Logic

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Tautological Implication

A compound proposition is said to tautologically imply (or) simply imply the compound proposition B if and only if $A \implies B$ is a tautology.

Example

1. $P \wedge Q$ tautologically implies Q
2. $(P \vee Q) \wedge (P \implies R) \wedge (Q \implies R)$ tautologically implies R

Note: All logical equivalences can be proved using truth table.

Logical Equivalence Laws

Equivalence	Name
$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity Laws
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination Laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent Laws
$\neg(\neg p) \equiv p$	Double Negation Law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative Laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative Laws

Equivalence	Name
$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$	Distributive Laws
$\neg(p \land q) \equiv \neg p \lor \neg q$ $\neg(p \lor q) \equiv \neg p \land \neg q$	De Morgan's Law
$p \lor (p \land q) \equiv p$ $p \land (p \lor q) \equiv p$	Absorption Laws
$p \lor \neg p \equiv T$ $p \land \neg p \equiv F$	Negation Laws

Examples

- Show that $\neg(p \implies q)$ and $p \wedge \neg q$ are logically equivalent.
- Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.
- Show that $(p \wedge q) \implies (p \vee q)$ is a tautology.

Argument:

An Argument in propositional logic is a sequence of propositions. All but the final proposition in the argument are called **premises** and the final proposition is called the **conclusion**.

An argument is valid if the truth of all its premises implies that the conclusion is true.

Argument Form:

An Argument Form in propositional logic is a sequence of compound propositions involving propositional variables. An argument form is **valid** if no matter what propositions are substituted for the propositional variables in its premises, the conclusion is true if the premises are all true.

We can conclude that, an argument form with premises $p_1, p_2, ...p_n$ and conclusion q is valid, when $(p_1 \wedge p_2 \wedge ...p_n) \implies q$ is a tautology.

Example:

Modus Ponens (Law of detachment)

$(p \wedge (p \implies q)) \implies q$ is a Tautology.

The tautology leads to the following valid argument form :

p

$p \implies q$

- $\therefore q$

Suppose that the statement "If it snows today then we will go skiing" and its hypothesis, "It is snowing today" are true. Then by modus ponens, it follows that the conclusion "We will go skiing" is true.

Rules of Inference

Table 1 Rules of inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$[p \wedge (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \wedge (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p q $\therefore p \wedge q$	$[(p) \wedge (q)] \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$	Resolution

Example 1 :

State which rule of inference is the basis of following argument "It is below freezing now. Therefore, it is either below freezing or raining now."

Example 2 :

State which rule of inference is the basis of following argument "If it rains today, then we will not have a barbecue today. If we do not have a barbecue today, then we will have a barbecue tomorrow. Therefore, if it rains today, then we will have a barbecue tomorrow."

Example 3:

Show that the hypothesis "It is not sunny this afternoon and it is colder than yesterday","We will go swimming only if it is sunny", "If we do not go swimming then we will take a trip" and "If we take a trip then we will be home by sunset" lead to the conclusion "We will be home by sunset"

Example 4:

Show that the following argument is valid. If Mohan is a lawyer, then he is ambitious. If Mohan is an early riser, then he does not like idlies. If Mohan is ambitious, then he is an early riser.
Then if Mohan is a lawyer, then he does not like idlies.

Try Yourself

Show that the hypotheses "If you send me an e-mail message, then I will finish writing the program. If you do not send me an email message, then I will go to sleep early. If I go to sleep early, then i will wake up feeling refreshed" leads to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed"

Try Yourself

Show that the following argument is valid. If today is Tuesday, I have a test in Mathematics or Economics. if my economics Professor is sick, I will not have a test in Economics. Today is Tuesday and my Economics Professor is sick. Therefore, I have a test in Mathematics.