Tutorial - 7 Solution

① Asymmetric:-

$\forall a \forall b (aRb \rightarrow (b,a) \notin R)$

To prove:-

$(\forall a \forall b (aRb \rightarrow (b,a) \notin R)) \rightarrow (\forall a \forall b ((aRb \wedge bRa) \rightarrow a=b))$

proof:-

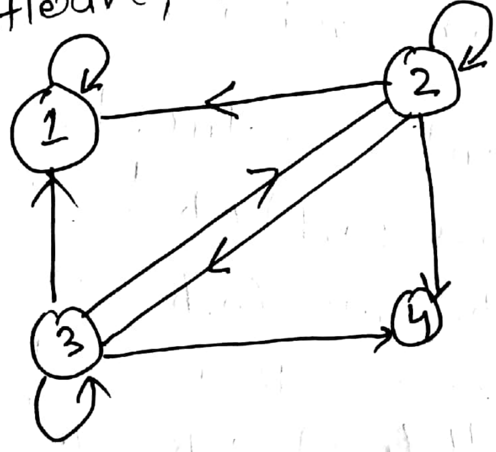
1) Assume R is asymmetric
2) $\forall a \forall b ((a,b) \notin R \vee (b,a) \notin R)$ (step 1 and by defn)
3) $\forall a \forall b ((aRb \wedge bRa) \longrightarrow a=b)$ implication's premise is false.
4) Therefore, asymmetry implies antisymmetry.

② Let $x, y, z \in N$ such that xRy and yRz. So there are integers m, n such that $y = mx$ and $z = ny$. Thus there $z = (mn)x$, so x divides z and xRz. Thus the relation is transitive.

③ (a) false

(b) false.

④ (b) Neither reflexive, nor irreflexive but transitive.



(5) Let n be the no. of element in a set

maximum no. of relations $= 2^{n^2}$

(6) Let $x \in Z$. Then $x - x = 0$ and 0 is divisible by 6

Therefore, xRx for all $x \in Z$

Hence, R is reflexive.

Again $xRy \Rightarrow (x-y)$ is divisible by 6

$\Rightarrow -(x-y)$ is divisible by 6

$\Rightarrow (y-x)$ is divisible by 6

$\Rightarrow yRx$.

Hence R is symmetric

xRy and $yRz \Rightarrow (x-y)$ is divisible by 6

and $(y-z)$ is divisible by 6.

$\Rightarrow [(x-y) + (y-z)]$ is divisible by 6

$\Rightarrow (x-z)$ is divisible by 6.

$\Rightarrow xRz$

R is transitive.

Thus R is an equivalence relation.

(7) Suppose R is antisymmetric. Let $(a,b) \in R \cap R^{-1}$. Then $(a,b) \in R$ and $(a,b) \in R^{-1}$. Again $(a,b) \in R^{-1}$ implies $(b,a) \in R$. Thus $(a,b) \in R$ and also $(b,a) \in R$. Hence, $b = a$ because R is antisymmetric. This is true for all $(a,b) \in R \cap R^{-1}$. Hence every elemen

of $R \cap R^{-1}$ is of the foorm (a,a) where $a \in A$, therefore $R \cap R^{-1} \subseteq I_A$.

Conversely, suppose $R \cap R^{-1} \subseteq I_A$. Let $(a,b) \in A \times A$ such that $(a,b) \in R$ and $(b,a) \in R$, i.e $(a,b) \in R$ and $(a,b) \in R^{-1}$. Then $(a,b) \in R \cap R^{-1}$. Since $R \cap R^{-1} \subseteq I_A$. it follows that $b=a$. Hence R is antisymmetric.

8)

$$M_R = \begin{array}{c|c} & \begin{matrix} a & b & c \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

$R = \{(1,a), (1,c), (2,c), (3,a), (4,b)\}$

$R^{-1} = \{(a,1), (c,1), (c,2), (a,3)\ (b,4)\}$

(i) $R \circ R^{-1} = \{(1,1), (2,2), (1,3)\ (1,2), (3,3), (3,1), (2,1), (4,4)\}$

it is symmetric.

