# Problem Set-1

Thursday, March 18, 2021 9:27 AM

$\bar{A} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\bar{B} = \hat{i} + \hat{j} - 2\hat{k}$

Angle between $\bar{A}$ & $\bar{B}$

$$\theta = \cos^{-1}\left(\frac{\bar{A}\cdot\bar{B}}{\|A\|\|B\|}\right)$$

$\bar{A}\cdot\bar{B} = \|A\|\|B\|\cos\theta$

$\cos\theta =$

$\bar{A}\cdot(\bar{B}\times\bar{A}) = \bar{B}\cdot(\underbrace{\bar{A}\times\bar{A}}_{0}) = 0$

$\bar{A}\cdot(\bar{B}\times\bar{C}) = \bar{B}\cdot(\bar{A}\times\bar{C}) = \bar{C}\cdot(\bar{B}\times\bar{A})$

1. Find out a unit vector which lies in the xy plane and which is perpendicular to A of previous problem

$\bar{A} = 2\hat{i} - \hat{j} + 3\hat{k}$

$C = x\hat{i} + y\hat{j}$

$\bar{A}\cdot\bar{C} = 0$

$\Rightarrow 2x - y = 0$

$\bar{C} = x(\hat{i} + 2\hat{j})$

$$\bar{C} = C\hat{C} \rightarrow \frac{\bar{C}}{\|C\|} = \frac{x(\hat{i}+2\hat{j})}{\sqrt{x^2+4x^2}}$$

$$= \frac{1}{\sqrt{5}}(\hat{i}+2\hat{j})$$

$$\cancel{A^2}+\cancel{B^2}+2\bar{A}\cdot\bar{B} = \cancel{A^2}+\cancel{B^2}-2\bar{A}\cdot\bar{B}$$

$$4\bar{A}\cdot\bar{B}=0$$

$$\bar{A}\cdot\bar{B}=0 \Rightarrow \bar{A}\perp\bar{B}$$

$$\bar{j}(t) = a\hat{i} + bt\hat{j} + ct^2\hat{k} = \frac{d\bar{a}}{dt}$$

$$j_x = \frac{da_x}{dt} = a$$

$$a_x = at + C_1$$

$$j_y = \frac{da_y}{dt} = bt$$

$$a_y = \frac{bt^2}{2} + C_2$$

$$j_z = \frac{da_z}{dt} = ct^2$$

$$a_z = \frac{c}{3}t^3 + C_3$$

$$C_1 = C_2 = C_3 = 0$$

$$\bar{a}(t) = at\hat{i} + \frac{b}{2}t^2\hat{j} + \frac{c}{3}t^3\hat{k}$$

$$\bar{a}(t) = -\omega^2\bar{r}(t)$$

$$a_x = \frac{d^2x}{dt^2} = -\omega^2 x \rightarrow \frac{dv_x}{dt} = -\omega^2 x$$

$$a_y = \frac{d^2y}{dt^2} = -\omega^2 y$$

$$\bar{a} = \frac{d\bar{v}}{dt} = -\omega^2\bar{r}(t)$$

$$\left(\frac{d^2x}{dt^2} = -\omega^2 x\right) \times 2\frac{dx}{dt}$$

$$2\frac{dx}{dt}\frac{d^2x}{dt^2} = -\omega^2\left(2x\frac{dx}{dt}\right) \rightarrow \frac{d}{dt}(x^2)$$

$$2x\frac{dx}{dt}$$

$$\cancel{\frac{d}{dt}}\left(\frac{dx}{dt}\right)^2 = -\omega^2\cancel{\frac{d}{dt}}(x^2)$$

$$dt$$

$$\cancel{\frac{d}{dt}}\left(\frac{dx}{dt}\right)^2 = -\omega^2\cancel{\frac{d}{dt}}\left(x^2\right) \qquad 2x\frac{dx}{dt}$$

$$\left(\frac{dx}{dt}\right)^2 = -\omega^2x^2 + C^2$$

$$v = \frac{dx}{dt} = \sqrt{C^2-\omega^2x^2} = f(x)$$

$$\frac{dx}{f(x)} = dt$$

$$\frac{dx}{\omega\sqrt{\alpha^2-x^2}} \Rightarrow dt$$

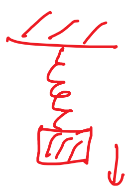
$$\frac{dx}{\sqrt{\alpha^2-x^2}} = \omega\, dt$$

