→ Argument

- An argument is a sequence of statements called premises followed by a conclusion.

- Consider a set of premises $(H_1, H_2, \ldots H_n)$ and another statement C, the conclusion.

  It is said that the conclusion C follows directly from the set of premises $(H_1, H_2, \ldots H_n)$ iff

$$H_1 \wedge H_2 \ldots \wedge H_n \Rightarrow C$$

or

$$(H_1 \wedge H_2 \wedge \ldots H_n) \rightarrow C \text{ is a tautology.}$$

- Such an argument is called a valid argument.

→ Rules of Inference

(a) Rule 1 :- Modus Ponens or Rule of detachment.

If the statement P is assumed as true and also the statement $P \rightarrow Q$ is accepted as true then Q must be true.

$P \rightarrow Q$	is T
P	is T
$\therefore Q$	must be T

Eg: If I study hard, then I get A's

I study hard

Therefore, I get A's

Let P: I study hard Q: I get A's

$P \to Q$	– T
P	– T
$\therefore Q$	– is also T

Hence, by modus ponens the argument is valid.

* Note: Another way of stating the above argument is that $[(P \to Q) \wedge P] \to Q$ is a tautology

(b) Rule 2:- Hypothetical Syllogism

Whenever two statements $P \to Q$ and $Q \to R$ are accepted as "True" then the statement $P \to R$ is accepted as True.

$P \to Q$	is T
$Q \to R$	is T
$\therefore P \to R$	is T

This rule is a valid argument because $[(P \to Q) \wedge (Q \to R)] \to (P \to R)$ is a tautology.

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$P \rightarrow Q$ is T
$\sim Q$ is T

$\therefore \sim P$ is T

Q Prove using truth table.

(d) Disjunctive Syllogism

$P \vee Q$ is T
$\sim P$ is T

$\therefore Q$ is T

Q Prove using truth table.

Eg: 1 Represent the argument symbolically and determine whether the argument is valid.

If it rains today, then we will not have a party today.

If we do not have party today, then we will have a party tomorrow.

Therefore, if it rains today, then we will have a party tomorrow.

Solution: Let us assume

P: It is raining today.

Q: We will not have a party today.

R: We will have a party tomorrow.

The given argument is of the form:

$$\begin{array}{l} P \to Q \\ Q \to R \\ \hline \therefore P \to R \end{array}$$

Hence, the argument is a hypothetical syllogism and thus the argument is valid.

Eg: 2 Represent the argument symbolically and determine whether the argument is valid.

If this number is divisible by 6, then it is divisible by 3.

This number is not divisible by 3.

This number is not divisible by 6.

Solution. Let us assume

P: The number is divisible by 6.

Q: It is divisible by 3.

The argument may be written as

$$P \rightarrow Q$$
$$\sim Q$$
$$\therefore \sim P$$

Thus, by modus tollens the argument is valid.

Eg:3 Represent the argument symbolically and determine whether the argument is valid.

Either Ram is not guilty or Shyam is telling the truth.

Shyam is not telling the truth.

Ram is not guilty.

Solution. Let us assume

P: Ram is not guilty.

Q: Shyam is telling the truth.

$$P \vee Q$$
$$\sim Q$$
$$\therefore P$$

Thus, by disjunctive syllogism, the argument is valid

Eg:4 Show that T is a valid conclusion from the premises.

$P \to Q$, $Q \to R$, $R \to S$, $\sim S$ and $P \vee T$

Solution.

1.	$P \to Q$	(Premise given)
2.	$Q \to R$	(Premise given)
3.	$R \to S$	(Premise given)
4.	$P \to R$	(By hypothetical syllogism using 1 and 2)
5.	$P \to S$	(By hypothetical syllogism using 4 and 3).
6.	$\sim S$	(Premise given)
7.	$\sim P$	(Modus Tollens using 5 and 6)
8.	$P \vee T$	(Premise given)
9.	$T \vee P$	(using commutative law)
10.	T	(By disjunctive syllogism using 9 and 7)

Thus, we conclude $\boxed{T}$ from the given premises.

Eg: 5 Prove the validity of the following argument:

"If I get the job and work hard, then I will get promoted.

If I get promoted, then I will be happy.

I will be not be happy.

Therefore, either I will not get the job or I will not work hard".

Solution. Assume, P: I get the job.
Q: I work hard.
R: I get promoted.
S: I will be happy.

Then, the given argument can be written in the symbolic from as

$$(P \wedge Q) \rightarrow R$$
$$R \rightarrow S$$
$$\sim S$$

So,

1.	$(P \wedge Q) \rightarrow R$	(Premise given)
2.	$R \rightarrow S$	(Premise given)
3.	$(P \wedge Q) \rightarrow S$	(By hypothetical Syllogism using 1 and 2)
4.	$\sim S$	(Premise given)
5.	$\sim (P \wedge Q)$	(By Modus Tollens using 3 and 4)
6.	$(\sim P) \vee (\sim Q)$	(using De Morgan's Law)

Hence, the argument is valid.