PARCIAL # 1
Señales y Sistemas
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① Se tiene un microprocesador de 5 bits con entrada análogo de -3.3 a 5 [V]. Diseñe el sistema de acondicionamiento y digitalización para la señal

$$x(t) : 20 \sin(7t - \pi/2) - 3\cos(5t) + 2\cos(10t).$$

$W_1 = 7 = \frac{2\pi}{T_1} = 2\pi F$

$F_1 = \frac{7}{2\pi} = 1.11 \, Hz$

$W_2 = 5 = \frac{2\pi}{T_2} = 2\pi F$

$F_2 = \frac{5}{2\pi} = 0.79 \, Hz$

$W_3 = 10 = \frac{2\pi}{T_3} = 2\pi F$

$F_3 = \frac{10}{2\pi} = \frac{5}{\pi} = 1.59 \, Hz$

$Fs \geq 2 \max(F_1, F_2, F_3)$

$Fs \geq 2 \cdot 1.59$

$Fs = 3.18$

$x(t) = 20 \, sen(7t - \pi/2) - 3\cos(5t) + 2\cos(10t)$

$= sen(7t - \pi/2)$

$= sen(7t) \cdot \cos(\pi/2) - \cos(7t) \cdot sen(\pi/2)$

$= -\cos(7t).$

$$x(t) = -20\cos(7t) - 3\cos(5t) + 2\cos(10t).$$

$Y_{min} = -3.3\,V$ $\qquad$ $X_{max} = 25\,V$ Suma de Amplitudes

$Y_{max} = 5\,V$ $\qquad$ $X_{min} = -25\,V$

$$m = \frac{Y_{max} - Y_{min}}{X_{max} - X_{min}} = \frac{5\,V - (-3.3)}{25 - (-25)} = \boxed{0.17}$$

$c = y - mx$

$c = (-3.3) - [(0.17)\cdot(-25)]$

$\boxed{c = 0.95}$

X ajustada $\boxed{= mx + c}$

$x(0.17) + (0.95)$

X ajustada $= -3.4\cos(7t) - 0.51(5t) + 0.34\cos(10t)$

$\text{Bits} = 2^5 = 32$ Niveles Digitales

2. Cuál es la señal obtenida en tiempo discreto al utilizar un conversor análogo digital con frecuencia de muestreo de 5 KHZ aplicado a la señal

$$x(t) = 3\cos(1000\pi t) + 5\sin(2000\pi t) + 10\cos(11000\pi t)$$

d Realizar la simulación del proceso de discretización. En caso de que la discretizador no sea apropiada. diseñe un conversor adecuado. para la señal.

Punto #2 Hallar Frecuencias.

$$W_1 = 1000\pi = \frac{2\pi}{T_1} = 2\pi F_1$$

$$F_1 = \frac{1000\pi}{2\pi} = 500\,Hz$$

$$W_2 = 2000\pi = \frac{2\pi}{T_2} = 2\pi F_2$$

$$F_2 = \frac{2000\pi}{2\pi} = 1000\,Hz$$

$$W_3 = 11000\pi = \frac{2\pi}{T_3} = 2\pi F_3$$

$$F_3 = \frac{11000}{2\pi} = 5500\,Hz$$

$$F_s \geq 2\max(F_1, F_2, F_3)$$

$$F_s \geq 2 \quad 5500\,Hz$$

$$5\,kHz < 11000\,Hz$$

No es apropiada

$$\boxed{X_a(t)} \xrightarrow[F_s = 1/T]{} X(n) = X_a\boxed{(nT)}$$

$$t = nT$$

$$T = \frac{1}{F_s} \qquad F_s = \frac{1}{T} \qquad T = \frac{1}{F_s}$$

PUNTO 2:

$$x(t) = 3\cos(1000\pi t) + 5\sin(2000\pi t) + 10\cos(11000\pi t).$$

$$\Omega = \frac{2\pi}{N} = 2\pi F$$

$$f = \frac{1}{N} = \frac{F}{F_s} \qquad \Omega = 2\pi F.$$

$W_1 = 500\,Hz$ $\qquad W_2 = 1000\,Hz$ $\qquad W_3 = 5500\,Hz$

$$x(n) = 3\cos(1000\pi nT) + 5\sin(2000\pi nT) + 10\cos(11000\pi nT)$$

$$X(n) = 3\cos\left(\frac{1000\pi n}{F_s}\right) + 5\sin\left(\frac{2000\pi n}{F_s}\right) + 10\cos\left(\frac{11000\, n}{F_s}\right)$$

