Lecture 11

Class Note

Dimension: The number of vectors in a basis of a vector space V is said to be the dimension of V and denoted by dim V.

Example: let $V = \mathbb{R}^n$

since $S = \{(1,0,0,\dots,0), (0,1,0,\dots,0), \dots, (0,0,0,\dots,1)\}$

is a basis of $\mathbb{R}^n$.

$\therefore$ dim (V) = number of element is S

$= n$

Example: let $V = \mathbb{R}^3$

$\therefore$ dim V = 3.

Theorem: Any two bases of a finite-dimensional vector space V have the same number of vectors.

Example: let $V = \mathbb{R}^3$

then $S_1 = \{(1,0,0), (0,1,0), (0,0,1)\}$ is basis of $\mathbb{R}^3$

Also we can show that

$S_2 = \{(0,1,1), (1,0,1), (1,1,0)\}$ is a basis of $\mathbb{R}^3$

$\therefore$ dim $(\mathbb{R}^3)$ = no of element in S_1 = 3.

dim $(\mathbb{R}^3)$ = no of element in S_2 = 3.

$\Rightarrow$ dim $(\mathbb{R}^3)$ = no of element in S_1

= no of element in S_2

$\Rightarrow$ an two bases of an finite-dimensional vector space V have the same number of vectors.

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Theorem: let V be a vector space of dimension n over a field F. Then any linearly independent set of n vectors of V is a basis of V

Theorem: let V be a vector space of dimension n over a field F. Then any subset of n vectors of V that generates V is a basis of V.

Result: The vector space consisting only zero element i.e if $V = \{0\}$, then dimension of that vector space is 0 i.e $\dim(V) = 0$

Example: Consider the following system of equations

$$\left.\begin{aligned} x_1 + 2x_2 + 2x_3 &= 0 \\ 2x_1 + x_2 + 2x_3 &= 0 \\ 2x_1 + 2x_2 + x_3 &= 0 \end{aligned}\right\} = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

$$\Rightarrow \quad AX_3 = 0$$

Determine the Null space of the matrix $A_{3\times3}$. and its dimension.

Solⁿ: $\mathcal{N}(A) = \{X \mid AX = 0\}$.

$$= \{(x_1, x_2, x_3) \mid AX = 0\}$$

$\because$ determinat of $A = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = 5 \neq 0$

$\Rightarrow$ Rank of A is 3

$\Rightarrow$ The homogeneous system $AX = 0$ has unique soln i.e only zero solution

$\therefore$ $\mathcal{N}(A) = \{(0,0,0)\}$

Now from the above result,

$$\dim(\mathcal{N}(A)) = 0$$

($\because$ null space of A contains only zero element, so that its dimension is zero).