01/06/2021

Functions

*) A Function may be defined by a formula that tells how to calculate the output for a given Input.

$$\xrightarrow[\text{Input } x]{} \boxed{\text{Function}} \xrightarrow[\text{Output } f(x)]{}$$

Eg. $f(x) = x - 1$

(I) Definition of a function

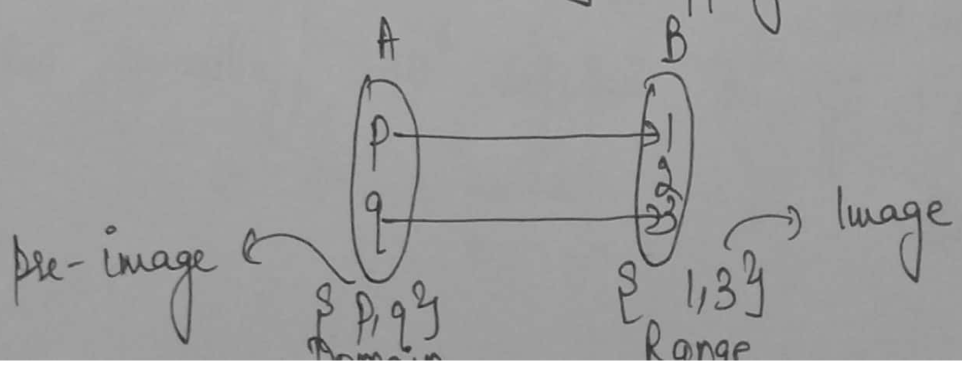
*) It is a special type of Relation, with the following properties

a) $\forall x \in$ Domain, there is a mapping.

b) Unique image $\forall x \in$ Domain

Eg. $\underbrace{A = \{p, q\}}_{\text{Domain}}$ $\underbrace{B = \{1, 2, 3\}}_{\text{Co-domain}}$

$f: A \to B \}$ Mapping



* Let X and Y be two non-empty sets. ②

* A function $f: X \rightarrow Y$ where,

  Set $X \rightarrow$ Domain

  Set $Y \rightarrow$ Co-Domain

* f maps every element $x \in X$ to the element $y \in Y$ and can be written as →

$$y = f(x)$$

  * The element $y \in Y$ is called Image of $x \in X$

  * The element $x \in X$ is called Pre-Image of $y \in Y$

  * The set of all image values $\{f(x) : x \in X\}$ is called the Range of f

Range is always a Subset of Co-domain

ⓐ Relation Vs Function

A function $f: X \rightarrow Y$ is a special kind of Relation $R: X \rightarrow Y$, if it satisfies the following additional properties :→

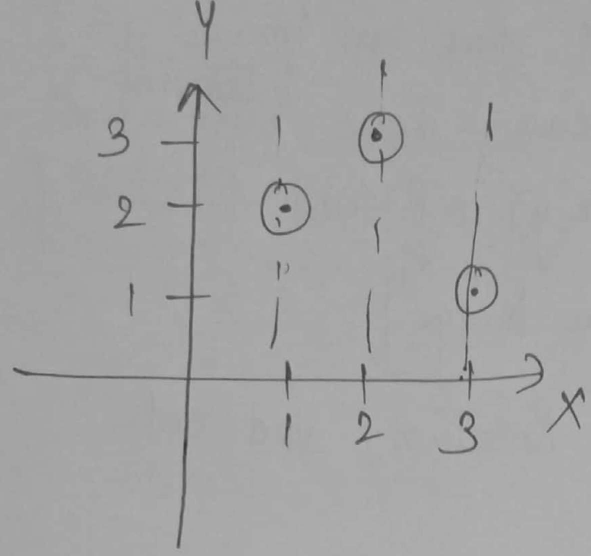
1) Every element $x \in X$ has an image $y \in Y$ [Domain $= X$]
2) One element of X can have only one image, that if $(x,y) \in f$ and $(x,z) \in f$, then $y = z$. [Unique Image]

[Every function is a Relation, but not Vice-versa]