$$X(n) = 3\cos\left(\frac{2\pi 500\,Hz\ n}{5000\,Hz}\right) + 5\sin\left(\frac{2\pi 1000\,Hz\, n}{5000\,Hz}\right) + 10\cos\left(\frac{2\pi 5500\,Hz\, n}{5000\,Hz}\right)$$

$$X(n) = 3\cos\left(\frac{\overset{\Omega n}{2\pi n}}{\cancel{10}}\right) + 5\sin\left(\frac{2\pi n}{5}\right) + 10\cos\left(\frac{22\pi n}{\cancel{10}}\right)$$

$$\rightarrow X(n) = 3\cos\left(\frac{\pi n}{5}\right) + 5\sin\left(\frac{2\pi n}{5}\right) + 10\cos\left(\frac{11\pi n}{5}\right).$$

Restamos 2π para dejar el intervalo

(Ω original Ω copia $\pm k\,2\pi$)

$$\Omega_3 = \frac{11\pi}{5} \notin [-\pi, \pi]$$

$$\Omega_3 - 2\pi = \frac{11\pi}{5} - \frac{10\pi}{5} = \frac{\pi}{5}$$

$$x[n/F_s] = 3\cos\left[\frac{\pi}{5}n\right] + 5\sin\left[\frac{2\pi}{5}n\right] + 10\cos\left[\frac{\pi}{5}n\right]$$

Podemos sumar las frecuencias angulares iguales

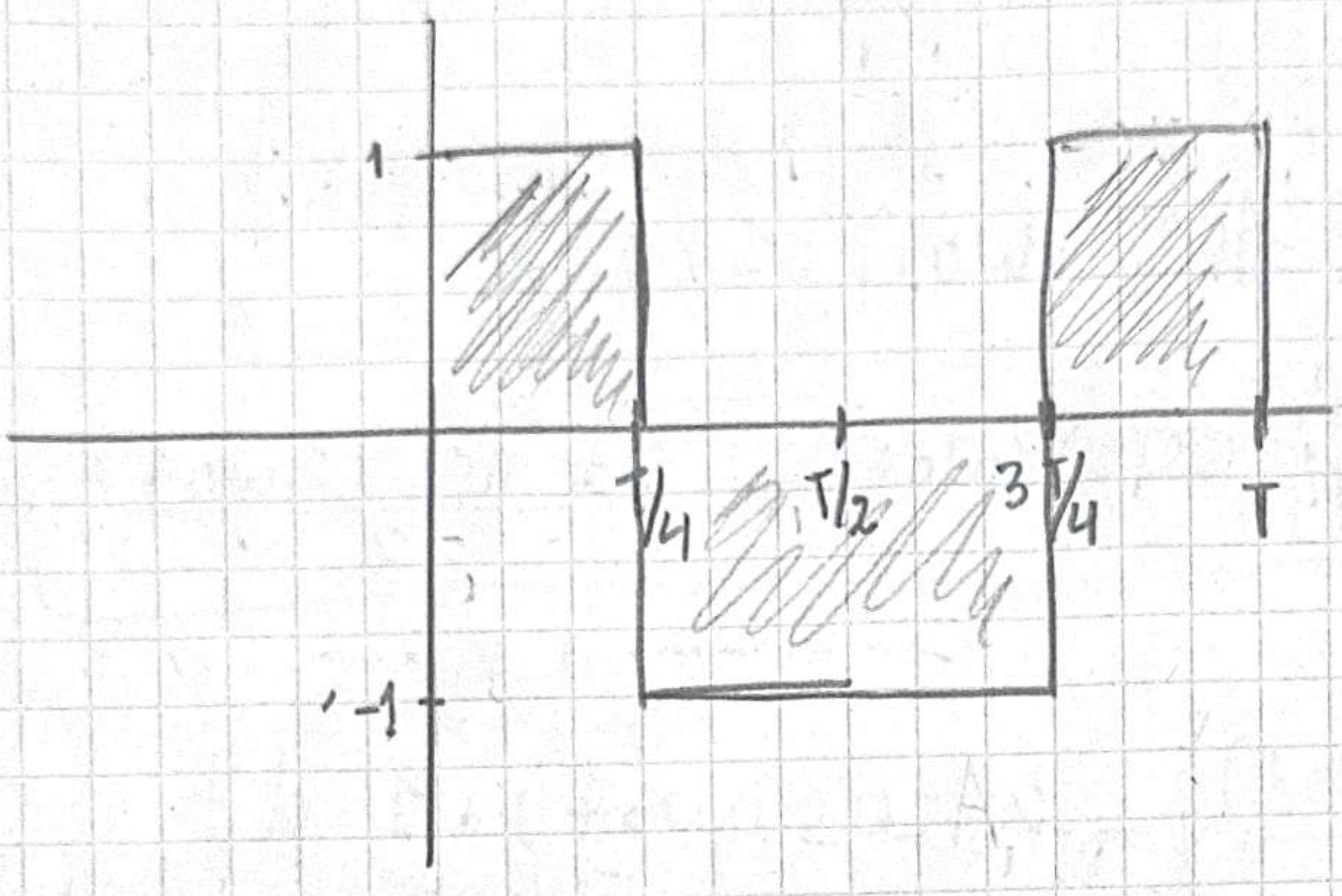
$$x[n/F_s] = 13\cos\left[\frac{\pi}{5}n\right] + 5\sin\left[\frac{2\pi}{5}n\right]$$

