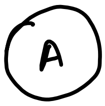
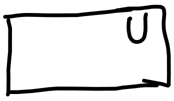
Venn Diagram

→ It is a schematic or diagrammatic representation of sets.

→ Symbols used are as follows:

(a) 

 - Set

(b) 

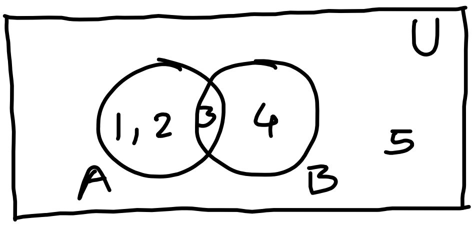
 - Universal Set

(c) Shade the region to show elements of the set.

$U = \{1, 2, 3, 4, 5\}$

$A = \{1, 2, 3\}$

$B = \{3, 4\}$

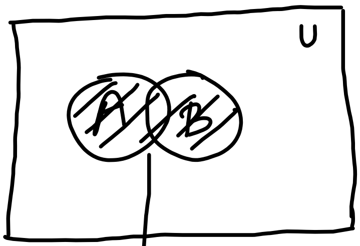


→ Operations on Sets

(a) Union ($\cup$)

$A \cup B$, set of all elements of set A as well as set B.

$A \cup B = \{x \mid x \in A \vee x \in B\}$

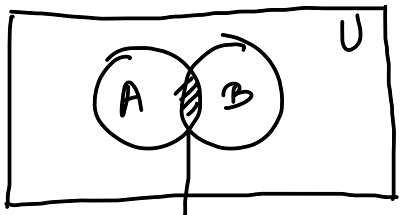


$A \cup B = \{1, 2, 3, 4\}$

(b) Intersection ($\cap$)

$A \cap B$, set of elements which belong to both A and B (common elements)

$A \cap B = \{x \mid x \in A \wedge x \in B\}$



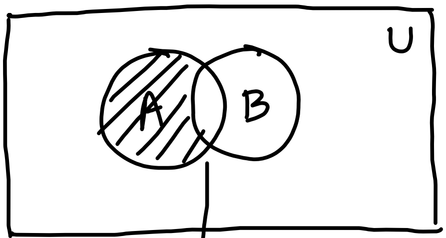
$A \cap B = \{3\}$

If $A \cap B = \emptyset$, then A and B are called disjoint sets.

(c) Set Difference (A - B)

A - B, set with elements of A that are not in B.

$A - B = \{x \mid x \in A \wedge x \notin B\}$

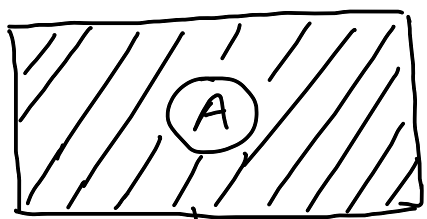


$A - B = \{1, 2\}$

(d) Complement (A^c)

A^c, set with elements that are not in A.

$A^c = U - A$
$= \{x \mid x \in U \wedge x \notin A\}$



$A^c = \{4, 5\}$

(e) Cartesian Product ($A \times B$)

$A \times B = C$, where $C = \{(x, y)\} \rightarrow$ ordered pair

- Pair of 2 elements wrapped in a bracket.
- Called ordered pair because they follow an order

$$(2,3) \neq (3,2)$$

$A = \{1,2,3\}$ $\qquad$ $B = \{a, b\}$

$$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$$

$A \times B$, set of all ordered pairs such that first member of the ordered pair is from set A and the second is from set B.

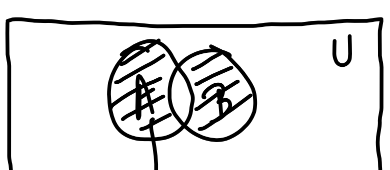
$$A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$$

$$\text{Cardinality of } A \times B = n(A) \times n(B) = |A| \times |B|$$

(f.) Symmetric Difference ($A \triangle B$)

$A \triangle B$, set with elements of A or B but not in both.

$$A \triangle B = \{x \mid (x \in A \vee x \in B) \wedge (x \notin (A \cap B))\}$$



$\rightarrow A \triangle B = \{1,2,4\}$

(g) Partition

Let X be a set and $S = \{ A_i \mid A_i \subseteq X, i \in \mathbb{N} \}$ be the set of the subsets of X. S is said to be a partition of X if the elements of S hold the following properties:

1. The union of all A_i's is the set X.

   ie $\boxed{\bigcup_i A_i = X}$

2. All A_i's are disjoint

   ie if $A_i, A_j \in S$ then

   $$\boxed{A_i \cap A_j = \emptyset}$$

Q) Check whether the following are partitions of X:

Let $X = \{1, 2, 3, 4, 5, 6, 7\}$

(i) $S = \{\{1,2,3\}, \{3,4,5\}, \{6,7\}\}$ - $\boxed{\text{No}}$ since $\{1,2,3\} \cap \{3,4,5\} = \{3\}$

(ii) $S = \{\{1,2\}, \{4,5\}, \{3,6,7\}\}$ - $\boxed{\text{Yes}}$

(iii) $S = \{\{1,2\}, \{3\}, \{5,6,7\}\}$ - $\boxed{\text{No}}$ since $\{1,2\} \cup \{3\} \cup \{5,6,7\} \neq X$

→ Set Identities

	Union	Intersection
① Commutative	$A \cup B = B \cup A$	$A \cap B = B \cap A$
② Associative	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
③ Distributive	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
④ Absorption	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
⑤ De Morgan's	$(A \cup B)^c = A^c \cap B^c$	$(A \cap B)^c = A^c \cup B^c$

Symmetric Difference

① Commutative $\quad A \Delta B = B \Delta A$

② Associative $\quad (A \Delta B) \Delta C = A \Delta (B \Delta C)$

Set Difference

① Commutative $\quad A - B \neq B - A$

② Associative $\quad (A - B) - C \neq A - (B - C)$

Cartesian Product

① Commutative $\quad A \times B \neq B \times A$

② Associative $\quad (A \times B) \times C \neq A \times (B \times C)$

Formulas for cardinality

① $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

② $n(A \cup B \cup C) = n(A) + n(B) + n(C) -$

$n(A \cap B) - n(A \cap C) - n(B \cap C)$

$+ n(A \cap B \cap C)$

③ $n(A - B) = n(A) - n(A \cap B)$