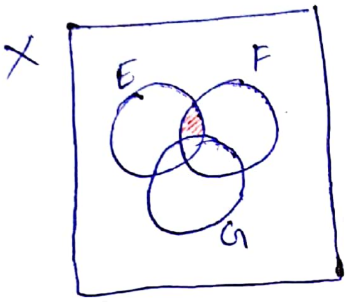
Tutorial Solution - 06

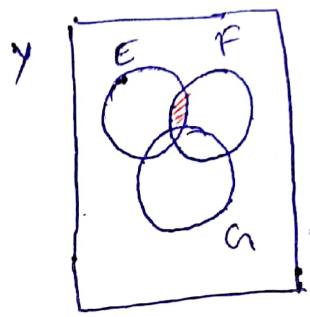
Q1) E, F and G are three finite sets where

$X = (E \cap F) - (F \cap G)$ and

$Y = (E - (E \cap G)) - (E - F)$

Using Venn Diagram, X and Y can be represented as





Therefore, the answer is (c) $X = Y$.

Q2) Let U denote the universal set, $P \rightarrow$ Set of students who took programming language, $D \rightarrow$ Set of students who took data structures and $C \rightarrow$ Set of students who took computer organization.

Therefore,

$U = 200$

$n(P) = 125$, $n(D) = 85$, $n(C) = 65$.

$n(P \cap D) = 50$, $n(D \cap C) = 35$, $n(P \cap C) = 30$

$n(P \cap D \cap C) = 15$

$n(P \cup D \cup C) = 125 + 85 + 65 - 50 - 35 - 30 + 15$
$= 175$

$\therefore n(P \cup D \cup C)^c = 200 - 175 = 25$ Ans.

Q3) universal set $X = \{a, b, c, d, e\}$

$\tilde{A} = \{(1, a), (0.3, b), (0.2, c), (0.8, d), (0, e)\}$

$\tilde{B} = \{(0.6, a), (0.9, b), (0.1, c), (0.3, d), (0.2, e)\}$

ⓐ $\text{Supp}(\tilde{A}) = \{a, b, c, d\}$

$\text{Supp}(\tilde{B}) = \{a, b, c, d, e\}$

ⓑ $\text{Core}(\tilde{A}) = \{a\}$

$\text{Core}(\tilde{B}) = \phi$

ⓒ $n(\tilde{A}) = 1 + 0.3 + 0.2 + 0.8 + 0 = 2.3$

$n(\tilde{B}) = 0.6 + 0.9 + 0.1 + 0.3 + 0.2 = 2.1$

ⓓ $\neg(\tilde{A}) = \{(0, a), (0.7, b), (0.8, c), (0.2, d), (1, e)\}$

$\neg(\tilde{B}) = \{(0.4, a), (0.1, b), (0.9, c), (0.7, d), (0.8, e)\}$

ⓔ $\tilde{A} \cup \tilde{B} = \{(1, a), (0.9, b), (0.2, c), (0.8, d), (0.2, e)\}$

ⓕ $\tilde{A} \cap \tilde{B} = \{(0.6, a), (0.3, b), (0.1, c), (0.3, d), (0, e)\}$

ⓖ $a\tilde{A} = \{(0.5, a), (0.15, b), (0.1, c), (0.4, d), (0, e)\}$

when $a = 0.5$

$a\tilde{B} = \{(0.3, b), (0.45, b), (0.05, c), (0.15, d), (0.1, e)\}$

For $a = 2$

(h) $\tilde{A}^a = \{(1,a), (0.09, b), (0.04, c), (0.64, d), (0, e)\}$

$\tilde{B}^a = \{(0.36, a), (0.81, b), (0.01, c), (0.09, d), (0.04, e)\}$

(i) $\tilde{A}_{0.3} = \{a, b, d\}$, $\tilde{A}_{0.9} = \{a\}$

$\tilde{B}_{0.3} = \{a, b, d\}$, $\tilde{B}_{0.9} = \{b\}$

(j) $h(\tilde{A}) = 1$, $h(\tilde{B}) = 0.9$

(k) $\tilde{A}$ is a normal fuzzy set.

