

Tutorial - 12

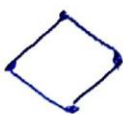
- Q1) Show that if every element of a Group (G, \circ) be its own inverse, then it is an abelian group.
- Q2) Determine whether the set together with the binary operation is a semi-group, a monoid or neither. If it is a monoid, specify the identity element:
- (a) \mathbb{N} , where $*$ is defined as ordinary addition
 - (b) \mathbb{Z}^+ , where $a * b = \max(a, b)$ for all $a, b \in \mathbb{Z}^+$

Q3) The algebraic structure $(\{1, 2, 3, \dots, p-1\}, X_p)$ is an Abelian Group, where p is a prime number and X_p represents multiplication modulo p

- A) True
- B) False

- Q4) Do the following sets form an integral domain under ordinary addition and multiplication? If so state whether they are fields:
- (a) the set of even integers.
 - (b) the set of positive integers.

Q5) Consider the following Hasse Diagrams



(i)



(ii)



(iii)



(iv)

Which of the above represent a lattice?

- (A) (i) and (iv) only
- (B) (ii) and (iii) only

Q6) Check whether the following are Posets?

(a) (R, \leq)

(b) $(Z, >)$

Q7) The following is the incomplete operation table of 4-element group.

\times	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b				
c				

The last row of the table is :

(a) c a e b

(b) c b a e

(c) e b e a

(d) c e a b

Q8) The set $\{1, 2, 4, 7, 8, 11, 13, 14\}$ is a group under multiplication modulo 15.

The inverse of 4 and 7 are respectively

(a) 3 and 13

(b) 2 and 11

(c) 4 and 13

(d) 8 and 14

Q9) Show that the matrices

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ form a multiplicative abelian group.

Q 10) For the composition table of a cyclic group shown below which of the choice is correct?

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

- (a) a, b are generators
- (b) b, c are generators
- (c) c, d are generators
- (d) d, a are generators

Q 11) Let $(R, *)$ be an algebraic structure. In each of the following identify which algebraic structure is defined, where $*$ is defined as-

- A) $a*b = a+b$ (where a and b are elements of R)
- B) $a*b = \min(a, b)$
- C) $a*b = a^b$

Q 12) Identify whether the given algebraic structure $(\{1, w, w^2\}, *)$ (i.e. 3rd roots of unity) represents a cyclic group. If yes, find all generators. (Here $*$ represents multiplication)