3. Sean $x_1(t)$ y $x_2(t)$,

$$x_1(t) = A\cos(W_0 t) \qquad W_0 = \frac{2\pi}{T} \qquad T, A \in \mathbb{R}^+$$

$$x_2(t) = \begin{cases} 1 & \text{si } 0 < t < T/4 \\ -1 & \text{si } T/4 \leq t < \frac{3T}{4} \\ 1 & \text{si } \frac{3T}{4} \leq t < T. \end{cases}$$

¿Cual es la distancia media entre las señales, corrobore sus desarollos con Sympy.



x_2 dividir la integral en tres partes.

$$\overline{P}_{x_1-x_2} = \lim_{T\to\infty} \left[\int_0^{T/4} |Acos(w_0t) - 1|^2 dt + \int_{T/4}^{3/T4} |Acos(w_0t) + 1|^2 dt \right.$$

$$+ \left[\int_{3T/4}^{T} |Acos(w_0t) - 1|^2 \right.$$

Factorizar.

$$\overline{P}_{x_1-x_2} = \lim_{T\to\infty} \frac{1}{T} \int_0^{T/4} (A^2cos^2(w_0t) - 2Acos(w_0t) + 1)\, dt$$

$$+ \int_{T/4}^{3T/4} (A^2cos^2(w_0t) + 2Acos(w_0t + 1))\, dt$$

$$+ \int_{3T/4}^{T} (A^2cos^2(w_0t) - 2Acos(w_0t) + 1)\, dt$$

Resuelvemos par $0 \leq t < T/4$

$$\int_0^{T/4} A^2\cos^2(w_0t) - 2A\cos(w_0t) + 1)\,dt$$

Propiedad. $\boxed{\cos^2(w_0t) = \frac{1+\cos(2w_0t)}{2}}$

$$\int_0^{T/4} A^2\left(\frac{1+\cos(2w_0t)}{2}\right) - 2A\cos(w_0t) + 1\,dt$$

$$\int_0^{T/4} \frac{A^2}{2} + \frac{A^2}{2}\cos(2w_0t) - 2A\cos(w_0t)\,dt + 1\,dt$$

$$\int_0^{T/4}\left(\frac{A^2}{2}\right) + \int_0^{T/4}\left(\frac{A^2}{2}\cos(2w_0t)\right)dt - \int_0^{T/4} 2A\cos(w_0t)\,dt + \int_0^{T/4} 1\,dt$$

Primer termino.

$$\int_0^{T/4} \frac{A^2}{2}\,dt = \frac{A^2}{2}\cdot\frac{T}{4} = \frac{TA^2}{8}$$

Segundo Termino

$$\int_0^{T/4} \frac{A^2}{2}\cos(2w_0t)\,dt = \frac{A^2}{2}\cdot\left[\frac{\sin(2w_0t)}{2w_0}\right]_0^{T/4}$$

$$= \frac{A^2}{4w_0}\left(\sin\left(\frac{T}{2}w_0\right) - \sin(0)\right)$$

$\sin\left(\frac{T}{2}w_0\right) = 0$ asumimos que $T = \frac{2\pi}{w_0} = \emptyset$

Soo

Tercer termino:

$$\int_0^{T/4} 2A^2\cos(w_0 t)\,dt = 2A\left[\frac{\sin(2w_0 t)}{w_0}\right]_0^{T/4}$$

$$\frac{W_0 T}{4} = \frac{2\pi}{4} = \frac{\pi}{2}\ A \quad \sin\left(\frac{\pi}{2}\right) = 1$$

$$= 2A/W_0$$

Cuarto termino.

$$\int_0^{T/4} 1\,dt = 1\frac{T}{4}$$

-0

$$\boxed{\frac{AT}{8}^2 = \frac{2A}{W_0} + \frac{T}{4}}$$ PARTE A.

