Department of Mathematics
Bennett University
EMAT102L: Ordinary Differential Equations
Tutorial Sheet-1

1) Classify each of the following differential equation as linear or nonlinear. Also find the order and degree of differential equation:

$(a)\quad x^2dy + y^2dx = 0; \quad (b)\quad \dfrac{d^2y}{dx^2} + x\sin y = 0; \quad (c)\quad \dfrac{d^6y}{dx^6} + \dfrac{d^4y}{d^4x}\dfrac{d^3y}{d^3x} + y = x;$

$(d)\quad \left(\dfrac{dy}{dx}\right)^3 = \sqrt{\dfrac{d^2y}{d^2x} + 1}.$

2) Verify that y is a solution of the ODE. Determine from y the particular solution of the IVP.

$(a)\quad \dfrac{dy}{dx} = y - y^2; \quad y = \dfrac{1}{1 + ce^{-x}}, \quad y(0) = \dfrac{1}{4},$

$(b)\quad \dfrac{dy}{dx} = y + e^x; \quad y = (x + c)e^x, \quad y(0) = \dfrac{1}{2}.$

Hint: $(a)\, y_p = \frac{1}{1+3e^{-x}}, \quad (b)\, y_p = (x + \frac{1}{2})e^x.$

3) Consider the differential equation $\frac{dy}{dx} = y^2 + 4$.

(a) Show that there exist no constant solutions of the DE.

(b) Can a solution curve have any relative extrema?

4) Solve the following ODEs:

$(a)\quad \dfrac{dy}{dx} = (x + 1)e^{-x}y^2; \quad (b)\quad \dfrac{dy}{dx} = \sec^2 y;$

$(c)\quad 2xy\dfrac{dy}{dx} = y^2 - x^2; \quad (d)\quad x\dfrac{dy}{dx} = y + 3x^4\cos^2(y/x);\ y(1) = 0.$

Hint: $(a)\quad y = \frac{1}{(x+2)e^{-x}-c}, \quad (c)\quad x^2 + y^2 = cx, \quad (d)\quad y = x\tan^{-1}(x^3 - 1).$

5) Solve the following ODEs:

$(a)\quad (x\tan(y/x) + y)dx - xdy = 0; \quad (b)\quad (5x + 2y + 1)dx + (2x + y + 1)dy = 0.$

Hint: $(a)\quad \sin(y/x) = cx, \quad (b)\quad 5x^2 + 4xy + y^2 + 2x + 2y = c.$