$$\sin^{-1}\left(\frac{x}{\alpha}\right) = \omega t + C_1$$

$$x = \alpha\sin(\omega t + C_1)$$

$$= A\sin\omega t + B\cos\omega t$$

$$y = C\sin\omega t + D\cos\omega t$$



$$m\ddot{x} = -kx$$

$$x(0) = 0$$
$$y(0) = a$$
$$v_x(0) = a\omega$$
$$v_y(0) = 0$$

$$\vec{r}(0) = a\hat{j}$$

$$x(0)\hat{i} + y(0)\hat{j} = a\hat{j}$$

$$v_x(0) = A\omega\cos\omega t - B\omega\sin\omega t$$

$$v_y(0) = C\omega\cos\omega t - D\omega\sin\omega t$$

$$x(0) = 0 \Rightarrow B = 0$$

$$x(t) = A\sin\omega t + B\cos\omega t$$

$$x(t=0) = A\sin 0 + B\cos 0$$
$$= 0 + B = 0$$

$$v_x(t) = A\omega\cos\omega t - \underset{0}{B}\omega\cancel{\sin\omega t}$$

$$v_x(t=0) = A\omega\times 1 = a\omega$$

$$A = a$$

$$y(t) = C\sin\omega t + D\cos\omega t$$

$$y(0) = a = D$$

$$C = 0$$

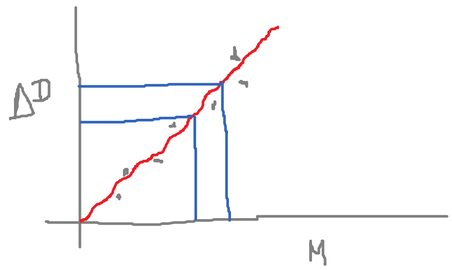
$$x(t) = a\sin\omega t$$

$$y(t) = a\cos\omega t$$

$$\bar{r}(t) = a(\sin\omega t\,\hat{i} + \cos\omega t\,\hat{j})$$

# Lab

Monday, April 5, 2021 5:19 PM



# Problem Set-II

Thursday, April 8, 2021 10:45 AM

# PQ6, $\vec{F}$, $\vec{\nabla}\times\vec{F} = 0$

$$\vec{F} = (y^2z^3 - 6xz^2)\hat{i} + 2xyz^3\hat{j} + (3xy^2z^2 - 6x^2z)\hat{k}$$

$\Rightarrow$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2z^3 - 6xz^2 & 2xyz^3 & 3xy^2z^2 - 6x^2z \end{vmatrix}$$

$$= (6xyz^2 - 6xyz^2)\hat{i} + (3y^2z^2 - 12xz - 3y^2z^2 + 12xz)\hat{j} + (2yz^3 - 2yz^3)\hat{k} = 0$$

$$\hat{r}\times\hat{\theta} = (\cos\theta\,\hat{i} + \sin\theta\,\hat{j})\times(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \end{vmatrix} = (\cos^2\theta + \sin^2\theta)\hat{k}$$

$= \hat{k}$

$\hat{\theta}\times\hat{k} = \hat{r}$

$\hat{r}, \hat{\theta}, \hat{k} \rightarrow$ cylindrical co-ordinate system

$\hat{i}\times\hat{j} = \hat{k}$

$\hat{r}\times\hat{\theta} = \hat{k}$

$\hat{\theta}\times\hat{k} = \hat{r}$

$\hat{k}\times\hat{r} = \hat{\theta}$

$\hat{r}, \hat{\theta}, \hat{\varphi}$ $\quad \hat{r}\times\hat{\theta} = \hat{\varphi}$

Q8

$$\bar{r} = a$$
$$\dot{r} = 0, \quad \ddot{r} = 0$$
$$\dot{\theta} = \omega_0 + \alpha t$$
$$\ddot{\theta} = \alpha$$
$$\bar{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} = a(\omega_0 + \alpha t)\hat{\theta}$$
$$\bar{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$
$$= -a(\omega_0 + \alpha t)^2\hat{r} + a\alpha\hat{\theta}$$

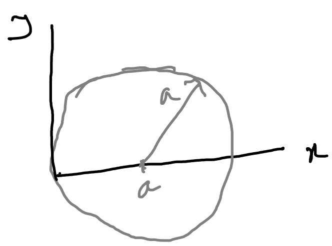
9//
$$\hat{i} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$

10//
$$\dot{\theta} = \omega, \quad \ddot{\theta} = 0$$
$$\cancel{\dot{r} =} \quad r(t) = r_0 e^{\beta t}$$
$$\dot{r} = r_0 \beta e^{\beta t}$$
$$\ddot{r} = r_0 \beta^2 e^{\beta t}$$
$$\bar{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} = r_0\beta e^{\beta t}\hat{r} + r_0 e^{\beta t}\omega\hat{\theta}$$
$$= r_0 e^{\beta t}(\beta\hat{r} + \omega\hat{\theta})$$
$$\bar{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$
$$= (\underline{r_0\beta^2 e^{\beta t} - r_0 e^{\beta t}\omega^2})\hat{r} \rightarrow 0$$
$$+ (2r_0\beta e^{\beta t}\omega\hat{\theta})$$
$$\beta^2 = \omega^2 \rightarrow \beta = \pm\omega$$

$x = r\cos\theta$

11//

$r^2 = x^2 + y^2$




$$(x-a)^2 + y^2 = a^2$$
$$x^2 + \cancel{a^2} - 2ax + y^2 = \cancel{a^2}$$
$$x^2 + y^2 = 2ax$$
$$r^{\cancel{2}} = 2a\cancel{r}\cos\theta$$
$$r = 2a\cos\theta$$

12//
$$r = A\theta, \quad A = 1/\pi, \quad \theta = \alpha t^2$$
$$r = A\alpha t^2 \qquad \dot{\theta} = 2\alpha t$$

$\dot{r} = 2A\alpha t \qquad \dot{\theta} = 2\alpha$

$\ddot{r} = 2A\alpha$

$\bar{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$

$\bar{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$

$= \underbrace{2A\alpha(1-2\alpha^2t^4)}\hat{r} + \underbrace{6A\alpha^2t^2}\hat{\theta}$

$\cancel{2}\cancel{A}\cancel{\alpha}(1-2\alpha^2t^4) = \overset{3}{\cancel{6}}\cancel{A}\alpha^{\cancel{2}}t^2$

