Assignment - 1

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***Abstract*—This document contains the solution to Exercise 3.21(a) of Oppenheim.**

Problem 1. Consider an LTI system with impulse response

$$h[n] = \begin{cases} a^n & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \tag{1}$$

$$x[n] = \begin{cases} 1 & , 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Determine the output $h[n]$ by explicitly evaluating the discrete convolution of $x[n]$ and $h[n]$.
Solution: Calculating the convolution function piecewise,

$$y[n] = 0 \quad n < 0 \tag{3}$$

$$y[n] = \sum_{k=0}^{n} x[k]h[n-k] = \sum_{k=0}^{n} a^{n-k} = \frac{1-a^{n+1}}{1-a} \quad 0 \leq n \leq N-1 \tag{4}$$

$$y[n] = \sum_{k=0}^{N-1} x[k]h[n-k] = \sum_{k=0}^{N-1} a^{n-k} = a^{n+1}\frac{1-a^{-N}}{a-1} \quad n \geq N \tag{5}$$

For the ROC $|z| > \frac{1}{6}$.