

## **Tutorial Solution - 5**

**Solution 1:** Here, we can see that C, D and E have the terms which are there in  $U$ . Therefore, C, D and E are the subsets of  $U$ .

**Solution 2:** Answer: d

Explanation: Set =  $\{0\}$  non-empty and finite set.

**Solution 3:** Answer: c

Explanation: Empty set is a subset of every set.

**Solution 4:** The power set has  $2^n$  elements. For  $n = 11$ , size of power set is 2048.

**Solution 5:** A powerset of a set is the set of all subsets of that set. For example, the power set of  $\{w, x, y\}$  is  $\{w\}, \{x\}, \{y\}, \{w, x\}, \{w, y\}, \{x, y\}, \{w, x, y\}$ , and  $\{\}$  as given by the question stem. The cardinality of a powerset (the number of subsets of the powerset) is calculated by  $2^N$ , where  $N$  is the number of elements in the set. Since  $\{w, x, y\}$  contains 3 elements, the cardinality of the powerset of that set contains  $2^3$  subsets.

The powerset of the set  $\{w, x, y, z\}$  contains  $2^4$  subsets. The powerset of the set  $\{x, y, z\}$  contains  $2^3$  subsets; these 8 subsets don't have  $w$ . So the number of subsets of the set  $\{w, x, y, z\}$  that contain  $w$  is the total number of subsets minus number of subsets that don't contain  $w$ ,  $16 - 8 = 8$ .

**Therefore, the answer is 8.**

**Solution 6:** Let us assume set  $S = \{1, 2\}$ . Therefore  $P(S) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

Option (a) is false as  $P(S)$  has  $2^2 = 4$  elements and  $P(P(S))$  has  $2^4 = 16$  elements and they are not equivalent.

Option (b) is true as intersection of  $S$  and  $P(S)$  is empty set.

Option (c) is false as intersection of  $S$  and  $P(S)$  is empty set.

Option (d) is false as  $S$  is an element of  $P(S)$ .

**Solution 7:** (a) Subset:  $\forall x(x \in A \rightarrow x \in B)$

(b) Equal set:  $\forall x(x \in A \leftrightarrow x \in B)$

(c) Proper subset:  $\exists x(x \notin A \wedge x \in B)$

**Solution 8:** (a)  $\{1, -1\}$

(b)  $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

(c)  $\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$

(d)  $\emptyset$  ( $\sqrt{2}$  is not an integer)

**Solution 9:** (a)  $\{x \mid x \text{ is a multiple of } 3 \text{ and } 0 \leq x \leq 12\}$

(b)  $\{x \mid x \in \mathbb{Z} \text{ and } -3 \leq x \leq 3\}$

(c)  $\{x \mid x \text{ is an english alphabet and lies between } m \text{ and } p, \text{ including } m \text{ and } p\}$

**Solution 10:** (a) Since 2 is an integer greater than 1, 2 is an element of this set.

(b) Since 2 is not a perfect square ( $1^2 < 2$ , but  $n^2 > 2$  or  $n > 1$ ), 2 is not an element of this set.

(c) This set has two elements, and as we can clearly see, one of those elements is 2.

(d) This set has two elements, and as we can clearly see, neither of those elements is 2. Both of the elements of this set are sets; 2 is a number, not a set.

(e) This set has two elements, and as we can clearly see, neither of those elements is 2. Both of the elements of this set are sets; 2 is a number, not a set.

(f) This set has just one element, namely the set  $\{\{2\}\}$ . So, 2 is not an element of this set. Note that  $\{2\}$  is not an element either, since  $\{2\} \neq \{\{2\}\}$ .

**Solution 11:** Let A = set of persons who got medals in dance. B = set of persons who got medals in dramatics. C = set of persons who got medals in music.

Given,

$$n(A) = 36$$

$$n(B) = 12$$

$$n(C) = 18$$

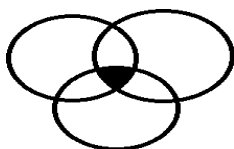
$$n(A \cup B \cup C) = 45$$

$$n(A \cap B \cap C) = 4$$

We know that number of elements belonging to exactly two of the three sets A, B, C

$$= n(A \cap B) + n(B \cap C) + n(A \cap C) - 3n(A \cap B \cap C)$$

$$= n(A \cap B) + n(B \cap C) + n(A \cap C) - 3 \times 4 \text{ (i)}$$



$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

Therefore,  $n(A \cap B) + n(B \cap C) + n(A \cap C) = n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cup B \cup C)$  From (i) required number

$$= n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cup B \cup C) - 12$$

$$= 36 + 12 + 18 + 4 - 45 - 12$$

$$= 70 - 57$$

$$= 13$$

**Solution 12:** Let A be the set of students who play chess B be the set of students who play scrabble

C be the set of students who play carrom Therefore, We are given  $n(A \cup B \cup C) = 40$ ,  $n(A) = 18$ ,  $n(B) = 20$   $n(C) = 27$ ,

$$n(A \cap B) = 7, n(C \cap B) = 12 \quad n(A \cap B \cap C) = 4$$

We have,

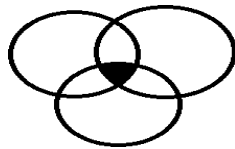
$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

$$\text{Therefore, } 40 = 18 + 20 + 27 - 7 - 12 - n(C \cap A) + 4$$

$$40 = 69 - 19 - n(C \cap A)$$

$$40 = 50 - n(C \cap A) \quad n(C \cap A) = 50 - 40 \quad n(C \cap A) = 10$$

Therefore, Number of students who play chess and carrom are 10. Also, number of students who play chess, carrom and not scrabble.



$$= n(C \cap A) - n(A \cap B \cap C)$$

$$= 10 - 4$$

$$= 6$$

### Solution 13

Let A be the set of odd integers between 1 and 100.

$$A = \{1, 3, 5, \dots\}$$

$$n(A) = 50$$

Let B be the set of integers between 1 and 100 that are squares of an integer.

$$B = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}.$$

$$n(B) = 10$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= 50 + 10 - 5$$

$$= 55.$$