$2\alpha^2t^4 + 3\alpha t^2 - 1 = 0$

$t^2 = \dfrac{-3 \pm \sqrt{17}}{4\alpha}$

$\theta = \alpha t^2$

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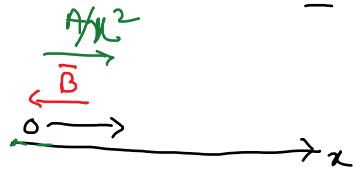



$\bar{F} = \left(-B + \frac{A}{x^2}\right)\hat{i} \quad , \quad x>,0$

$-\frac{dV}{dx} = -B + \frac{A}{x^2}$

$V(x) = Bx + \frac{A}{x} + C$

C is a constant, and I can set

$C = 0$

$V(x) = Bx + \frac{A}{x}$

$B = A = 1$

$x = 0.1, 0.2, 0.5, 1, 2, 5$

$V(x) = 0.1 + 10$

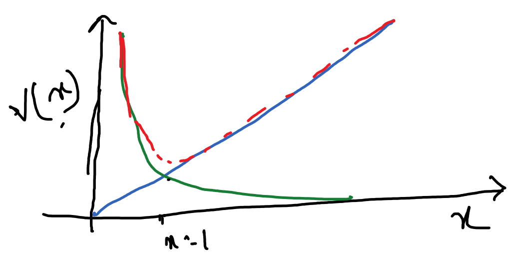
$= 0.2 + 5$

$= 0.5 + 2$

$= 1 + 1$

$= 2 + 0.5$

$= 5 + 0.2$




$x<1, \frac{A}{x}$ dominate

$x>1$, $Bx$ dominate

$$\frac{dV}{dx} = 0 \rightsquigarrow B - \frac{A}{x^2} = 0$$

$$\Rightarrow x_0 = \sqrt{\frac{A}{B}}$$

$$V(x)\Big|_{x=x_0} = +Bx\Big|_{x=x_0} + \frac{A}{x}\Big|_{x=x_0}$$

$$= +B \times \sqrt{\frac{A}{B}} + A\sqrt{\frac{B}{A}}$$

$$= 2\sqrt{AB}$$

15. $$V = -\frac{GMm}{r} = -\frac{GMm}{\sqrt{x^2+y^2+z^2}}$$

$$\bar{F}(\bar{r}) = -\bar{\nabla}V(r) = GMm\left(\frac{\partial}{\partial x}\frac{\hat{i}}{\sqrt{x^2+y^2+z^2}} + \frac{\partial}{\partial y}\frac{\hat{j}}{\sqrt{x^2+y^2+z^2}} + \frac{\partial}{\partial z}\frac{\hat{k}}{\sqrt{x^2+y^2+z^2}}\right)$$

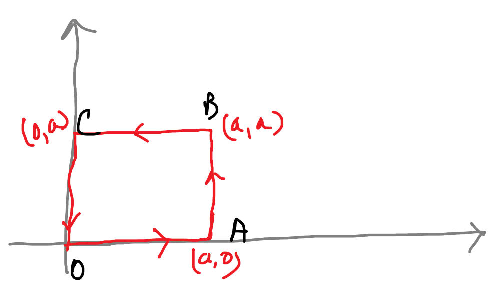
$$= -GMm\left(\frac{\cancel{2}x\,\hat{i}}{\cancel{2}(x^2+y^2+z^2)^{3/2}} + \frac{y\,\hat{j}}{(x^2+y^2+z^2)^{3/2}} + \frac{z\hat{k}}{(\ )}\right)$$

$$= -GMm\left(\frac{x\hat{i}+y\hat{j}+z\hat{k}}{(x^2+y^2+z^2)^{3/2}}\right) = -\frac{GMm\,\bar{r}}{r^3}$$

$$= -\frac{GMm}{r^2}\hat{r}$$

$$\bar{\nabla}\times\bar{F} = 0$$

16. $$\bar{F} = A(y^2\hat{i} + 2x^2\hat{j})$$




$$\oint \bar{F}\cdot d\bar{l} \qquad \int(\bar{\nabla}\times\bar{F})\cdot d\bar{s}$$

$\oint \bar{F}\cdot d\bar{l}$ , $\int (\bar{\nabla}\times\bar{F})\cdot d\bar{S}$

$\Updownarrow$

$\int_C \bar{F}\cdot d\bar{l}$

$$\oint \bar{F}\cdot d\bar{r} = \int_{OA} \bar{F}\cdot d\bar{r} + \int_{AB} \bar{F}\cdot d\bar{r} + \int_{BC} \bar{F}\cdot d\bar{r} + \int_{CO} \bar{F}\cdot d\bar{r}$$

$\uparrow 0$ F is conservative

$\bar{\nabla}\times\bar{F} \neq 0$ , $W = \oint \bar{F}\cdot d\bar{r}$

$\int_S (\bar{\nabla}\times\bar{F})\cdot d\bar{S}$

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$a_r = \ddot{r} - r\dot{\theta}^2$

$\ddot{r} = 0$

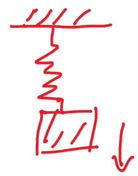
$a_r = -r\dot{\theta}^2 = -r\omega^2$

$F = T = -m\omega^2 r$

$W = \int_{R_0}^{R_1} F\,dr$

# Simple Harmonic Motion

Tuesday, April 13, 2021 9:24 AM



$m\ddot{x} = -kx$

Hooke's Law $F \propto x$

$F = -kx$

Newton's Law $F = ma$

$$ma = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$\Rightarrow \underbrace{\frac{d^2x}{dt^2} + \omega^2 x = 0}, \quad \omega^2 = k/m$$

2nd order linear differential eqn.

