

AI1103 : Assignment 2

Amaan - EP20BTECH11003

Download all python codes from

<https://github.com/amaan28/Assignment2/blob/main/Assignment2/codes/Assignment2.py>

and latex-tikz codes from

<https://github.com/amaan28/Assignment2/blob/main/Assignment2/Assignment2.tex>

TABLE 2: Markov states and Notations

Notation	State
$S = 1$	$(X, Y) = (0, 1)$
$S = 2$	$(X, Y) = (1, 1)$
$S = 3$	$(X, Y) = (0, 0)$
$S = 4$	$(X, Y) = (1, 0)$

GATE 2012 EE Q.47

A fair coin is tossed till head appears for the first time. The probability that the number of required tosses is odd, is,

- 1) $\frac{1}{3}$
- 2) $\frac{1}{5}$
- 3) $\frac{2}{3}$
- 4) $\frac{3}{4}$

GATE 2012 EE Q.47 - SOLUTION

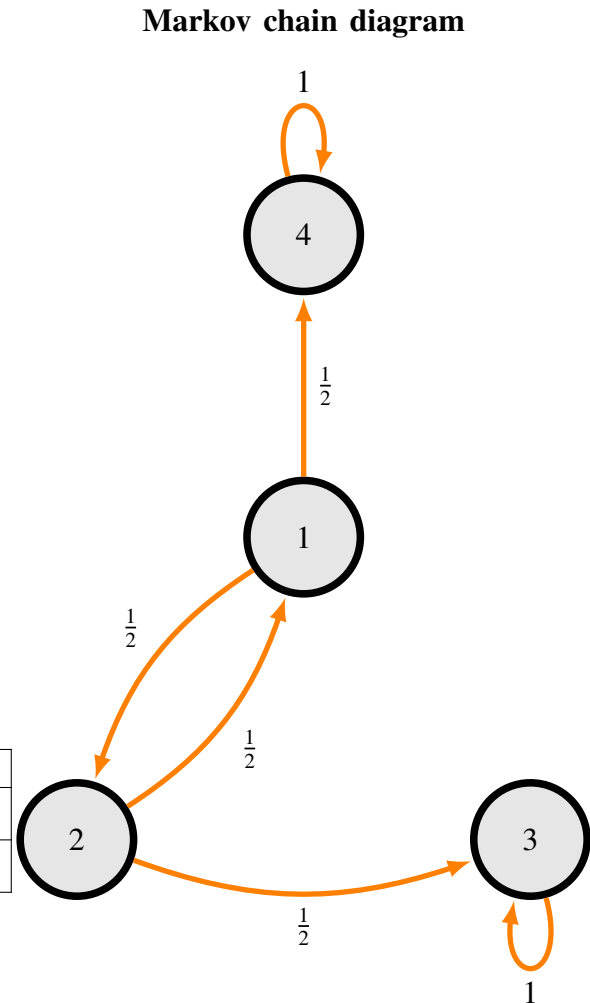
Given, a fair coin is tossed till heads turns up.

$$p = \frac{1}{2}, q = \frac{1}{2} \quad (47.1)$$

Let us define a Markov chain $\{X_0, X_1, X_2, \dots\}$, where $X_n \in S = \{1, 2, 3, 4\}$ where $n \in \{0, 1, 2, \dots\}$,

TABLE 1: Definition of Random Variables

R.V	Definition
X	tosses done till now is even($X = 0$)/odd($X = 1$)
Y	coin face right now, heads($Y = 0$)/tails($Y = 1$)



Definition 1. The standard form of a state transition

matrix is,

$$\mathbf{P} = \begin{matrix} & \begin{matrix} A & N \end{matrix} \\ \begin{matrix} A \\ N \end{matrix} & \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{R} & \mathbf{Q} \end{bmatrix} \end{matrix} \quad (47.2)$$

where,

TABLE 3: Notations and their meanings

Notation	Meaning
A	Absorbing states (3,4)
N	Non-absorbing states (1,2)
\mathbf{I}	Identity matrix
\mathbf{O}	Zero matrix
\mathbf{R}, \mathbf{Q}	Other sub-matrices

Corollary 0.1. *The state transition matrix for the above Markov chain is,*

$$\mathbf{P} = \begin{matrix} & \begin{matrix} 3 & 4 & 1 & 2 \end{matrix} \\ \begin{matrix} 3 \\ 4 \\ 1 \\ 2 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 \end{bmatrix} \end{matrix} \quad (47.3)$$

From (47.3),

$$\mathbf{R} = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}, \mathbf{Q} = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix} \quad (47.4)$$

Definition 2. *The limiting matrix for absorbing Markov chain is,*

$$\bar{\mathbf{P}} = \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{FR} & \mathbf{O} \end{bmatrix} \quad (47.5)$$

where,

$$\mathbf{F} = (\mathbf{I} - \mathbf{Q})^{-1} \quad (47.6)$$

is called the fundamental matrix of \mathbf{P} .

Corollary 0.2. *Limiting Matrix of the Markov chain*

under observation is,

$$\bar{\mathbf{P}} = \begin{matrix} & \begin{matrix} 3 & 4 & 1 & 2 \end{matrix} \\ \begin{matrix} 3 \\ 4 \\ 1 \\ 2 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{bmatrix} \end{matrix} \quad (47.7)$$

Definition 3. *A element \bar{p}_{ij} of $\bar{\mathbf{P}}$ denotes the absorption probability in state j , starting from state i .*

Corollary 0.3. *The required probability is,*

$$P = \bar{p}_{14} \quad (47.8)$$

From (47.7) and (47.8),

$$P = \frac{2}{3} \quad (47.9)$$

Therefore, option 3 is correct.