PHYS 375: Final Project - Gravity Group

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Equations of Stellar Structure

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} \qquad \frac{d\rho}{dr} = -\left[\frac{GM\rho}{r^2} + \frac{dP}{dT}\frac{dT}{dr}\right] \Bigg/ \frac{dP}{d\rho}$$

$$\frac{dT}{dr} = \min\left[\frac{3\kappa\rho L}{16\pi acT^3r^2}, \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{GM\rho}{r^2}\right]$$

$$\frac{dM}{dr} = 4\pi r^2\rho \qquad \frac{dL}{dr} = 4\pi r^2\rho\epsilon \qquad \frac{d\tau}{dr} = \kappa\rho$$

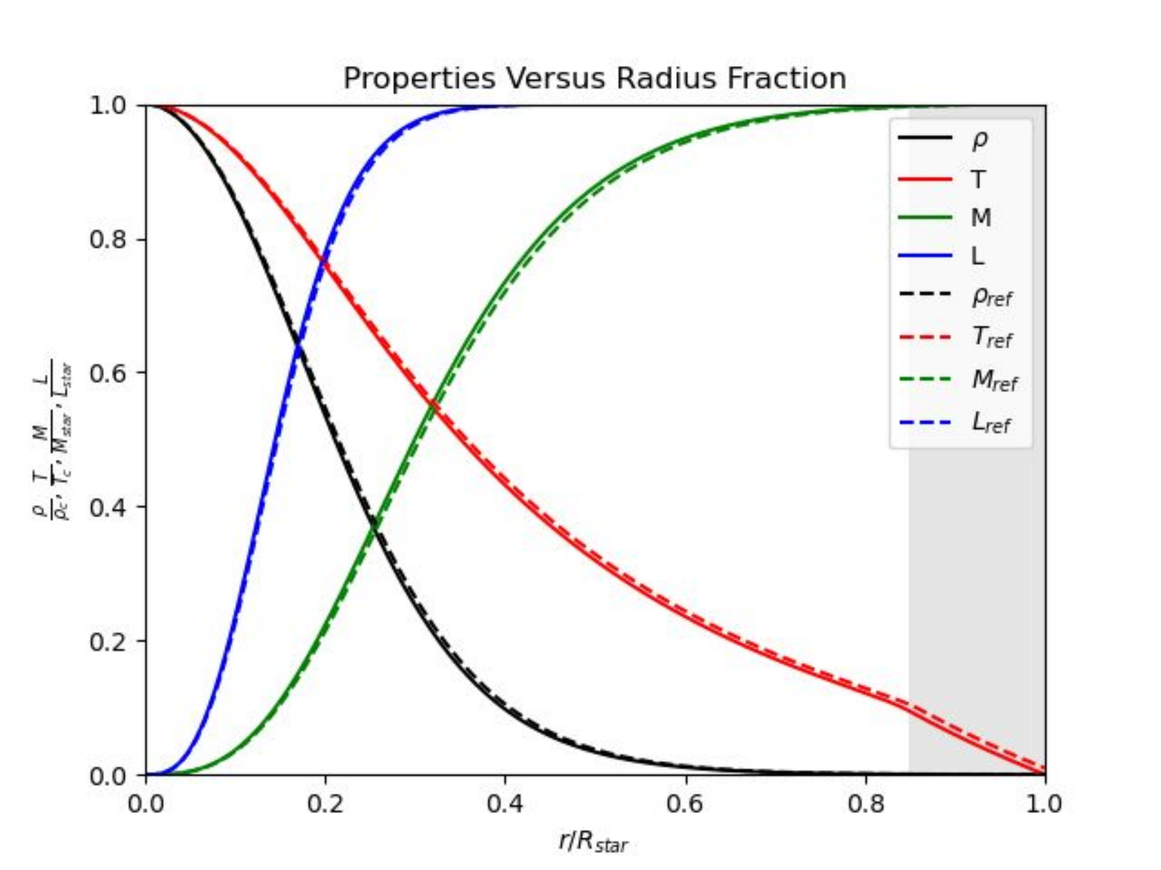
Verification Process of Results

M = 0.66 Solar Mass Star

R = 0.85 Solar Radius

Luminosity = 0.06 Solar Luminosity

Surface Temperature = 3096 K



Gravity Modification: Updated Stellar Equations

$$g = \frac{GM}{r^2}$$

$$g = \frac{GM}{r^2}\left(1 + \frac{\lambda}{r}\right)$$

Small Scale Adjustment

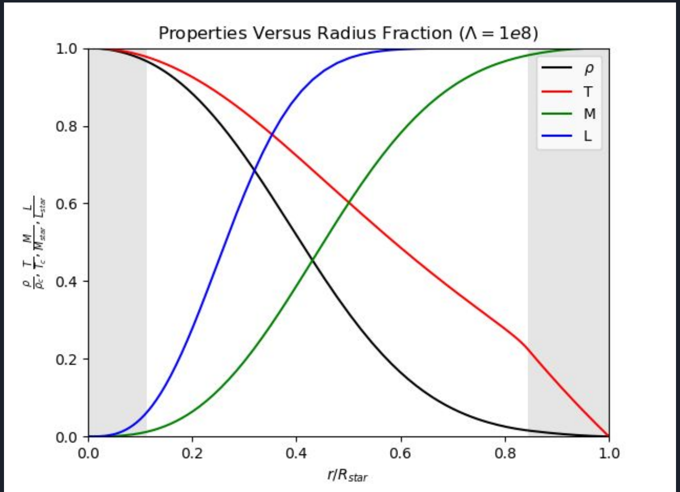
$$g = \frac{GM}{r^2}\left(1 + \frac{r}{\Lambda}\right)$$