$$\left(\frac{d^2x}{dt^2} = -\omega^2 x\right) \times 2\frac{dx}{dt}$$

$$\Rightarrow 2\frac{dx}{dt}\frac{d^2x}{dt^2} = -\omega^2 2x\frac{dx}{dx}$$

$$\Rightarrow \frac{d}{dt}\left(\dot{x}^2\right) = -\omega^2\frac{d}{dt}\left(x^2\right)$$

$$\dot{x}^2 = -\omega^2x^2 + c^2$$

$$\frac{dx}{dt} = \pm\sqrt{c^2 - \omega^2x^2}$$

$$\pm\frac{dx}{\omega\sqrt{\alpha^2 - x^2}} = dt, \quad \alpha = \frac{c}{\omega}$$

$$\int\frac{dx}{\sqrt{\alpha^2 - x^2}} = \pm\omega\int dt$$

$$\sin^{-1}\left(\frac{x}{\alpha}\right) = c \pm \omega t$$

$$x = \alpha\sin(c \pm \omega t)$$

$$= A\sin\omega t + B\cos\omega t \quad \text{— (1)}$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

characteristic eqn.

$$m^2 + \omega^2 = 0 \quad ; \rightarrow m_1 \; \& \; m_2 \text{ are roots of characteristic eqn.}$$

$$x(t) = C_1 e^{m_1 t} + C_2 e^{m_2 t}$$

$$m = \pm i\omega$$

$$x(t) = C_1 e^{i\omega t} + C_2 e^{-i\omega t} \quad \text{— (2)}$$

$$\cos\omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

$$\sin\omega t = \frac{e^{i\omega t} - e^{-i\omega t}}{2i}$$

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2 = 0$$

$$m^2 - 3m + 2 = 0$$

$$\Rightarrow (m-2)(m-1) = 0, \quad m = 2, 1$$

$$x(t) = C_1 e^{m_1 t} + C_2 e^{m_2 t}$$

$$= C_1 e^{2t} + C_2 e^{t}$$

$$\frac{d^3x}{dt^3} + 5\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 4 = 0$$

$$m^3 + 5m^2 - 3m + 4 = 0$$

$$\frac{d^2x}{dt^2} - \omega^2 x = 0, \quad m^2 - \omega^2 = 0 \rightarrow m = \pm\omega$$

$$x(t) = C_1 e^{\omega t} + C_2 e^{-\omega t}$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \longrightarrow x(t) = C_1 e^{i\omega t} + C_2 e^{-i\omega t}$$

$$= A\sin\omega t + B\cos\omega t$$

$$\frac{d^2x}{dt^2} - \omega^2 x = 0 \longrightarrow x(t) = C_1 e^{\omega t} + C_2 e^{-\omega t}$$

$$= A\sinh\omega t + B\cosh\omega t$$

$$\cosh\omega t = \frac{e^{\omega t} + e^{-\omega t}}{}$$

$$\sinh \omega t = \frac{e^{\omega t} - e^{-\omega t}}{2}$$

$F = -kx \rightarrow$ conservative force

$V = \frac{1}{2} k x^2$

Taylor Series

$$f(x)\Big|_{x=x_0} = \underbrace{f(x_0)}_{\text{constant potential}} + \underbrace{(x-x_0)\frac{df}{dx}\Big|_{x=x_0}}_{0} + \frac{1}{2!}\underline{(x-x_0)^2}\frac{d^2 f}{dx^2}\Big|_{x=x_0}$$

$\alpha x^2$

# Rigid Body Motion

Thursday, June 17, 2021 9:55 AM

$$\tau = \left(\frac{d\bar{L}}{dt}\right)_{inertial} = \left(\frac{d\bar{L}}{dt}\right)_{rot} + \bar{\omega} \times \bar{L}$$

$$\bar{L} = I_1\omega_1\hat{e}_1 + I_2\omega_2\hat{e} + I_3\omega_3\hat{e}_3$$

$$\bar{\omega} = \omega_1\hat{e}_1 + \omega_2\hat{e}_2 + \omega_3\hat{e}_3$$

$$\bar{\tau} = \tau_1\hat{e}_1 + \tau_2\hat{e}_2 + \tau_3\hat{e}_3$$

$$\tau_1 = I_1\dot{\omega}_1 + \omega_2\omega_3(I_3 - I_2)$$
$$= I_1\dot{\omega}_1 - \omega_2\omega_3(I_2 - I_3)$$

$$\bar{\omega} \times \bar{L} = \begin{vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ \omega_1 & \omega_2 & \omega_3 \\ I_1\omega_1 & I_2\omega_2 & I_3\omega_3 \end{vmatrix}$$

$$\Rightarrow (\omega_2 I_3\omega_3 - \omega_3 I_2\omega_2)\hat{e}_1 + (\omega_3 I_1\omega_1 - \omega_1 I_3\omega_3)\hat{e}_2$$
$$+ (\omega_1 I_2\omega_2 - \omega_2 I_1\omega_1)\hat{e}_3$$

$$\Rightarrow \omega_2\omega_3(I_3 - I_2)\hat{e}_1 + \omega_3\omega_1(I_1 - I_3)\hat{e}_2$$
$$+ \omega_1\omega_2(I_2 - I_1)\hat{e}_3$$

$$I_2\dot{\omega}_2 - \omega_1\omega_3(I_3 - I_1) = 0 \quad \text{— (1)}$$

$$I_3\dot{\omega}_3 - \omega_1\omega_2(I_1 - I_2) = 0 \quad \text{— (2)}$$

differentiate (1) w.r.t $t$

$$I_2\ddot{\omega}_2 - \omega_1\dot{\omega}_3(I_3 - I_1) = 0 \quad \text{— (3)}$$

from (2)

$$\dot{\omega}_3 = \omega_1\omega_2\frac{I_1 - I_2}{I_3} \quad \text{— (4)}$$

(4) in (3)

$$\ddot{\omega}_2 - \underbrace{\omega_1^2\frac{(I_1 - I_2)(I_3 - I_1)}{I_3 I_2}}_{A}\omega_2 = 0$$

