(10111)_2$ from $(1001101)_2$...

$$\begin{array}{r}
 \begin{array}{ccccccc}
 & & 1 & & 10 & & \\
 0 & \cancel{1}0 & 10 & 0 & \cancel{0} & 10 & \\
 \end{array}
 & \longleftarrow \text{borrows} \\
 \begin{array}{r}
 \cancel{1} \quad \cancel{0} \quad \cancel{0} \quad \cancel{1} \quad \cancel{1} \quad \cancel{0} \quad 1 \\
 - \qquad \qquad \qquad 1 \quad 0 \quad 1 \quad 1 \quad 1 \\
 \hline
 \qquad \qquad \qquad 1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0
 \end{array}
 \end{array}$$



Binary Multiplication

- Binary multiplication is much the same as decimal multiplication, except that the multiplication operations are much simpler...

$$\begin{array}{r} 1 0 1 1 1 \\ X 1 0 1 0 \\ \hline 0 0 0 0 0 \\ 1 0 1 1 1 \\ 0 0 0 0 0 \\ 1 0 1 1 1 \\ \hline 1 1 1 0 0 1 1 0 \end{array}$$

$$1 \times 0 \rightarrow 0$$

$$0 \times 1 \rightarrow 0$$

$$0 \times 0 \rightarrow 0$$

$$1 \times 1 \rightarrow 1$$



Addition

$$(123)_4 + (23)_4 = (?)_4$$

$$123 + 23$$

$$\begin{array}{r} (123)_{10} \\ (23)_{10} \\ \hline 146 \end{array}$$

$$\begin{array}{r} 123_4 \\ 23_4 \\ \hline (212)_4 \end{array}$$

$$(35)_5 + (23)_5 = (?)_5 \quad \leftarrow \text{Practice.}$$

↓ ↓ ↓
0, 1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23, ...



