Logic

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Overview

Propositions

Unary Operators

Binary Operators

Converse, Inverse and Contrapositive

Compound Propositions

Precedence of Logical Operators

Propositional Logic

A proposition is a declarative sentence that is either true or false, but not both.
true = T (or 1), false = F (or 0) (binary logic)

Example 1 Sentences that are propositions:

1. New Delhi is the capital of India.
2. Moon is made of green cheese.
3. $1+1=2$
4. $2+2=7$

Example 2 Sentences that are not propositions:

1. What time is it?
2. Read this carefully.
3. $x+1=2$
4. $x+y=z$

Propositional Logic (cont.)

Propositional Variables : p,q,r,s....
New Propositions from old: **calculus of propositions** - relate new propositions to old using TRUTH TABLES
Logical operators: unary, binary

1. Unary:
   - Negation
2. Binary:
   - Conjunction
   - Disjunction
   - Exclusive OR
   - Implication
   - Biconditional

Unary Operators

- Negation

not, denoted by: $\neg$

Example 3:

p: Today is Friday.

$\neg p$: It is not the case that today is Friday.

or

$\neg p$: Today is not Friday.

or

$\neg p$: It is not Friday today.

p	$\neg p$
0	1
1	0

Table 1: Truth Table : $\neg$

Binary Operators

- Conjunction

and, denoted by: $\wedge$

Example 4:
p: Today is Friday.
q: It is raining today.
$p \wedge q$ Today is Friday and it is raining today.
Note: Both p and q must be true

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

Table 2: Truth Table : $p \wedge q$

Binary Operators

- Disjunction

inclusive 'or', denoted by: $\vee$

Example 5:
p: Today is Friday.
q: It is raining today.
$p \vee q$: Today is Friday or it is raining today.
Note: only one of p and q must be true. Hence, the inclusive nature.

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

Table 3: Truth Table : $p \vee q$

Binary Operators

- Exclusive OR

Exclusive 'or', denoted by: $\oplus$

Example 6:
p: Today is Friday.
q: It is raining today.
$p \oplus q$: Today is Friday or it is raining today, but not both.
Note: p or q, but not both.

p	q	$p \oplus q$
0	0	0
0	1	1
1	0	1
1	1	0

Table 4: Truth Table : $p \oplus q$

Binary Operators

- Implication

'If.. then..', denoted by: $\implies$

Example 7:

p: I am elected.

q: I will lower taxes.

$p \implies q$: If I am elected, then I will lower taxes.

Note: The implication is false only when P is true and Q is false!

$p \implies q$ has the same truth value as $\neg p \vee q$

p	q	$p \implies q$
0	0	1
0	1	1
1	0	0
1	1	1

Table 5: Truth Table : $p \implies q$

Equivalent forms:

If p, then q

p implies q

If p, q

p only if q

p is a sufficient condition for q

q if p

q whenever p

q is a necessary condition for p

Terminology:

p = premise, hypothesis, antecedent

q = conclusion, consequence

More terminology:

$q \implies p$ is the CONVERSE of $p \implies q$

$\neg q \implies \neg p$ is the CONTRAPOSITIVE of $p \implies q$

$\neg p \implies \neg q$ is the INVERSE of $p \implies q$

Example 8 :

Find the converse, contrapositive and inverse of the following statement:

R: 'Raining tomorrow is a sufficient condition for my not going to town.'

Solution

Step 1: Assign propositional variables to component propositions
P: It will rain tomorrow
Q : I will not go to town
Step 2: Symbolize the assertion
$R : P \implies Q$
Step 3: Symbolize the converse
$Q \implies P$
Step 4: Convert the symbols back into words
'If I don't go to town then it will rain tomorrow'
or
'Raining tomorrow is a necessary condition for my not going to town.'
or
'My not going to town is a sufficient condition for it raining tomorrow.'

Binary Operators

- Biconditional

'If and only if', 'iff', denoted by: $\iff$

Example 9:

P - 'I am going to town', Q - 'It is going to rain'

$P \iff Q$: 'I am going to town if and only if it is going to rain.'

Note: Both P and Q must have the same truth value.

$p \iff q$ has exactly the same truth value as $(p \implies q) \wedge (q \implies p)$

p	q	$p \iff q$
0	0	1
0	1	0
1	0	0
1	1	1

Table 6: Truth Table : $p \iff q$

Truth Tables of Compound Propositions

Example 10 :

Construct truth table of the compound proposition
$(p \vee \neg q) \implies (p \wedge q)$

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \implies (p \wedge q)$
-	-	-	-	-	-
-	-	-	-	-	-
-	-	-	-	-	-
-	-	-	-	-	-

Table 7: Compound Proposition

Solution:

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \implies (p \wedge q)$
0	0	1	1	0	0
0	1	0	0	0	1
1	0	1	1	0	0
1	1	0	1	1	1

Table 8: Compound Proposition

Precedence of Logical Operators

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$, $\oplus$	3
$\implies$	4
$\iff$	5

Table 9: Operator Precedence Table

Contribution

- Aristotle
- George Boole
- Raymond Smullyan
- John Wilder Tukey

Queries?