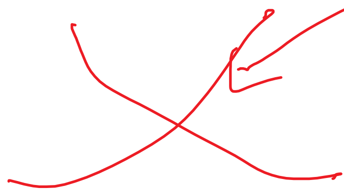
$$\ddot{\omega}_2 - A\omega_2 = 0$$

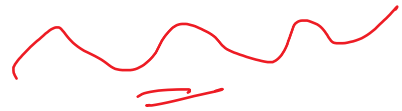
$$I_1 > I_2 > I_3$$

$$I_2 > I_1 > I_3$$

$$\ddot{\omega}_2 + A\omega_2 = 0 \rightarrow e^{\pm i\omega_2 t}$$

$$\ddot{\omega}_2 - A\omega_2 = 0 \rightarrow e^{\pm \omega_2 t}$$





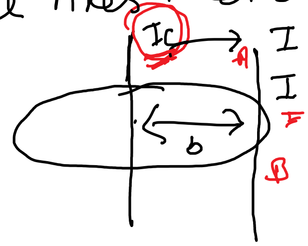
# Problem-3

Saturday, June 19, 2021 9:46 AM

Radius of Gyration — $K^2 = \frac{I}{M} = \frac{\sum_i m_i r_i^2}{\sum_i m_i}$

$$= \frac{\int dm\, r^2}{\int dm}$$

Parallel Axes theorem

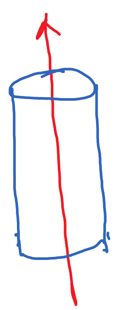



$I = I_c + Mb^2$

$I$

If $I$ be the moment of inertia of a system about axis AB, and $I_c$ be the moment of inertia about an axis parallel to AB and passing through the center of mass of the system.

S



$$I = \int_{r=0}^{a} 2\pi\sigma r^3 h\,dr$$

volume element $= 2\pi r\,dr \times h$

$dm = 2\pi\sigma r h\,dr$

$\sigma$ = mass/vol.

$$I = \int r^2\,dm$$

$$= \int_0^R 2\pi\sigma r^3 h\,dr$$

$$= \frac{1}{2}\pi\sigma h R^4 \rightarrow \underbrace{\sigma\pi R^2 h}_{M} \times \frac{1}{2} R^2$$

$$M = \int_0^R \underbrace{2\pi\sigma r h\,dr}_{dm} = \sigma\pi R^2 h$$

$$I = \frac{1}{2}MR^2$$

$$K^2 = \frac{I}{M} = \frac{\frac{1}{2}MR^2}{M} = \frac{R}{2}$$

$$K = \frac{R}{\sqrt{2}}$$

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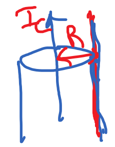


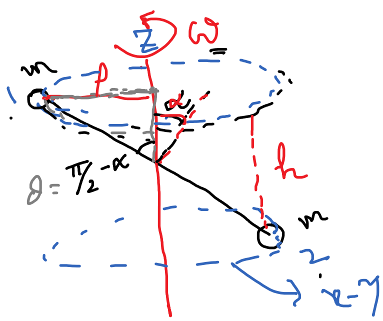

$$I = I_c + MR^2 = \frac{3}{2}MR^2$$

Perpendicular Axes theorem

If there is a mass distribution is the x-y plane of an xyz coordinate system, and $I_x, I_y, I_z$ are the moment of inertia about x, y, z,

$$I_z = I_x + I_y$$

3




$l\sin\theta$

$l\sin(\pi/2-\alpha)$

$l\cos\alpha$

$l\cos\theta = l\cos(\pi/2-\alpha) = l\sin\alpha$

$\bar{L}(t) =$

$\bar{L} = \bar{r}_2 \times \bar{p}$

$\bar{r}_1 = l\cos\alpha(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}) + l\sin\alpha\,\hat{k}$

$\bar{r}_2 = -l\cos\alpha(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}) - l\sin\alpha\,\hat{k}$

$\bar{r}_2 = -\bar{r}_1$

$$\bar{p}_1 = \frac{m\,dr_1}{dt} =$$

$$\bar{p}_2 = m\frac{dr_2}{dt} \rightarrow p_2 = \frac{d\bar{r}_2}{dt}$$

$$\bar{L} = \bar{r}_1 \times \bar{p}_1 + \bar{r}_2 \times \bar{p}_2$$

$$-\bar{r}_1 \times \left(-m\frac{d\bar{r}_1}{dt}\right)$$

$$= 2\bar{r}_1 \times \bar{p}_1$$

Example 7.14

$x_1 = \rho\cos\omega t$

$y_1 = \rho\sin\omega t$

$z_1 = -h$

$x_2 = -\rho\cos\omega t$

$y_2 = -\rho\sin\omega t$

$z_2 = h$

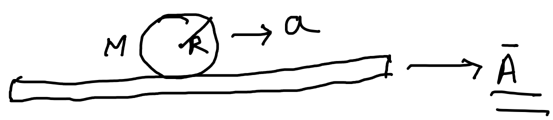
$z_1 = -h$

$$I_{xx} = m_1(y_1^2 + z_1^2) + m_2(y_2^2 + z_2^2)$$

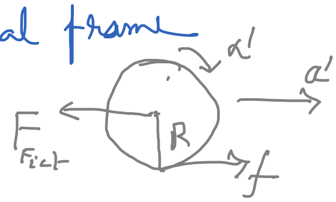
$$I_{yy} = m_1(x_1^2 + z_1^2) + m_2(x_2^2 + z_2^2)$$

