Lecture 8

Class Note

Vector Space in $\mathbb{R}^n$ (Sub-Spaces)

Sub-Spaces:

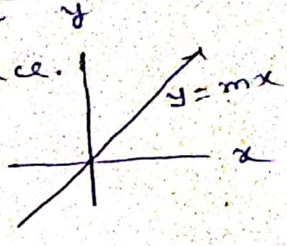
A Subset $W \subseteq \mathbb{R}^n$ is called ~~vector~~ sub-space ~~of~~ in $\mathbb{R}^n$ if $v, w \in W$, $a, b \in \mathbb{R} \Rightarrow av + bw \in W$

Note: If $a = b = 0$ the $av + bw = 0v + 0w = 0 \in W$

$\Rightarrow$ The zero vector will belong to every vector spaces in $\mathbb{R}^n$ (i.e sub-spaces)

Example: (1) $L = \{(x, y) \in \mathbb{R}^2 \mid y = mx\}$

Show that L is vector space in $\mathbb{R}^2$ i.e sub-space.



Sol^n: Let (x_1, y_1) & $(x_2, y_2) \in L$

$\therefore y_1 = mx_1$ & $y_2 = mx_2$

Now $ay_1 + by_2 = amx_1 + bmx_2$

$= m(ax_1 + bx_2) \in L$

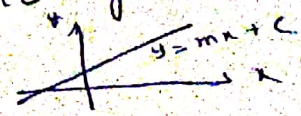
Also, ~~(0)~~ $(0, 0) \in L$

$\therefore$ ~~0~~ $0 = m \times 0$

$\Rightarrow L$ is a vector space in $\mathbb{R}^2$ i.e sub-space. (Here L is ~~a~~ line passing through origin)

Example: (2) $L = \{(x, y) \in \mathbb{R}^2 \mid y = mx + c,\ c \neq 0\}$

Show that L is not vector space in $\mathbb{R}^2$ i.e sub-space



Sol^n: Here $(0, 0)$ vector not belongs to L, since $c \neq 0$

But we know zero vector must be belong to vector space in $\mathbb{R}^2$ i.e sub-space

$\Rightarrow L$ is not vector space in $\mathbb{R}^2$ i.e sub-space

$\Rightarrow$ A line not passing through the origin is not a vector space in $\mathbb{R}^2$ (i.e sub-space)

Example 3: Show that null space $= \mathcal{N}(A) = \{X_n \in \mathbb{R}^n \mid A_{m\times n} X_n = O_m\}$ is a vector space in $\mathbb{R}^n$ i.e. sub-space.

Solution: We know that Homogeneous system always have a zero solution

$$\therefore O_n = (0, 0, \cdots, 0) \in \mathcal{N}(A)$$

Now let $X^0, X^1 \in \mathcal{N}(A)$

$$A X^0 = O, \quad A X^1 = 0 \qquad a, b \in \mathbb{R}$$

$$\therefore A(aX^0 + bX^1) = aAX^0 + bAX^1$$
$$= a\times 0 + b\times 0$$
$$= 0$$

$\Rightarrow aX^0 + bX^1 \in \mathcal{N}(A)$

$\Rightarrow$ Null space is a vector space. in $\mathbb{R}^n$ i.e sub-space

Range Space: $R(A) = \{B_m \in \mathbb{R}^m \mid \exists$ at least on X_n s.t $A_{m\times n} X_n = B_m\}$

$\downarrow$ Range space

Example 4: Show that range space $R(A)$ is a vector space in $\mathbb{R}^m$ i.e sub-space.

Solution: $\because A_{m\times n} O_n = O_m$

$\Rightarrow O_m \in R(A)$

Now let, B^0, B^1 are in $R(A)$

$\because \exists X^0, X^1 \in \mathbb{R}^n$ s.t $AX^0 = B^0$ & $AX^1 = B^1$

Now $aB^0 + bB^1 = aAX^0 + bAX^1$
$= A(aX^0 + bX^1)$

$\because X^0, X^1 \in \mathbb{R}^n$, then $aX^0 + bX^1 \in \mathbb{R}^n$

$\Rightarrow$ for $aB^0 + bB^1$, $\exists aX^0 + bX^1$ in $\mathbb{R}^n$ s.t

$A(aX^0 + bX^1) = aB^0 + bB^1$

$\Rightarrow R(A)$ is a vector space in $\mathbb{R}^m$ i.e sub-space