Tutorial 4

1. An odd number of people stand in a yard at mutually distinct distances. At the same time each person throws a pie at their nearest neighbor, hitting this person. Use mathematical induction to show that there is at least one survivor, that is, at least one person who is not hit by a pie.

2. Use mathematical induction to show that if S is a finite set with n elements where n is a nonnegative integer, then S has 2^n subsets.

3. Use mathematical induction to prove that $2^n < n!$ for every positive integer n with $n \geq 4$. (Note that this inequality is false for $n = 1,2$ and 3).

4. Use mathematical induction to show that $1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$ for all nonnegative integers n

5. Give a proof by contradiction of the theorem "If $3n + 2$ is odd, then n is odd."

6. Prove that for all non-negative real numbers x, y and z and if $x^2 + y^2 = z^2$, then $x + y \geq z$.

7. Let $a, b \in \mathbb{R}$ with $a \geq 0$ and $b \geq 0$. Use a proof by contradiction to show that $\frac{a+b}{2} > \sqrt{ab}$.

8. Prove the theorem "If n is a positive integer, then n is odd if and only if n^2 is odd".

9. Suppose an ATM machine has only two-dollar and five-dollar bills. You can type in the amount you want, and it will figure out how to divide things up into the proper number of two's and fives. Prove that the ATM machine can generate any output amount n >= 4, using these two-dollar and five-dollar bills.

10. Show that at-least 10 of any 64 days chosen must fall on the same day of the week.

11. Prove that if n is a positive integer, then n is odd if and only if 5n+6 is odd.

12. Show that the square of an even number is an even number using direct proof.

13. Prove the following proposition using direct proof:

Let a, b and c be integers, with $a \neq 0$. If $a|b$ and $a|c$ then $a|(b + c)$

(**Note:** The integer a is divisible by nonzero integer b if $a = bc$ for some integer c We denote this by $b|a$ and say that b divides a.