$$I_{zy} = I_{yz} = -m_1 y_1 z_1 - m_2 y_2 z_2$$

4




Non inertial frame




$$a' = \alpha' R \Rightarrow \alpha' = \frac{a'}{R}$$

$$f\cancel{R} = -I\alpha' = \frac{1}{2}MR^2\alpha' = \frac{1}{2}M\cancel{R}a'$$

$$f - F_{fict} = Ma'$$

$$f - MA = Ma'$$

$$a' = -\frac{2}{3}A$$

Inertial frame

$$a = a' + A$$

11

$$\bar{F}(\bar{r}) = -\frac{C}{r^3}\hat{r}$$

$$V(r) = -\int_{\infty}^{r} F(r')\,dr' = -\frac{C}{2r^2}$$

$$V_{eff}(r) = \frac{L^2}{2\mu r^2} - \frac{C}{2r^2} = \frac{1}{r^2}\underbrace{\left(\frac{L^2}{2\mu} - \frac{C}{2}\right)}_{A}$$

→ angular mom conservation

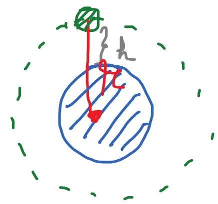
$r = r_{min}$

$$\frac{\partial V}{\partial r} = 0 \Rightarrow -\frac{A}{r} = 0$$

$$A = 0$$
$$L^2 = \mu C$$
$$L = \sqrt{\mu C}$$

12

$$\frac{G M_e \cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m} v^2}{\cancel{r}}$$



$$r = R_e + \underset{=}{h}$$
$$v = \Omega_e r$$
$$r = \frac{G M_e}{\Omega_e^2 r^2} \Rightarrow r^3 = \frac{G M_e}{\Omega_e^2}$$
$$\Rightarrow r = \left(\frac{G M_e}{\Omega_e^2}\right)^{1/3}$$

$$\frac{G M_e \cancel{m}}{R_e^2} = \cancel{m} g$$
$$\Rightarrow G M_e = g R_e^2$$
$$r = \left(\frac{g R_e^2}{\Omega_e^2}\right)^{1/3} = h + R_e$$

$9.8\ m/s^2$ ; $6400\ km$

$$\Rightarrow h = \left(\frac{g R_e^2}{\Omega_e^2}\right)^{1/3} - R_e$$

$\frac{2\pi}{86400}\ rad/s$

$$\simeq 35850\ km$$

10

$$V_{eff}(r) = V_{min} + (r - r_{min}) \left.\frac{\partial V_{eff}}{\partial r}\right|_{r = r_{min}} + \frac{1}{2}(r - r_{min})^2 \left.\frac{\partial^2 V_{eff}}{\partial r^2}\right|_{r = r_{min}}$$

$$\left.\frac{\partial V_{eff}}{\partial r}\right|_{r = r_{min}} = 0$$

$$V_{eff}(r) = \frac{L^2}{2\mu r^2} - \frac{C}{r}$$

$$\frac{\partial V_{eff}}{\partial r} = -\frac{L^2}{\mu r^3} + \frac{C}{r^2}$$

$$\frac{\partial V_{eff}}{\partial r} = -\frac{L^2}{\mu r^3} + \frac{C}{r^2}$$

$$\left.\frac{\partial^2 V_{eff}}{\partial r^2}\right|_{r=r_{min}} = \left[\frac{3L^2}{\mu r^4} - \frac{2C}{r^3}\right]_{r=r_{min}}$$

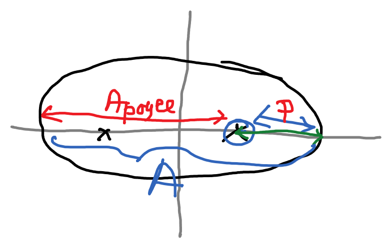
$$r_{min} = \frac{\mu C}{L^2}$$

13

$m = 2000$ kg

Perigee = 1100 km

Apogee = 35850 km




$$E = -\frac{C}{A}$$

$$C = GM_e m = R_e^2 g m$$

A = Earth diameter + Apogee + Perigee

$\simeq 5 \times 10^7$ m

$$E_{orb} = -\frac{C}{A} = -\frac{g\, m R_e^2}{A} = -1.61 \times 10^{10}\ \text{J}$$

$$E_{earth} = -\frac{GM_e m}{R_e} + \frac{1}{2} m (\Omega_e R_e)^2$$

$$\simeq -mgR_e + \frac{1}{2} m (\Omega_e R_e)^2$$

$$= -2000 \times 9.8 \times 6.4 \times 10^6 + \frac{1}{2} \times 2000 \times \left(\frac{2\pi}{86400} \times 6.4 \times 10^6\right)^2$$