Intervalo $T/4 \leq t \leq 3T/4$

$$\int_{T/4}^{3T/4} A^2\cos^2(W_0 t) + 2A\cos(W_0 t) + 1)\,dt$$

usamos identidad trigonometrica:

$$\boxed{\cos^2(W_0 t) = \frac{1+\cos(2W_0 t)}{2}}$$

$$\int_{T/4}^{3T/4} A^2\left(\frac{1+\cos(2w_0t)}{2} + 2A\cos(w_0t) + 1\right) dt$$

$$= \int_{T/4}^{3T/4} \left(\frac{A^2}{2} + \frac{A^2}{2}\cos(2w_0t) + 2A\cos(w_0t) + 1\right) dt$$

- Primer termino

$$= \int_{T/4}^{3T/4} \frac{A^2}{2} = \frac{A^2}{2}\left(\frac{3T}{4} - \frac{T}{4}\right) = \frac{A^2}{2}\cdot\frac{T}{2} = \boxed{\frac{TA^2}{4}}$$

- Segundo Termino

$$\int_{T/4}^{3T/4} \frac{A^2}{2}\cos(2w_0t)\, dt = \frac{A^2}{2}\cdot\left[\frac{\sin(2w_0t)}{2w_0}\right]_{T/4}^{3T/4}$$

$T = 2\pi/w_0 \rightarrow 2w_0\cdot 3T/4 = 3\pi \rightarrow 2w_0\cdot T/4 = \pi$

$$\frac{A^2}{2}\left(\frac{\sin(3\pi) - \sin(\pi)}{2w_0}\right) = \emptyset$$

- Tercer termino.

$$\int_{T/4}^{3T/4} 2A\cos(w_0t)\, dt = 2A\cdot\left[\frac{\sin(w_0t)}{w_0}\right]_{T/4}^{3T/4}$$

DD MM AA

$W_0 T = 2\pi$ • $W_0 \cdot 3T/4 = 3\pi/2 \Rightarrow \sin = -1$

• $W_0 \cdot T/4 = \pi/2 \Rightarrow \sin = 1$

$$2A\left(\frac{-1-1}{W_0}\right) = \frac{-4A}{W_0}$$

Parte 4

$$\int_{T/4}^{3T/4} 1\,dt = 1\left[\frac{3T}{4} - \frac{T}{4}\right] = \frac{2T}{4} = \frac{T}{2}$$

Entonces

$$\frac{TA^2}{4} + 0 - \frac{4A}{W_0} + \frac{T}{2} = \boxed{\frac{TA^2}{4} - \frac{4A}{W_0} + \frac{T}{2}}$$

PARTE B.

INTERVALO $3T/4 \leq t < T$.

$$\cos^2(W_0 t) = \left(\frac{1 + \cos(2W_0 t)}{2}\right)$$

$$\int_{3T/4}^{T} A^2\cos^2(W_0 t) - 2A\cos(W_0 t) + 1)\,dt$$

$$\int_{3T/4}^{T} \frac{A^2}{2} + \frac{A^2}{2}\cos(2W_0 t) - 2A\cos(W_0 t) + 1\,dt$$

$$= \int_{3T/4}^{T} \frac{A^2}{2}\,dt + \int_{3T/4}^{T} \frac{A^2}{2}\cos(2W_0 t)\,dt - \int_{3T/4}^{T} 2A\cos(W_0 t)\,dt + \int_{3T/4}^{T} 1\,dt$$

Scribe

Término A

$$\int_{3T/4}^{T} \frac{A^2}{2}\,dt = \frac{A^2}{2}\cdot\left(T - \frac{3T}{4}\right) = \frac{A^2}{2}\cdot\frac{T}{4} = \boxed{\frac{TA^2}{8}}$$

SEGUNDO TERMINO

$$\int_{3T/4}^{T} \frac{A^2}{2}\cos(2w_0t)\,dt = \frac{A^2}{2}\cdot\left[\frac{\sin(2w_0t)}{2w_0}\right]_{3T/4}^{T}$$

$w_0T = 2\pi \Rightarrow 2w_0T = 4\pi$

- $2w_0T = 4\pi \Rightarrow \sin(4\pi) = 0$
- $2w_0T\cdot 3T/4 \Rightarrow 3\pi \Rightarrow \sin(3\pi) = 0$

$$\Rightarrow \frac{A^2}{2}\left(\frac{0-0}{2w_0}\right) = \emptyset$$

TERMINO C.

$$\int_{3T/4}^{T} 2A\cos(w_0t)\,dt = 2A\left[\frac{\sin(w_0t)}{w_0}\right]_{3T/4}^{T}$$

