Department of Mathematics
Bennett University
EMAT102L: Ordinary Differential Equations
Tutorial Sheet-3

1) Consider the linear differential equation

$$x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 0.$$

   (a) Show that x^3 and $|x^3|$ are two linearly independent solutions of the differential equation on $x \in (-\infty, \infty)$.
   (b) x^3 and $|x^3|$ are two linearly independent solutions of the differential equation but $W(x^3, |x^3|) = 0, \forall x \in \mathbb{R}$. Does it violate any result? Explain.
   (c) x^2 and x^3 are also two linearly independent solutions of the differential equation. Can we write general solution of the differential equation in terms of these solutions?

2) Use reduction of order method to find the second linearly independent solution of the following differential equations

   (a) $(x-1)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0; \quad y_1(x) = x.$
   (b) $x^2\frac{d^2y}{dx^2} - (2a-1)x\frac{dy}{dx} + a^2y = 0;\ a \neq 0,\ x > 0, \quad y_1(x) = x^a.$
   (c) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = 0; \quad y_1(x) = x\sin(logx).$

   Also write the general solution for each differential equation.
   Solutions: (a) $y_2 = e^x$. (b) $y_2 = x^a logx$. (c) $y_2 = -x\cos(logx)$.

3) Find the second order differential equation corresponding to given linearly independent solutions

   (a) $y_1 = \cos 2\pi x,\ y_2 = \sin 2\pi x.$
   (b) $y_1 = e^{-\sqrt{2}x},\ y_2 = xe^{-\sqrt{2}x}.$
   (c) $y_1 = e^{(-1+i\sqrt{2})x},\ y_2 = e^{(-1-i\sqrt{2})x}.$

   Solutions: (a) $y'' + 4\pi^2 y = 0$. (b) $y'' + 2\sqrt{2}y' + 2y = 0$. (c) $y'' + 2y' + 3y = 0$.

4) Solve the IVP's

   (a) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0; \quad y(-1) = e,\ y'(-1) = -\frac{e}{4}.$
   (b) $\frac{d^2y}{dx^2} - k^2y = 0;\ k \neq 0, \quad y(0) = 1,\ y'(0) = 1.$
   (c) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13y = 0; \quad y(0) = 3,\ y'(0) = -1.$
   (d) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0; \quad y(0) = 2,\ y'(0) = -3.$
   (e) $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 0; \quad y(0) = 1,\ y'(0) = -8,\ y''(0) = -4.$

5) Use method of Undetermined Coefficients to find the particular integral of the following differential equations

(a) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = x^3 + x + e^{-2x}$.

(b) $\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = x^2 \cos x$.

Hint: (a) $y_p = Ax^3 + Bx^2 + Cx + D + Ee^{-2x}$. (b) $y_p = Ax^4 \sin x + Bx^4 \cos x + Cx^3 \sin x + Dx^3 \cos x + Ex^2 \sin x + Fx^2 \cos x$.

6) Solve the following non-homogeneous differential equation

(a) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = \frac{e^{-3x}}{x^3}$.

Solutions: $y = \left(c_1 + c_2 x + \frac{1}{2x}\right) e^{-3x}$.