Sets

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Overview

Sets

Definition: A set is an unordered collection of objects (distinct) of any sort.
Eg. $A = \{1, 2, 3\}$ is a set containing three elements in it.
Often objects in a set have similar properties (numbers,alphabets), but is is not always true. eg.$\{a, 3, Ram\}$is also a set (heterogeneous set).
Elements or Members: objects in a set.
An element a belongs to set A is denoted by $a \in A$
A set may have other set as member. eg. A= {{1,2},{3}}

Example of Standard Sets

Some of the important sets are-
N : Set of Natural Numbers{1,2,3...}
Z : Set of integers{... -2,-1,0,1,2,....}
Z^+ : Set of positive integers{0,1,2...}
Q : Set of rational numbers
R: Set of real numbers

Representation of Sets

1. List/Roster Form : List all the elements of set when possible between between braces.
   eg. set of vowels in English alphabet $\{a, e, i, o, u\}$
   eg. set of integers less than 100 $\{1, 2, 3, ..., 99\}$
2. Set builder notation : In this notation, we characterize the elements in the set by stating the properties they must have to be members.
   eg. set O, of odd positive integers less than 10 can be written as :
   O=$\{x \mid x$ is a odd positive integer less than 10 $\}$
   or
   O=$\{x \in Z^+ \mid x$ is odd and $x<10\}$
3. Graphical (using Venn Diagram)

Cardinality : Number of elements in the set
Denoted as n(S) or $\mid S \mid$
eg. A={1,2,3,4,5}
n(A)=5
eg. B={a,b,c,..z}
n(B)=26
eb. C={a,b,{c,d}}
n(C)=3

Singelton Set

A set with only one element is a singleton

Example:

H $= \{ 4\}$, $\mid H \mid= 1$, H is a singleton

Subset

"X is a subset of Y" is written as $X \subseteq Y$

Predicate- $\forall x(x \in X \implies x \in Y)$

"X is not a subset of Y" is written as $X \nsubseteq Y$

Example:

X = {a,e,i,o,u}, Y = {a, i, u} and Z= {b,c,d,f,g}

$Y \subseteq X$, since every element of Y is an element of X

$Y \nsubseteq Z$, since $a \in Y$, but $a \notin Z$

P={{a,b},c,d}

$\{a, b\} \subseteq P$ False

$\{a, b\} \in P$ True

$\{\{a, b\}\} \subseteq P$ True

Subset Properties

- $A \subseteq A$ (Reflexive)
- $A \subseteq B, B \subseteq C \implies A \subseteq C$ (Transitive)

Superset

X and Y are sets. If $X \subseteq Y$, then "X is contained in Y" or "Y contains X" or Y is a superset of X, written as $Y \supseteq X$

Proper Subset

X and Y are sets. If X is a subset of Y and X does not equal Y, we say that X is a proper subset of Y and write $X \subset Y$.

i.e Y has atleast 1 element more than X

n(Y)>n(X)

Example:

X = {a,e,i,o,u}, Y = {a,e,i,o,u,y} $X \subset Y$, since $y \in Y$, but $y \notin X$

$X \subset Y \implies \exists x(x \notin X \wedge x \in Y)$

Properties

$X \subset Y$ False (Not Reflexive)

$X \subset Y \implies Y \subset X$ False (Not Symmetric)

$X \subset Y, Y \subset Z \implies X \subset Z$ True (Transitive)

Equal Sets

Two sets A and B are equal if and only if they have the same elements, i.e. $\forall x(x \in A \iff x \in B)$ or $A \subseteq B$ and $B \subseteq A$

We write A= B if A and B are equal sets.

eg. Sets {1,3,5},{3,1,5} and {1,3,3,5,5,5} are equal sets as all contain the same elements.

Properties of Equal Sets

- Reflexive (A=A)
- Symmetric ($A = B \implies B = A$)
- Transitive ($A = B, B = C \implies A = C$)

Empty (Null) Set

A Set is Empty (Null) if it contains no elements.
The Empty Set is written as $\emptyset$ or $\{\}$
The Empty Set is a subset of every set

Power Set

For any set X ,the power set of X ,written P(X),is the set of all subsets of X

Example:
If X $= \{red, blue, yellow\}$,
then P(X) $=$
$\{\emptyset, \{red\}, \{blue\}, \{yellow\}, \{red, blue\}, \{red, yellow\}, \{blue, yellow\},$
$\{$red, blue, yellow$\}$ $\}$
Cardinality of Power set of $X = 2^n$, if X contains n elements

Universal Set

An arbitrarily chosen, but fixed set

A set is called Universal Set if it includes every set under consideration.

eg. $A = \{1, 2, 3\}$

B=$\{4, 5\}$

C=$\{6, 7\}$

U=$\{1, 2, 3, 4, 5, 6, 7\}$

Finite and Infinite Sets

X is a set. If there exists a non-negative integer n such that X has n elements, then X is called a finite set with n elements.
If a set is not finite, then it is an infinite set.
Examples:
Y = $\{1,2,3\}$ is a finite set
P = $\{$red, blue, yellow$\}$ is a finite set
E $=\{2,4,6,...\}$the set of all even integers, is an infinite set
$\emptyset$, the Empty Set, is a finite set with 0 elements

Countable and Uncountable Sets

A set X is said to be countable if there exists a one to one correspondence from X to a subset of the set of natural numbers.Else if it is not countable, it is uncountable.

eg. The set of positive even numbers is a countable and infinite set.

The set of real numbers between any 0 and 1 is uncountable.