Rules of Inference for Quantified Statements

Rule of Inference	Name
$\dfrac{\forall x P(x)}{\therefore\ P(c)}$	Universal instantiation
$\dfrac{P(c) \text{ for an arbitrary } c}{\therefore\ \forall x P(x)}$	Universal generalization
$\dfrac{\exists x P(x)}{\therefore\ P(c) \text{ for some element } c}$	Existential instantiation
$\dfrac{P(c) \text{ for some element } c}{\therefore\ \exists x P(x)}$	Existencial generalization

Show that the premises:

- A student in Section A of the course has not read the book.
- Everyone in Section A of the course passed the first exam.

imply the conclusion

- Someone who passed the first exam has not read the book.

$A(x)$: "x is in Section A of the course"
$B(x)$: "x read the book"
$P(x)$: "x passed the first exam."

Hypotheses: $\exists x(A(x) \wedge \neg B(x))$ and $\forall x(A(x) \to P(x))$.
Conclusion: $\exists x(P(x) \wedge \neg B(x))$.

Hypotheses: $\exists x(A(x) \wedge \neg B(x))$ and $\forall x(A(x) \rightarrow P(x))$.
Conclusion: $\exists x(P(x) \wedge \neg B(x))$.

Step	Reason
1. $\exists x(A(x) \wedge \neg B(x))$	Hypothesis
2. $A(a) \wedge \neg B(a)$	Existencial instantiation from (1)
3. $A(a)$	Simplification from (2)
4. $\forall x(A(x) \rightarrow P(x))$	Hypothesis
5. $A(a) \rightarrow P(a)$	Universal instantiation from (4)
6. $P(a)$	Modus ponens from (3) and (5)
7. $\neg B(a)$	Simplification from (2)
8. $P(a) \wedge \neg B(a)$	Conjunction from (6) and (7)
9. $\exists x(P(x) \wedge \neg B(x))$	Existential generalization from (8)