$w_0T = 2\pi$

- $w_0T = 2\pi \Rightarrow \sin(2\pi = 0$

$w_0\cdot 3T/4 = 3\pi/2 \Rightarrow \sin(3\pi/2) = -1$

$$= 2A\left(\frac{0-(-1)}{w_0}\right) = 2A\frac{1}{w_0} = \frac{2A}{w_0}.$$

Cuarto termino.

$$\int_{3T/4}^{T} 1dt = 1\left[T - \frac{3T}{4}\right] = \frac{T4 - 3T}{4} = \frac{T}{4}$$

$$\frac{3TA^2}{8} + 0 - \frac{2A}{W_0} + \frac{T}{4} = \boxed{\frac{TA^2}{8} - \frac{2A}{W_0} + \frac{T}{4}}$$

$$\overline{P}_{x_1-x_2} = \lim_{T\to\infty} \frac{1}{T}\left[\left(\frac{TA^2}{8} - \frac{2A}{W_0} + \frac{T}{4}\right) + \frac{TA^2}{4} - \frac{4A}{W_0} + \frac{T}{2} + \frac{TA^2}{8} - \frac{2A}{W_0} + \frac{T}{4}\right]$$

$$P_{x_1-x_2} \lim_{T\to\infty} = \frac{1}{T}\left[\frac{TA^2}{8} - \frac{2A}{\frac{2\pi}{T}} + \frac{T}{4} + \frac{TA^2}{4} - \frac{4A}{\frac{2\pi}{T}} + \frac{2T}{2} + \frac{TA^2}{8} - \frac{2A}{\frac{2\pi}{T}} + \frac{T}{4}\right]$$

$$P \lim_{T\to\infty} = \frac{1}{T}\left[\frac{TA^2}{8} - \frac{2AT}{2\pi} + \frac{T}{4} + \frac{TA^2}{4} - \frac{4AT}{2\pi} + \frac{T}{2} + \frac{TA_2}{8} - \frac{2AT}{2\pi} + \frac{T}{4}\right]$$

$$\lim_{T\to\infty} \frac{1}{\cancel{T}}\cancel{T}\left[\frac{A^2}{8} - \frac{2A}{2\pi} + \frac{1}{4} + \frac{A^2}{4} - \frac{4A}{2\pi} + \frac{1}{2} + \frac{A^2}{8} - \frac{2A}{2\pi} + \frac{1}{4}\right]$$

$$= \left[\frac{\cancel{4}A^2}{\cancel{8}} \frac{\,}{2} - \frac{\cancel{8}A}{\cancel{2}\pi} + 1\right]$$

$$= \frac{A^2}{2} - \frac{4A}{\pi} + 1$$

4. Demuestre que los coeficientes de la serie exponencial de Fourier se pueden calcular según:

$$C_n = \frac{1}{(t_i - t_F)\, n^2 w_0^2} \int_{t_i}^{t_f} x''(t)\, e^{-jnw_0 t}\, dt \quad n \in \mathbb{Z}$$

¿Cómo se pueden calcular los coeficientes a_n y b_n desde $x''(t)$ en la serie trigonométrica de Fourier?

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, e^{-jnw_0 t}\, dt$$

$u = x(t)$ $\quad du = x'(t)\, dt$

$dv = e^{-jnw_0 t}$ $\quad v = \frac{j e^{-jnw_0 t}}{n w_0}$

$$\boxed{\int u\, dv = uv - \int v\, du}$$

$$C_n = \frac{1}{T}\left(\left[x(t)\, \frac{j e^{-jnw_0 t}}{n w_0}\right|_{-T/2}^{T/2} - \frac{1}{n w_0} \int_{-T/2}^{T/2} x'(t)\, e^{-jnw_0 t}\, dt\right)$$

$u = x'(t)$ $\quad du = x'(t)\,dt$

$dv = e^{-jnw_0t}$ $\quad v = \frac{je^{-jnw_0t}}{nw_0}$

$$\int_{-T/2}^{T/2} x'(t)e^{-jnw_0t}\,dt = \left[x'(t)\frac{je^{-jnw_0t}}{nw_0}\right|_{-T/2}^{T/2} - \frac{j}{nw_0}\int_{-T/2}^{T/2} x''(t)e^{-jnw_0t}\,dt$$

$$C_n = \frac{1}{T}\left(\left[x(t)\frac{je^{-jnw_0t}}{nw_0}\right|_{-T/2}^{T/2} - \frac{j}{nw_0}\left(\left[x'(t)\frac{je^{-jnw_0t}}{nw_0}\right|_{-T/2}^{T/2} - \frac{j}{nw_0}\int_{-T/2}^{T/2} x''(t)e^{-jnw_0t}\,dt\right)\right)$$

$$= \frac{1}{T}\left[jx(t)\frac{e^{-jnw_0t}}{nw_0}\right|_{-T/2}^{T/2} + \frac{1}{T}\left[x'(t)\frac{e^{-jnw_0t}}{n^2w_0^2}\right|_{-T/2}^{T/2} - \frac{1}{Tn^2w_0^2}\int_{-T/2}^{T/2} x''(t)e^{-jnw_0t}\,dt$$