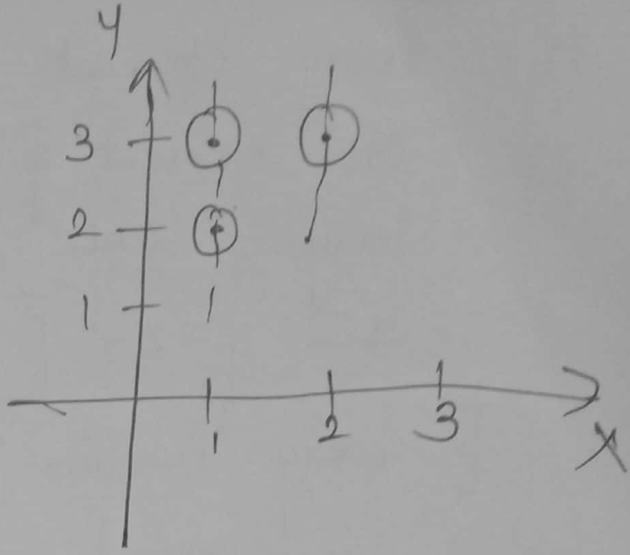
(4) Graphical Determination of a Relation as a function

*) Use Vertical Line Test
*) If the line intersects the graph of the relation at more than one point, then not a function, else a function.

Eg.
$X = \{1, 2, 3\}$
$R_1 = \{(1,2), (2,3), (3,1)\}$
$R_2 = \{(1,2), (2,3), (1,3)\}$



$R_1 \rightarrow$ is a function



$R_2 \rightarrow$ Not a function

(C) Why do we need functions?

ADD (2, 3) $\rightarrow$ Guaranteed Result

Unambigous Output

$\downarrow$

5 Ans

and not

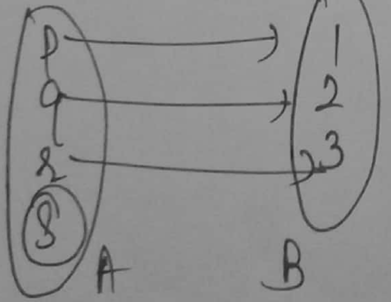
1, 10, 20 etc X

Examples of Functions

$A = \{p, q, r, s\}$ $B = \{1, 2, 3\}$

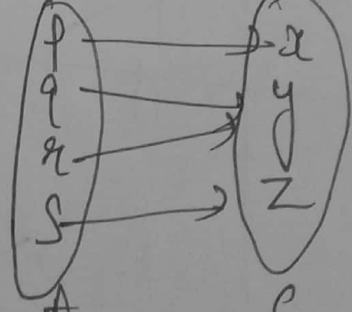
$C = \{x, y, z\}$

$f_1 : A \rightarrow B$



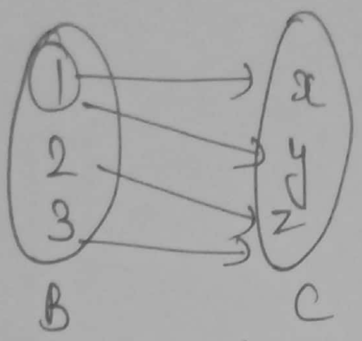
Not a function

$f_2 : A \rightarrow C$



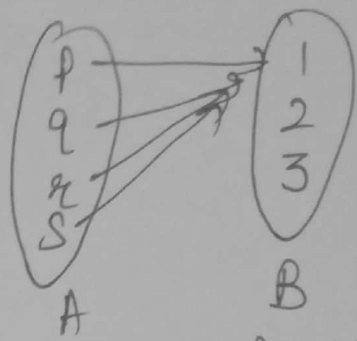
Yes a function

$F_3: B \to C$



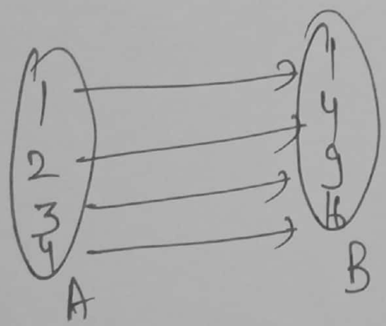
Not a function

$f_4: A \to B$



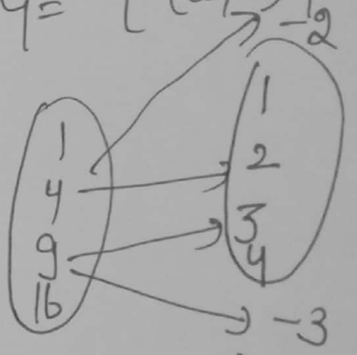
Yes, a function, Called a Constant function

