

Tutorial 2 Solution

Q1

- | | |
|--------------------------------|--|
| 1. $p \vee s$ | Premise (Given) |
| 2. $s \vee r$ | Commutative Law |
| 3. $\sim s$ | Premise (Given) |
| 4. r | Disjunctive Syllogism using 2 and 3 |
| 5. $\sim t \rightarrow \sim r$ | Premise (Given) |
| 6. $\sim(\sim t)$ | Modus tollens using 5 and 4
(since $\sim(\sim P) \Leftrightarrow P$) |
| 7. t | |
| 8. $t \rightarrow w$ | Premise (Given) |
| 9. w | Modus Ponens using 8 and 7 |

Q2

$$\begin{aligned}
 & [(p \vee q) \wedge \sim(\sim p \wedge (\sim q \vee \sim r))] \vee (\sim p \wedge \sim q) \vee (\sim p \wedge \sim r) \\
 \equiv & [(p \vee q) \wedge \sim(\sim p \wedge \sim(q \wedge r))] \vee (\sim(p \vee q)) \vee \sim(p \vee r) \\
 & \quad \text{(using De Morgan's Law)} \\
 \equiv & [(p \vee q) \wedge (p \vee (q \wedge r))] \vee \sim((p \vee q) \wedge (p \vee r)) \\
 \equiv & [(p \vee q) \wedge (p \vee q) \wedge (p \vee r)] \vee \sim((p \vee q) \wedge (p \vee r)) \\
 & \quad \text{(using Distributive Law)} \\
 \equiv & [(p \vee q) \wedge (p \vee q) \wedge (p \vee r)] \vee \sim((p \vee q) \wedge (p \vee r)) \\
 \equiv & ((p \vee q) \wedge (p \vee r)) \vee \sim((p \vee q) \wedge (p \vee r)) \\
 \equiv & x \vee \sim x \text{ where } x = (p \vee q) \wedge (p \vee r) \\
 \equiv & T \quad \text{(complement law)}
 \end{aligned}$$

Q3

$$(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \equiv R$$

$$(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$$

$$\equiv (\neg P \wedge (\neg Q \wedge R)) \vee ((Q \vee P) \wedge R) \quad \text{By Distributive Law}$$

$$\equiv ((\neg P \wedge \neg Q) \wedge R) \vee ((Q \vee P) \wedge R) \quad \text{By Associative Law}$$

$$\equiv ((\neg P \wedge \neg Q) \wedge R) \vee ((P \vee Q) \wedge R) \quad \text{By Commutative Law}$$

$$\equiv (\neg(P \vee Q) \vee (P \vee Q)) \wedge R \quad \text{By Distributive Law}$$

$$\equiv T \wedge R \quad \text{By Negation Law}$$

$$\equiv R \quad \text{By Identity Law}$$

Hence Proved

Q4

If we derive a contradiction by using the given premises, then they are inconsistent.

1. P Premise (Given)

2. $P \rightarrow Q$ Premise (Given)

3. Q Modus Ponens using 1 and 2

4. $Q \rightarrow \neg R$ Premise (Given)

5. $\neg R$ Modus Ponens using 3 and 4.

6. $P \rightarrow R$ Premise (Given)

7. R Modus Ponens using 1 and 6.

8. $R \wedge \neg R$ Conjunction using 7 and 5

9. F

Hence, $P \rightarrow Q$, $P \rightarrow R$, $Q \rightarrow \neg R$ and P are inconsistent

Q5

Let P : Roli has completed MBA
 Q : She is assured of a good job
 R : She is happy.

Given: $P \rightarrow Q$ — H_1
 $Q \rightarrow R$ — H_2
 $\neg R$ — H_3

 $\neg P$ — C

Solution: $P \rightarrow Q$
 $Q \rightarrow R$

 $P \rightarrow R$

using hypothetical syllogism.

$P \rightarrow R$
 $\neg R$

 $\neg P$

using Modus Tollens.

Hence, Valid.

Q6

Let p be "I study", q be "I fail Mathematics", and r be "I play basketball".
Then the given argument is as follows:

The hypothesis are:

$P \rightarrow \neg q, \neg r \rightarrow p, q$

And the Conclusion is :

r

<u>Step</u>	<u>Reason</u>
1. $p \rightarrow \neg q$	Hypothesis
2. q	Hypothesis
3. $\neg p$	Using Modus Tollens in 1 & 2
4. $\neg r \rightarrow p$	Hypothesis
5. $\neg(\neg r)$	Using Modus Tollens in 3 & 4
$\equiv \underline{r}_c$	Using Double Negation Law
Hence the given Argument is Valid.	