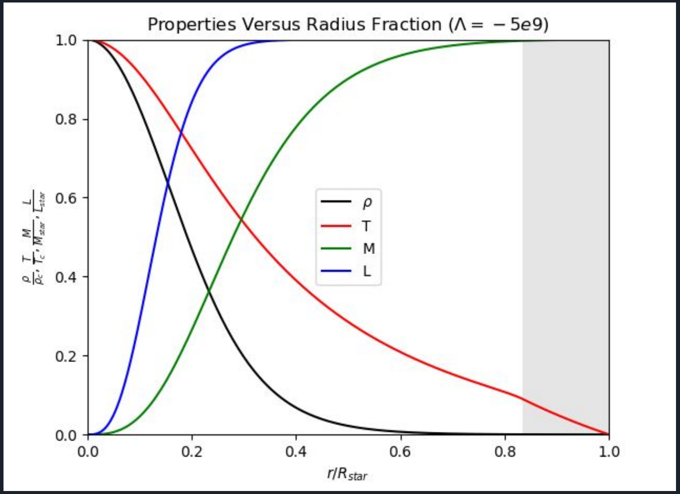
Large Scale Adjustment

$$[\lambda] = [\Lambda] = \mathrm{m}$$

Note: This only affects the density gradient and convective temperature gradient equations

Properties' Plots

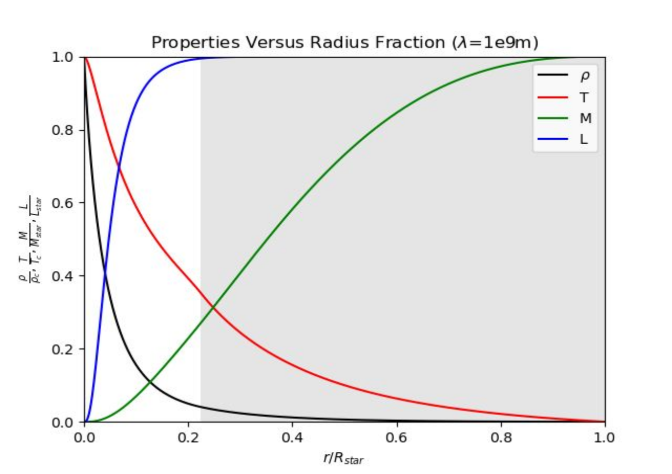


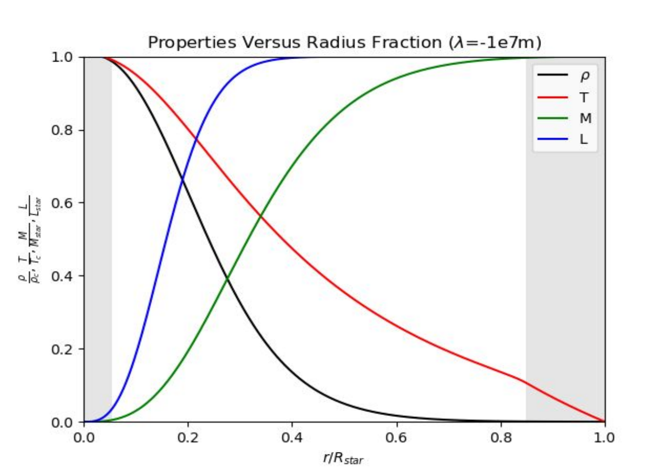


Convection Zones Shown in Gray

Observations:

- λ tends to compress/expand stellar core
- Positive λ creates a cusp at stellar center