Addition using 1's complement

$$\underline{(12)_{10}} + \underline{(23)_{10}} = (?)_{10}$$

$$12 \rightarrow 0000 \overset{1}{1} \overset{1}{1} 00$$

$$23 \rightarrow 00010111 +$$

→

$$\underline{00100011}$$

$$\boxed{\quad}$$

→ Decimal

$$(35)_{10}$$

$$168421$$
$$000 \cancel{00} | 100$$

$$000 | 0111$$

$$00 | 0001$$

$$\underline{32168421}$$



~~_____~~

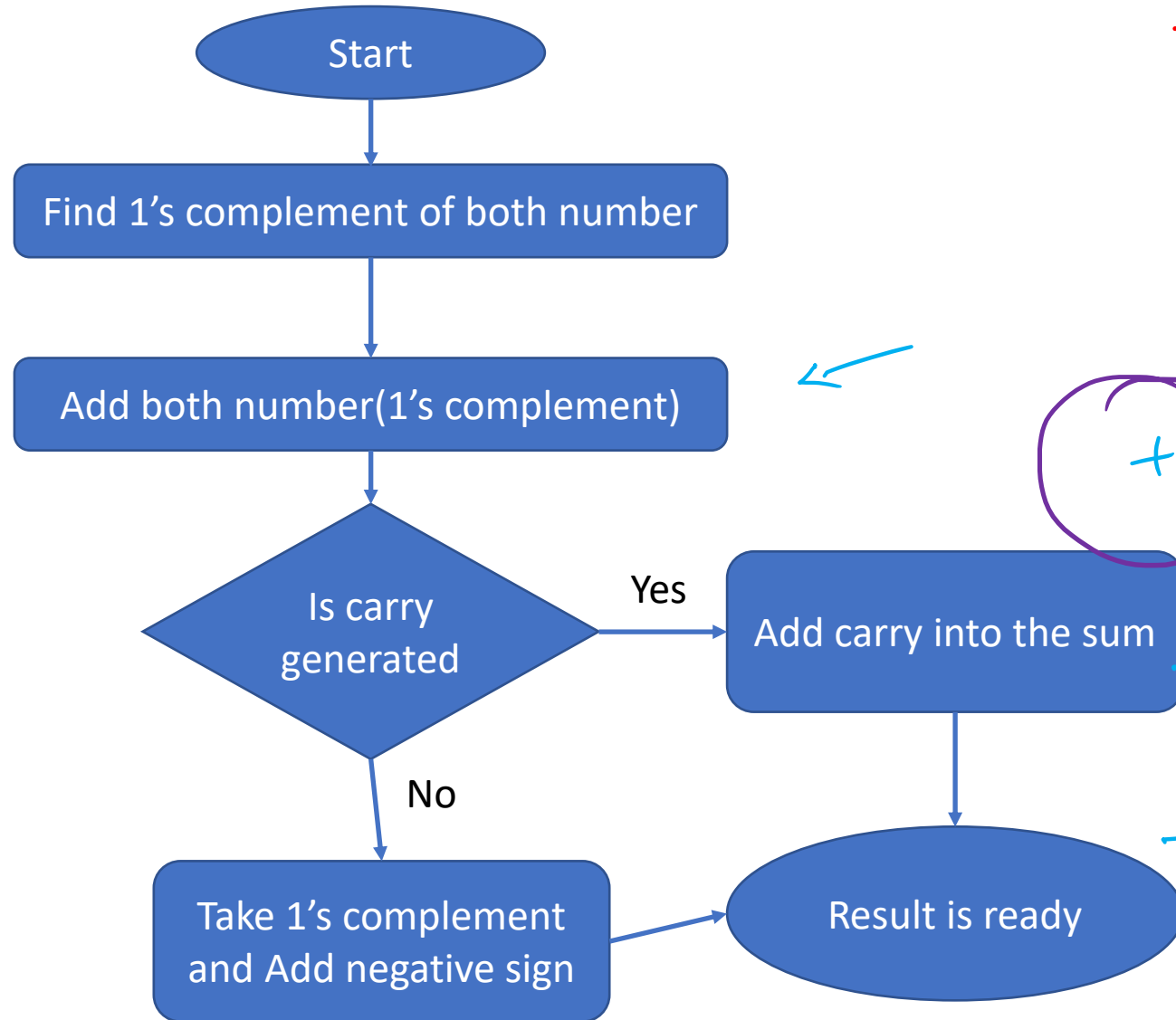
$$(+35)_{10}$$

A man with dark hair and glasses, wearing a black shirt, is speaking. Behind him is a large screen displaying a table with the following content:

NAME	ROLL NO.	MARKS
VIJAY KUMAR	1001	80
VIJAY KUMAR	1002	70
VIJAY KUMAR	1003	60
VIJAY KUMAR	1004	50
VIJAY KUMAR	1005	40
VIJAY KUMAR	1006	30
VIJAY KUMAR	1007	20
VIJAY KUMAR	1008	10
VIJAY KUMAR	1009	0

Below the table, the text "Vijay Kumar Mishra" is visible.

Subtraction using 1's complement



Handwritten calculations for $168421 - 2813$ using 1's complement:

168421
 $- 2813$

Conversion to 8-bit binary (assuming 28 and 13 are the numbers to subtract from 168421):

$+28 \rightarrow 11100$
 $-13 \rightarrow 01101$

1's complement of 13 is 1111010.

Adding 28 and 1's complement of 13:

$$\begin{array}{r} 00011100 \\ + 1111010 \\ \hline 10001100 \end{array}$$

Carry 1 is generated. Add it to the sum:

$$\begin{array}{r} 10001100 \\ + 1 \\ \hline 10001101 \end{array}$$

Result is 10001101, which is 8421 in decimal.

Alternatively, using 9-bit binary:

$+28 \rightarrow 00011100$
 $-13 \rightarrow 00001101$

1's complement of 13 is 11110010.

Adding 28 and 1's complement of 13:

$$\begin{array}{r} 00011100 \\ + 11110010 \\ \hline 10001110 \end{array}$$

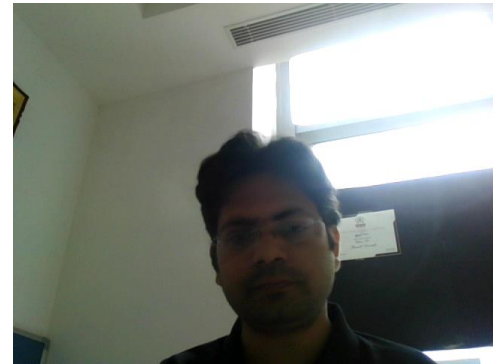
Carry 1 is generated. Add it to the sum:

$$\begin{array}{r} 10001110 \\ + 1 \\ \hline 10001111 \end{array}$$

Result is 10001111, which is 8421 in decimal.



