Tutorial 3

1. Symbolize the expression "Every mother loves her children" in predicate logic.

2. Write down the negation of the following predicate:

   "for every number x there is a number y such that y < x"

3. Write the following statement in symbolic form using quantifiers:
   (i) All students have taken a course in mathematics.
   (ii) Some students are intelligent, but not hardworking.

4. Let A = {1, 2, 3, 4, 5}, determine the truth value of the following:
   i) $(\forall x \in A)(x + 3 = 10)$
   ii) $(\exists x \in A)(x + 3 < 5)$.

5. Let W(x, y) mean that student x has visited website y, where the domain for x

   consists of all students in your school and the domain for y consists of all websites. Express each

   of these statements by a simple English sentence.

   a. W (Sarah Smith, www.att.com).

   b. $\exists x W$ (x, www.imdb.com).

   c. $\exists y W$ (Jose Orez, y).

   d. $\exists y$ (W(Ashok Puri, y) $\wedge$ W (Cindy yoon, y).

   e) $\exists y \forall z$ (y $\neq$ (David Belcher) $\wedge$ (W(David Belcher, z) $\rightarrow$ W(y, z))).

   f) $\exists x \exists y \forall z ((x \neq y) \wedge (W(x, z) \leftrightarrow W(y, z)))$

   Note: In part e and f, the predicate W(a,b) represents a i.e the first variable has the domain as the set of all students in your school and b I.e the second variable has the domain as the set of all websites.

6. What is the first order predicate calculus statement equivalent to the following?
   "Every teacher is liked by some student"

   i) $\forall(x) [teacher\ (x) \rightarrow \exists(y) [student(y) \rightarrow likes\ (y,x)]]$
   ii) $\forall(x) [teacher\ (x) \rightarrow \exists(y) [student\ (y) \wedge likes\ (y,x)]]$
   iii) $(\exists(y)\forall(x) [teacher\ (x) \rightarrow [student\ (y) \wedge likes\ (y, x)]]$
   iv) $(\forall(x)[teacher\ (x) \wedge \exists(y) [student(y) \rightarrow likes\ (y,x)]]$

7. Give the symbolic form of the following statements:
   (i) Some men are genius.
   (ii) For every x, there exists a y such that $x^2 + y^2 \geq 100$

8. Negate each of the following statements:
   i.   If the teacher is absent, then some students do not complete their homework.
   ii.   All the students completed their homework, and the teacher is present.
   iii.   Some of the students did not complete their homework or the teacher is absent.