Digital Design ECSE108L

$f(A, B, C)$

#	A	B	C	Minterms	Maxterm
0	0	0	0	$\bar{A} \cdot \bar{B} \cdot \bar{C}$	$A + B + C$
1	0	0	1	$\bar{A} \cdot \bar{B} \cdot C$	$A + B + \bar{C}$
2	0	1	0	$\bar{A} \cdot B \cdot \bar{C}$	$A + \bar{B} + C$
3	0	1	1	$\bar{A} \cdot B \cdot C$	$A + \bar{B} + \bar{C}$
4	1	0	0	$\vdots$	$\vdots$
5	1	0	1		
6	1	1	0		
7	1	1	1		

Canonical Sum of Minterms

$$F(A,B,C) = \sum_m (0,1,3,5)$$

$$\boxed{F = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot\bar{B}\cdot C + \bar{A}\cdot B\cdot C + A\cdot\bar{B}\cdot C}$$

Standard Sum-of-Products (SOP)

(Canonical Sum of minterms)

$F = \Sigma_m(0, 1, 2, 6)$

$$\boxed{F = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot\bar{B}\cdot C + \bar{A}B\bar{C} + AB\bar{C}}$$

$$= \bar{A}\bar{B}(\cancel{\bar{C}+C}) + B\bar{C}(\cancel{\bar{A}+A})$$

$x + \bar{x} = 1$

$$\boxed{F = \bar{A}\bar{B} + B\bar{C}}$$

Sum-of-product

SOP

Canonical Product of Maxterm

$$F(A,B,C) = \prod_M (0, 1, 4, 5)$$

$$\boxed{F = (A+B+C)\cdot(A+B+\bar{C})\cdot(\bar{A}+B+C)\cdot(\bar{A}+B+\bar{C})}$$

Standard Product-of-Sums (POS)

$$F(A,B,C) = \Pi_m(0,1,2,6)$$

Canonical product of maxterm

$$\boxed{F = (A+B+C)\cdot(A+B+\bar{C})\cdot(A+\bar{B}+C)\cdot(\bar{A}+\bar{B}+C)}$$

$$= \overline{\overline{(A+B+C)\cdot(A+B+\bar{C})\cdot(A+\bar{B}+C)\cdot(\bar{A}+\bar{B}+C)}}$$

De morgans law.

$$= \overline{(\bar{A}\cdot\bar{B}\cdot\bar{C}) + (\bar{A}\cdot\bar{B}\cdot C) + (\bar{A}\cdot B\cdot\bar{C}) + (A\cdot B\cdot\bar{C})}$$

$$= \overline{\bar{A}\bar{B}(\cancel{\bar{C}+C}) + B\cdot\bar{C}(\cancel{\bar{A}+A})}$$

$$= \overline{\bar{A}\cdot\bar{B} + B\bar{C}} = \boxed{(A+B)\cdot(\bar{B}+C)}$$

POS

Standard Forms

- **<u>Standard Sum-of-Products (SOP) form:</u> equations are written as an OR of AND terms**
- **<u>Standard Product-of-Sums (POS) form:</u> equations are written as an AND of OR terms**
- **Examples:**
  - **SOP:** $\mathbf{A\,B\,C + \overline{A}\,\overline{B}\,C + B}$
  - **POS:** $\mathbf{(A+B)\cdot(A+\overline{B}+\overline{C})\cdot C}$
- **These "mixed" forms are <u>neither SOP nor POS</u>**
  - $\mathbf{(A\,B + C)\,(A + C)}$
  - $\mathbf{A\,B\,C + A\,C\,(A + B)}$

Standard Sum-of-Products (SOP)

- **A Simplification Example:**
- **$F(A, B, C) = \Sigma m(1,4,5,6,7)$**
- **Writing the minterm expression:**

$F = \overline{A}\,\overline{B}\,C + A\,\overline{B}\,\overline{C} + A\,\overline{B}\,C + AB\overline{C} + ABC$