Q4) (a) $A = \{1, 2\}$ $\therefore n(A) = 2$

(b) $A = \{1, 2, 3\}$ $\therefore n(A) = 3$

(c) $A = \{1, 2\}$ $\therefore n(A) = 2$

Q5) (a) Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let (x, y) be any element of $A \times (B \cap C)$. Then,

$(x, y) \in A \times (B \cap C) \Rightarrow x \in A$ and $y \in (B \cap C)$

$\Rightarrow x \in A$ and $(y \in B$ and $y \in C)$

$\Rightarrow (x \in A$ and $y \in B)$ and $(x \in A$ and $y \in C)$

$\Rightarrow (x, y) \in A \times B$ and $(x, y) \in A \times C$

$\Rightarrow (x, y) \in (A \times B) \cap (A \times C)$

$\therefore A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ — (1)

Now, let R.h.s = $(x,y) \in (A \times B) \cap (A \times C)$

$\Rightarrow (x,y) \in A \times B$ and $(x,y) \in A \times C$

$\Rightarrow (x \in A, y \in B)$ and $(x \in A, y \in C)$

$\Rightarrow x \in A$ and $(y \in B$ and $y \in C)$

$\Rightarrow x \in A$ and $y \in B \cap C$

$\Rightarrow (x,y) \in A \times (B \cap C)$

Hence, $(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$ — ②

from ① and ②, we get

$$\boxed{A \times (B \cap C) = (A \times B) \cap (A \times C)}$$

⑥ Prove that $A - B = A \cap \overline{B}$

L.h.s = let $x \in A - B$ then,

$x \in A - B \Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in A$ and $x \in \overline{B}$

$\Rightarrow x \in A \cap \overline{B}$

$A - B \subseteq A \cap \overline{B}$ — ①

R.h.s = let $x \in A \cap \overline{B} \Rightarrow x \in A$ and $x \in \overline{B}$

$\Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in A - B$

$A \cap \overline{B} \subseteq A - B$ — ②

Hence from ① and ② $A - B = A \cap \overline{B}$

(c) Prove $A - (B \cap C) = (A - B) \cup (A - C)$

L.H.S = Let $x \in A - (B \cap C) \Rightarrow x \in A$ and $x \notin (B \cap C)$

$\Rightarrow x \in A$ and $(x \notin B$ or $x \notin C)$

$\Rightarrow (x \in A$ and $x \notin B)$ or $(x \in A$ and $x \notin C)$

$\Rightarrow x \in (A - B)$ or $x \in (A - C)$

$\Rightarrow x \in (A - B) \cup (A - C)$

So, $A - (B \cap C) \subseteq (A - B) \cup (A - C)$ — (1)

R.H.S = Let $x \in (A - B) \cup (A - C)$

$\Rightarrow (x \in A$ and $x \notin B)$ or $(x \in A$ and $x \notin C)$

$\Rightarrow (x \in A$ and $x \notin (B \cap C))$

$\Rightarrow x \in A - (B \cap C)$

So, $(A - B) \cup (AC) \subseteq A - (B \cap C)$ — (2)

Hence, from (1) and (2) $A - (B \cap C) = (A - B) \cup (A - C)$

Q6) $S_1 = \{1, 2, 3\}$

$S_2 = \{x \mid x^2 - 2x + 1 = 0\} = \{1\}$ $(\because \{x \mid (x-1)^2 = 0\})$

$S_3 = \{x \mid x^3 - 6x^2 + 11x - 6 = 0\} = \{1, 2, 3\}$

$(\because \{x \mid (x-1)(x-2)(x-3) = 0\})$

From the above calculation, we can see that

$\boxed{S_1 = S_3}$ Ans.

Q7) $A = \{1, 2, 3\}$, $B = \{4, 5\}$, $C = \{1, 2, 3, 4, 5\}$

(a) $A \times B = \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\}$

(b) $C \times B = \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5), (4,4), (4,5), (5,4), (5,5)\}$

(c) $B \times B = \{(4,4), (4,5), (5,4), (5,5)\}$

Prove that $(C \times B) - (A \times B) = (B \times B)$

$L.H.S = (C \times B) - (A \times B) = \{(4,4), (4,5), (5,4), (5,5)\}$

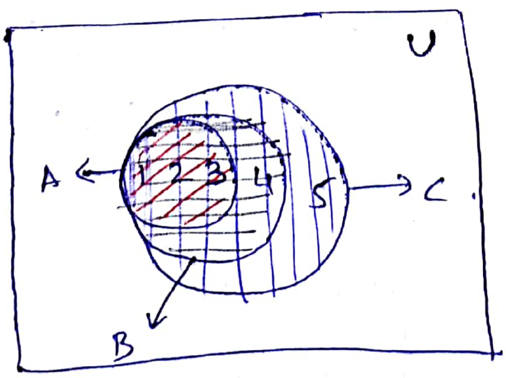
$R.H.S = (B \times B) = \{(4,4), (4,5), (5,4), (5,5)\}$

