$\pm$ binary

0, 1

± 25

± 16

Sign Number and Addition, Subtraction

How To Represent Signed Numbers

- Plus and minus sign used for decimal numbers: 25 (or +25), -16, etc.
- For computers, desirable to represent everything as *bits*.
- Three types of signed binary number representations:
  - signed magnitude,
  - 1's complement,
  - 2's complement.
- In each case: left-most bit indicates sign: positive (0) or negative (1).

Signed Magnitude

8 bit Signed mag:

$\underset{\text{sign}}{1}\ \underbrace{0000111}_{\text{mag}} = -7$

Consider ***signed magnitude***:

$\underset{\text{Sign bit}}{0}\underbrace{0001100_2}_{\text{Magnitude}} = 12_{10}$

$\underset{\text{Sign bit}}{1}\underbrace{0001100_2}_{\text{Magnitude}} = -12_{10}$

0 → + Sign

1 → - Sign

One's Complement Representation

- **The one's complement of a binary number involves inverting all bits.**
  - **1's comp of 00110011 is 11001100**
  - **1's comp of 10101010 is 01010101**
- **To find negative of 1's complement number take the 1's complement.**

+ve
Same
-ve

15 →
+15 → 01111
-15 → 10000

→ $00001100_2 = 12_{10}$

Sign bit Magnitude

$11110011_2 = -12_{10}$

Sign bit Magnitude

One's complement

8 bit →

+20

↓

−20

16 8 4 2 1

0 0 0 1 0 1 0 0

sign

mag

+15 →

−15

0 0 0 1 0 1 0 0

↓ 1's complement

1 1 1 0 1 0 1 1

↑

Sign

+ values

Signed ma

−ve

one's comp

Two's Complement Representation

- **The two's complement of a binary number involves inverting all bits and adding 1.**
  - **2's comp of 00110011 is 11001101**
  - **2's comp of 10101010 is 01010110**
- **To find negative of 2's complement number take the 2's complement.**

$0\underline{0001100}_2 = 12_{10}$

Sign bit Magnitude

$1\underline{1110100}_2 = -12_{10}$

Sign bit Magnitude

-12

00001100

↓ 1's Com

11110011 1's Com

+ 1

11110100 → -12 in 2's comb

Two's Complement Representation

+17

16 8 4 2 1

Signed mag → $\underset{\text{sign}}{0}\underbrace{001\ 0001}_{\text{mag}}$

1's comp → 0001 0001

2's comp → 0001 0001

−17

Signed no → $\underset{\text{sign}}{1}\underbrace{001\ 0001}_{\text{mag}}$

1's complemt → 0001 0001

↓

(1110 1110) → −17

2's Complement →

1110 1110

+1

(1110 1111) → −17

Binary Addition

$\begin{array}{r}0\\0\\\hline 0\end{array}\quad\begin{array}{r}0\\1\\\hline 1\end{array}\quad\begin{array}{r}1\\0\\\hline 1\end{array}\quad\begin{array}{r}1\\1\\1\\\hline 11\end{array}\quad\begin{array}{r}1\\1\\\hline 10\end{array}$

- Binary addition is very simple.
- This is best shown in an example of adding two binary numbers…

```
     1   1   1   1   1   1  <------------- carries
         1   1   1   1   0   1
+            1   0   1   1   1
------------------------------
     1   0   1   0   1   0   0
```

Binary Subtraction

- **We can also perform subtraction (with borrows in place of carries).**
- **Let's subtract $(10111)_2$ from $(1001101)_2$…**

$$
\begin{array}{cccccccc}
 & & 1 & & & 10 & & \\
 & 0 & \cancel{10} & 10 & 0 & \cancel{0} & 10 & \\
 & \cancel{1} & \cancel{0} & \cancel{0} & \cancel{1} & \cancel{1} & \cancel{0} & 1 \\
- & & & 1 & 0 & 1 & 1 & 1 \\
\hline
 & & 1 & 1 & 0 & 1 & 1 & 0
\end{array}
$$

$\longleftarrow$ borrows

Binary Multiplication

- Binary multiplication is much the same as decimal multiplication, except that the multiplication operations are much simpler…

```
                 1   0   1   1   1
X                    1   0   1   0
-----------------------------------
                 0   0   0   0   0
             1   0   1   1   1
         0   0   0   0   0
     1   0   1   1   1
-----------------------------------
     1   1   1   0   0   1   1   0
```

