Demostrar los transformada de Laplace.

1.) $L\{e^{bt}u(t)\} = ?$
2.) $L\{\cos(w_0t)u(t)\} = ?$
3) $L\{\sin(w_0t)u(t)\} = ?$
4.) $L\{e^{bt}\cos(w_0t)u(t)\}$
5.) $L\{e^{bt}\sin(w_0t)u(t)\}$

1.) $L\{e^{bt}u(t)\}$.

$$= \int_0^{\infty} e^{bt} \cdot e^{-st} dt = \int_0^{\infty} e^{t(b-s)} dt$$

$$= \int_0^{\infty} e^{-t(s-b)} dt$$

$u = -t(s-b)$
$du = -(s-b)dt$
$dt = \frac{-1}{s-b} du$

$$= \int_0^{\infty} e^{u} \cdot \frac{1}{s-b} du$$

$$= -\frac{1}{s-b}\left[e^{u}\right]_0^{\infty} = \frac{-1}{s-b}\left[e^{-\infty} - e^{0}\right]$$

$$= -\frac{1}{s-b}[0-1] = \frac{1}{s-b}$$

Region de convergenaa

$$X(s) = \frac{1}{s-b} \; ; \; e^{-t(s-b)}$$

$$\bar{e}^{t(\sigma-b)} \longleftarrow e^{-t(\sigma+jw-s)}$$

$$\cdot\, e^{-tjw}$$

seno y cos
complejo $\in [-1,1]$

- $\sigma - b > 0 \rightarrow \sigma > b$

$$e^{-\infty(+\mathbb{N})} \rightarrow \emptyset$$

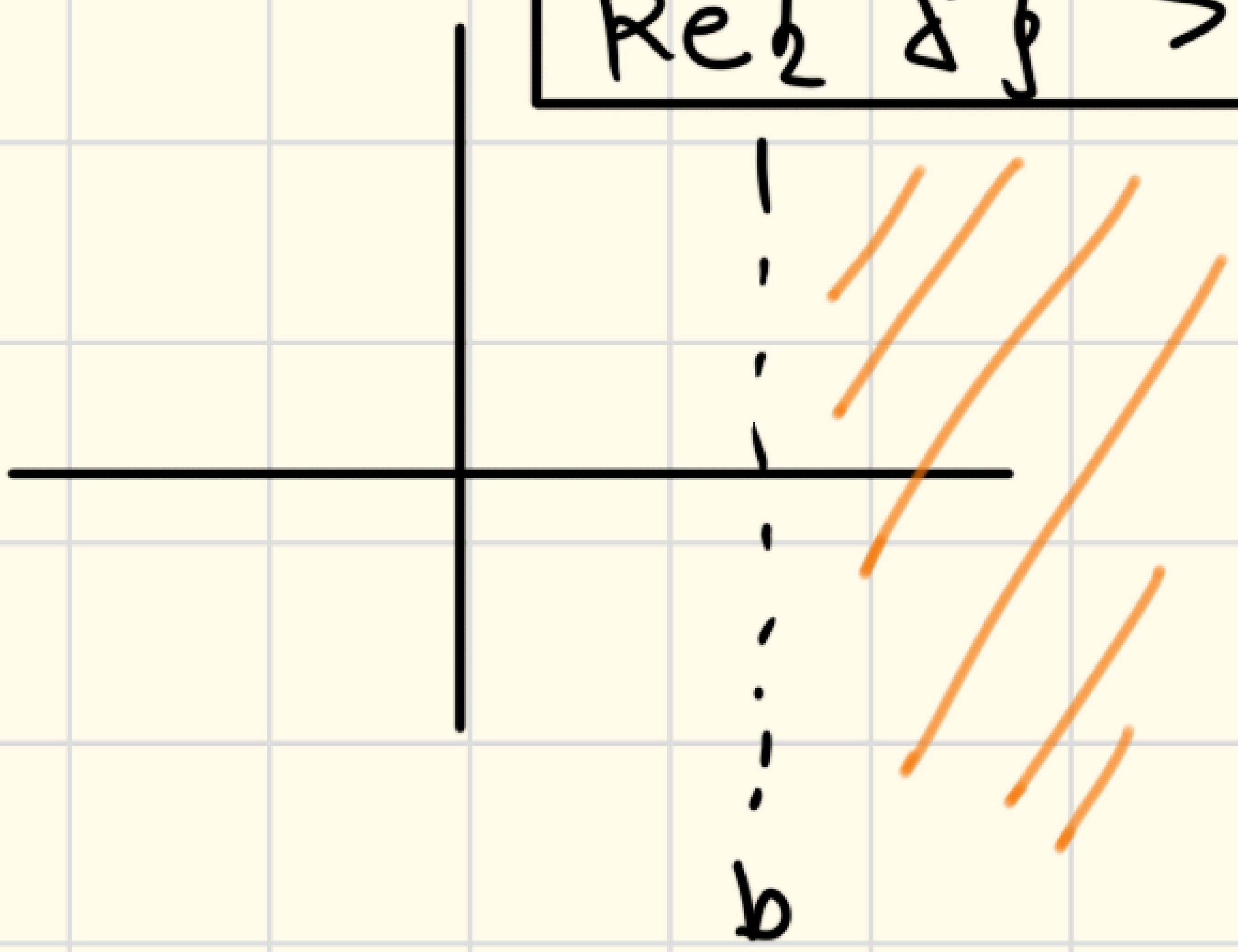
- $\sigma - b < 0 \rightarrow \sigma < b$