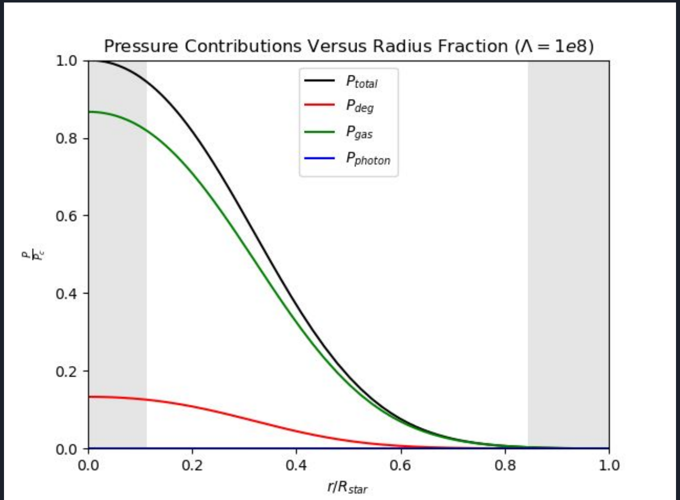


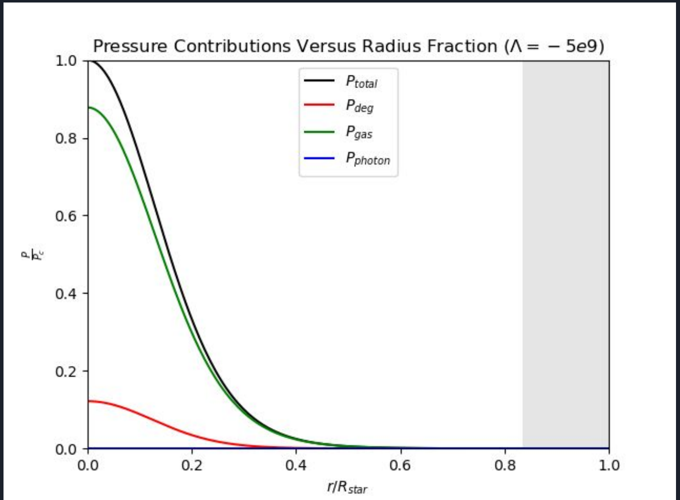


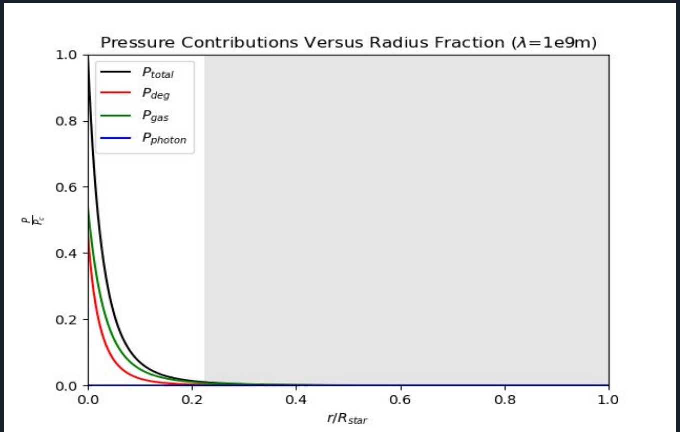
Pressure Plots

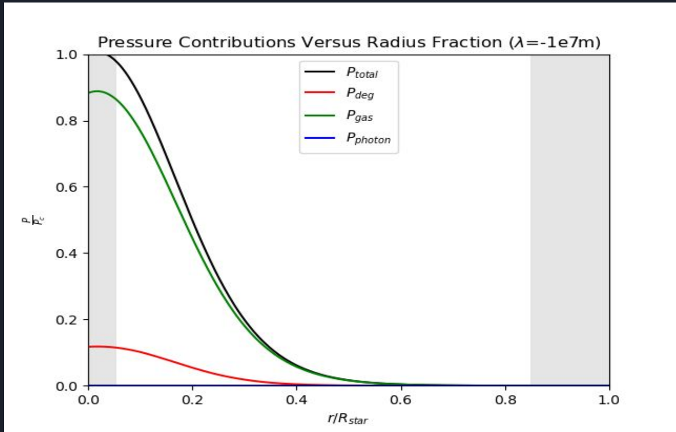
Observations:

- Degenerate Pressure dominates in all cases
- The pressure decreases rapidly at various radii, dependent on Λ and λ