$1 \times 0 \to 0$

$0 \times 1 \to 0$

$0 \times 0 \to 0$

$1 \times 1 \to 1$

Addition

$(123)_4 + (23)_4 = (?)_4$

123 + 23

$(123)_{10}$
$(23)_{10}$
146

$$\begin{array}{r} 1\,1 \\ 123_4 \\ 23_4 \\ \hline (212)_4 \end{array}$$

$(35)_5 + (23)_5 = (?)_5$ ← Practice

0, 1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23 ---

Addition using 1's complement

$(12)_{10} + (23)_{10} = (?)_{10}$

1 1 1

12 → 00001100

23 → 00010111 +

00100011

→ Decimal

$(35)_{10}$

16 8 4 2 1

00001100

00010111

00100011

32 16 8 4 2 1

Addition using 2's complement

$(12)_{10} + (23)_{10} = (?)_{10}$

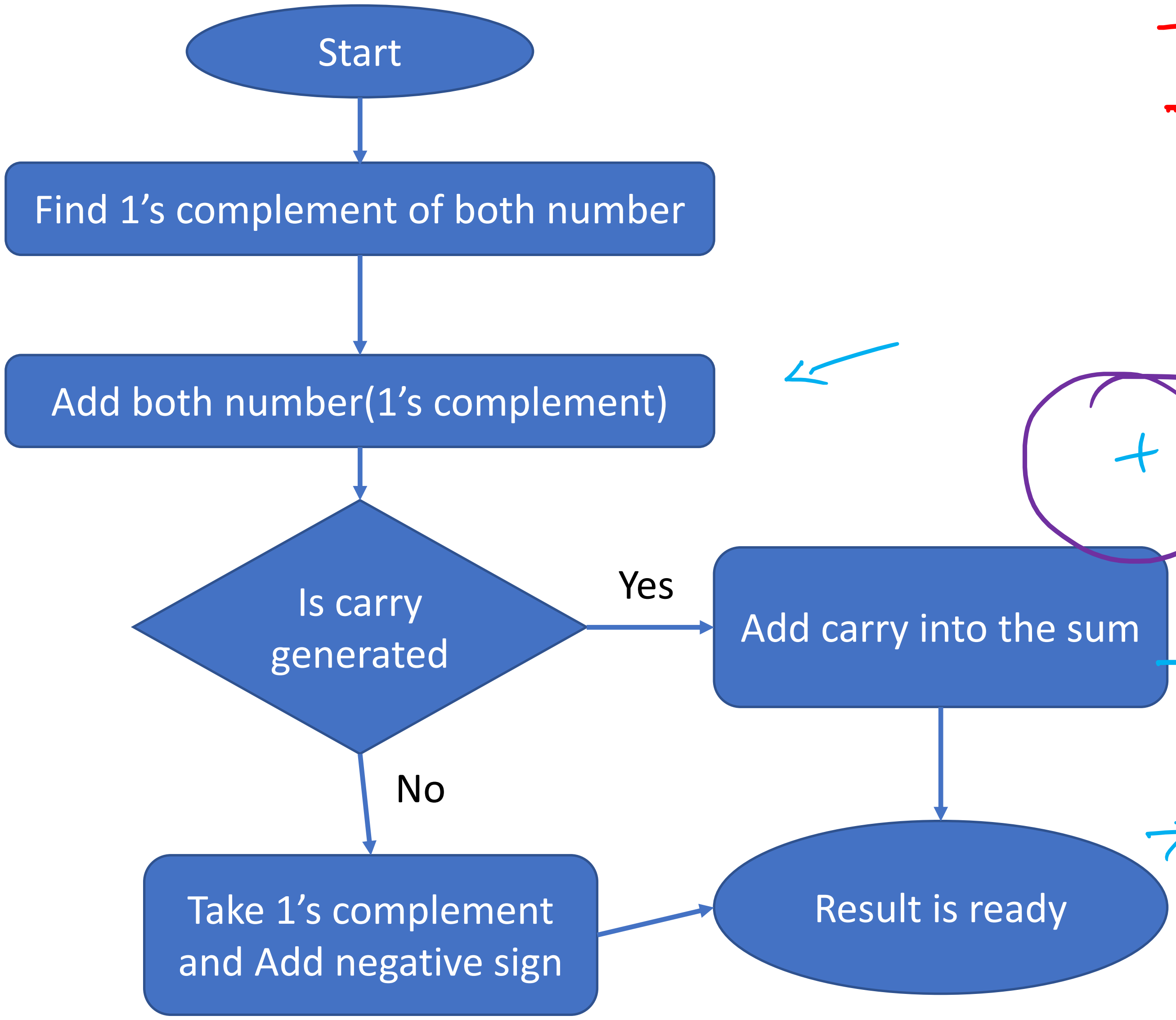
$12 \rightarrow$

```
   111
  00001100
  00010111
  --------
  00100011
+
```