$$= -1.25 \times 10^{11}\ \text{J}$$

$$\Delta E = E_{orb} - E_{earth} = 1.09 \times 10^{11}\ \text{J}$$

$$r_{min} = \frac{r_0}{1+\epsilon}, \quad r_{max} = \frac{r_0}{1-\epsilon}$$

$$\epsilon = \frac{r_{max} - r_{min}}{r_{max} + r_{min}} = \frac{(35850+6400) - (1100+6400)}{(35850+6400) + (1100+6400)}$$

$$= 0.7$$

$$\epsilon^2 = 1 + \frac{2 E_{orb} L^2}{m C^2} \quad , \quad \frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

$$\underline{\underline{\mu}} = \frac{m_1 m_e}{m_1 + m_e}$$

$$= \frac{m_e}{m_e}\left(\frac{m_1}{1 + \frac{m_1}{m_e}}\right)$$

$$\frac{m_1}{m_e} \simeq 0$$

$$L = 1.43 \times 10^{14} \text{ kg m}^2/\text{s}$$

$$L = m r_p v_p = m r_a v_a$$

$$r_p = (1100 + 6400) \text{ km}$$

$$r_a = (35850 + 6400) \text{ km}$$

$$v_a = 1690 \text{ m/s}$$

$$v_p = 9530 \text{ m/s}$$

14

$$R_{geo} = (35850 + 6400) \text{ km} \simeq 4.2 \times 10^7 \text{ m}$$

$$\Delta E = -\frac{C}{A_{geo}} - E_{orb}$$

$$A_{geo} = 2 R_{geo} \qquad \simeq 6.6 \times 10^9 \text{ J}$$

# Central Force

Tuesday, June 22, 2021 9:56 AM

$$\bar{R} = \frac{m_1\bar{r}_1 + m_2\bar{r}_2}{m_1+m_2}$$

$$\bar{r} = \bar{r}_1 - \bar{r}_2 \Rightarrow \bar{r}_2 = \bar{r}_1 - \bar{r}$$

$$(m_1+m_2)\bar{R} = m_1\bar{r}_1 + m_2\bar{r}_2$$

$$\bar{r}_1 = \frac{m_1+m_2}{m_1}R - \frac{m_2}{m_1}r_2$$

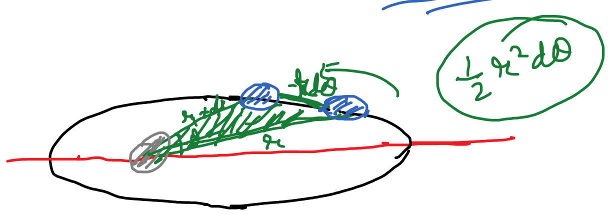
$$= \frac{m_1+m_2}{m_1}R - \frac{m_2}{m_1}(r_1 - r)$$

$$= \frac{m_1+m_2}{m_1}R + \frac{m_2}{m_1}r$$

$$\bar{r}_1\left(1+\frac{m_2}{m_1}\right) = \frac{m_1+m_2}{m_1}R + \frac{m_2}{m_1}r$$

$$r_1\left(\frac{m_1+m_2}{m_1}\right) = \frac{m_1+m_2}{m_1}R + \frac{m_2}{m_1}r$$

$$r_1 = \bar{R} + \frac{m_2}{m_1+m_2}r$$




$$A = \frac{1}{2}r^2 d\theta$$

$$\frac{dA}{dt} = \frac{1}{2}r^2\dot{\theta}$$

$$\mu r^2\dot{\theta} = L$$

$$\dot{\theta} = \frac{L}{\mu r^2}$$

$$E = \frac{1}{2}\mu\dot{r}^2 + \frac{1}{2}\mu r^2\dot{\theta}^2 + V(r)$$

$$= \frac{1}{2}\mu\dot{r}^2 + \frac{1}{2}\mu r^2\left(\frac{L}{\mu r^2}\right)^2 + V(r)$$

$$= \frac{1}{2}\mu\dot{r}^2 + \frac{1}{2}\cancel{\mu}\cancel{r^2}\frac{L^2}{\mu^{\cancel{2}} r^{\cancel{4}}} + V(r)$$

$$= \frac{1}{2}\mu\dot{r}^2 + \underbrace{\frac{1}{2}\frac{L^2}{\mu r^2} + V(r)}_{V_{eff}(r)}$$

$$\Rightarrow \frac{1}{2}\mu\dot{r}^2 = E - V_{eff}(r)$$

$$\Rightarrow \dot{r}^2 = \frac{2}{\mu}\left(E - V_{eff}(r)\right)$$

$$\Rightarrow \dot{r}^2 = \frac{2}{\mu}\left(E - V_{eff}(r)\right)$$

$$\Rightarrow \dot{r} = \sqrt{\frac{2}{\mu}\left(E - V_{eff}(r)\right)}$$

$$\Rightarrow \frac{dr}{dt} = \sqrt{\frac{2}{\mu}\left(E - V_{eff}(r)\right)}$$

$$\Rightarrow dt = \frac{dr}{\sqrt{\frac{2}{\mu}\left(E - V_{eff}(r)\right)}}$$

$$\Rightarrow t - t_0 = \int_{r_0}^{r} \frac{dr}{\sqrt{\frac{2}{\mu}\left(E - V_{eff}(r)\right)}}$$

$$r \equiv r(t)$$

$$s = a\sin\phi$$

$$a = \sqrt{\frac{2E\mu L^2 + (\mu C)^2}{L^4}}$$

$$\theta - \theta_0 = -\int \frac{ds \rightarrow a\cos\phi\, d\phi}{\sqrt{\underbrace{\frac{2\mu E L^2 + (\mu C)^2}{L^4}}_{a^2} - s^2}} \quad s^2 \rightarrow a\sin\phi$$

