Set Theory

* **Def.:** A set is an unordered collection of different well defined objects.

Consider the following examples:-

(1) Collection of all the tall persons of a class.

(2) Collection of all the educated persons of a Society.

(3) Collection of all the old items of your house.

(4) Collection of all the students whose height is more than 185 cms

(5) Collection of all those people of your Society who have atleast a Bachelor's degree

(6) Collection of all the items of the house which were bought more than 10 yrs ago

Q) Check whether the following are sets:-

(a) $S = \{1, 2, 3, 4\}$

$T = \{a, b, c\}$

Can sets be a collection of heterogeneous elements?

→ A set should be well defined i.e there should be a membership clause.

Eg:- $N = \{2, 4, 6, 8, 10\}$ elements/object/member

Clause: Even numbers between 1 to 10

$\therefore$ $2 \in N$ (belongs to) and $5 \notin N$ (doesn't belong to)

(I) Representation of Sets:

(a) Roaster Notation (b) Set Builder Notation

(a) Roaster Notation:

$A = \{a, b, c, \underbrace{\ldots\ldots}_{\text{many elements}}, z\}$

$B = \{1, 2, 3, 4, \ldots\}$ → Infinite Set.

(b) Set Builder Notation

$S_1 = \{x \mid x \text{ is an even number between } 1 \text{ to } 10\}$

$S_2 = \{x \mid x \text{ is a Chocolate}\}$.

(C) Cardinality: The no. of elements in the set is called the cardinality of the set.

Denoted as $\boxed{|S| \text{ or } n(S)}$

Ex:

$A = \{1, 2, 5, 9, 20\}$, $|A| = 5$

$B = \{a, b, c, \ldots, z\}$, $|B| = 26$

$S = \{a, b, \underbrace{\{m, n\}}_{\text{single element}}, z\}$, $|S| = 4$

Does $a \in S$ ✓ True

$\{m, n\} \in S$ ✓ True

$z \in S$ ✓ True

$m \in S$ × False

$n \in S$ × False.

(d) **Subset**: If we have two sets A and B and every element of A is an element of B, then A is called the subset of B.

Denoted as $A \subseteq B$

$\subseteq$ ↓ Subset symbol

Predicate: $\forall x\,(x \in A \rightarrow x \in B)$

Ex: $A = \{1, 2, 3, 4, 5\}$

$B = \{1, 2, 3\}$, $B \subseteq A$

$C = \{3, 4\}$, $C \subseteq A$

$D = \{5, 6\}$, $D \nsubseteq A$

$E = \{7\}$, $E \nsubseteq A$

$F = \{3\}$, $F \subseteq A$

Note:- If $M = \{a, b, c\}$, $N = \{a\}$

then $\boxed{N \subseteq M}$ is true but $\boxed{a \subseteq N}$ is false.

$a \notin N$ instead $a \in N$ is true.

b $P = \{\{a, b\}, c, d\}$

$\{a, b\} \subseteq P$ ×

$\{a, b\} \in P$ ✓

$\{\{a, b\}\} \subseteq P$ ✓

→ **Subset Properties**

(i) $A \subseteq A$ – Reflexive

(ii) $A \subseteq B,\ B \subseteq C \rightarrow A \subseteq C$ – Transitive

(e) Equal Sets

Two sets A and B are said to be equal if $A \subseteq B$ and $B \subseteq A$ i.e. both A and B should have the same elements.

Denoted as $\boxed{A = B}$

Ex:- $\{1, 2, 3\} = \{1, 2, 2, 3, 3\}$

$\{1, 2, 4\} = \{4, 2, 1\}$

Note:
- Order does not matter.
- Repetition does not matter

Q) Check whether the following are equal sets.

(i) $\{1, \{2,3\}\} = \{1,2,3\}$

(ii) $\{\{1\}\} = \{1\}$

Equal Set Properties

(i) $A = A$ — Reflexive

(ii) $A = B \rightarrow B = A$ — Symmetric

(iii) $A = B,\ B = C \rightarrow A = C$ — Transitive.

(f) Proper Subset

A set 'A' is called a proper subset of 'B' if

$$\boxed{A \subseteq B \text{ and } A \neq B}$$

ie B has atleast 1 element more than A

$$\boxed{n(B) > n(A)}$$

Denoted as $\boxed{A \subset B}$

Proper Subset Properties

$A \subset A$ ✗ — Not reflexive

$A \subset B \rightarrow B \subset A$ ✗ — Not Symmetric

$A \subset B,\ B \subset C \rightarrow A \subset C$ ✓ — Transitive

(g) Universal Set (U)

A set is called a universal set if it includes every set under discussion.

$$\left.\begin{array}{l} A = \{1,2,3\} \\ B = \{4,5\} \\ C = \{6,7\} \end{array}\right] \rightarrow U = \{1,2,3,4,5,6,7\}$$

(h) Null set or Empty Set ($\emptyset$ or $\{\ \}$)

A set which does not contain any element.

Note :- $\emptyset$ Cardinality – 0

$\{\emptyset\}$ Cardinality – 1

$$\emptyset \neq \{\emptyset\}$$

$$\emptyset \in \{\emptyset\}$$

Also, remember $\boxed{\emptyset \subseteq A}$

(i) Singleton Set

A set with one element is called a singleton set.

Eg:- $A = \{x \mid x \in Z \text{ and } 3 < x < 5\}$

(j) Finite and Infinite Set

A set is said to be finite if it has a finite no. of elements else it is infinite.

Eg:- (i) $A = \{x \mid x \in Z^+ \text{ and } x < 10\}$

$\hookrightarrow$ Finite Set

(ii) The set of real numbers is an infinite set.

(k) Countable and Uncountable Set

A set X is said to be countable if there exists a one-one correspondance from X to a subset of the set of natural numbers else it is uncountable.

Eg:- (i) The set of positive even numbers is a countable and infinite set.

(ii) the set of real nos between 0 and 1 is uncountable.

(l) Power Set $P(A)$

For a set A, a collection of all subsets of A is called the power set of A.

$P(A) = \{x \mid x \subseteq A\}$

$A = \{1, 2, 3\}$

$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

So, $\emptyset \subseteq P(A)$ and $A \subseteq P(A)$

Cardinality of $P(A) = 2^n$ where n is the no. of elements in A.

So, if $A = \{1,2,3\}$, here $n = 3$

$$|P(A)| = 2^n = 2^3 = 8$$