$(+35)_{10}$

```
   16 8 4 2 1
0 0 0 0 1 1 0 0
0 0 0 1 0 1 1 1
```

Subtraction using 1's complement



16 8 4 2 1

28 − 13

+28 → 11100 → 00011100 +28

−13 → 01101 → 00001101 → +13

+13

11110010 −13

  00011100
+ 11110010

1 00001110

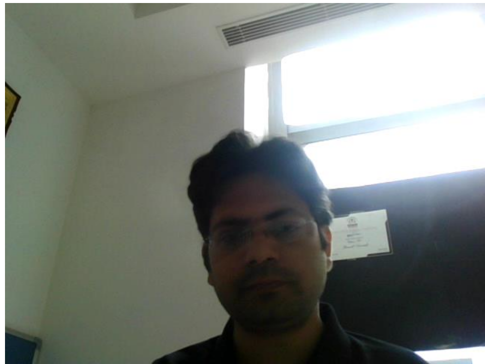
+ 1

00001111

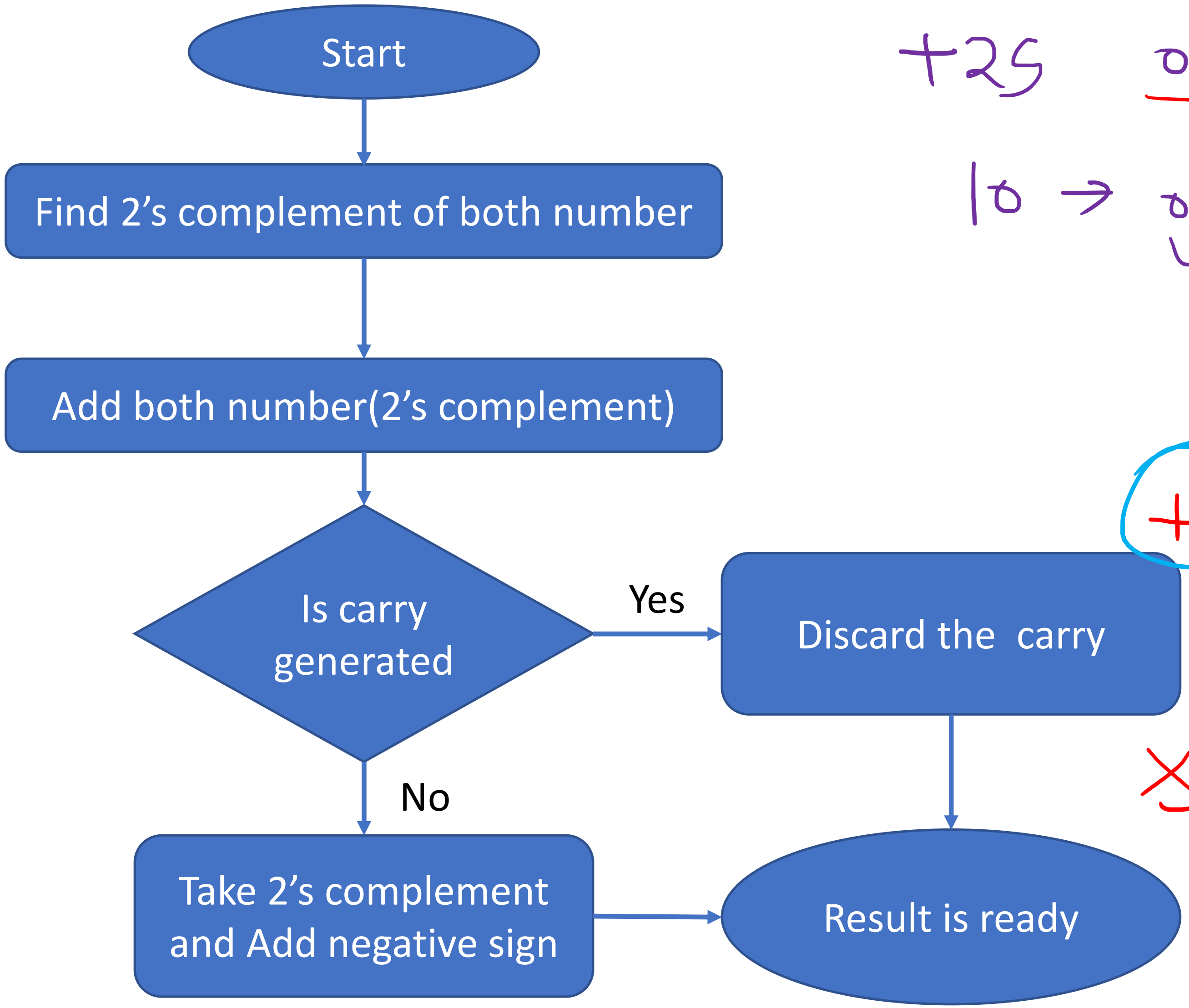
8421

+(15)

Subtraction using 1’s complement



Subtraction using 2's complement



16 8 4 2 1

25 - 10

+25 00011001

10 → 00001010 → 11110101

+10 +1

$\boxed{11110110}$ -10

  00011001
+ 11110110

~~1~~ $\boxed{00001111}$ → +15