$x + \bar{x}y$

$= x + y$

- **Simplifying:**

F = $\bar{A}\bar{B}C + A\bar{B}(\cancel{\bar{C}+C}) + AB(\cancel{C+\bar{C}})$

$= \bar{A}\bar{B}C + A\bar{B} + AB = \bar{A}\bar{B}C + A(\cancel{B+\bar{B}})$

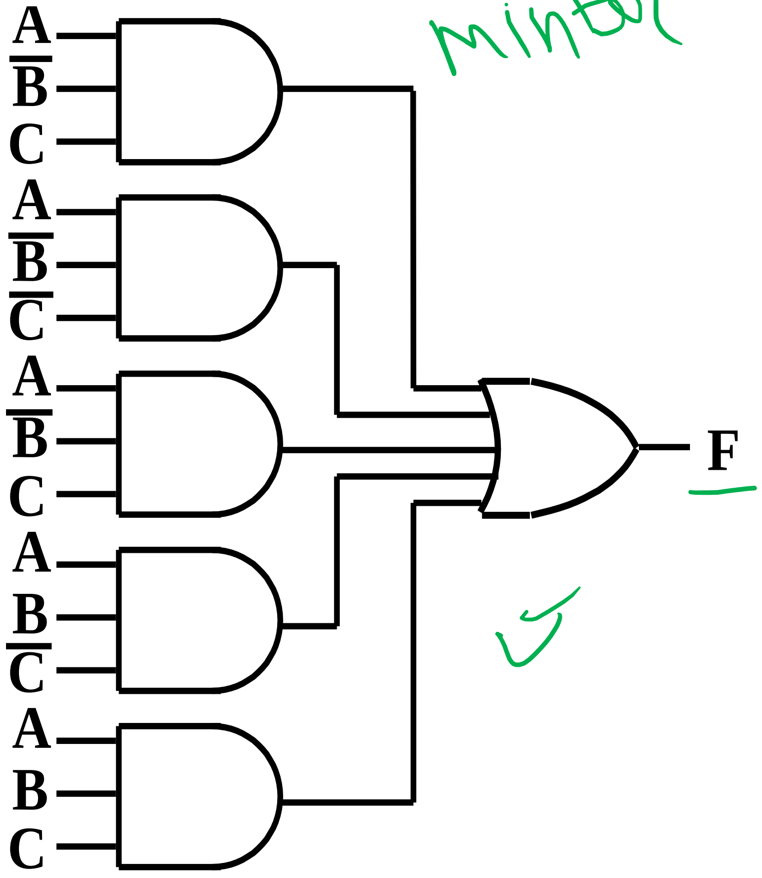
$= \boxed{A + \bar{B}C}$ ← SOP

- **Simplified F contains 3 literals compared to 15 in minterm F**

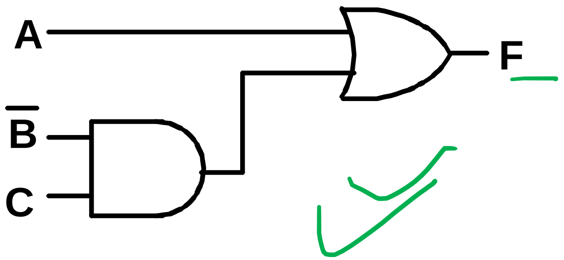
AND/OR Two-level Implementation of SOP Expression

- **The two implementations for F are shown below – it is quite apparent which is simpler!**

minter



SOP



Circuit Optimization

- **Goal: To obtain the simplest implementation for a given function**
- **Optimization is a more formal approach to simplification that is performed using a specific procedure or algorithm**
- **Optimization requires a cost criterion to measure the simplicity of a circuit**
- **Distinct cost criteria we will use:**
  - **Literal cost (L)**
  - **Gate input cost (G)**
  - **Gate input cost with NOTs (GN)**

Literal Cost

- Literal – a variable or it complement
- Literal cost – the number of literal appearances in a Boolean expression corresponding to the logic circuit diagram
- Examples:
  - $F = BD + A\bar{B}C + A\bar{C}\bar{D}$ $L = 8$
  - $F = BD + A\bar{B}C + A\bar{B}\bar{D} + AB\bar{C}$ $L = 11$
  - $F = (A + B)(A + D)(B + C + \bar{D})(\bar{B} + \bar{C} + D)$ $L = 10$
  - Which solution is best?

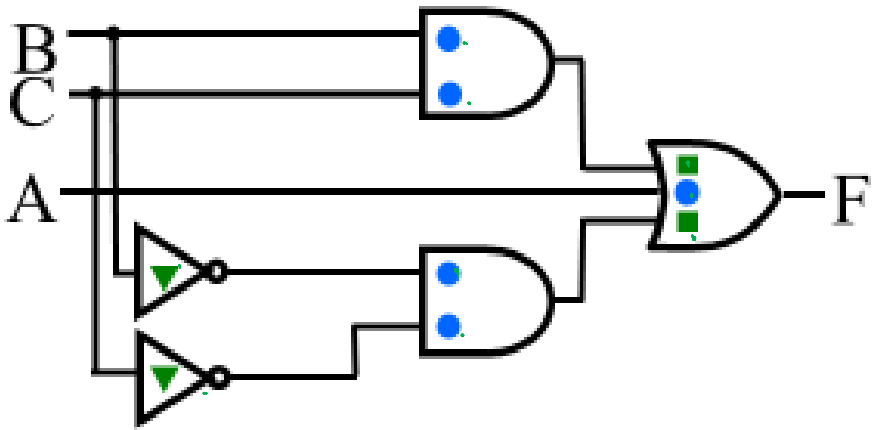
Cost Criteria (continued)

