Solution of Assignment 2

1) $A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 3 & 6 & 6 & 3 \end{pmatrix}$

$$\xrightarrow[R_3' \to R_3 - 3R_1]{R_2' \to R_2 - 2R_1} \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 6 & 6 \\ 0 & 0 & 3 & 3 \end{pmatrix}$$

$$\xrightarrow{R_3' \to R_3 - \frac{1}{2}R_2} \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 6 & 6 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \text{row echelon matrix}$$

It has 2 non-zero rows

$\therefore$ Rank of $A = 2$

2) $B = \begin{pmatrix} 3 & 12 & 9 \\ 2 & 10 & 12 \\ 1 & 12 & 2 \end{pmatrix}$

$$\xrightarrow{R_1' \to \frac{1}{3}R_1} \begin{pmatrix} 1 & 4 & 3 \\ 2 & 10 & 12 \\ 1 & 12 & 2 \end{pmatrix}$$

$$\xrightarrow[R_3' \to R_3 - R_1]{R_2' \to R_2 - 2R_1} \begin{pmatrix} 1 & 4 & 3 \\ 0 & 2 & 6 \\ 0 & 8 & -1 \end{pmatrix}$$

$$\xrightarrow{R_3' \to R_3 - 4R_2} \begin{pmatrix} 1 & 4 & 3 \\ 0 & 2 & 6 \\ 0 & 0 & -25 \end{pmatrix} = \text{row - echelon matrix}$$

It has 3 non-zero rows

$\therefore$ Rank of $B = 3$

3) $C = \begin{pmatrix} 0 & 0 & 1 & 2 & 1 \\ 1 & 3 & 1 & 0 & 3 \\ 2 & 6 & 4 & 2 & 8 \\ 3 & 9 & 4 & 2 & 10 \end{pmatrix}$

$$\xrightarrow{R_{12}} \begin{pmatrix} 1 & 3 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 & 1 \\ 2 & 6 & 4 & 2 & 8 \\ 3 & 9 & 4 & 2 & 10 \end{pmatrix}$$

$$\xrightarrow[R_4' \to R_4 - 3R_1]{R_3' \to R_3 - 2R_1} \begin{pmatrix} 1 & 3 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 1 & 2 & 1 \end{pmatrix}$$

$$\xrightarrow[R_4' \to R_4 - R_2]{R_3' \to R_3 - 2R_2} \begin{pmatrix} 1 & 3 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

= row - echelon matrix.

It has 3 - non zero rows

$\therefore$ Rank of $C = 3$

4)

~~Example~~ Find the inverse of A where $A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 4 \\ 3 & 3 & 7 \end{pmatrix}$ using Gauss-Jordan method.

Solution: Now $(A \mid I_3)$

{ let us form 3×6 matrix $(A \mid I_3)$ and perform elementary row operation to reduce A to a row-~~reduced~~ reduced echelon matrix }

$$= \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 2 & 4 & 4 & 0 & 1 & 0 \\ 3 & 3 & 7 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[R_3 - 3R_1]{R_2 - 2R_1} \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{\frac{1}{2}R_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_1 - R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & -1/2 & 0 \\ 0 & 1 & 0 & -1 & 1/2 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_1 - 2R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & -1/2 & -2 \\ 0 & 1 & 0 & -1 & 1/2 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array}\right)$$

$$= \left(I_n \mid A^{-1} \right)$$

$$\therefore A^{-1} = \begin{pmatrix} 8 & -1/2 & -2 \\ -1 & 1/2 & 0 \\ -3 & 0 & 1 \end{pmatrix}$$

5) Using Gauss - Jordan method, find the inverse of the matrix $A = \begin{pmatrix} 3 & 12 & 9 \\ 2 & 10 & 12 \\ 1 & 12 & 2 \end{pmatrix}$.

Sol^n: Let us form 3×6 matrix $(A \mid I_3)_{3\times 6}$ and perform elementary row operation to reduce A to a row-reduced echelon matrix.

