Tutorial - 12

Q1) Show that if every element of a Group $(G, \circ)$ be its own inverse, then it is an abelian group.

Q2) Determine whether the set together with the binary operation is a semi-group, a monoid or neither. If it is a monoid, specify the identity element:

(a) N, where $*$ is defined as ordinary addition

(b) Z^{+}, where $a * b = \max(a, b)$ for all $a, b \in Z^{+}$

Q 3) The algebraic structure ({1,2,3,…p-1}, Xp) is an Abelian Group, where p is a prime number and Xp represents multiplication modulo p

A) True

B) False

Q4) Do the following sets form an integral domain under ordinary addition and multiplication? If so state whether they are fields:

(a) the set of even integers.

(b) the set of positive integers.

Q5) Consider the following Hasse Diagrams

(i) (ii) (iii) (iv)

Which of the above represent a lattice?

(A) (i) and (iv) only

Q6) Check whether the following are POSETS?

(a) $(R, \leq)$

(b) $(Z, >)$

Q7) The following is the incomplete operation table of 4-element group.

*	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b				
c				

The last row of the table is:

(a) c a e b

(b) c b a e

(c) c b e a

(d) c e a b

Q8) The set $\{1, 2, 4, 7, 8, 11, 13, 14\}$ is a group under multiplication modulo 15.
The inverse of 4 and 7 a respectively

(a) 3 and 13

(b) 2 and 11

(c) 4 and 13

(d) 8 and 14

Q9) Show that the matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$ from a multiplicative abelian group.

Q10) For the composition table of a cyclic group shown below which of the choice is correct?

*	a	b	c	d
a	**a**	**b**	**c**	**d**
b	**b**	**a**	**d**	**c**
c	**c**	**d**	**b**	**a**
d	**d**	**c**	**a**	**b**

(a) a, b are generators
(b) b, c are generators
(c) c, d are generators
(d) d, a are generators

Q 11) Let (R,*) be an algebraic structure. In each of the following identify which algebraic structure is defined, where * is defined as-

A) a*b=a+b (where a and b are elements of R)

B) a*b=min(a,b)

C) a*b=a^b

Q 12) Identify whether the given algebraic structure ({1,w,w2},*) (i.e. 3rd roots of unity) represents a cyclic group. If yes, find all generators. (Here * represents multiplication)