Linear Algebra (EMAT102L)

Lecture No 2: Linear Algebra (EMAT102L)

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Topics cover in Lecture 2

- Invertible matrix
- Cramers rule
- Elementary Row operation

Row Equivalence

Two $m \times n$ matrices A and B are said to be row equivalent if B can be obtained from A by a finite sequence of elementary row operations.

If A_{mn} and B_{mn} are row equivalent, we write $A_{mn} \cong B_{mn}$

Note: If A and B are row-equivalent $m \times n$ matrices, then the homogeneous system of linear equation $AX = 0$ and $BX = 0$ have exactly the same solution.

Row-reduced matrix

An $m \times n$ matrix A is called row-reduced matrix if

- the first non-zero element in each non-zero row is 1 (called the leading 1)
- in each column containing the leading 1 of same row, the leading 1 is the only non-zero element.

Examples: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Row-reduced echelon matrix

An $m \times n$ matrix A is called row-reduced echelon matrix if

- A is row-reduced.
- All zero rows, if any, belongs at the bottom of the matrix.
- Every leading 1 is to the right of the one above it

Examples: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Note: a matrix A can be now equivalent to a row reduced echelon matrix B by elementary row operations.

Rank of a Matrix

If a matrix $A_{m\times n}$ is equivalent to row reduced echelon matrix $B_{m\times n}$, then number of non-zero rows in a row-reduced echelon matrix $B_{m\times n}$ is the rank of matrix $A_{m\times n}$.

$$\text{Rank of } A_{m\times n} \leq \min(m,n)$$


Thank you!