4 Solving Ordinary Differential Equations

The goal is to solve

$$y'(t) = \frac{dy}{dt} = f(t, y),\ a \leq t \leq b,\ y(a) = \alpha$$

Euler's Explicit Method

We first discretive the domain $[a, b]$, into $n + 1$ points $t_i = a + ih$ where $h = \frac{b-a}{n}$, then

$$y(t_{i+1}) = y(t_i) + hf(t_i, y_i)$$

Euler's Implicit Method

We first discretive the domain $[a, b]$, into $n + 1$ points $t_i = a + ih$ where $h = \frac{b-a}{n}$, then

$$y(t_{i+1}) = y(t_i) + hf(t_{i+1}, y_{i+1})$$

Runge Kutta of Order 4

We first discretive the domain $[a, b]$, into $n + 1$ points $t_i = a + ih$ where $h = \frac{b-a}{n}$, then

$$\begin{aligned}
k_1 &= hf(t_i, y_i) \\
k_2 &= hf(t_i + \frac{h}{2}, y_i + \frac{k_1}{2}) \\
k_3 &= hf(t_i + \frac{h}{2}, y_i + \frac{k_2}{2}) \\
k_3 &= hf(t_i + h, y_i + k_3) \\
y(t_{i+1}) &= y(t_i) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)
\end{aligned}$$

See **scipy.integrate.RK45**.