ECE-18847 Homework 2
Due 11:59 PM, 03/19/2025

February 25, 2025

1. Implement `SparseMatrix.cpp`, get `TestMatrix.exe` to execute without errors for a wide range of input values [0...150]. The operations specified should be exact in floating point arithmetic. Expected output of `make run` should generate

```
  > make run
./TestMatrix.exe 10
linear phi passed
quadratic phi passed

./TestMatrix.exe 25
linear phi passed
quadratic phi passed

./TestMatrix.exe 46
linear phi passed
quadratic phi passed

./TestMatrix.exe 150
linear phi passed
quadratic phi passed
```

2. Implement `JacobiSolver.cpp`. Jacobi iteration can operate with a relaxation parameter α as high as 0.85 for these grids. Tolerance refers to the ratio of norm(residual)/norm(rhs), where the norm is the max norm. have your Jacobi solver print the initial norm(rhs), the final norm(residual)/norm(rhs), and the number of iterations it takes to converge. You are expected to write your own test harness for this class, and update the `GNUmakefile` to verify correct behavior.

3. Implement `FEPoissonOperator.cpp`. As a handy item for building the rhs vector, keep in mind that the value of any linear basis function at the centroid of a triangular element is 1/3. This can be derived from barycentric coordinates. Using these, you should be able to build `fesolver`.

- `testFEsolver.exe` verifies you have built a symmetric matrix with positive non-zero elements along the diagonal.
- `reinsert` puts the internal nodal solution back into the global ordering, with zero in all the boundary value locations.
- You are being provided two sets of input files: `A.{node,ele} and B.2.{node,ele}`. `A` is a small problem suitable for debugging, while `B.2` is a slightly larger problem for the final run.

4. along with a setup to build and execute `testFEsolver.exe`, submit `.png` files of plots from visit of `solution.vtk` for both the `A` and `B.2` meshes provided with the default color range and color scheme. Each plot should have both the mesh and pseudocolor for the "nodeData" variable field.

   Some notes on the structure of the homework 2 code.

   - The main `homework2` directory has several subdirectories.
     - `/src/fe` contains the files for the finite element classes, as well as the header file `FEPoisson.H`. When you implement `FEPoisson.cpp`, you will put it here. `/src/spmat` has the header files for the sparse matrix classes `SparseMatrix.H` and `JacobiSolver.H`, and the `.cpp` classes will be put in the same directory.
     - `/testmatrix` contains a test program for the `SparseMatrix` class. As soon as you have implemented it, you should be able to build and run the test program from this directory.
     - `/testFEGrid` contains a test program for the finite element code that we have provided. You should be able to run it and look at the results using VisIt without writing any code yourself. If that is not the case, let us know.
     - `/exec` contains the main program for the finite element solver and a makefile. When you have written all of your classes, you should build and run the solver from this directory.
     - `/FEData` contains the files with the finite element data for the two test cases here. When running `fesolver.exe`, you input the root for the input files, e.g. to run the case corresponding to `A.ele , A.node` the command line is
       `> ./fesolver.exe ../FEData/A`
       i.e. you need to include the path in your input.
   - the main `homework2` directory also contains a `GNUmakefile` that is included in all of the makefiles in the subdirectories. testMatrix does not use the common source since it doesn't depend on dimension.

   Expected outputs from `fesolver.exe` can be seen in the `make run` command

```
  ./fesolver.exe ../FEData/A
0 , 0
Intial RHS norm 1.93797
the number of iterations = 81
 Final Solver residual was 9.73923e-07
```

```
./fesolver.exe ../FEData/B.2
0 , 0
Intial RHS norm 0.482459
the number of iterations = 601
 Final Solver residual was 9.9861e-07
```