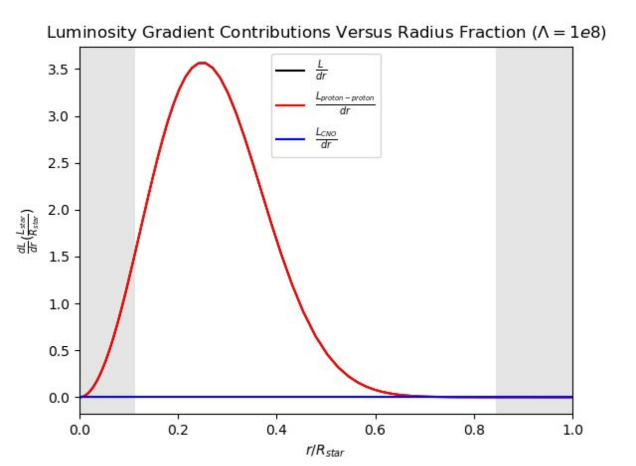


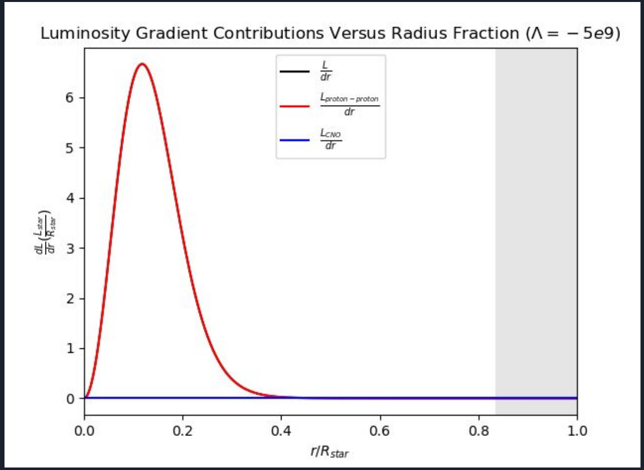


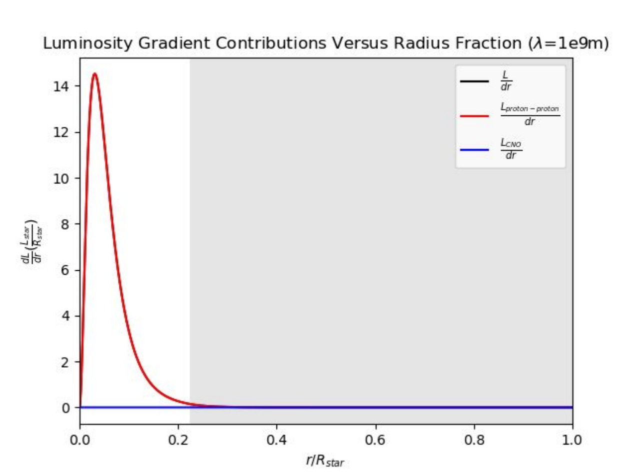
Energy Generation Plots

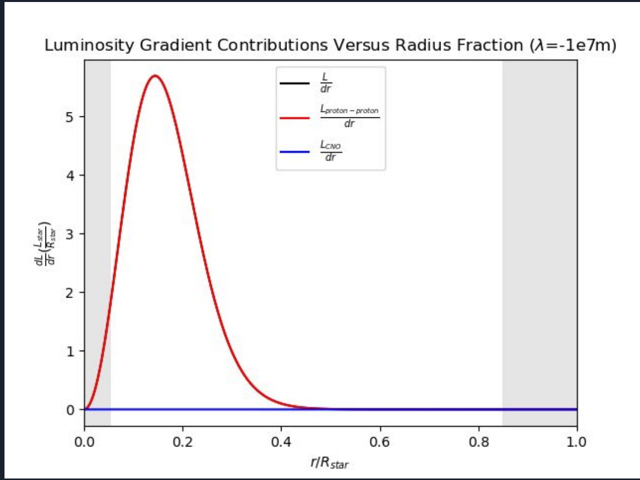
Observations:

- Proton-Proton Chain dominates in all cases
- The gradient is maximized at various radii, dependent on Λ and λ

