Ordinary Differential Equations

(Lecture-5)

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Learning Outcome of the Lecture

We learn

- Equations Reducible to Separable Form (Contd..)
- Examples

Equations Reducible to Separable Form (Contd..)

Consider the DE

$$(a_1x + b_1y + c_1)dx + (a_2x + b_2y + c_2)dy = 0.$$

where a_1, a_2, b_1, b_2, c_1 and c_2 are constants.

- If $\frac{a_2}{a_1} \neq \frac{b_2}{b_1}$, then the transformation

  $$x = x_1 + h \quad \text{and} \quad y = y_1 + k$$

  reduces the above DE to the homogenous equation in variables x_1 and y_1 and (h, k) is the solution of the system

  $$a_1h + b_1k + c_1 = 0 \ \text{ and } \ a_2h + b_2k + c_2 = 0.$$

- If $\frac{a_2}{a_1} = \frac{b_2}{b_1} = k$, then the transformation

  $$z = a_1x + b_1y$$

  reduces the above DE to a separable equation in the variables x and z.

Examples

Example-1: Solve the DE

$$(5x + 2y + 1)dx + (2x + y + 1)dy = 0.$$

Solution: Here $\frac{a_2}{a_1} \neq \frac{b_2}{b_1}$. Take $x = x_1 + h$ and $y = y_1 + k$, we get,

$$\{(5x_1 + 2y_1) + (5h + 2k + 1)\}dx_1 + \{(2x_1 + y_1) + (2h + k + 1)\}dy_1 = 0.$$

Choose (h, k) such that

$$5h + 2k + 1 = 0 \text{ and } 2h + k + 1 = 0 \Rightarrow h = 1 \;\; k = -3.$$

Now reduced ODE is homogenous,

$$(5x_1 + 2y_1)dx_1 + (2x_1 + y_1)dy_1 = 0.$$

Put $y_1 = vx_1$ and solve ODE. We get

$$y_1^2 + 4x_1y_1 + 5x_1^2 = c$$

Substitute $x_1 = x - h = x - 1$ and $y_1 = y - k = y + 3$, solution is

$$y^2 + 4xy + 5x^2 + 2x + 2y = c$$

Example-2: Solve the DE

$$(2x+3y+1)dx+(4x+6y+1)dy=0, \;\; y(-2)=2.$$

Solution: Here $\frac{a_2}{a_1}=\frac{b_2}{b_1}=2$. Take
$2x+3y=z \Rightarrow 2dx+3dy=dz \Rightarrow dy=\frac{dz-2dx}{3}$. We get

$$(z+1)dx+(2z+1)\frac{dz-2dx}{3}=0,$$

$$3(z+1)dx+(2z+1)(dz-2dx)=0 \Rightarrow (2z+1)dz+(3z+3-4z-2)dx=0,$$

$$dx-\frac{2z+1}{z-1}dz=0 \Rightarrow dx-(2+\frac{3}{z-1})dz=0,$$

$$x-2z+3\ln|z-1|=c \Rightarrow x+2y-\ln|2x+3y-1|-2=0.$$

Total Differential

The first order differential equation may be expressed in either the derivative form

$$\frac{dy}{dx} = f(x, y)$$

or the differential form

$$M(x, y)dx + N(x, y)dy = 0.$$

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Example-1:

$$\frac{dy}{dx} = \frac{x^2 + y^2}{x - y}$$

Example-2:

$$(\sin x + y)dx + (x + 3y)dy = 0$$

Total Differential

Definition

Let F be a function of two variables such that F has continuous first order partial derivatives in a domain D. The total differential dF of the function F is defined by the formula

$$dF(x,y) = \frac{\partial F(x,y)}{\partial x}dx + \frac{\partial F(x,y)}{\partial y}dy$$

for all $(x,y) \in D$.

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Example: $F(x,y) = xy^2 + 2x^3 y$

$$\frac{\partial F(x,y)}{\partial x} = y^2 + 6x^2 y \text{ and } \frac{\partial F(x,y)}{\partial y} = 2xy + 2x^3$$

The total differential dF is

$$dF(x,y) = (y^2 + 6x^2 y)dx + (2xy + 2x^3)dy$$

for all real (x,y).