

Activity 3.2. Problem Solving with Patterns

Answer the following.

I. True or False. Determine whether each of the following statements about Fibonacci numbers is true or false.

1. If n is even then F_n is an odd number. **FALSE**
2. F_{3n} is an even number **TRUE**
3. $3F_n - F_{n-2} = F_{n+2}$ **TRUE**
4. $F_n F_{n+3} = F_{n+1} F_{n+2}$ **FALSE**
5. $F_{n+3} = 5F_n - 2F_{n-2}$ for $n \geq 3$ **TRUE**

II. Answer the following as indicated and show your complete solution.

1. Use the definition of Fibonacci numbers to find the seventh and eighth Fibonacci numbers.

$$\begin{aligned}
 F_n &= F_{n-1} + F_{n-2} & F_0 &= 0; F_1 = 1 \\
 F_2 &= 0 + 1 = 1 & F_6 &= F_5 + F_4 \\
 & & &= 5 + 3 = 8 \\
 F_3 &= F_2 + F_1 & F_7 &= F_6 + F_5 \\
 &= 1 + 1 = 2 & &= 8 + 5 = 13 \\
 F_4 &= F_3 + F_2 & F_8 &= F_7 + F_6 \\
 &= 2 + 1 = 3 & &= 13 + 8 = 21 \\
 F_5 &= F_4 + F_3 & & \\
 &= 3 + 2 = 5 & &
 \end{aligned}$$

2. Use the definition of Fibonacci numbers to find the tenth Fibonacci number.

Binet's Formula for the n th Fibonacci number is given by

$$\begin{aligned}
 F_n &= \frac{1}{\sqrt{5}} \left[\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n \right] \\
 F_{10} &= \frac{1}{\sqrt{5}} \left[\left(\frac{1 + \sqrt{5}}{2} \right)^{10} - \left(\frac{1 - \sqrt{5}}{2} \right)^{10} \right] \\
 &= \frac{\left(\frac{1 + \sqrt{5}}{2} \right)^{10} - \left(\frac{1 - \sqrt{5}}{2} \right)^{10}}{\sqrt{5}} = 55
 \end{aligned}$$

3. Use Binet's formula and a calculator to find the 25th, 50th and 75th Fibonacci numbers.

$$\begin{aligned}
 F_n &= \frac{\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n}{\sqrt{5}} \\
 &= \frac{\left(\frac{1 + \sqrt{5}}{2} \right)^{25} - \left(\frac{1 - \sqrt{5}}{2} \right)^{25}}{\sqrt{5}} \\
 &= 75025
 \end{aligned}$$

$$\begin{aligned}
 F_n &= \frac{\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n}{\sqrt{5}} \\
 &= \frac{\left(\frac{1 + \sqrt{5}}{2} \right)^{50} - \left(\frac{1 - \sqrt{5}}{2} \right)^{50}}{\sqrt{5}} \\
 &= \frac{1.259616902 \times 10^{11} - 1.26 \times 10^{-11}}{2.236067977} \\
 &= 12586269025
 \end{aligned}$$

$$\begin{aligned}
 F_n &= \frac{\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n}{\sqrt{5}} \\
 &= \frac{\left(\frac{1 + \sqrt{5}}{2} \right)^{75} - \left(\frac{1 - \sqrt{5}}{2} \right)^{75}}{\sqrt{5}} \\
 &= \frac{2.111485078 \times 10^{15} - 2.11 \times 10^{-15}}{2.236067977} \\
 &= 2111485078 \times 10^{14}
 \end{aligned}$$

4. Find the third, fourth, fifth and sixth terms of the sequence defined by $a_1 = 2$, $a_3 = 3$ and $a_n = (-1)^n a_{n-1} + a_{n-2}$ for $n \geq 3$.

