Assignment 7

1. For f and g in $C[a,b]$ (= The space of continuous functions on $[a,b]$) define:

$$< f.g >= \int_a^b f(t)g(t)\, dt \tag{1}$$

Show that (1) defines an inner-product in $C[a,b]$

2. For the vectors $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in R^2, define:

$$< u.v >= 4u_1v_1 + 5u_2v_2 \tag{2}$$

Show that (2) defines an inner-product in R^2

3. Let $C[0,1]$ is an inner product space with

$$< f.g >= \int_0^1 f(t)g(t)\, dt$$

and let W be the subspace spanned by the polynomials $\{P_1(t) = 1, P_2(t) = 2t - 1, P_3(t) = 12t^2\}$. Use Gram-Schmidt process to find an orthogonal basis for W.

4. Let W =Span$\{x_1 = (1,1,1), x_2 = (\frac{1}{3}, \frac{1}{3}, \frac{-2}{3})\}$. Construct an orthonormal basis for W.

5. Let W be the subspace of R^2 spanned by $x = (\frac{2}{3}, 1)$. Find a unit vector that is a basis for W.

6. The set $S = \{(3,1,1), (-1,2,1), (\frac{-1}{2}, -2, \frac{7}{2})\}$. Express the vector $y = (6,1,-8)$ as a linear combination of the vector in S.

7. Let $y = (7,6)$ and $u = (4,2)$. Find the orthogonal projection of y onto u.

8. Show that $\{(\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}), (\frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}), (\frac{-1}{\sqrt{66}}, \frac{-4}{\sqrt{66}}, \frac{7}{\sqrt{66}})\}$ **is an orthonormal basis of** R^3**.**

9. Find a least-squares solution of $Ax = b$ **for**

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \ b = \begin{pmatrix} -3 \\ -1 \\ 0 \\ 2 \\ 5 \\ 1 \end{pmatrix}$$