Assignment 1

1. Show that every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew-symmetric matrix.

2. If
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{pmatrix}$$
 and $B = \begin{pmatrix} 0 & 6 & 1 \\ 3 & 8 & -2 \end{pmatrix}$, then what is AB ?

3. Show that
$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

Evaluate without expanding
$$\begin{vmatrix} 1 & a & a^2 & a^3 + bcd \\ 1 & b & b^2 & b^3 + cda \\ 1 & c & c^2 & c^3 + abd \\ 1 & d & d^2 & d^3 + abc \end{vmatrix}$$

Let $A = \begin{pmatrix} 3 & 5 & 7 \\ 6 & 9 & 5 \\ 4 & 6 & 8 \end{pmatrix}$, then find the co-factor of an element $7(=a_{13})$ in a matrix A?

6. If
$$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{pmatrix}$$
, find $adj(A)$

and inverse of the matrix A (using determinant method)?

7. Solve the system by Cramer's Rule

$$3x + y + z = 4$$
$$x - y + 2z = 6$$
$$x + 2y - z = -3$$