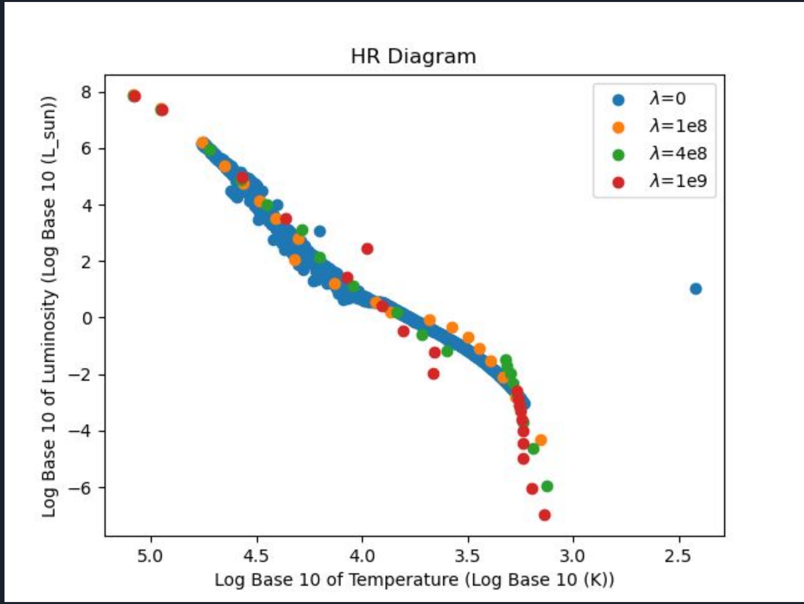


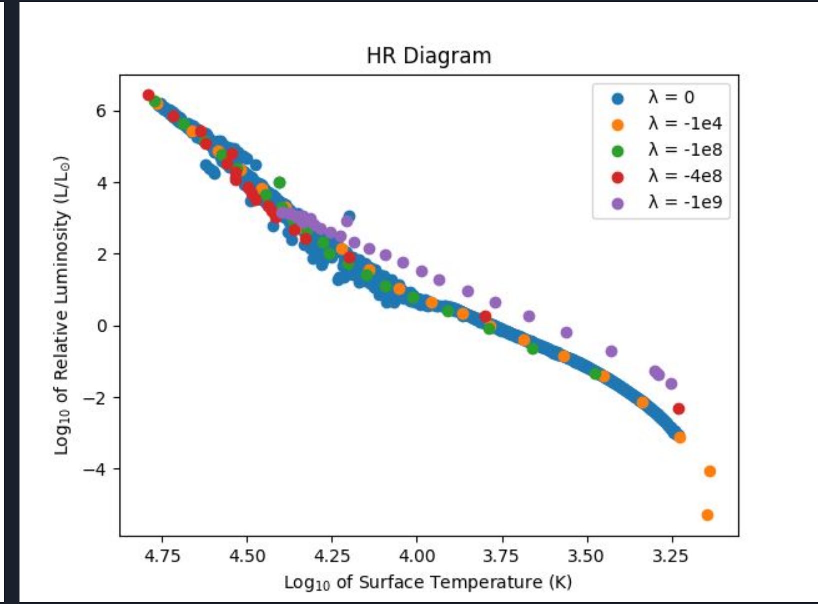






H-R Diagrams - Small Scale

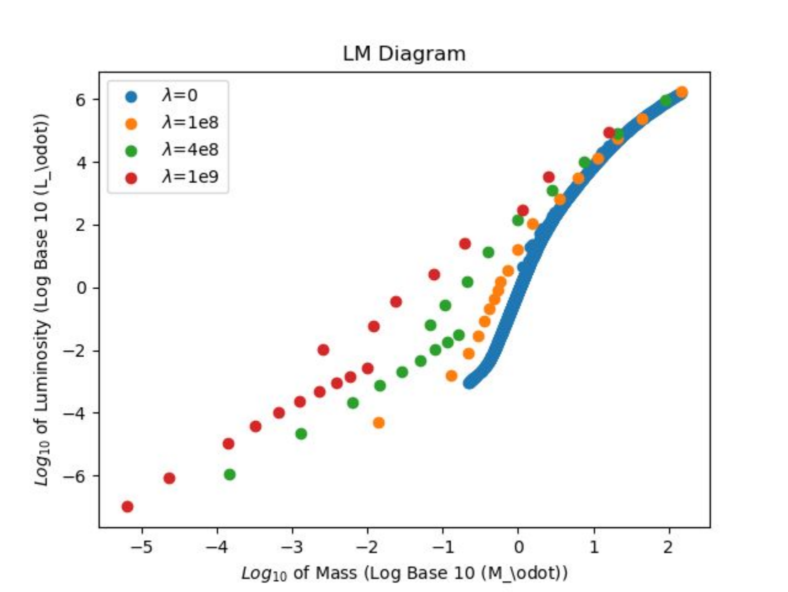


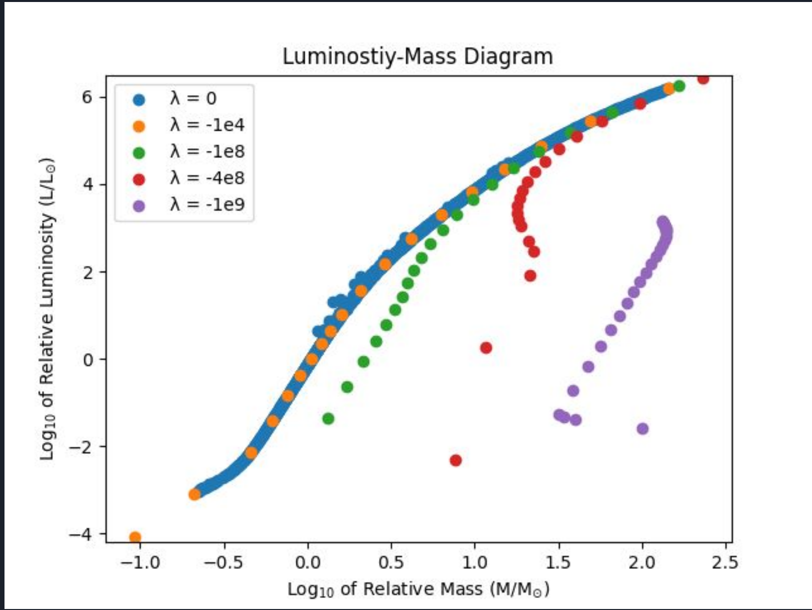


Observations:

- Breaks down above $|\lambda| = 1 \times 10^8$
- Radius of the sun is ~7×10^8

L-M Diagrams - Small Scale

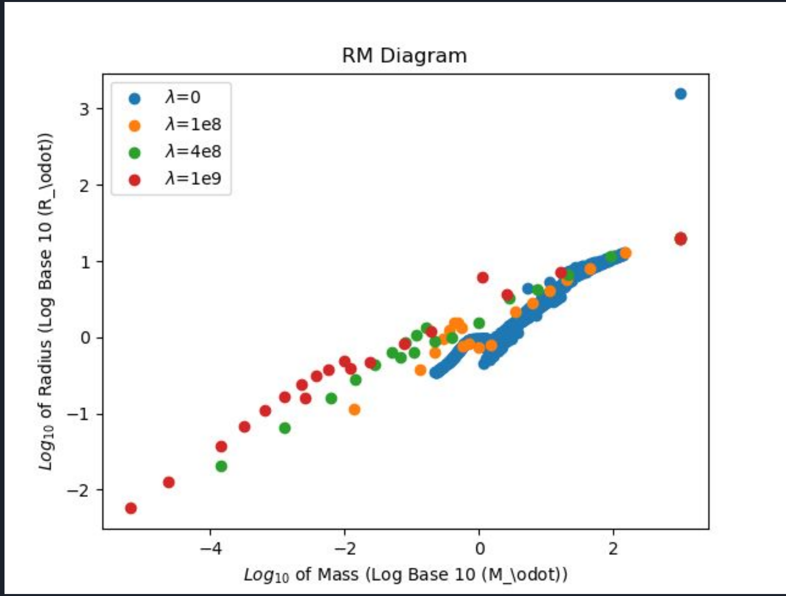


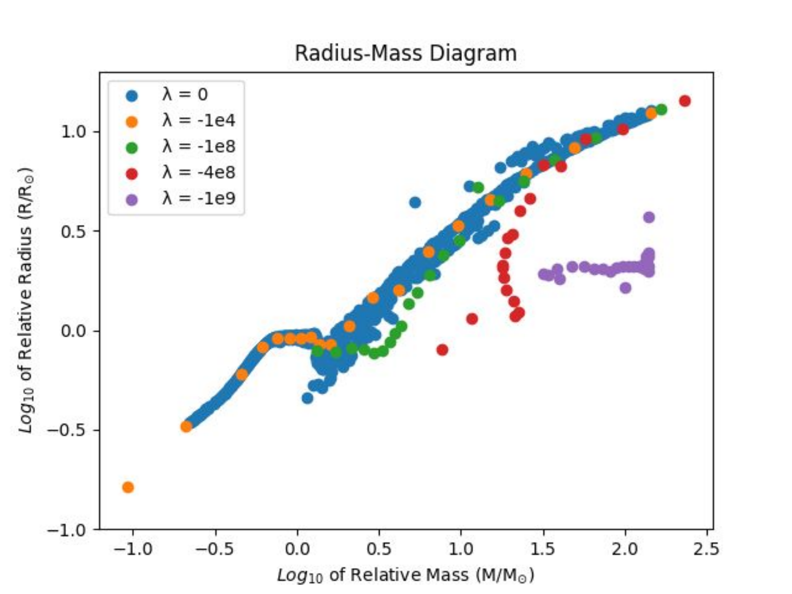


Observations:

- Positive λ increases luminosity for a given mass since compressed core has greater energy generation
- Varying |λ| above 10^8 changes L-M Diagram curve
- Breaks down above |λ| = 1 x 10^8
- Note: Radius of the sun is ~7x10^8

R-M Diagrams - Small Scale

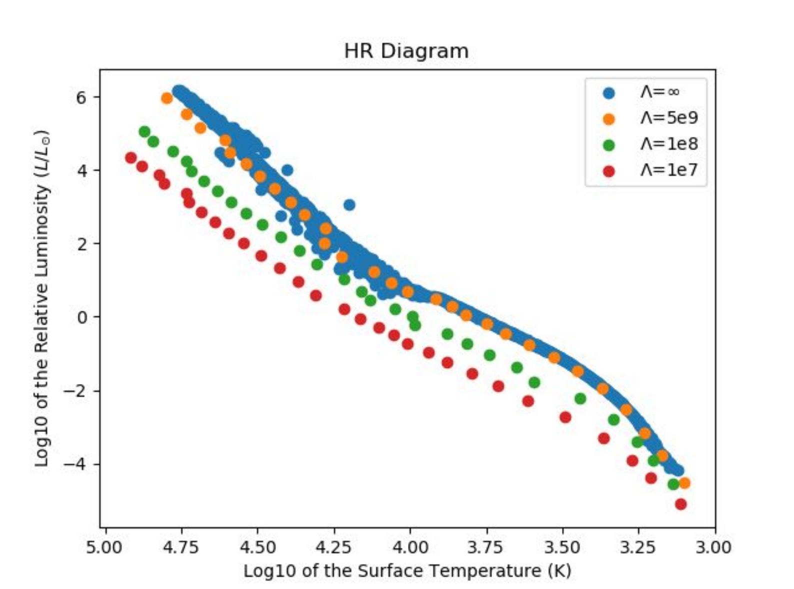


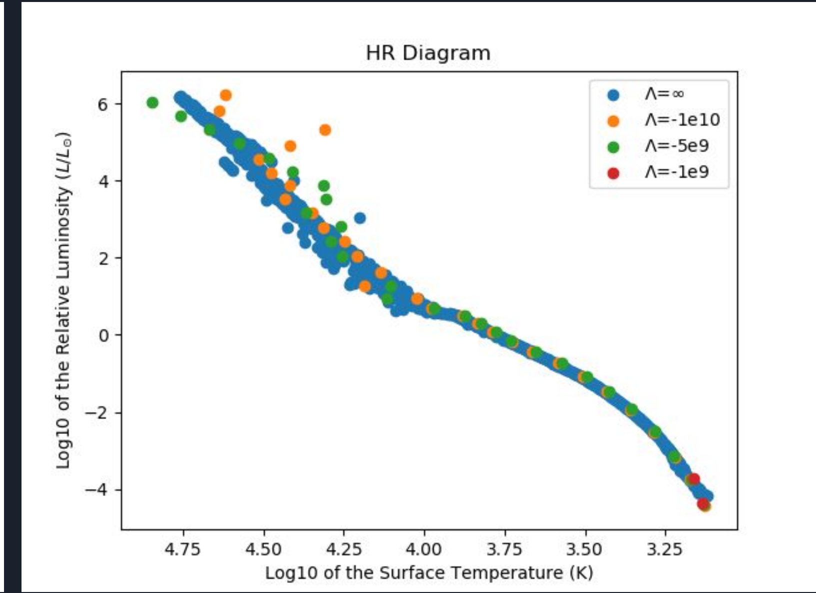


Observations:

- Greater lambda compresses the core, which expands the radius based on the Mirror principle
- Varying lambda near 10^8 moves the trend
- Breaks down below $|\Lambda| = 1 \times 10^9$
- Radius of the sun is ~7×10^8

