Lacture-18.

Clars Note

Real Inner Product:

Let V be a real vector space. A real inner product

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let V be a real vector space. A real inner product on V is a mapping t: VXV -> IR that arrights to each. ordered pair of vectors (x,B) of V a scaal number +(x,B), generally denoted by (x.B), satistying the following proposities:-(1) $\langle \alpha, \beta \rangle = \langle \beta, \alpha \rangle + \alpha, \beta \in V$ (symmetry). (8) \(ex+dB, y) = e(x, y) + d(B, y) + d(B, y) e, d \(ex+dB, y) = e, d \(ex+dB, y) \\
e, d \(ex+dB, y) = e(x, y) + d(B, y) \\
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e, d \(ex+dB, y) = e, d \(ex+dB, (3) <x.x>>0 if x = D (posetivity) let V be a complex vector space. A complex Complex inner producto inner product is a mapping tivxv -> c +hat arrights each ordered pair of rectors (x, B). of v a complex number f(x,B), generally. denoted by $\langle \alpha, \beta \rangle$, satisfying the following is the conjugate (1) (x.B) = (B.x) where (B.x) proposeties: of the complexme. < B. x>. (2) (ex+dB.8) = e(x.2) + d (B.2) x x, B,8 €~ c, 4 € C (3) $\langle x.x \rangle > 6$ if $x \neq \theta$

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Ex amples: let & = (a1, a2, ... an) & B = (b1, b2, --- bn) E 12 1) fc+ V = 112" let us define:-< x.B> = a,b,+a2 b2+---+ anbn Then (x.B) satisfies all the conditions for.

a steal inner product.

let & 6 (V, (.)) Where (.) is the inner product Norm of a vectors Then norm of $\alpha = 11 \times 11 = \sqrt{\langle \alpha, \alpha \rangle}$

Theorem & let & E (Vo(.)) and IIdII be its (i) 11ex11 = 1e1 11x11, e boing a read number.

(ii) 11x11 >0 unless x = 0 and 11011 = 0

broof: (i) 11ex11 = \(\langle ex.ex\rangle = \(\frac{1}{2}\langle \langle \alpha\rangle \rangle \langle \langle \alpha\rangle \rangle \langle \alpha\rangle \rangle \langle \alpha\rangle \rangle \langle \alpha\rangle \alpha\rangle \rangle \alpha\rangle \alpha\rangle \rangle \alpha\rangle \alpha\rangle \rangle \alpha\rangle \alpha\rangle \alpha\rangle \rangle \alpha\rangle \alpha\rangl = Je(x.ex) = Jekeria> = Jer(d. x) = 101/(x.x) = 101/1011

" < x x > > 0 if x = 0 (ii) 11×11 = \(\lambda \times \rangle \circ\) => 11×11 /0 Cavely-sehwartz inequality? For any any two rectors x, B (V, <.>). 1<x. 0>1 = 11 x 11 11 B 11 For any two rectors x, B & (v, <->) Triangle inequality & 11x+B11 = 11x11 + 11B11 proof: 11x+B11 = \(\alpha + B \cdot \alpha + B \rangle $\Rightarrow 11 \times + \beta 11 = \langle \times + \beta \cdot \times + \beta \rangle$ $= \langle x \cdot x + \beta \rangle + \langle \beta \cdot x + \beta \rangle$ = (x+B.x) + (x+B.B) = < x · x> + < B · x> + < x · B> + < B · B) = < x. x> + < x. B> + < x. B> + < B.B) = 11x11 + 2 (x.B) + 11B11 < 11x11+211x1111B11+ 11B17 (c.5) = (11×11+11BII)

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Orthogonal : Then x is said to be orthogonal to a vector B if, < x . B> = 0 We express this by writting XIB.

Orthogonal Set of vectors & A set of rectors {B1,B2,--,Bn} is said. to be orthogonal if $\langle \beta_i, \beta_j \rangle = 0$ Whenever $i \neq j$

Orthonormal set of rectors & A set of rectors { B1, B2, --, Bn} is said to be orthonormal it

 $\langle Bi, Bj \rangle = 0$ to $i \neq j$ = $\pm j$ for i = j

Notes An orthogonal set of rectors may contain the mull rector & but an orthonormal set contains only non-null