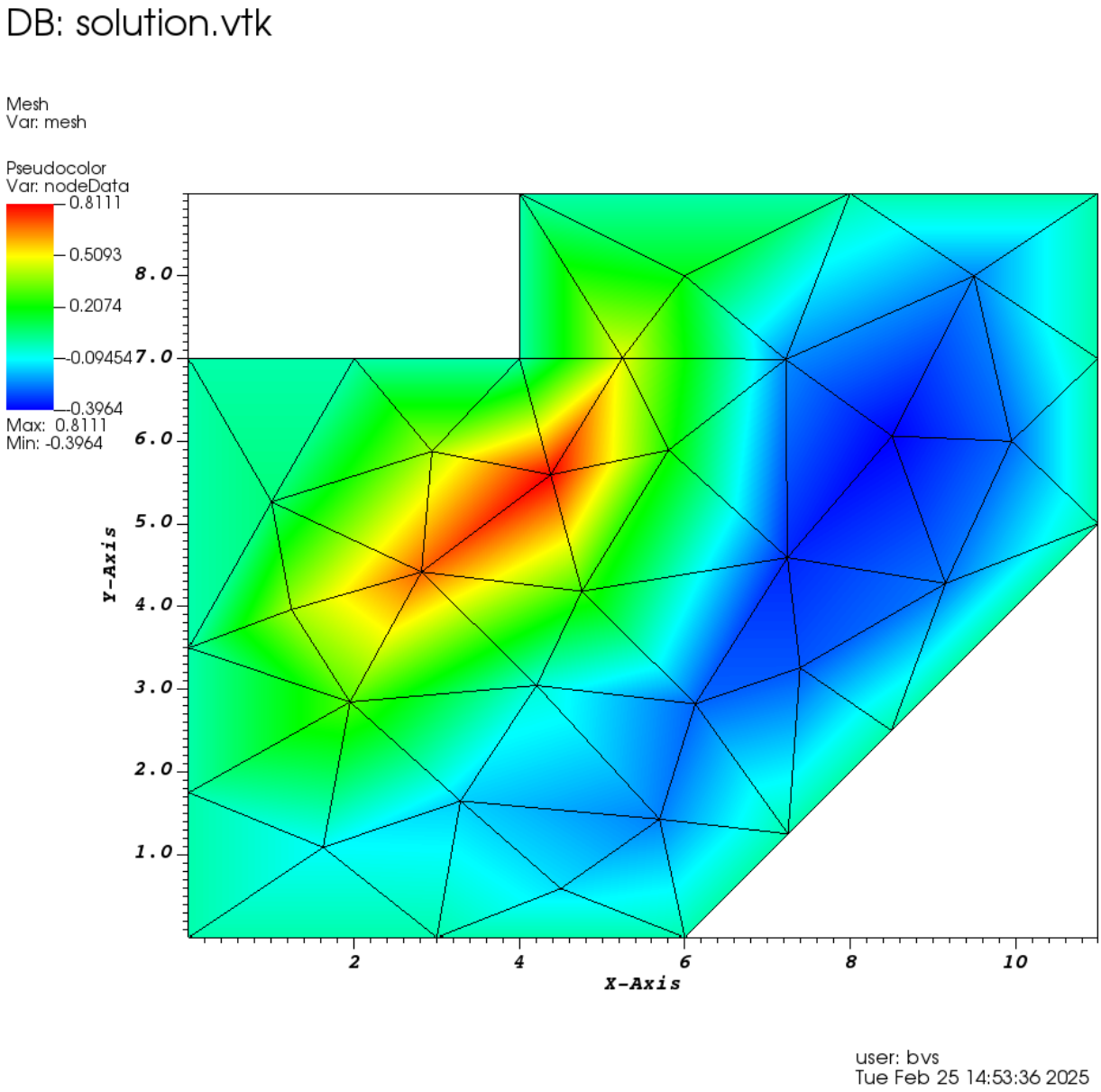

Figure 1: Expected VisIt output for data set FEData/A

