(c) Proof by Mathematical Induction

→ Following are the steps:-

(i) Show true for the first term mostly $n=1$ } Base Case

(ii) Assume true for $n=k$ } Induction Hypothesis

(iii) Prove/Show true for $n=k+1$ } Conclusion

(iv) Restate ∴ by the process of mathematical induction given statement.

Eg 1:- Prove: $3+6+9+12+\ldots\ldots+3n = \dfrac{3n(n+1)}{2}$

(i) Show true for $\boxed{n=1}$

$$3(1) = \frac{3(1)(1+1)}{2}$$

$$\Rightarrow 3 = \frac{3(\cancel{2})}{\cancel{2}}$$

$$\Rightarrow 3 = 3 \quad ✓$$

(ii) Assume true for $\boxed{n=k}$

$$3+6+9+12+\ldots\ldots+3k = \frac{3k(k+1)}{2}$$

(iii) Show true for $\boxed{n=k+1}$

$$\underbrace{3+6+9+12+\ldots\ldots+3k}_{\frac{3k(k+1)}{2}} + 3(k+1) = \frac{3(k+1)(k+1+1)}{2}$$

$\dfrac{3k(k+1)}{2}$ (as shown in step (ii))

$$\Rightarrow \frac{3k(k+1)}{2} + 3(k+1) = \frac{3(k+1)(k+2)}{2}$$

$$\Rightarrow \frac{3k(k+1) + 6(k+1)}{2} = \frac{3(k+1)(k+2)}{2}$$

$$\Rightarrow \frac{3(k+1)[k+2]}{2} = \frac{3(k+1)(k+2)}{2}$$

$\therefore$ L.H.S = R.H.S

Hence, by the process of mathematical induction,

$$3+6+9+12+ \cdots + 3n = \frac{3n(n+1)}{2}$$

Eg 2:- Using mathematical induction, prove that for every natural number

$$1+2+3+ \cdots + n = \frac{n(n+1)}{2}$$

(i) Show true for $\boxed{n=1}$

$$\Rightarrow 1 = \frac{1(1+1)}{2}$$

$$\Rightarrow 1 = \frac{1(\cancel{2})}{\cancel{2}}$$

$$\Rightarrow 1 = 1 \checkmark$$

(ii) Assume true for $\boxed{n=k}$

$$1+2+3+ \cdots + k = \frac{k(k+1)}{2}$$

(iii) Show true for $\boxed{n=k+1}$

$$\underbrace{1+2+3+ \cdots + k}_{k(k+1)/2} + k+1 = \frac{(k+1)(k+1+1)}{2}$$

$$\Rightarrow \frac{k(k+1)}{2} + (k+1) = \frac{(k+1)(k+2)}{2}$$

$$\Rightarrow \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$$

$$\Rightarrow \frac{(k+1)(k+2)}{2} = \frac{(k+1)(k+2)}{2}$$

$\therefore$ By the process of mathematical induction

$$1+2+3+\ldots\ldots+n = \frac{n(n+1)}{2}$$

(VII) Mechanization of Reasoning

→ Every X is Y
A is X

Therefore, A is Y

→ Every politician is clever
Manoj is a politician

Therefore, Manoj is clever

→ This is an effort of mechanizing the rules of inferences or simply reasoning.

→ Mechanization of reasoning leads to automated deduction.

(a) Satisfiable :- A set of formula is called satisfiable if for a set of truth values of the variables in the formula, all the formula are true.

Eg:- 1. $\{P, Q\}$ is satisfiable as both the formula are true when P and Q is true.

2. $\{P, \sim P\}$ is not satisfiable.

3. $\{P, \sim P \vee Q\}$ is satisfiable.

(b) Consistence :- A set of formula is called consistent if we cannot derive a contradiction from the set.

or

A set of formula is called consistent if there is no formula P such that both P and $\sim P$ can be proved from the given premises and deductive system of formula.

(c) Applications of propositional logic

(i) Excel, (ii) Programming languages,

(iii) Digital logic, (iv) Artificial Intelligence

(v) Web search engines (vi) relational calculus.

(a) Russell's Paradox

Let X be a set containing all sets that do not contain themselves,

$$X = \{x : x \notin X\}$$

Now, consider two cases:

(i) If $X \in X$, then the set X contains itself.
↳ Contradiction

(ii) If $X \notin X$, then the set X does not contain itself, but according to the definition X must contain all the sets that do not contain themselves.
↳ Contradiction

This is a paradox.

Eg:- There is a city X, where a barber does the shave for all those men in the city who do shave themselves.
Now, the question is who does the shave for the barber?