Now $(A \mid I_3)$

$$= \left(\begin{array}{ccc|ccc} 3 & 12 & 9 & 1 & 0 & 0 \\ 2 & 10 & 12 & 0 & 1 & 0 \\ 1 & 12 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_1' \to \frac{1}{3}R_1} \left(\begin{array}{ccc|ccc} 1 & 4 & 3 & \frac{1}{3} & 0 & 0 \\ 2 & 10 & 12 & 0 & 1 & 0 \\ 1 & 12 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[R_3' \to R_3 - R_1]{R_2' \to R_2 - 2R_1} \left(\begin{array}{ccc|ccc} 1 & 4 & 3 & \frac{1}{3} & 0 & 0 \\ 0 & 2 & 6 & -\frac{2}{3} & 1 & 0 \\ 0 & 8 & -1 & -\frac{1}{3} & 0 & 0 \end{array}\right)$$

$$\xrightarrow{R_2' \to \frac{1}{2}R_2} \left(\begin{array}{ccc|ccc} 1 & 4 & 3 & \frac{1}{3} & 0 & 0 \\ 0 & 1 & 3 & -\frac{1}{3} & \frac{1}{2} & 0 \\ 0 & 8 & -1 & -1/3 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[R_3' \to R_3 - 8R_2]{R_1' \to R_1 - 4R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -9 & \frac{5}{3} & -2 & 0 \\ 0 & 1 & 3 & -1/3 & 1/2 & 0 \\ 0 & 0 & -25 & 7/3 & -4 & 1 \end{array}\right)$$

$$\xrightarrow{R_3' \to -\frac{1}{25} R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & -9 & \frac{5}{3} & -2 & 0 \\ 0 & 1 & 3 & 1/3 & 1/2 & 0 \\ 0 & 0 & 1 & -\frac{7}{75} & \frac{4}{25} & -\frac{1}{25} \end{array}\right)$$

$$\xrightarrow[R_2' \to R_2 - 3R_3]{R_1' \to R_1 + 9R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{62}{75} & \frac{14}{25} & \frac{9}{25} \\ 0 & 1 & 0 & \frac{4}{75} & \frac{1}{50} & \frac{3}{25} \\ 0 & 0 & 1 & \frac{7}{75} & \frac{4}{25} & \frac{1}{25} \end{array}\right)$$

$$\sim \left(I_3 \mid A^{-1} \right)$$

$$\therefore \quad A^{-1} = \begin{pmatrix} \frac{62}{75} & \frac{14}{25} & \frac{9}{25} \\ \frac{4}{75} & \frac{1}{50} & \frac{3}{25} \\ \frac{7}{75} & \frac{4}{25} & \frac{1}{25} \end{pmatrix}$$

6)
$$\left.\begin{array}{l} x_1 + x_2 = 4 \\ x_2 - x_3 = 1 \\ 2x_1 + x_2 + 4x_3 = 7 \end{array}\right\} \longrightarrow (1)$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 2 & 1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 7 \end{pmatrix} \Rightarrow A_{3\times 3} X_3 = B_3$$

Augmented matrix $= \overline{A} = \left(\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 1 & -1 & 1 \\ 2 & 1 & 4 & 7 \end{array}\right)$

$$\xrightarrow{R_3' \to R_3 - 2R_1} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & -1 & 4 & -1 \end{array}\right)$$

$$\xrightarrow{R_3' \to R_3 + R_2} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 3 & 0 \end{array}\right) = \text{row-echelon form}$$

$\therefore$ Rank of $A = 3$

Rank of $\overline{A} = 3$

$\therefore$ Rank A = Rank $\overline{A} = 3 \Rightarrow$ System is consistent

$\therefore$ The system of equation (1) is transformed to the system:

$$\left.\begin{array}{l} x + y = 4 \\ y - z = 1 \\ 3z = 0 \end{array}\right\} (2)$$

$\Rightarrow z = 0,\ y = z + 1 = 1,\ x = 4 - 1 = 3$ $\quad\therefore (x, y, z) = (3, 1, 0)$

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