Evaluamos $\quad w_0 = \frac{2\pi}{T}$

$$\frac{1}{T}\left[jx(t)\frac{e^{-jnw_0t}}{nw_0}\right|_{-T/2}^{T/2} = \frac{1}{T}\left(jx(T/2)\frac{Te^{-jn\pi}}{2n\pi} - jx(-T/2)\frac{Te^{jn\pi}}{2n\pi}\right)$$

$$= \frac{1}{T}\left(\frac{jT}{2n\pi}\left(x(T/2)e^{-jn\pi} - x(-T/2)e^{jn\pi}\right)\right)$$

$$= \frac{1}{n\pi}\left(x(T/2) + x(-T/2)\sin(n\pi)\right)$$

$= \emptyset$

Intercambio $x(t)$ por $x'(t)$

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-Jnw_0t}\,dt = -\frac{1}{Tn^2w_0^2}\int_{-T/2}^{T/2} x''(t)e^{-Jnw_0t}\,dt$$

$$T = t_F - t_i$$

$$-\frac{1}{(t_F - t_i)n^2w_0^2}\int_{t_i}^{t_F} x''(t)e^{-Jnw_0t}\,dt$$

$$\boxed{C_n = \frac{1}{(t_i - t_F)n^2w_0^2}\int_{t_i}^{t_f} x''(t)e^{-Jnw_0t}\,dt.}$$

Coeficientes de la serie trigonometrica

→ con el espectro de la serie de Fourier se calcular como:

$$C_n = \frac{1}{(t_i - t_F)n^2w_0^2}\int_{t_i}^{t_f} x''(t)e^{-Jnw_0t}\,dt$$

$$C_n = \frac{1}{(t_i - t_F)n^2w_0^2}\int_{t_i}^{t_f} x''(t)(\cos(nw_0t) - J\sin(nw_0t))\,dt$$

$$C_n = \frac{1}{(t_i - t_F)n^2w_0^2}\int_{t_i}^{t_f} x''(t)\cos(nw_0t) - \frac{J}{(t_i - t_F)n^2w_0^2}\int_{t_i}^{t_f} x''(t)\sin(nw_0t)$$