$$= -\int \frac{a\cos\phi\, d\phi}{\sqrt{a^2 - a^2\sin^2\phi}} = -\int \frac{\cancel{\cos\phi}\, d\phi}{a\cancel{\cos\phi}}$$

$$= -\int d\phi = \phi$$

$$r = \frac{1}{s - \alpha}$$

$$s - \alpha = \frac{1}{r}$$

$$s = \frac{1}{r} + \alpha$$

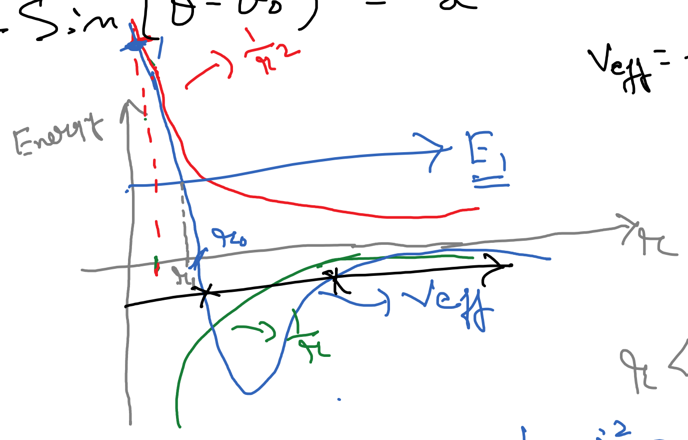
$$\theta - \theta_0 = -\phi = -\sin^{-1}\frac{s}{a}$$

$$= -\sin^{-1}\left(\frac{1}{a}\left(\frac{1}{r} + \alpha\right)\right)$$

$$-\sin(\theta - \theta_0) = \frac{s}{a} = \frac{1}{a}\left(\frac{1}{r} + \alpha\right)$$

$$V_{eff} = \frac{L^2}{2\mu r^2} + V(r)$$

$$= \frac{L^2}{2\mu r^2} - \frac{C}{r}$$




$E > 0$

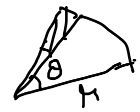
$E < 0$

$r < r_1$

$$\frac{1}{2}m\dot{r}^2 = E - V_{eff} < 0$$

$$L = \mu r^2 \dot{\theta} \Rightarrow \mu r^2 \frac{d\theta}{dt}$$

$\frac{1}{2} r^2 d\theta$




$$\Rightarrow \frac{1}{2}\frac{L}{\mu} dt = \boxed{\frac{1}{2} r^2 d\theta}$$

$$\frac{1}{2}\frac{L}{\mu} T = \pi a b$$

# Problem Set 4

Tuesday, July 13, 2021 10:09 AM

1/ $100\ \text{km/h} = 28\ \text{m s}^{-1}$

$v << c$

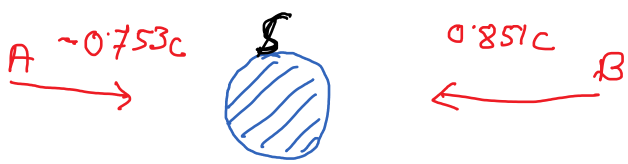
$$\gamma = \sqrt{1-\frac{v^2}{c^2}} = \left(1-\frac{v^2}{c^2}\right)^{1/2} \approx 1 - \frac{1}{2}\frac{v^2}{c^2}$$

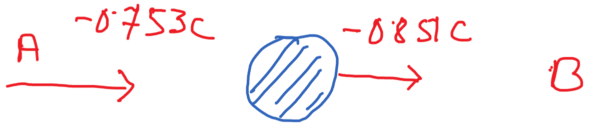
$$L = L_0\sqrt{1-v^2/c^2}$$

$$= L_0\left(1-\frac{v^2}{2c^2}\right)$$

$$L_0 - L = L_0\frac{v^2}{2c^2} = 2.6\times10^{-8}\ \text{m}$$

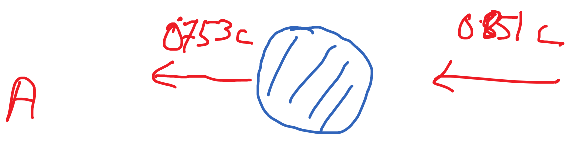
3/




$$v = \frac{v'+u}{1+\frac{v'u}{c^2}} = \frac{-(0.851+0.753)c}{1+0.851\times0.753}$$

$$= -0.978c$$




$= 0.978c$

4/ $\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$, $\Delta t = 0.465\ \mu s$

$\Delta x = 53.4\ \text{m}$

4

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right),$$

$\Delta t = 0.465\,\mu s$

$\Delta x = 53.4\,m$

$v = 0.762\,c$

$$\simeq 0.508\,\mu s$$

$$\Delta x' = \gamma\left(\Delta x - v\Delta t\right) \simeq -81.5\,m$$

5

KE of proton $= 4.25\,GeV$

$$KE + m_0c^2 = mc^2$$

$$KE = (m - m_0)c^2$$

$$m - m_0 = \frac{KE}{c^2} = \frac{4.25\times10^{9}\times1.6\times10^{-19}}{9\times10^{16}}$$

$$= 0.75\times10^{-26}\,kg$$

$$m_0 = 1.674\times10^{-27}\,kg$$

$$\frac{m}{m_0} = 1 + 0.75\times10^{-26}/m_0$$

$$= 5.51$$