Ordinary Differential Equations (Lecture-1)

Neelam Choudhary

Department of Mathematics Bennett University India

June 2, 2021



Books

Reference Books

- S. L. Ross, "Differential Equations", 3rd Edition, Wiley India, 1984.
- Erwin Kreyszig, "Advanced Engineering Mathematics", Tenth Edition, Wiley India, 2016.



Learning Outcome of the Lecture

We learn

- Differential Equation
 - Definition
 - Examples
- Classification of DE's
 - Type: ODE/PDE
 - Order
 - Linear/Nonlinear





Differential Equation

Definition

An equation involving derivatives of one or more dependent variables with respect to one or more independent variables is called a differential equation.

Differential equations occur naturally in real life problems encountered in science and engineering.

- Radioactive decay
- Motion of Projectile, rocket, satellite, or planet
- Current in the electric Circuit
- Heat conduction in a rod
- Vibration of a wire
- Motion of simple pendulum





Types of Differential Equations

A DE is classified based on its type as Ordinary Differential Equation (ODE) or Partial Differential Equation (PDE).

Definition

Let y(x) denotes a function in the variable x. An ordinary differential equation (ODE) is an equation containing one or more derivatives of an unknown function y.

In general, a differential equation involving ordinary derivatives of one or more dependent variables with respect to a single independent variable is called an ordinary differential equation.

Definition

A differential equation involving partial derivatives of one or more dependent variables with respect to more than one independent variable is called a partial differential equation.

Examples

(i)
$$\frac{d^2y}{dx^2} + xy\left(\frac{dy}{dx}\right)^2 = 0$$
, Ordinary Differential Equation

(ii)
$$\frac{d^4x}{dt^4} + 5\frac{d^2x}{dt^2} + 3x = \sin t, \quad \text{Ordinary Differential Equation}$$

$$(iii) \qquad \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} = v,$$

Partial Differential Equation

$$(iv) \qquad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

Partial Differential Equation



Order of Differential Equation

Definition

The order of the highest ordered derivative involved in a differential equation is called the order of the differential equation.

(i)
$$\frac{d^2y}{dx^2} + xy\left(\frac{dy}{dx}\right)^2 = 0$$
, DE of second order

(ii)
$$\frac{d^4x}{dt^4} + 5\frac{d^2x}{dt^2} + 3x = \sin t, \quad \text{DE of fourth order}$$

(iii)
$$\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} = v$$
, DE of first order

(iv)
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$
 DE of second order





Linear Differential Equations

Definition

Linear ODE: A linear DE of order n, in the dependent variable y and the independent variable x, is an equation that is in, or can be expressed in, the form

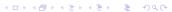
$$a_0(x)\frac{d^ny}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \dots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)y = b(x),$$

where $a_0(x)$ is not identically zero.

Check list: If the dependent variable is *y*, derivatives occur upto first degree only, no products of *y* and/or its derivatives are there.

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0,$$
$$\frac{d^4y}{dx^4} + x^2\frac{d^3y}{dx^3} + x^3\frac{dy}{dx} = xe^x.$$





Nonlinear Differential Equation

Definition

A nonlinear ordinary differential equation is an ordinary differential equation that is not linear.

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y^2 = 0,$$

$$\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 + 6y = 0,$$

$$y\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0,$$

$$\frac{d^2y}{dx^2} + 5y\frac{dy}{dx} + 6y = 0.$$

