**Department of Mathematics**
**Bennett University**
**EMAT102L: Ordinary Differential Equations**
**Tutorial Sheet-2**

1) Solve the following exact/reducible to exact ODEs:

$$
\begin{aligned}
&(a) \quad 2xye^{x^2}dx + e^{x^2}dy = 0, \quad y(0) = 2;\\
&(b) \quad \cos(x+y)dx + (3y^2 + 2y + \cos(x+y))dy = 0;\\
&(c) \quad (1+2x)\cos y dx + \sec y dy = 0;\\
&(d) \quad 3x^2 y dx + 4x^3 dy = 0.
\end{aligned}
$$

**Hint:** $(a) \quad y = 2e^{-x^2}, \quad (b) \quad \sin(x+y) + y^3 + y^2 = c, \quad (c) \quad \tan y = -x - x^2 + c.$

2) Solve the following linear/reducible to linear ODEs:

$$
\begin{aligned}
&(a) \quad \frac{dy}{dx} + 3x^2 y = x^2,\ y(0) = 2;\\
&(b) \quad y^2 dx + (3xy - 1)dy = 0;\\
&(c) \quad \frac{dy}{dx} + y = f(x),\ y(0) = 0,\ \text{where } f(x) = \begin{cases} 2 & 0 \le x < 1, \\ 0 & x \ge 1. \end{cases},\\
&(d) \quad dy + (4y - 8y^{-3})x dx = 0.
\end{aligned}
$$

3) Under what conditions for the constants $a, b, k, l$, is $(ax+by)dx + (kx+ly)dy = 0$ exact? Solve the exact ODE.

4) Find the orthogonal trajectories of the family of circles which are tangent to the y axis at the origin.
**Hint:** $x^2 + y^2 = my$.

5) Find the value of $n$ such that the curves $x^n + y^n = c$ are orthogonal trajectories of the family $y = \frac{x}{1-c_1 x}$.
**Hint:** $n = 3$.

6) Does the IVP $(x-2)\frac{dy}{dx} = y;\ y(2) = 1$ have a solution? Justify your answer.

7) Show that existence and uniqueness theorem guarantees the existence of a unique solution of the IVP-

$$
\begin{aligned}
&(a) \quad \frac{dy}{dx} = e^{2y};\ y(0) = 0.\\
&(b) \quad \frac{dy}{dx} = y^{4/3};\ y(x_0) = y_0.
\end{aligned}
$$