B. Tech, Spring-2021

EPHY108L

Problem Set-3

1. A particle is rotating in the *xy*-plane, along a circular path in counter-clockwise direction, with angular speed ω, about the z-axis.

   (a) Write down the angular velocity of the particle in the vector form, i.e., in terms of components and unit vectors.

   (b) If the particle is moving along a circle of radius a, write down its position vector $\vec{r}(t)$, as a function of time, assuming that $\vec{r}(0) = a\,\hat{\imath}$

   (c) Express its velocity both in Cartesian, and plane polar coordinates

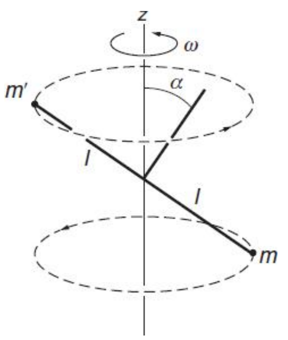
   (d) Compute the acceleration of the particle both in Cartesian, and plane polar coordinates.

2. A vector $\vec{A}$ of magnitude a is rotating in the *y-z* plane in a counter-clockwise manner,with a uniform angular velocity ω. It is given that $\vec{A}(t = 0) = a\,\hat{\jmath}$.

   (a) Obtain $\vec{A}(t)$, as a function of time.

   (b) Show that $\frac{d\vec{A}}{dt}$ calculated directly and computed using $\vec{\omega} \times \vec{A}$ are the same.

3. Consider a simple rigid body consisting of two particles of mass m separated by a massless rod of length $2l$. The midpoint of the rod is attached to a vertical axis that rotates at angular speed ω around the z axis. The rod is skewed at angle α, as shown in the figure.



   (a) Calculate the angular momentum $\vec{L}(t)$ of the system, in Cartesian coordinates.

(b) Verify that $\frac{d\vec{L}}{dt}$ is same as $\vec{\omega} \times \vec{L}$.

4. A cylinder of mass M and radius R rolls without slipping on a plank which is moving with an acceleration $\vec{A}$. Calculate the acceleration of the cylinder by analyzing the problem both in the inertial frame and the non-inertial frames. You can use the fact that moment of inertia of a cylinder about its axis is $\frac{1}{2}MR^2$.
5. Find the moment of inertia and radius of gyration of a solid circular cylinder of radius a, height h and mass M about the axis of the cylinder.
6. Use parallel axes theorem to find the moment of inertia of the above circular cylinder about a line on the surface of the cylinder and parallel to the axis of the cylinder.
7. Find the moment of inertia and radius of gyration of a rectangular plate whose sides are a and b about a side.
8. Find the moment of inertia of the above plate about an axis perpendicular to the plate and passing through a vertex.

9. In the lectures, we argued that the effective potential for the central force problem is $V_{eff}(r) = \frac{L^2}{2\mu r^2} + V(r)$, where $V(r)$ is the potential energy corresponding to the central force, and L is the angular momentum. Consider the case of gravitational motion so that $V(r) = -\frac{C}{r}$, with $C > 0$. Plot the effective potential as a function of r, and argue based upon the plot that for $E \geq 0$ orbits will be unbound, while for $E < 0$, we will obtain bound orbits, where E is the total energy of the system.

10. Suppose a satellite is moving around a planet in a circular orbit of radius r_0. Due to a collision with another object, satellite's orbit gets perturbed. Show that the radial position of the satellite will execute simple harmonic motion with $\omega = \frac{L}{mr_0^2}$, where L is the angular momentum of the satellite.

11. A particle of mass m is moving under the influence of a central force $\vec{F}(r) = -\frac{C}{r^3}\hat{r}$, with $C > 0$. Find the nonzero values of angular momentum L for which the particle will move in a circular orbit.

12. A geostationary orbit is one in which a satellite moves in a circular orbit at the given height in the equatorial plane, so that its angular velocity of rotation around earth is same as earth's angular velocity, thereby, making it look stationary when seen from a point on equator. Assuming that the earth's rotational velocity, and radius, respectively, are $\Omega_e = \frac{2\pi}{86400}\ rad/s$, and $R_e = 6400\ km$, calculate the altitude of the satellite, and its orbital velocity.

13. A space company wants to launch a satellite of mass $m = 2000\ kg$, in an elliptical orbit around earth, so that the altitude of the satellite above earth at perigee is $1100\ kms$, and at apogee it is $35{,}850\ kms$. Assuming that the launch takes place at the equator, calculate:

    (a) energy of the satellite in the elliptical orbit,

    (b) energy required to launch the satellite,

    (c) eccentricity of the orbit,

    (d) angular momentum of the satellite, and

    (e) speeds of the satellite at apogee and perigee.

    Use the values of R_e and Ω_e specified in the previous problem.

14. The ultimate aim of the space company of the previous problem is to put the satellite ina geostationary orbit. Therefore, after launching it in the elliptical orbit, the company wants to transfer it in a geostationary orbit by firing rockets at the apogee to increase its speed to the required one. How much change in speed is needed to put the satellite in the geostationary orbit, and how much energy will be required to achieve that change?