(ii) $R^{-1} \circ R = \{(a,a), (a,c), (c,a), (c,c), (b,b)\}$.

it is Symmetric.

(iii) Reflexive $= \{(a,a)\ (b,b)\ (c,c)\}$

$= \{(1,1)\ (2,2)\ (3,3)(4,4)\}$

transitive $= \{\ \{(1,3)\ (1,2), (2,1)\ (1,3), (1,1)\}\}$

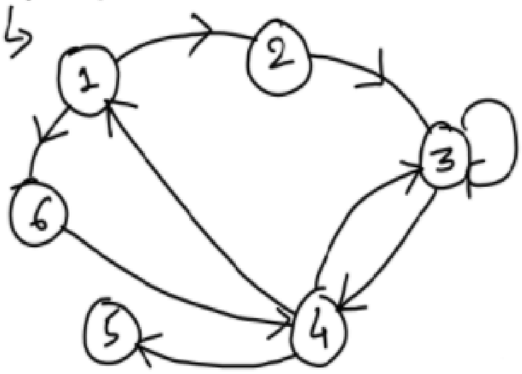
$\{(a,c)\ (c,c)\}$

$\therefore$ It is equivalence relations.

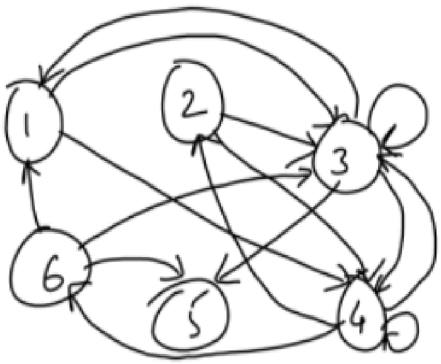
9) $A = \{1,2,3,4,5,6\}$ and $R = \{(1,2), (1,6), (2,3), (3,3), (3,4), (4,1), (4,3), (4,5), (6,4)\}$

$R^2 = \{(1,3), (1,4), (2,3), (2,4), (3,4), (3,1), (3,3), (3,5), (4,2), (4,6), (4,3), (4,4), (6,1), (6,3), (6,5)\}$

Graph of R



Graph of R^2



$$M_R^2 = \begin{array}{c|cccccc|} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 1 & 0 & 0 \\ 3 & 1 & 0 & 1 & 1 & 1 & 0 \\ 4 & 0 & 1 & 1 & 1 & 0 & 1 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 1 & 0 & 1 & 0 & 1 & 0 \end{array}$$

10) $R^{(r)} = \{(a,a), (b,b), (c,c), (d,d), (a,b), (b,c), (d,c), (d,a), (a,d)\}$

$R^{(s)} = \{(a,b), (b,a), (b,c), (c,b), (d,c), (c,d), (d,a), (a,d), (d,d)\}$

$R^{*} = \{(a,a), (a,b), (a,c), (a,d), (b,c), (d,a), (d,b), (d,c), (d,d)\}$

11) 2^{n^2-n}

12) A. Reflexive-No, Symmetric-No, Anti-symmetric-Yes, Transitive-Yes
B. Reflexive-No, Symmetric-No, Anti-symmetric-No, Transitive-No

13) Reflexive-No, Symmetric-No, Anti-symmetric-No, Transitive-Yes, Equivalence Relation-No, Partial ordering Relation-No

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$$\begin{array}{c c} & \begin{matrix} 1 & 2 & \cdots & n \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ \vdots \\ n \end{matrix} & \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix}_{n \times n} \end{array}$$

$n^2 - n$ positions to fill
Each can be filled in 2 ways

$\therefore 2^{n^2-n}$ ways

$[2 \times 2 \times \ldots n^2 - n \text{ times}]$