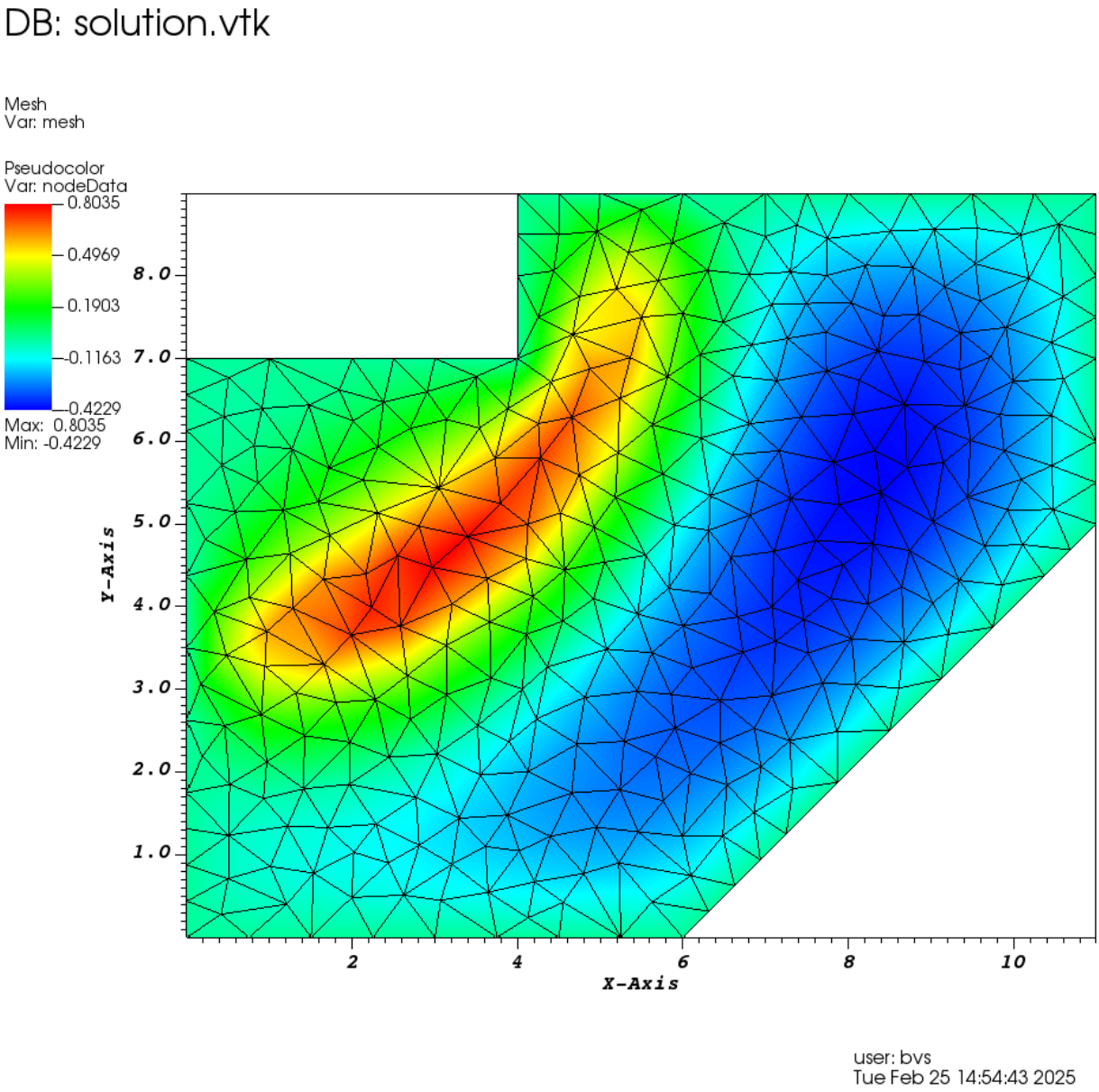

Figure 2: Expected VisIt output for data set FEData/B.2