$$a_1 = 2 \quad a_3 = 3 \quad a_n = (-1)^n a_{n-1} + a_{n-2}$$

$$a_3 = (-1)^3 a_2 + a_1$$

$$a_3 = 3$$

$$a_5 = (-1)^5 a_4 + a_3$$

$$3 = (-1)^3 a_2 + 2$$

$$a_4 = (-1)^4 a_3 + a_2$$

$$a_5 = (-1)^5 2 + 3$$

$$3 = -a_2 + 2$$

$$a_4 = 3 + (-1)$$

$$a_5 = 1$$

$$a_2 = 2 - 3$$

$$a_4 = 2$$

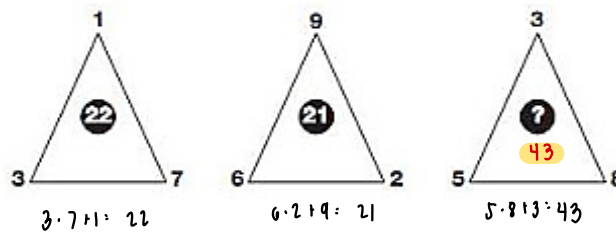
$$a_6 = (-1)^6 a_5 + a_4$$

$$a_6 = 1 + 2$$

$$= -1$$

$$a_6 = 3$$

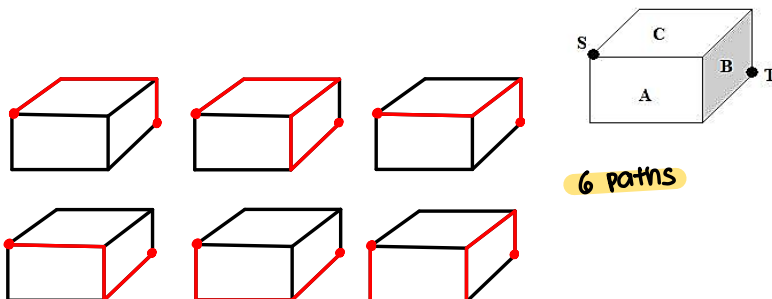
5. Can you solve the picture puzzle by replacing the question mark with the correct number?



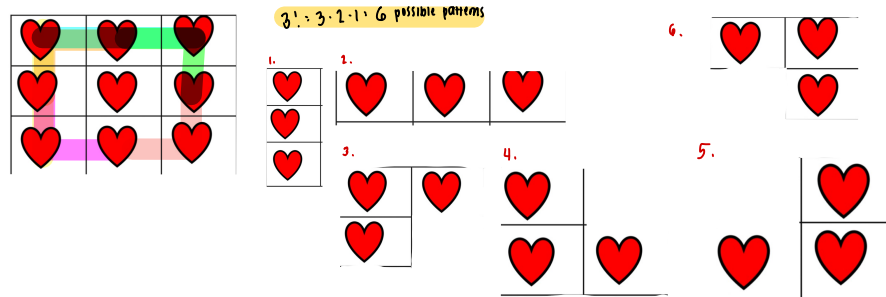
1. What's the next number in the given sequence?

1
11
21
1211
111221
312211
13112221
1113213211

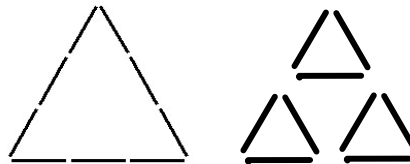
2. A plumber has to connect a pipe from a storage tank at the corner, S, of the roof to a tap at the diagonally opposite corner, T, in the figure below. Find the number of paths for the pipe if the pipe can only run along the edges of walls A, B, or roof C.



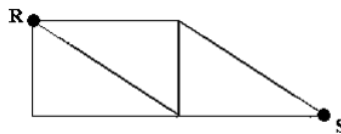
3. Three stamps are to be torn from a sheet of nine stamps as shown below. The three stamps must be intact so that each stamp is joined to another stamp along at least one edge. Find the possible patterns for these three stamps.



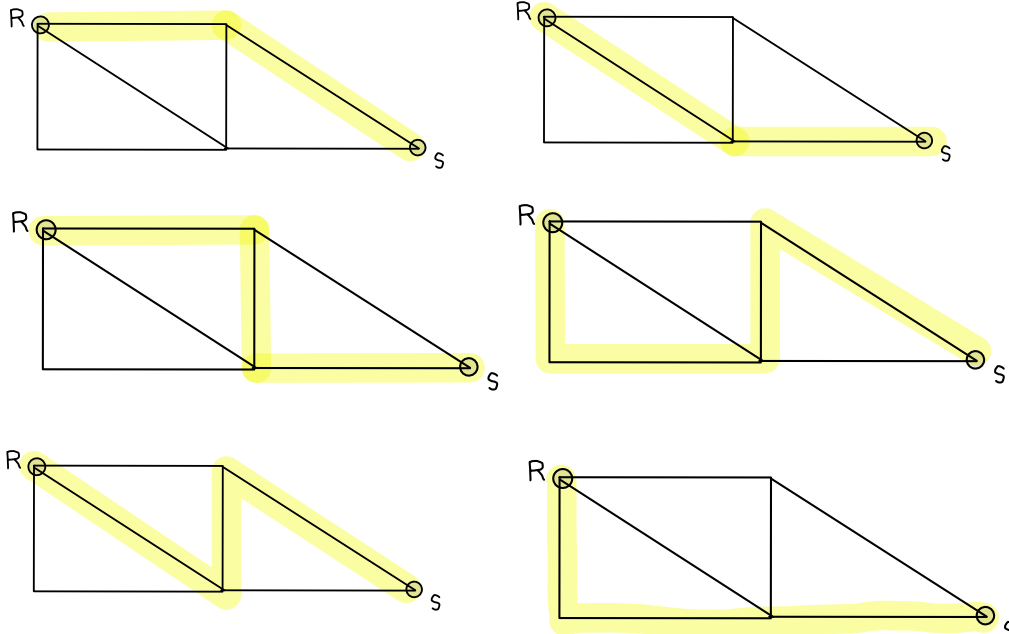
4. The figure below shows 9 matchsticks arranged as an equilateral triangle. Rearrange exactly 5 of the matchsticks to form 5 equilateral triangles, without leaving any stray matchsticks.