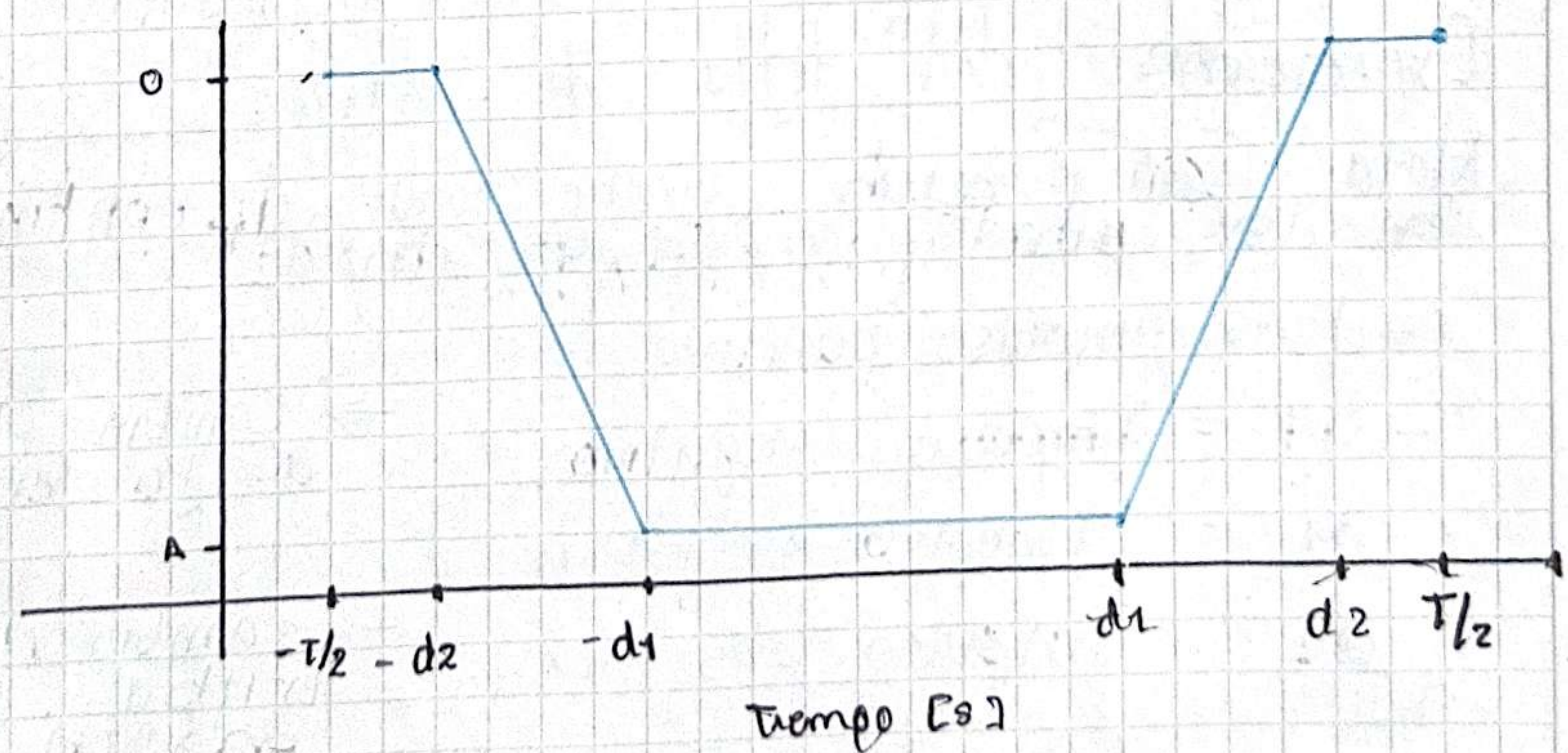
Utilizando igualdad de $a_n = 2\,\mathrm{Re}\{C_n\}$

$$\boxed{a_n = \frac{2}{(t_i - t_F)\, n^2 w_0^2} \int_{t_i}^{t_f} x''(t) \cos(n w_0 t)}$$

Utilizando igualdad de $b_n = 2\,\mathrm{Im}\{C_n\}$

$$\boxed{b_n = \frac{2}{(t_i - t_F)\, n^2 w_0^2} \int_{t_i}^{t_f} x''(t) \sin(n w_0 t)}$$

Encuentre el espectro de Fourier, su magnitud fase, parte real e imaginaria y el error relativo de reconstrucción para $n \in \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$ a partir de $x''(t)$ para la señal $x(t)$ en la Figura 1. Compruebe el espectro obtenido con la estimación a partir de $x(t)$ y presente las respectivas simulaciones sobre python



Intervalo | Función

$$x(t) = \begin{cases} -T/2 \le t < -d_2 & x(t) = 0 \\ -d_2 \le t < -d_1 & x(t) = m_1 t + b_1 \\ -d_1 \le t < d_1 & x(t) = -A \\ d_1 \le t < d_2 & x(t) = m_2 t + b_1 \\ d_2 \le t < T/2 & x(t) = 0 \end{cases}$$

$0 - A$

derivamos para obtener pendientes m_1 e m_2.

$$m_1 = \frac{\Delta x}{\Delta t} = \frac{-A - 0}{-d_2 - (-d_1)} = \frac{-A}{d_1 - d_2}$$

$$m_2 = \frac{\Delta x}{\Delta t} = \frac{0 - (-A)}{d_1 - d_2} = \frac{A}{d_1 - d_2}$$

Expresemos con delta de dirac

Nota: Esta función tiene una discontinuidad en los puntos $-d_1, -d_2, d_1$ y d_2

$-d_2$ = impulso positivo.

$-d_1$ = Impulso negativo → cambio hacia abajo en positivo $x''(t)$

d_1 = Impulso negativo

d_2 = impulso positivo. → cambio brusco hacia arriba es negativo en $x''(t)$.