H-R Diagrams - Large Scale

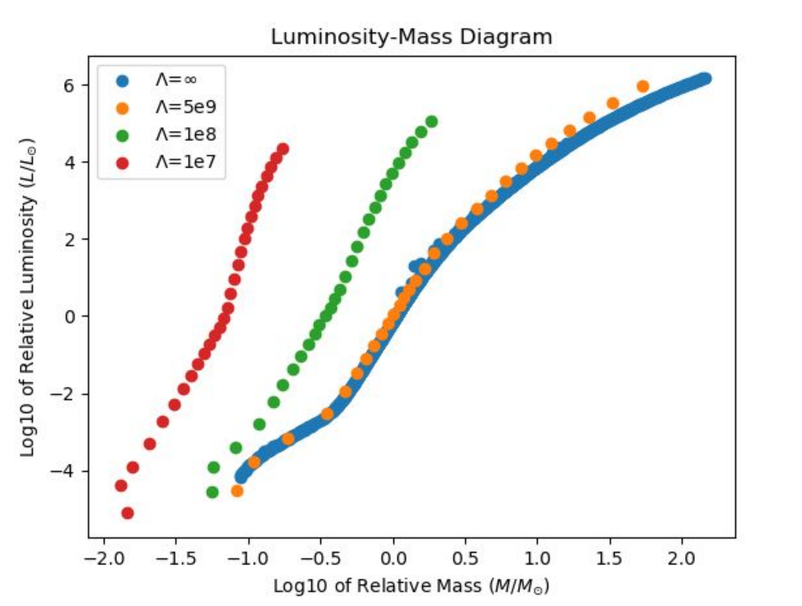


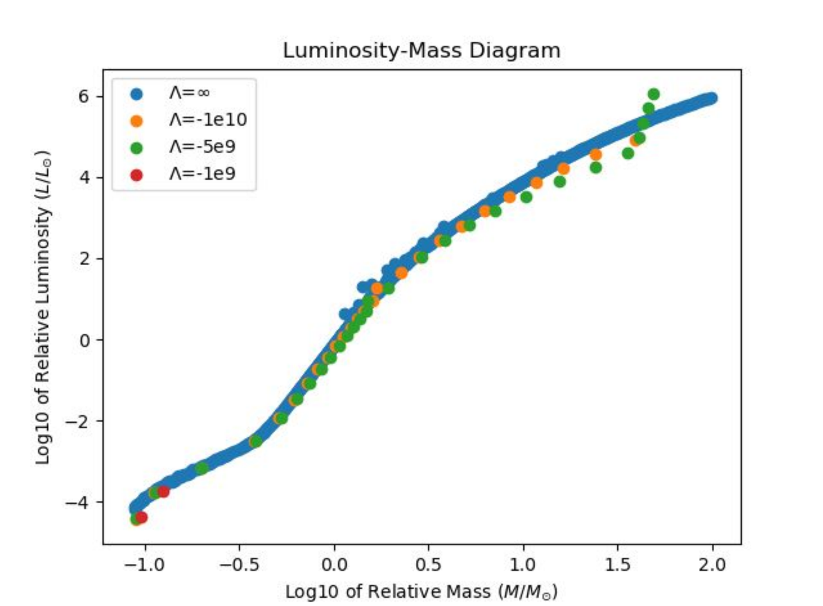


Observations:

- Varying Λ near 10^8 moves the trend line
- Breaks down below |Λ| = 1 x 10^9
- Radius of the sun is ~$7x10^8$