$$e^{-\infty(-\mathbb{N})} = e^{\infty} \rightarrow \infty$$

ROC debe estar en $\sigma - b > 0$
parte real

$$Re\{s\} - b > 0$$

$$\boxed{Re\{\sigma\} > b}$$



* $\mathcal{L}\{\cos(w_0 t) \cdot u(t)\}$

$$= \int_0^{\infty} \cos(w_0 t) \cdot e^{-bt}\, dt$$

$$= \int_0^{\infty} \frac{e^{jw_0 t} + e^{-jw_0 t}}{2} \cdot e^{-st}\, dt$$

$$= \frac{1}{2}\int_0^\infty e^{jw_0t - st} + e^{-jw_0t - st}\, dt$$

$$= \frac{1}{2}\int_0^\infty e^{-t(s-jw_0)} + e^{-t(s+jw_0)}\, dt$$

$$= \frac{1}{2}\left[\int_0^\infty e^{-t(s-jw_0)}\, dt + \int_0^\infty e^{-t(s+jw_0)}\, dt\right]$$

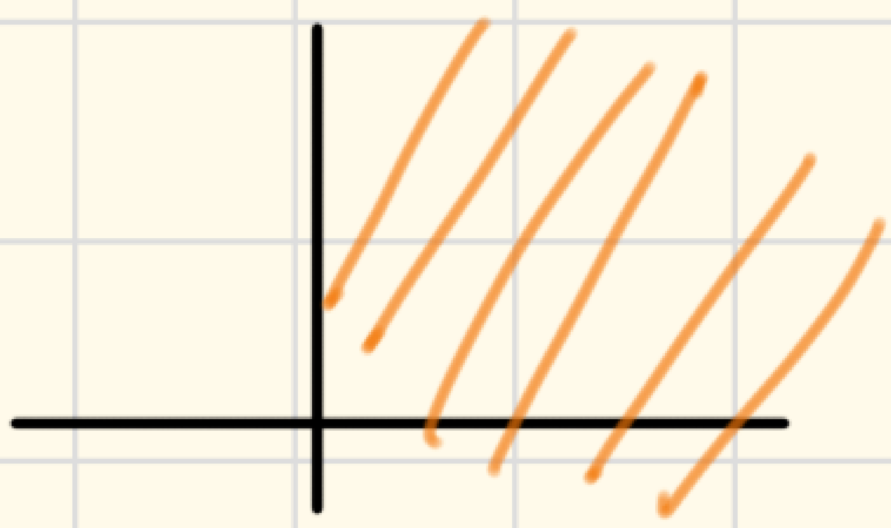
$u = -t(s - jw_0)$
$du = -(s - jw_0)\, dt$
$dt = -\frac{1}{s - jw_0}\, du$