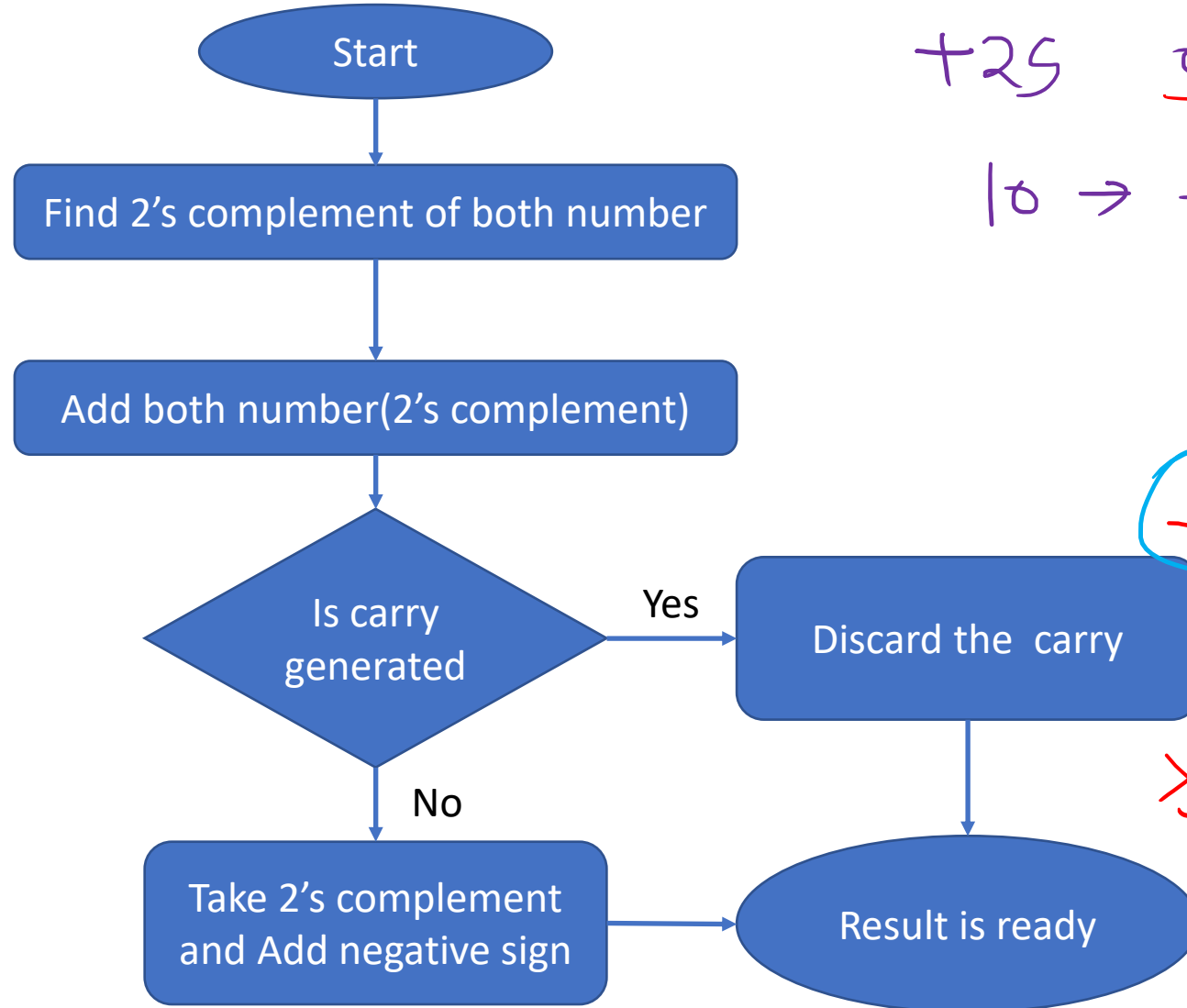
Subtraction using 1's complement



Subtraction using 2's complement

16 8 4 2 1

$$\underline{25} = \underline{10}$$



$$+25 \quad \underline{00011001}$$

$$10 \rightarrow \underline{00001010} \rightarrow \begin{array}{r} 11110101 \\ +1 \end{array}$$

$$\begin{array}{r} 00011001 \\ + 11110101 \\ \hline \end{array}$$

$$\boxed{11110101} - 10$$

$$\begin{array}{r} 10001111 \\ \times 10001111 \\ \hline \end{array} \rightarrow +15$$