- **Example 1:**
- $F = A + B\,C + \bar{B}\,\bar{C}$

$GN = G + 2 = 9$

$L = 5$

$G = L + 2 = 7$



- **L (literal count) counts the AND inputs and the single literal OR input.**
- **G (gate input count) adds the remaining OR gate inputs**
- **GN(gate input count with NOTs) adds the inverter inputs**

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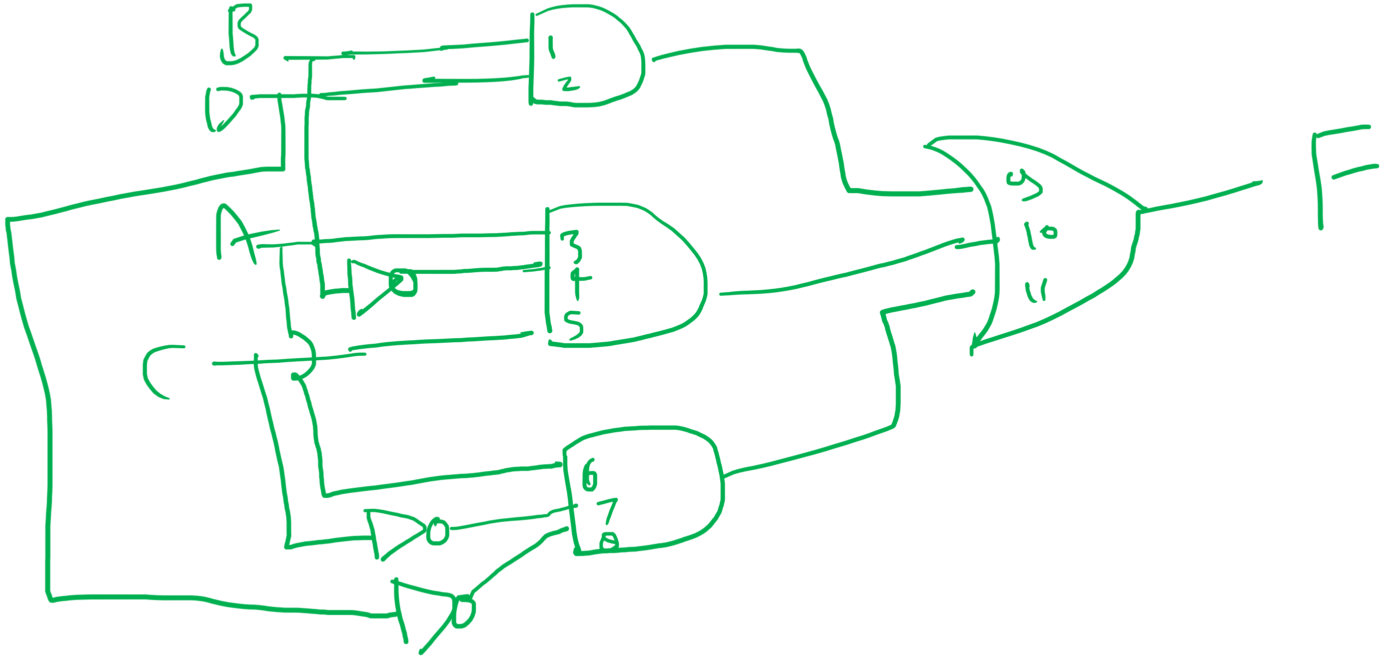
F = BD + A B'C + AC'D'   - find L, G, GN

