

Assignment 5

1. Verify whether T is a linear mapping. If T is linear, find $\text{Ker}(T)$ and $\text{Im}(T)$. Also verify rank and nullity of that linear mapping.

(i) $T : R^2 \rightarrow R^2$ defined by $T(x, y) = (x + y, x - y)$

(ii) $T : R^2 \rightarrow R^3$ defined by $T(x, y) = (x + 2y, 2x + y, x + y)$

(iii) $T : R^3 \rightarrow R^3$ defined by $T(x, y, z) = (yz, zx, xy)$

2. Determine the linear mapping $T : R^{33}$ which maps the basis vectors $(0, 1, 1), (1, 0, 1), (1, 1, 0)$ of R^3 to $(1, 1, 1), (1, 1, 1), (1, 1, 1)$ respectively. Verify that $\dim(\text{Ker } T) + \dim(\text{Im } T) = \dim(R^3)$.

3. A linear mapping $T : R^3 \rightarrow R^3$ is defined by

$T(x_1, x_2, x_3) = (3x_1 - 2x_2 + x_3, x_1 - 3x_2 - 2x_3)$. Find the matrix of T relative to the ordered bases

(i) $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ of R^3 and $\{(1, 0), (0, 1)\}$ of R^2

(ii) $\{(0, 1, 0), (1, 0, 0), (0, 0, 1)\}$ of R^3 and $\{(0, 1), (1, 0)\}$ of R^2

(iii) $\{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}$ of R^3 and $\{(1, 0), (0, 1)\}$ of R^2