$f = \{(x, x^2) \mid x \in Z\}$



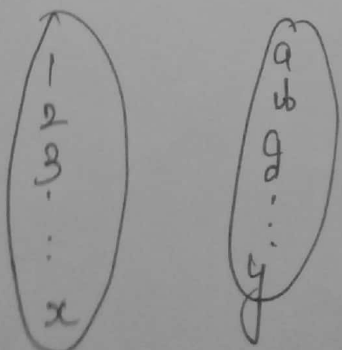
Yes a function

$g = \{(x^2, x) \mid x \in Z\}$



Not a function

(d) How many functions are possible from a set of 'x' elements to a set of 'y' elements?



$n(f) = \underbrace{y \times y \times y \ldots}_{x} \; x \text{ times}$

$= y$

$n(f) = \boxed{y^x}$

(II) Types of Functions

(a) One - One function

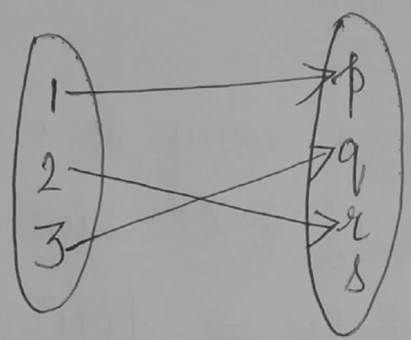
$f : X \to Y$ is one to one if,

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

or

$$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$$

Eg



one-one
(Injection)
[Unique Image]

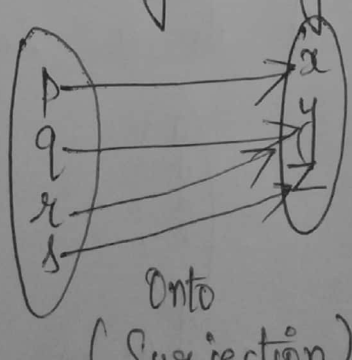
(b) Onto function

$f : X \to Y$ is an Onto function if,

$Ran(f) = Y$ i.e for each $y \in Y$, there is an $x \in X$ such that,

$$f(x) = y$$

Eg.



Onto
(Surjection)

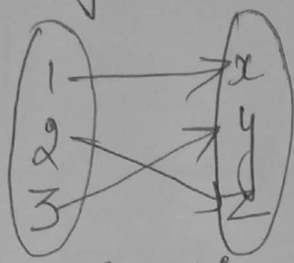
$\boxed{\text{Range} = \text{Codomain}}$

(c) One-One Onto Function

→ It is both one-one as well as Onto

→ Also called Bijection.

Eg.

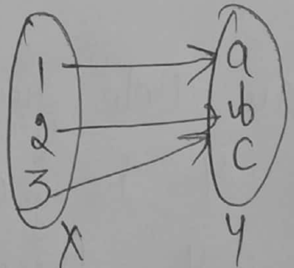


Bijection

(d) Many-One Function

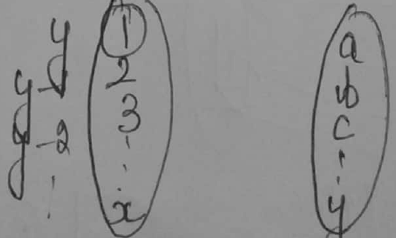
$f: X \to Y$ is a many to one if,
$\exists\, x_1, x_2 \in X$ such that
$x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)$

Example



Many-One

Q How many One-One functions are possible from a set of 'n' elements to a set of 'y' elements?



$$^nP_r = \frac{n!}{(n-r)!}$$

$$n(f) = y \times (y-1) \cdots y-(x-1))$$
$$= {}^yP_x = \frac{y!}{(y-x)!}$$

Q How many one - one functions are there from a set A with 'n' elements onto itself.

$f : X \to Y$
$\quad n \to n$

$n(f) = {}^{y}P_{x}$

$n(f) = {}^{n}P_{n} = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$