$\therefore$ L.H.S = R.H.S. Hence Proved

Q8) Let $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4\}$, $C = \{1, 2, 3, 4, 5\}$

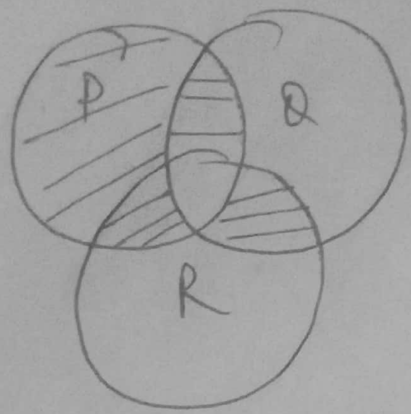
Here, $A \subseteq B$ and $B \subseteq C$.

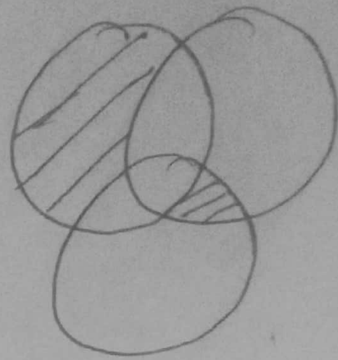
$\therefore$ Since all the elements of A are also present in C, we can say that $A \subseteq C$.



From, the venn diagram, it is clear that if $A \subseteq B$, $B \subseteq C$, then $A \subseteq C$.

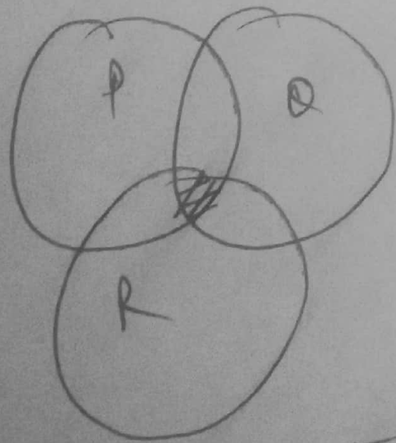
9. $P \Delta (Q \cap R)$ $(P \Delta Q) \cap (P \Delta R)$

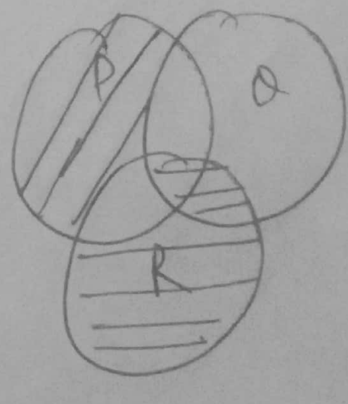




They are not equivalent

$P \cap (Q \Delta R)$ $(P \cap Q) \Delta (P \Delta R)$





They are also not equivalent

Hence option C

(10) Let Salary/month = x_1

Interest in Job = x_2

Distance in Km = x_3

Stability = x_4

A

$$\tilde{A} = \{(x_1, 0.5), (x_2, 0.4), (x_3, 0.24), (x_4, 1.0)\}$$

$$\tilde{B} = \{(x_1, 0.87), (x_2, 0.8), (x_3, 0.42), (x_4, 0.5)\}$$

$$\tilde{C} = \{(x_1, 0.37), (x_2, 1.0), (x_3, 0.73), (x_4, 0.7)\}$$

$$\tilde{D} = \{(x_1, 0.75), (x_2, 0.1), (x_3, 0.56), (x_4, 0.3)\}$$

$$\mu_{x_1 \wedge x_2 \wedge x_3 \wedge x_4}(\tilde{A}) = 0.24$$

$$\mu_{x_1 \wedge x_2 \wedge x_3 \wedge x_4}(\tilde{B}) = 0.42$$

$$\mu_{x_1 \wedge x_2 \wedge x_3 \wedge x_4}(\tilde{C}) = 0.37$$

$$\mu_{x_1 \wedge x_2 \wedge x_3 \wedge x_4}(\tilde{D}) = 0.1$$

$\therefore$ Max of this is 0.42 which is present in fuzzy set B

He will chose job at location B.

Note: The intersection operator, tries to optimize all the objectives simultaneously, with equal priority to each.
For example, Salary, interest, distance, stability all is given equal priorities in the above example.

If the objectives have different priorities, then the intersection operator can not be used. In that case a weighted approach can be used, wherein we assign weight to each objective (between 0 and 1).
A higher weight to objective with higher priority, and then instead of the intersection, we can multiply each objective with its weight and add them all and chose the one with the highest value.