Tutorial 10 Solution

Q1) Let S denote the set of students who participate in the singing competition and D denote the set of students who participate in the dancing competition.

Given that $|S \cup D| = 70$ and $|S| = 50$

$|D|$ = No. of students who get a chance to perform during the annual function.

$|S \cap D|$ = No. of students who get 10 additional points in general proficiency.

Given that $|(S \cap D)'| = 30$, $|S \cap D| = 70 - 30 = 40$

We know that $|S \cup D| = |S| + |D| - |S \cap D|$.

Hence, $|D| = 70 - 50 + 40 = 60$

Thus, the number of students who get the chance to perform during the annual function but do not get additional point is

$|D| - |S \cap D| = 60 - 40 = 20$ Ans.

Q2) In Caesar cipher, the alphabets A to Z are represented by numbers 0 to 25.

Every alphabet of the given text needs to be replaced using the given equation

$$c = f(p) = p + 3 \pmod{26}$$

B O O K

1 14 14 10

After applying the function.

4 17 17 13

E R R N

BOOK $\rightarrow$ ERRN Ans.

Similarly, PARK $\rightarrow$ SDUN Ans.

Q3) The stated problem can be solved using Chinese Remainder Theorem.

The equations are as follows:-

$$x \equiv 3 \pmod 4$$

$$x \equiv 2 \pmod 5$$

$$x \equiv 4 \pmod 7$$

Here, let x denote the minimum no. of pens in the bag.

Here, $m_1 = 4$, $m_2 = 5$, $m_3 = 7$, $a_1 = 3$, $a_2 = 2$, $a_3 = 4$

$M = 4 \times 5 \times 7 = 140$

Now, $M_1 = \frac{140}{4} = 35$ $\quad M_1^{-1} = 35 \times (?) \equiv 1 \pmod 4$
$= 3$

$M_2 = \frac{140}{5} = 28$ $\quad M_2^{-1} = 28 \times (?) \equiv 1 \pmod 5$
$= 2$

$M_3 = \frac{140}{7} = 20$ $\quad M_3^{-1} = 20 \times (?) \equiv 1 \pmod 7$
$= 6$

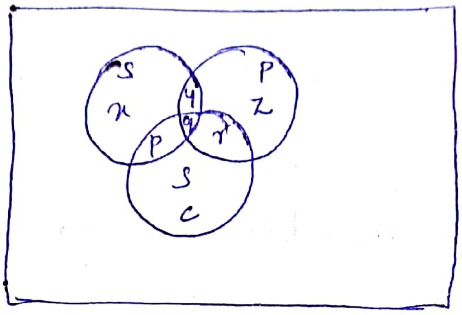
$\therefore x = (3 \times 35 \times 3 + 2 \times 28 \times 2 + 4 \times 20 \times 6) \bmod 140$

$x = (315 + 112 + 480) \bmod 140$

$= 907 \bmod 140$

$= \cancel{67}\ 67$ Ans.

Q5) The problem can be understood with the help of the Venn Diagram



In this Venn Diagram, the sets S, P, and C represent the sets of persons who like singing, playing and cooking, respectively. The variable written inside a portion shows the number of persons in that subset.

Given that $x = 35$, $z = 20$, $s = 25$, $y + q = 10$, $q + r = 15$, $p + q = 8$ and $q = 5$. Solving these equations, we get,

$p = 3$, $r = 12$ and $y = 5$.

(a) No. of people who like singing $= x + y + p + q = 35 + 5 + 8$
$= 48.$

(c) No. of people who like playing $= y + z + q + r = 5 + 20 + 15 = 40$

(b) No. of people who like cooking $= s + r + p + q =$ 25+15+3=43

(d) No. of people included in the survey $= x + y + z + s + p + q + r$
$=$ 103

Q4 a) $2x \equiv 3 \pmod 5$

$a = 2,\ b = 3,\ m = 5$

Let $d = \gcd(2,5)$

$d = 1$

$\therefore$ There is one solution.

$2x \equiv 3 \pmod 5$

Let $x = 1 \Rightarrow 2 \bmod 5 = 3 \pmod 5$ ×

Let $x = 2 \Rightarrow 4 \bmod 5 = 3 \pmod 5$ ×

Let $x = 3 \Rightarrow 6 \bmod 5 = 1$ ×

Let $x = 4 \Rightarrow 8 \bmod 5 = 3 \pmod 5$ ✓

so $x = 4$.

b) $3x \equiv 2 \pmod 8$

$a = 3 \quad b = 2 \quad c = 8$

Let $d = \gcd(3,8) \Rightarrow d = 1$

$\therefore$ There is one solution

$3x \equiv 2 \pmod 8$

Let $x = 1 \Rightarrow 3 \bmod 8 = 2$ ~~mod8~~ ×

Let $x = 2 \Rightarrow 6 \bmod 8 = 6$ ~~mod8~~ ×

Let $x = 3 \Rightarrow 9 \bmod 8 = 1$ ×

Let $x = 4 \Rightarrow 12 \bmod 8 = 4$ ×

Let $x = 5 \Rightarrow 15 \bmod 8 = 7$ ×

Let $x = 6 \Rightarrow 18 \bmod 8 = 2$ ✓

so $x = 6$.

6) Let A be the number of people who own a dog &
Let B be the number of people who own a Cat

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A) = 30, \quad n(B) = 25$$

$n(A \cup B)$ cannot be greater than 50, because there are 50 people in total.

$$\therefore \quad 50 \geq 30 + 25 - n(A \cap B)$$

$$n(A \cap B) \geq 30 + 25 - 50 - 5$$

$\therefore$ There are atleast 5 people, who own both a dog and a cat.

7) Let x be the length in inches of the dining room.

$$x \equiv 3 \pmod{5}$$

$$x \equiv 3 \pmod{7}$$

$$x \equiv 3 \pmod{9}$$

Since 5, 7, 9 are relatively prime, we can apply the Chinese Remainder Theorem.

$$a_1 = a_2 = a_3 = 3$$

$$m_1 = 5, \quad m_2 = 7, \quad m_3 = 9$$

$M = 5 \times 7 \times 9 = 315$

$M_1 = M / m_1 = 315/5 = 63$

$M_2 = M / m_2 = 315/7 = 45$

$M_3 = M / m_3 = 315/9 = 35$

$$x = \left((a_1 \times M_1 \times M_1^{-1}) + (a_2 \times M_2 \times M_2^{-1}) + (a_3 \times M_3 \times M_3^{-1}) \right) \bmod M$$

$M_1^{-1} = 2$ $\quad\quad$ $63 \times (\ ) \bmod 5 = 1$

$M_2^{-1} = 5$

$M_3^{-1} = 8$

$\therefore\ x = (3 \times 63 \times 2 + 3 \times 45 \times 5 + 3 \times 35 \times 8) \bmod 315$

$= 3$

$\because$ x is length of room

3 inches cannot be the possible answer

Next Nearest answer will be

$3 + 315 = 318$

$\therefore$ 318 inches

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8)

Let number of integers divisible by 3 be A &
number of integers divisible by 5 be B

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A) = \left\lfloor \frac{500}{3} \right\rfloor = 166$$

$$n(B) = \left\lfloor \frac{500}{5} \right\rfloor = 100$$

$$n(A \cap B) = \left\lfloor \frac{500}{3 \times 5} \right\rfloor = 33$$

$$\therefore \quad n(A \cup B) = 166 + 100 - 33$$

$$= 233$$

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