Lecture No 2: Linear Algebra (EMAT102L)

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Topics cover in Lecture 1

- What is Linear Algebra?
- System of linear equation in 2 variable and then 3 variable,
- Review of Matrices,
- Review of Basic properties of determinant,
- Co-factor expansion,
- Adjoint of a matrix.

Invertible matrices

Inverse of a matrix: Let A be a square matrix. Another matrix, say B of same size is said to be inverse of A if $AB = I = BA$, where I is identity matrix of same size.

If B is inverse of A , we write $B = A^{-1}$.

Invertible matrices: A matrix which has inverse is called an invertible matrices.

Properties of invertible matrices:

- $(AB)^{-1} = B^{-1}A^{-1}$
- $(A^t)^{-1} = (A^{-1})^t$

Determinant method for finding inverse of a matrix: If A be a square matrix such that $det(A) \neq 0$, then it is invertible and its inverse

$A^{-1} = \frac{1}{det(A)} adj(A)$

Singular and non-singular matrix

Singular Matrix: A square matrix A_n is said to be singular if $|A| = 0$

Non-singular matrix A square matrix A_n is said to be non-singular if $|A| \neq 0$

Note 1: If A is a square invertible matrix then it is non-singular.

Proof: A is a invertible matrix, then $\exists\ B$ suct that

$$AB = BA = I$$
$$\Rightarrow |AB| = |I|$$
$$\Rightarrow |A||B| = 1 \text{ (Since } |AB| = |A||B| \text{ and } |I| = 1)$$
$$\Rightarrow |A| \neq 0$$
$$\Rightarrow A \text{ is non -singular.}$$

Cramer's rule for solution of linear equations

Let

$$a_1x + b_1y + c_1z = d_1, \tag{1}$$

$$a_2x + b_2y + c_2z = d_2. \tag{2}$$

$$a_3x + b_3y + c_3z = d_3. \tag{3}$$

be a system of three linear equations with the three unknows x, y, z.

If the co-efficient determinant D = $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$

then, $\exists$ unique solution $x = \frac{D_1}{D}$, $y = \frac{D_2}{D}$, $z = \frac{D_3}{D}$,

where D_1 = $\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$, D_2 = $\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$, D_3 = $\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

Elementary row operation

An elementary row operation on a matrix A_{mn} is an operation of the following three types:

Type 1: The interchange of the i^{ih} and j^{ih} row is denoted by R_{ij}

Type 2: Multiplication of the i^{ih} row by a non-zero scalar c is denoted by cR_i

Type 3: Addition of c times the j^{ih} row to the i^{ih} row is denoted by $R_i + cR_j$


Thank you!