$u = -t(s + jw_0)$
$du = -(s + jw_0)\, dt$
$dt = -\frac{1}{s + jw_0}\, dt$

$$= \frac{1}{2}\left[\frac{1}{jw_0 - s}\left[e^{-t(s-jw_0)}\right]_0^\infty - \frac{1}{s+jw_0}\left[e^{-t(s+jw_0)}\right]_0^\infty\right]$$

$-t(\sigma + jw - jw_0)$
$e^{-t\sigma} \cdot e^{-tjw} \cdot e^{tjw_0}$
$\in [-1, 1]$
$\circ\ e^{-t\sigma}$ $R\{s\} > 0$

$-t(\sigma + jw + jw_0)$
$e^{-t\sigma} \cdot e^{-tjw} \cdot e^{-tjw_0}$
$\in [-1, 1]$
$\bullet\ e^{-t\sigma}$ — $Re\{s\} > 0$



Entonces

$$= \frac{1}{2}\left[\frac{1}{jw_0 - s}\left[e^{-\infty} - e^{0}\right] - \frac{1}{s+jw_0}\left[e^{-\infty} - e^{0}\right]\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-jw_0} + \frac{1}{s+jw_0}\right]$$

$$= \frac{1}{2}\left[\frac{s+jw_0 + s - jw_0}{s^2 - (jw_0)^2}\right]$$

$$= \frac{1}{2}\left[\frac{2s}{s^2 - (jw_0)^2}\right] = \boxed{\frac{s}{s^2 - (jw_0)^2}}$$

3.) $\mathcal{L}\{\operatorname{Sen}(w_0 t) \cdot u(t)\}$

$$= \int_0^{\infty} \operatorname{sen}(w_0 t)\, e^{-st}\, dt = \int_0^{\infty} \frac{e^{jw_0 t} - e^{-jw_0 t}}{2j} \cdot e^{-st}\, dt$$

$$= \frac{1}{2j}\int_0^{\infty} e^{-t(s-jw_0)} - e^{-t(s+jw_0)}\, dt$$

$u = -t\,(s - jw_0)$ $\qquad$ $u = -t\,(s+jw_0)$

$du = -(s - jw_0)\, dt$ $\qquad$ $du = -(s + jw_0\, dt)$

$dt = \frac{-1}{s-jw_0}\, du$ $\qquad$ $dt = -\frac{1}{s+jw_0}\, du$

$$= \frac{1}{2j}\left[-\frac{1}{s-jw_0}\left(e^{-t(s-jw_0)}\right)_0^{\infty} + \frac{1}{s+jw_0}\left(e^{-t(s+jw_0)}\right)_0^{\infty}\right]$$

Tenemos

$-t\,(s - jw - jw_0)$ y $-t\,(s + jw + jw_0)$

$$e^{-t\sigma} - e^{t\sigma}\cdot e^{-tjw}\cdot e^{tjw_0}$$

$$e^{-t\sigma}\cdot \underbrace{e^{-tjw}\cdot e^{tjw_0}}_{\in[-1\,.\,1]}$$

$$\mathrm{Re}\{s\}>0.$$

$$-t(\sigma+jw+jw_0)$$

$$e^{-t\sigma}\cdot \underbrace{e^{-tjw}\cdot e^{-tjw_0}}_{\in[-1\,.\,1]}$$

$$\mathrm{Re}\{s\}>0$$

ROC.

$$=\frac{1}{2j}\left[-\frac{1}{sjw_0}\left(e^{-\infty}-e^{0}\right)^{1}+\frac{1}{sjw_0}\left(e^{-\infty}-e^{0}\right)\right]$$

$$=\frac{1}{2j}\left[\frac{1}{s-jw_0}-\frac{1}{s+jw_0}\right]$$

$$=\frac{1}{2j}\left[\frac{s+jw_0-s+jw_0}{s^2-(jw_0)^2}\right]=\frac{1}{\cancel{2}j}\left[\frac{\cancel{2}jw_0}{s^2-(jw_0)^2}\right]$$

$$=\frac{w_0}{s^2-(jw_0)^2}=\frac{w_0}{s^2-(-1)w_0^2}=\boxed{\frac{w_0}{s^2+w_0^2}}$$

