→ Well formed formula

- A statement or proposition may consist of variables, parenthesis and connective symbols.

- A grammatically correct expression is called a well formed formula, pronounced as "woof".

(a) Every atomic statement is a wff.
(b) If P is a wff then $\sim P$ is also wff.
(c) If P and Q are wff, then $(P \wedge Q)$, $(P \vee Q)$ and $(P \to Q)$ are also wff.

Q Check the following :-

(i) $\neg(P \vee Q)$ →

(ii) $(P \to (P \wedge Q))$ —

(iii) $(P \wedge Q) \to (\ \wedge P)$

→ Tautological Implication

- A compound proposition is said to be tautologically implied if and only if $A \to B$ is a tautology where A and B are 2 propositions.

- It is denoted as $A \Rightarrow B$ and read as 'A tautologically implies B'.

Eg:- $(P \wedge Q) \Rightarrow Q$. Verify

P	Q	$P \wedge Q$	$P \wedge Q \rightarrow Q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Q Prove that $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R) \Rightarrow R$

→ Laws of Logical Equivalence

- All the logical equivalences can be proved using truth tables.

Q Prove the De Morgan's Law using a truth table.

Eg:-① Without constructing the truth table show that

$$P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$$

L.H.S = $P \rightarrow (Q \rightarrow R) \Leftrightarrow P \rightarrow (\sim Q \vee R)$ [since $P \rightarrow Q \Leftrightarrow \sim P \vee Q$]

$\Leftrightarrow \sim P \vee (\sim Q \vee R)$ [since $P \rightarrow Q \Leftrightarrow \sim P \vee Q$]

$\Leftrightarrow (\sim P \vee \sim Q) \vee R$ [using associative law]

$\Leftrightarrow \sim (P \wedge Q) \vee R$ [using De Morgan's law]

$\Leftrightarrow (P \wedge Q) \rightarrow R$ [since $P \rightarrow Q \Leftrightarrow \sim P \vee Q$]

Hence Proved

Eg:- ② without constructing the truth table, show that $(\sim P \wedge (P \vee Q)) \rightarrow Q$ is a tautology.

$$(\sim P \wedge (P \vee Q)) \rightarrow Q$$

$\Leftrightarrow \sim(\sim P \wedge (P \vee Q)) \vee Q$ [since $P \rightarrow Q \Leftrightarrow \sim P \vee Q$]

$\Leftrightarrow (P \vee \sim(P \vee Q)) \vee Q$ [using De Morgan's law]

$\Leftrightarrow (P \vee (\sim P \wedge \sim Q)) \vee Q$ [using De Morgan's law]

$\Leftrightarrow ((P \vee \sim P) \wedge (P \vee \sim Q)) \vee Q$ [using Distributive law]

$\Leftrightarrow (T \wedge (P \vee \sim Q)) \vee Q$ [since $P \vee \sim P \Leftrightarrow T$]

$\Leftrightarrow (P \vee \sim Q) \vee Q$ [since $T \wedge P \Leftrightarrow P$]

$\Leftrightarrow P \vee (\sim Q \vee Q)$ [using associative law]

$\Leftrightarrow P \vee T$ [since $Q \vee \sim Q \Leftrightarrow T$]

$\Leftrightarrow T$ [since $P \vee T \Leftrightarrow T$]

Hence, $(\sim P \wedge (P \vee Q)) \rightarrow Q$ is a Tautology.