L-M Diagrams - Large Scale

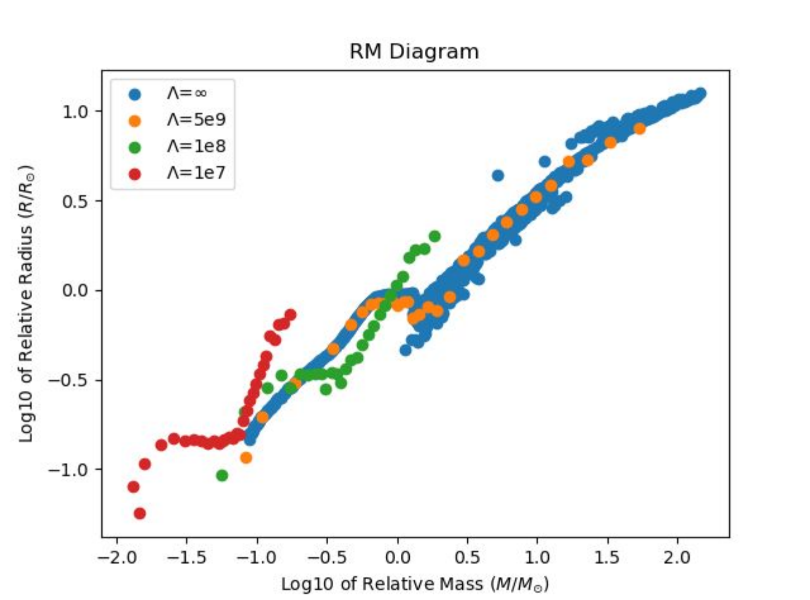


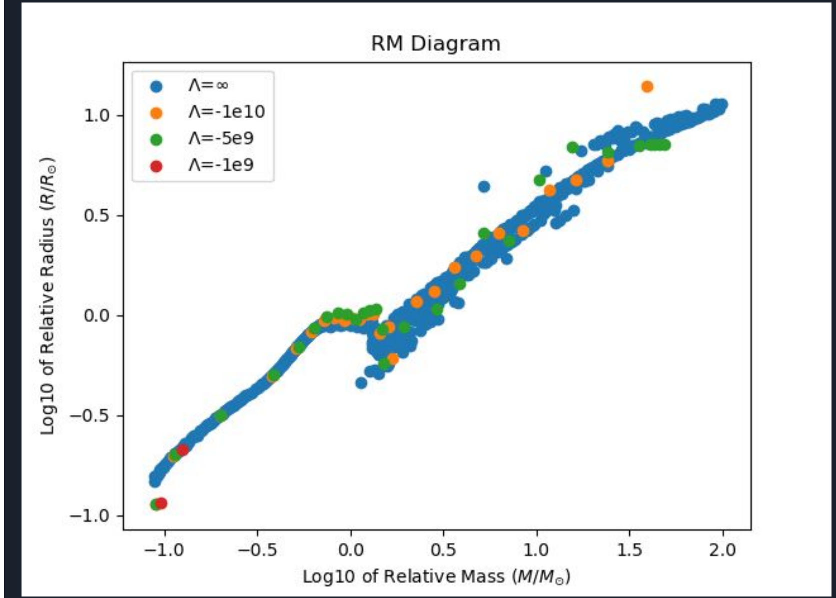


Observations:

- Varying Λ below 10^9 moves the trend line towards smaller relative masses
- Breaks down at and below Λ = - 1 x 10^9
- Note: Radius of the sun is ~7×10^8

R-M Diagrams - Large Scale





Observations:

- As lambda decreases, the strength of the gravity on large scales increases so the outer material is pulled inwards and the radii decrease
- Decreasing positive Λ below 10^9 has a large impact on stellar radii
- Negative Lambda breaks down easily beyond $\Lambda = -1 \times 10^9$

Trends Observed

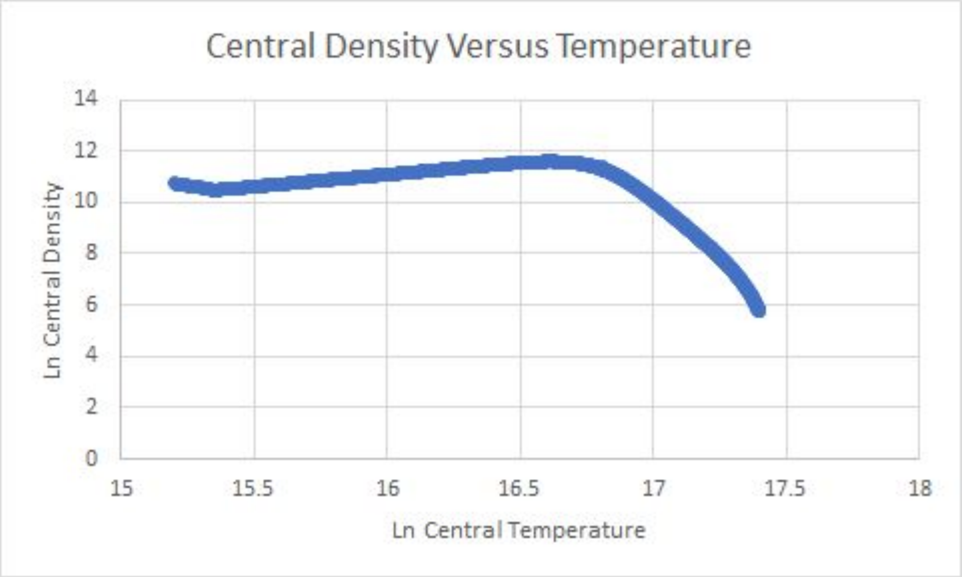
- Non-Convergent H-R, L-M, and R-M plots at sufficiently high λ and low Λ
- Largely Negative Λ and λ values caused the plot to break down immediately upon deviating from the $\Lambda = \infty$ and $\lambda = 0$ lines
- The behaviour of the H-R Diagrams remained unchanged, unlike the positioning
- Increasing positive λ and decreasing positive Λ produced larger luminosities at smaller masses
- Increasing positive λ and decreasing positive Λ produced larger radii at smaller masses
- For minimal changes to observed stellar relationships, $|\lambda| < 10^8$m or $|\Lambda| > 10^8$m

Algorithms and Analytical Techniques Used

- Adaptive step RK45, with variable local error
- Simple bisection algorithm
  - Adaptively increase local error requirements as rho_c converges
  - Speed up convergence by estimating rho_c: ln(10)/ln(2)=3.32
- Generating a sequence of stars with varying T_c
  - Use information from previous stars to prediction rho_c
  - Prediction overhead is negligible
  - Linear prediction in log-log graph based on last 2 data points
  - Could likely be improved with quadratic fit

Rho_c Prediction Accuracy

Rho_c Prediction Absolute Error	Proportion
<0.01	13.1%
< 0.1	39.1%
<1	56.8%
<10	87%



Mean Percentage Error: 0.25%

Thank you for listening!