L → 8

G → 11

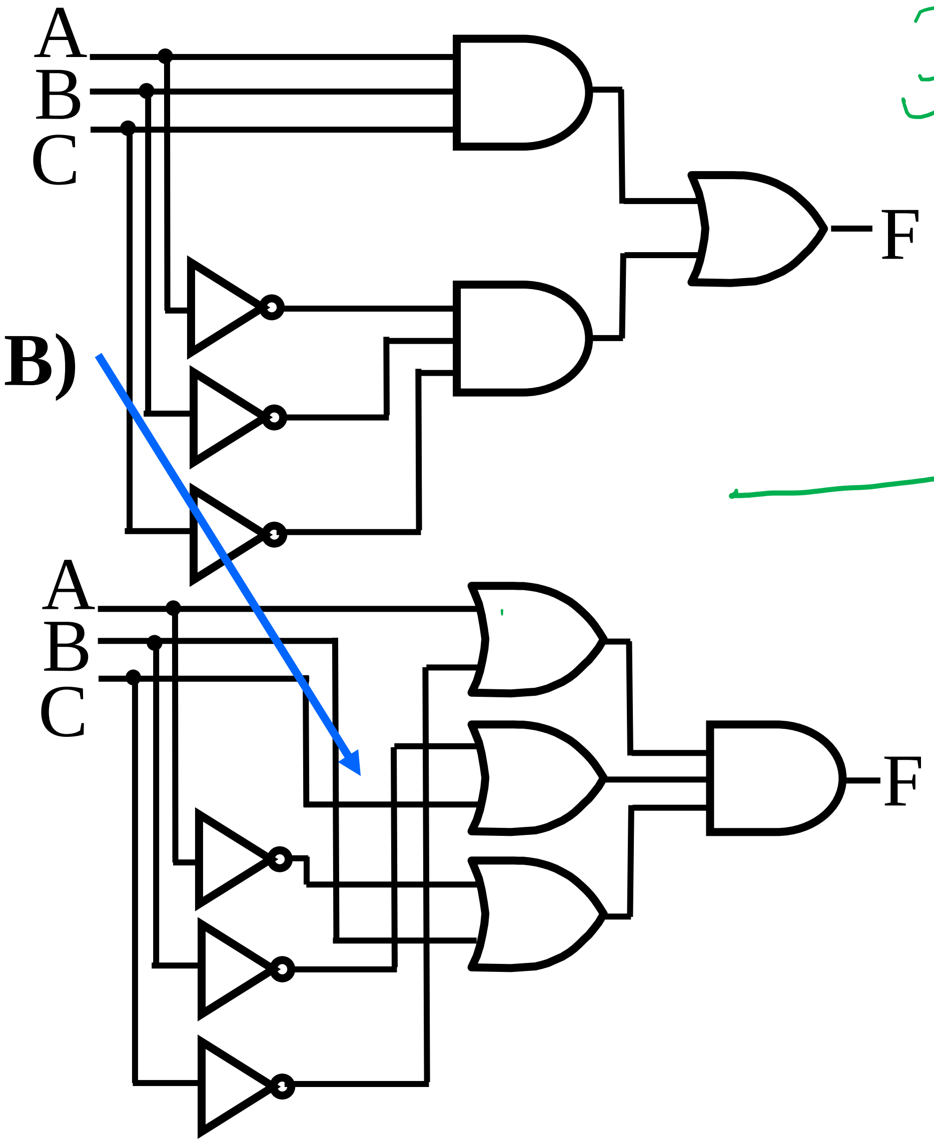
GN → 11+3

= 14



Cost Criteria (continued)

- Example 2:
- $F = A\,B\,C + \bar{A}\,\bar{B}\,\bar{C}$
- $L = 6 \quad G = 8 \quad GN = 11$
- $F = (A + \bar{C})(\bar{B} + C)(\bar{A} + B)$
- $L = 6 \quad G = 9 \quad GN = 12$
- Same function and same literal cost
- But first circuit has better gate input count and better gate input count with NOTs
- Select it!



Simplify - F(A,B,C,D) = m2 + m3 + m6 + m7 + m10 +m11

0010 0011 0110 1010

$F = \bar{A}\cdot\bar{B}\cdot C\cdot\bar{D} + \bar{A}\cdot\bar{B}\cdot C\cdot D + \bar{A}\cdot B\cdot C\cdot\bar{D} + \bar{A}\cdot B\cdot C\cdot D + A\cdot\bar{B}\cdot C\cdot\bar{D} + A\cdot\bar{B}\cdot C\cdot D$

$= \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C$

$= \bar{A}C + A\bar{B}C$

$C(\bar{A} + A\bar{B})$

$C(\bar{A} + \bar{B})$

$x + \bar{x}y = x + y$

$\boxed{= \bar{A}C + \bar{B}C}$ ← SOP

Boolean Function Optimization

- **Minimizing the gate input (or literal) cost of a (a set of) Boolean equation(s) reduces circuit cost.**
- **We choose gate input cost.**
- **Boolean Algebra and graphical techniques are tools to minimize cost criteria values.**
- **Some important questions:**
  - **When do we stop trying to reduce the cost?**
  - **Do we know when we have a minimum cost?**
- **Treat optimum or near-optimum cost functions for two-level (SOP and POS) circuits first.**
- **Introduce a graphical technique using Karnaugh maps (K-maps, for short)**