BENNETT UNIVERSITY

B. Tech, Spring-2021

EPHY108L

Problem Set-1: Answer

1. Consider two vectors $\vec{A} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{B} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$. Find a third vector $\vec{C}$ (say), which is perpendicular to both $\vec{A}$ and $\vec{B}$. Further find the angle between $\vec{A}$ and $\vec{B}$.

   Ans

   $\vec{C} = \pm(-\hat{\imath} + 7\,\hat{\jmath} + 3\,\hat{k})$.

   $\theta \simeq 123.06°$.

2. Find a unit vector, which lies in the $x - y$ plane, and which is perpendicular to $\vec{A}$ of previous problems. Similarly, find a unit vector which is perpendicular to $\vec{B}$, and lies in the $x - z$ plane.

   Ans

   $\hat{C} = \pm\frac{1}{\sqrt{5}}(\hat{\imath} + 2\,\hat{\jmath})$, $\hat{C}$ is $\perp$ to $\vec{A}$

   $\hat{D} = \pm\frac{1}{\sqrt{5}}(2\,\hat{\imath} + \hat{k})$, $\hat{D}$ is $\perp$ to $\vec{B}$

3. Calculate $\vec{A} \cdot (\vec{B} \times \vec{A})$ for the vectors of the previous problem. Does this result hold only for the above defined vectors only?

   Ans

   0, the result is general.

4. Consider two distinct general vectors $\vec{A}$ and $\vec{B}$. Show that $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ implies that $\vec{A}$ and $\vec{B}$ are perpendicular.

5. If three sides of the rectangular box are $\vec{A}, \vec{B}, \vec{C}$. What is the volume of the box?

   Ans

   $\vec{A} \cdot (\vec{B} \times \vec{C})$.

6. A particle moves along the space curve $\vec{r} = (t^2 + t)\hat{\imath} + (3t - 2)\hat{\jmath} + (2t^3 - 4t^2)\hat{k}$. Find the velocity at time $t = 2$.

   Ans

   $\vec{v}(t = 2) = 5\hat{\imath} + 3\hat{\jmath} + 8\hat{k}$.

7. Due to a force field, a particle of mass *5* units moves along a space curve whose position vector is given as a function of time *t* by $\vec{r} = (2t^3 + t)\hat{\imath} + (3t^4 - t^2 + 8)\hat{\jmath} - 12t^2\hat{k}$. Find the velocity, momentum, acceleration and force field at any time *t*.

   Ans

   Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = (6t^2 + 1)\hat{\imath} + (12t^3 - 2t)\hat{\jmath} - 24t\hat{k}$.

   Momentum, $\vec{p} = m\vec{v} = 5\vec{v} = (30t^2 + 5)\hat{\imath} + (60t^3 - 10t)\hat{\jmath} - 120t\hat{k}$

   Acceleration, $\vec{a} = \frac{d\vec{v}}{dt} = 12t\hat{\imath} + (36t^2 - 2)\hat{\jmath} - 24\hat{k}$

   Force, $\vec{F} = m\vec{a} = 5\vec{a} = 60t\hat{\imath} + (180t^2 - 10)\hat{\jmath} - 120\hat{k}$

8. A particle of mass *2* units moves in a force field depending on time *t* given by $\vec{F} = 24t^2\hat{\imath} + (36t - 16)\hat{\jmath} - 12t\hat{k}$. Assuming that at $t = 0$ the particle is located at $\vec{r}_0 = 3\hat{\imath} - \hat{\jmath} + 4\hat{k}$ and has velocity $\vec{v}_0 = 6\hat{\imath} + 15\hat{\jmath} - 8\hat{k}$. Find the velocity and position at any time *t*

   Ans

   $\vec{v}(t) = (4t^3 + 6)\hat{\imath} + (9t^2 - 8t + 15)\hat{\jmath} - (3t^2 + 8)\hat{k}$.

   $\vec{r}(t) = (t^4 + 6t + 3)\hat{\imath} + (3t^3 - 4t^2 + 15t - 1)\hat{\jmath} - (t^3 + 8t - 4)\hat{k}$.

9. Position of a particle in xy plane is given by $\vec{r}(t) = A(e^{\alpha t}\hat{\imath} + e^{-\alpha t}\hat{\jmath})$, where α and A are constants. Calculate the velocity and acceleration of the particle, as functions of time t.

   Ans

   $\vec{v}(t) = A\alpha(e^{\alpha t}\,\hat{\imath} - e^{-\alpha t}\,\hat{\jmath})$,

   $\vec{a}(t) = A\alpha^2(e^{\alpha t}\,\hat{\imath} + e^{-\alpha t}\,\hat{\jmath}) = \alpha^2\vec{r}$.

10. Acceleration of a particle in the xy plane is given by $\vec{a}(t) = -\omega^2\vec{r}(t)$, where $\vec{r}(t)$ denotes its position, and ω is a constant. If $\vec{r}(0) = a\hat{\jmath}$ and $\vec{v}(0) = a\omega\hat{\imath}$. Then obtain an expression for $\vec{r}(t)$ in cartesian coordinates.

    Ans

    $\vec{r}(t) = a(\sin\omega t\ \hat{\imath} + \cos\omega t\ \hat{\jmath})$.

11. The rate of change of acceleration of a particle is called jerk which can be defined as $\vec{\jmath}(t)$. If the jerk of a particle is given by, $\vec{\jmath}(t) = a\hat{\imath} + bt\hat{\jmath} + ct^2\hat{k}$, where a, b, c are constants. Assuming that at time $t = 0$, the particle was located at the origin, and its velocity and acceleration were zero, obtain its position $\vec{r}(t)$, as a function of time in cartesian coordinates.

    Ans

    $\vec{r}(t) = \frac{at^3}{6}\,\hat{\imath} + \frac{bt^4}{24}\,\hat{\jmath} + \frac{ct^5}{60}\hat{k}$.