Eg 21:- Prove that there is no largest integer that is a multiple of 5 using proof by contradiction.

Soln :- Let P: There is no largest integer that is a multiple of 5.

We assume $\sim P$ to be true ie there is a largest integer that is a multiple of 5 and suppose that the integer is $\boxed{m}$.

Thus, $m = 5k$ for some $k \in \mathbb{Z}$

Now, consider the integer $m+5$

$$m+5 = 5k+5 = 5(k+1)$$

This shows that $\boxed{m+5}$ is also a multiple of $\boxed{5}$ and $\boxed{m+5}$ is greater than $\boxed{m}$ as well.

Therefore, this is a contradiction that $\boxed{m}$ is the largest integer that is a multiple of 5 and our assumption is not true.

Hence, there is no largest integer that is a multiple of 5.

→ To prove the conditional statement $P \to Q$ We assume both P and $\sim Q$ are true.

→ Then considering $\sim Q$ as a premise, we draw the Conclusion $\sim P$.

→ Thus, we get the contradiction $\boxed{P \wedge \sim P}$.

→ Therefore, we say that our initial assumption is not true ie $\sim Q$ is false as P is assumed to be true.

→ Finally, $\sim Q$ is false implies that Q is true and hence $P \to Q$.

→ Steps are as follows:

(a) Assume both P and $\sim Q$ are true.

(b) Use $\sim Q$ and show that P is false, which is a Contradiction.

Eg 3:- Prove the statement :-

'If $3n+1$ is even, then n is odd'

by using the method of proof by contradiction.

Soln.:- Here, P: $3n+1$ is even

Q: n is odd

We shall assume that P is true and $\sim Q$ is true.

$\therefore$ Let $3n+1$ is even and n is even.

We can say, $\boxed{n = 2k}$ where k is some integer then

then, $3n+1 = 3(2k)+1 = 6k+1$

since, $6k = 2(3k)$, $\therefore$ $6k$ is an even no.

$\Rightarrow$ $6k+1$ is an odd number.

$\Rightarrow$ $3n+1$ is an odd number.

So, this is a contradiction to the assumption that $3n+1$ is even.

Hence, n is not even ie n is odd.

This proves the statement 'if $3n+1$ is even, then n is odd'.

Eg 4:- Prove that the sum of two consecutive integers is odd.

Soln: Let a and b be two integers.

Here P: a and b are two consecutive integers.

Q: $a+b$ is odd.

We shall assume that P and $\sim Q$ is true.

Thus, a and b are consecutive integers and

$\boxed{a+b \text{ is even}}$

$\therefore$ $a = k$ and $b = k+1$ for some integer k.

Thus $a+b = k+k+1 = 2k+1$ which is an odd no.

Contradiction

$\therefore$ "sum of 2 consecutive integers is odd" by contradiction