DD MM AA

"$x(t)''$ = se puede representar por la discontinuidad

$x''(t)$ igual a:

$$x''(t) = \delta(t+d_2) - \delta(t+1) + \delta(t-d_1) - \delta(t-d_2)$$

$$C_n = \frac{1}{-Tn^2 W_o^2}\left[\int \delta(t+d_2)e^{-jnw_o t}dt - \int \delta(t+d_1)e^{-jnw_o t}dt + \int \delta(t-d_1)e^{-jnw_o t}dt - \int \delta(t-d_2)e^{-jnw_o t}dt\right]$$

Recodemos

$$\int_{-\infty}^{\infty} x(\pm t_0)\,\delta(t \pm t_0)\,dt = x(\pm t_0)$$

propiedad Delta Dirac

$$C_n = \frac{1}{-Tn^2 W_o^2}\left[e^{jnw_o d_2} - e^{jnw_o d_1} + e^{-jnw_o d_1} - e^{-jnw_o d_2}\right]$$

Descomponemos

$$= \cancel{\cos(nw_o d_2)} + j\,\mathrm{sen}(nw_o d_2) - (\cos(nw d_1) + j\,\mathrm{sen}(nw_o d_1))$$
$$+ \cancel{\cos(nw_o d_1)} - j\,\mathrm{sen}(nw_o d_1) - (\cancel{\cos(nw_o d_2)} - j\,\mathrm{sen}(nw_o d_2))$$

$$= 2j\,\mathrm{sen}(nw_o d_2) - 2j\,\mathrm{sen}(nw_o d_1)$$

$$C_n = \frac{1}{-Tn^2 W_o^2}\left[2j\,\mathrm{sen}(nw_o d_2) - 2j\,\mathrm{sen}(nw_o d_1)\right]$$

Scribe

Magnitud

$C_n = a_n + b_n j$ con $a_n = 0$

Entonces

$C_n = b_n - j$

$|C_n| = |b_n j|$

$|C_n| = \sqrt{0^2 + b_n^2} = b_n$

$$|C_n| = \frac{2(\operatorname{sen}(n w_0 d_2) - \operatorname{sen}(n w_0 d_1))}{-T n^2 W_0^2}.$$

. Fase

$$\theta = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

$a_n = 0$. $\theta = \pm \pi/2$ por que esta en los imaginarios

$$\theta_{cn} = \begin{cases} \pi/2 & \text{si } d_1 > d_2 \\ -\pi/2 & \text{si } d_1 < d_2 \\ 0 & \text{si } d_1 = d_2. \end{cases}$$

- Parte Real: No tiene parte real
- Parte Imaginaria Es el mismo bn

$$bn = \frac{2}{-Tn^2 w_0^2} \cdot \left(Sen(n w_0 d_2) - Sen(n w_0 d_1) \right)$$

Error Relativo:

Al conocer los coeficientes de $x''(t)$ hayamos los coeficientes de $x(t)$

$$C_n'' = -n^2 w_0^2 \cdot C_n$$

$$C_n = \frac{C_n''}{-n^2 w_0^2} \rightarrow \text{Coeficientes } x(t) \; n \neq 0.$$

$$C_n'' = -n^2 w_0^2 \overbrace{\left(\frac{2J}{-Tn^2 w_0^2} (Sen(n w_0 d_2) - Sen(n w_0 d_1)) \right)}^{C_n}$$

$$C_n'' = \frac{2J}{T} (Sen(n w_0 d_2) - Sen(n w_0 d_1))$$

$$\lim_{N \to \infty} Px = \sum_{n=-N}^{N} |C_n|^2 \cdot P_n = 0.$$

$$P_x = \sum_{n=-\infty}^{\infty} |C_n|^2$$

$|C_n|^2$ entre los n $[-5, 5]$.

$$\sum_{n=-5}^{5} |C_n|^2$$

Que por simetria tenemos

$$\sum_{n=1}^{5} 2 \cdot |C_n|^2$$

$$|C_n| = \frac{2}{-Tn^2 w_0^2} (\operatorname{sen}(n w_0 d_2 - \operatorname{sen}(n w_0 d_1))$$

$$|C_n|^2 = \frac{4}{-T^2 n^4 w_0^4} (\operatorname{sen}(n w_0 d_2 - \operatorname{sen}(n w_0 d_1))$$

Error Relativo.

$$\% \, Er = \left(1 - \frac{\sum_{n=1}^{5} 2 |C_n|^2}{\sum_{n=-\infty}^{\infty} |C_n|^2} \right) \times 100\%$$

$$\% \, Er = \left[1 - \frac{\sum_{n=1}^{5} 2 \left(\frac{4}{-Tn^4 w_0^4} (\operatorname{sen}(n w_0 d_2) - \operatorname{sen}(n w_0 d_1)) \right)}{\sum_{n=-\infty}^{\infty} 2 \left(\frac{4}{-Tn^4 w_0^4} (\operatorname{sen}(n w_0 d_2) - \operatorname{sen}(n w_0 d_2) \right.} \right] \times 100\%$$