$$*\ \mathcal{L}\{e^{bt}\cdot\cos(w_0t)\cdot u(t)\}.$$

$$=\int_0^\infty e^{bt}\cdot\frac{e^{jw_0t}+e^{-jw_0t}}{2}\cdot e^{-dt}\,dt$$

$$=\int_0^\infty \frac{1}{2}\cdot e^{-t(s-b-jw_0)}+e^{-t(s-b+jw_0)}\,dt$$

$$=\frac{1}{2}\int_0^\infty e^{-t(s-(b+jw_0))}+e^{-t(s-(b-jw_0))}\,dt$$

$u = -t\,(s-(b+jw_0))$ $\qquad u = -t\,(s-(b-jw_0))$

$du = -(s-(b+jw_0))$ $\qquad du = -(1-(b-jw_0))\,dt$

$dt = -1/s-(b+jw_0)\,du$ $\qquad dt = -1/s-(b-jw_0)\,du$

$$= 1/2\left[-\frac{1}{s-(b+jw_0)}\left(e^{-t(s-(b+jw_0))}\right)_0^{\infty} - \ldots\right.$$

$$\left.-\frac{1}{s-(b-jw_0)}\left(e^{-t(s-(b-jw_0))}\right)_0^{\infty}\right]$$

ROC

$e^{-t(s-(b+jw_0))}$

$e^{-t(\sigma+jw-b-jw_0)}$

$e^{-t(\sigma-b)}$ $\qquad e^{tj(w-w_0)} \in [-1,1]$

$e^{-t(s-(b-jw_0))}$ $\qquad e^{-t(\sigma-b)}$

$e^{-t(\sigma+jw-b+jw_0)}$ $\qquad e^{-tj(w+w_0)}$

$\sigma - b > 0 \rightarrow Re\{s\} > b$ $\qquad \in [-1,1]$

$$= 1/2\left[-\frac{1}{s-(b+jw_0)}\left(e^{-\infty}-e^{0}\right) - \frac{1}{s-(b-jw_0)}\left(e^{-\infty}-e^{0}\right)^{1}\right]$$

$$= 1/2\left[\frac{1}{s-(b+jw_0)} + \frac{1}{s+(b-jw_0)}\right]$$

$$= 1/2\left[\frac{s-(b-jw_0+s-(b+jw_0)}{s^2-s(b-jw_0)-s(b+jw_0)+(b^2+w_0^2)}\right]$$

$$= 1/2\left[2s-2b/s^2+(b^2+w_0^2)-2sb\right]$$

$$= 1/2\left[2(s-b)/(s^2-2sb+b^2+w_0^2\right]$$

$$\boxed{= s-b\,/(s-b)^2+w_0^2}$$

* $\mathcal{L}\{e^{bt}\ \text{sen}(w_0 t)\, u(t)\}$.

$$= \int_0^\infty e^{bt} \cdot \frac{e^{jw_0 t} + e^{-jw_0 t}}{2} \cdot e^{-dt} \quad dt$$

$$= \frac{1}{2j} \left[e^{-t(s-b-jw_0)} - e^{-t(s-b+jw_0)} \right] dt$$

$$= \frac{1}{2j} \int_0 e^{-t(s-(b+jw_0))} - e^{-t(s-(b-jw_0))} \quad dt$$

$u = -t(s-(b+jw_0))$ $\qquad$ $u = -t(s-(b-jw_0))$

$du = -(s-(b+jw_0))\,dt$ $\qquad$ $du = -(s-(b-jw_0))\,dt$

$dt = -\frac{1}{s-(b+jw_0)}\,dt$ $\qquad$ $dt = -\frac{1}{s-(b-jw_0)}\,du$

$$\frac{1}{2j}\left[-\frac{1}{s-(b+jw_0)} \left(e^{-t(s-(b+jw_0))} \right)_0^\infty + \dots \right.$$

$$\left. \dots + \frac{1}{s-(b-jw_0)} \left(e^{-t(1-(b-jw_0))} \right)_0^\infty \right]$$

ROC $\qquad$ sero

* $e^{-t(s-(b+jw_0))}$

$e^{-t(\sigma+jw-b-jw)}$

$e^{-t(\sigma-b)}$ $\qquad$ $-tj(w-w_0)$

$\downarrow$ $\qquad$ $\in [-1, 1]$

$\sigma - b > 0 \rightarrow \mathrm{Re}\{s\} > b$