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Solving Ordinary Differential Equations

The goal is to solve

$$y'(t) = \frac{dy}{dt} = f(t, y), \quad a \leq t \leq b, \quad y(a) = \alpha$$

Euler's Explicit Method

We first discretize the domain $[a, b]$, into $n + 1$ points $t_i = a + ih$ where $h = \frac{b-a}{n}$, then

$$y(t_{i+1}) = y(t_i) + hf(t_i, y_i)$$

Euler's Implicit Method

We first discretize the domain $[a, b]$, into $n + 1$ points $t_i = a + ih$ where $h = \frac{b-a}{n}$, then

$$y(t_{i+1}) = y(t_i) + hf(t_{i+1}, y_{i+1})$$

Runge Kutta of Order 4

We first discretize the domain $[a, b]$, into $n + 1$ points $t_i = a + ih$ where $h = \frac{b-a}{n}$, then

$$k_1 = hf(t_i, y_i)$$

$$k_2 = hf\left(t_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(t_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$$

$$k_4 = hf(t_i + h, y_i + k_3)$$

$$y(t_{i+1}) = y(t_i) + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

See `textbfscipy.integrate.RK45`.