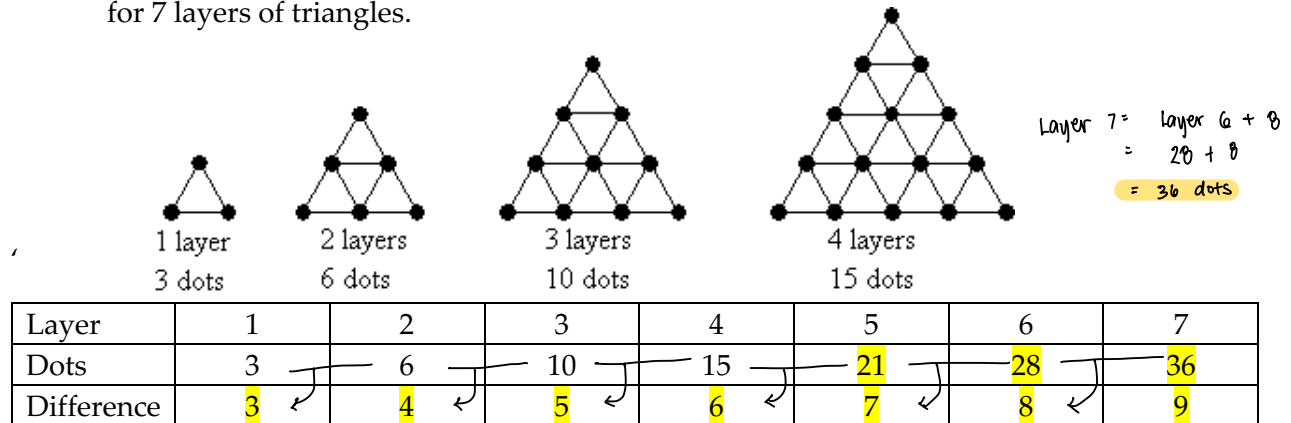
5. The figure below shows the roads linking cities **R** and **S**. What are the different routes to travel from **R** to **S**?



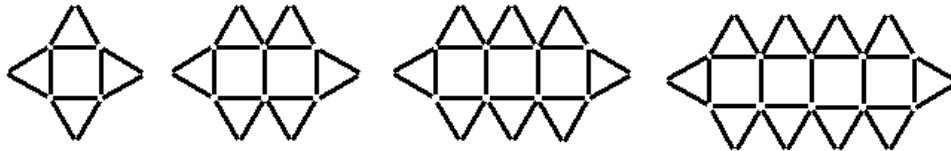
travel from **R** to **S**?



6. Each triangle in the figure below has 3 dots. Study the pattern and find the number of dots for 7 layers of triangles.



7. The following figures were formed using matchsticks.



- a) Based on the above series of figures, complete the table below.

Number of squares	1	2	3	4	5	6	7	8
Number of triangles	4	6	8	10	12	14	16	18
Number of matchsticks	12	19	26	33	40	47	54	61

- b) How many triangles are there if the figure in the series has 9 squares?

$$a_n = a_{n-1} + 2 \quad a_9 = a_8 + 2$$

$$= 18 + 2$$

$$a_9 = 20$$

- c) How many matchsticks would be used in the figure in the series with 11 squares?

$$a_n = a_{n-1} + 7 \quad a_9 = 61 + 7 \quad a_{11} = a_{10} + 7$$

$$= 68 \quad = 75 + 7$$

$$a_{10} = 68 + 7 \quad a_{11} = 82$$

$$= 75$$

8. The printer in the media center can print 1 page every 30 seconds. The printer in the office can print 4 pages every 30 seconds. If both printers are printing, how many pages will the office printer have printed by the time the media center printer prints 5 pages?

$$x = \text{media center}$$

$$y = \text{office}$$

$$y = 4x ; x = 5$$

$$y = 4(5)$$

$$= 20 \rightarrow \text{The office printer will have printed 20 pages by the time the media center prints 5 pages}$$

9. What should follow in the lower right corner? **A**

