Heterogeneity in imperfect inflation expectations: theory and evidence from a novel survey*

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Abstract

Using novel survey data from Germany, we study heterogeneity in how households form inflation expectations. We elicit (i) uncertainty in perceptions of current inflation, and (ii) how persistent households perceive inflation to be. Combining these with standard survey questions on inflation, we infer laws of motion for expectations at the individual level. Based on averages alone, a standard model calibrated to our data predicts inflation shocks generate small and transitory responses in expectations and consumption. The considerable heterogeneity we observe in expectation formation, however, amplifies the transmission to aggregate consumption by an order of magnitude, and substantially increases its persistence.

JEL codes: D83, D84, E31, E71

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1 Introduction

Households' information and subjective models of inflation shape how their inflation expectations respond to macroeconomic shocks. However, commonly used survey data on expectations is consistent with various combinations of information and subjective models, with contrasting implications. The well-known regressions in Coibion and Gorodnichenko (2015a), for example, are consistent with models in which information is noisy but households know the true law of motion for inflation (Coibion and Gorodnichenko, 2015a), or models with full information but misspecified forecasting rules (Gabaix, 2020; Hajdini, 2020). How then do households form their expectations?

This paper uses novel survey data to answer that question. We add new questions to the Bundesbank's Survey on Consumer Expectations to elicit the uncertainty in household perceptions of current inflation, and how persistent households perceive inflation to be. When combined with responses to other standard survey questions, this allows us to infer subjective laws of motion and details of information processing at the individual level.

We find that, on average, uncertainty about current inflation is low, and the perceived persistence of inflation is close to that of realized inflation. However, these averages mask considerable heterogeneity, which is important for understanding aggregate behavior. Calibrating a standard consumption-saving model to our data, heterogeneity in the expectations process amplifies the aggregate consumption response to inflation shocks by an order of magnitude, relative to the representative-agent case. Expectations may therefore play a substantially larger role in business cycle fluctuations than implied by models based on aggregate expectations alone.[1]

How do these questions pin down households' subjective models of inflation? Suppose that each household i believes inflation follows the AR(1) process:[2]

$$\begin{aligned} \pi_{t+1} &= \tilde{\rho}_i \pi_t + \varepsilon_{t+1} \\ \varepsilon_{t+1} &\sim N(0, \tilde{\sigma}^2_{\varepsilon,i}) \end{aligned} \tag{1}$$

Uncertainty in the inflation forecast can then be decomposed into: (i) uncertainty about current inflation, and (ii) uncertainty arising from future shocks:[3]

$$\tilde{Var}_{i,t}(\pi_{t+1}) = \tilde{\rho}_i^2 \tilde{Var}_{i,t}(\pi_t) + \tilde{\sigma}^2_{\varepsilon,i} \tag{2}$$

Of the terms in this equation, existing survey questions only measure subjective uncer-

[1] e.g. Fuster et al. (2010), Bhandari et al. (2019), Angeletos et al. (2020), and many others.

[2] Numerous studies suggest household forecasts are well characterized by such simple forecasting rules (e.g. Adam, 2007).

[3] We abstract from uncertainty about $\tilde{\rho}_i$.

tainty in the forecast $\tilde{Var}_{i,t}(\pi_{t+1})$. However, different combinations of the two components of this uncertainty imply very different aggregate dynamics, for the same observed forecast uncertainty. To illustrate, consider three ways of decomposing a given $\tilde{Var}_{i,t}(\pi_{t+1})$, assuming that the true law of motion for inflation is persistent but stationary:

1. $\tilde{\rho}_i = 1$, $\tilde{\sigma}^2_{\varepsilon,i} = 0$, $\tilde{Var}_{i,t}(\pi_t) = \tilde{Var}_{i,t}(\pi_{t+1})$. Agents are uncertain in their perceptions, suggesting imperfect information about current inflation. This implies their inflation perceptions respond sluggishly to shocks. Agents overextrapolate from their perceptions to expectations, generating 'delayed overshooting' (Angeletos et al., 2020).
2. $\tilde{\rho}_i = 1$, $\tilde{\sigma}^2_{\varepsilon,i} = \tilde{Var}_{i,t}(\pi_{t+1})$, $\tilde{Var}_{i,t}(\pi_t) = 0$. Agents overextrapolate, but are certain about perceived inflation, so there is no delay in the response of perceptions and expectations to realized inflation.
3. $\tilde{\rho}_i = 0$, $\tilde{\sigma}^2_{\varepsilon,i} = \tilde{Var}_{i,t}(\pi_{t+1})$, $\tilde{Var}_{i,t}(\pi_t)$ undefined. Agents underextrapolate. Forecasts do not respond to current inflation.

A given level of uncertainty in inflation forecasts could be consistent with a continuum of models between these extremes. Our questions pin down the correct model at the household level by eliciting the variance of the inflation perception $\tilde{Var}_{i,t}(\pi_t)$, and the perceived persistence $\tilde{\rho}_i$. Combined with existing questions eliciting $\tilde{Var}_{i,t}(\pi_{t+1})$, we then calculate the perceived variance of the innovations $\tilde{\sigma}^2_{\varepsilon,i}$.

To measure uncertainty in perceptions, we start with an existing survey question eliciting a point estimate of the respondent's inflation perception. We then add a new question, asking for the probability that the inflation rate lies within a specified range around that point estimate. Fitting a triangular distribution to these responses yields an estimate of $\tilde{Var}_{i,t}(\pi_t)$. To measure $\tilde{\rho}_i$, we present respondents with hypothetical scenarios of macroeconomic shocks, as in Andre et al. (2022). We specify that the shock has increased current inflation by one percentage point, and ask how the respondent would update their inflation expectations as a result. These novel questions are critical to answer our question: even with panel data on $\tilde{Var}_{i,t}(\pi_{t+1})$, as in the US Survey of Consumer Expectations, it is not possible to jointly identify $\tilde{Var}_{i,t}(\pi_t)$, $\tilde{Var}_{i,t}(\pi_{t+1})$, and $\tilde{\rho}_i$ at the individual level without restrictions on other aspects of the expectations process.

Our finding that, on average, respondents are less uncertain about current inflation than future inflation suggests that much of the uncertainty in expectations comes from perceived noise in the inflation process, not a lack of information about current inflation. However, a minority of households are very uncertain in their inflation perceptions. Perceived persistence is similarly heterogeneous: while two-thirds of households perceive no inflation persistence at the one-year horizon, those who do update their expectations after the hypothetical shock often do so by a large amount.

This heterogeneity has a large impact on the dynamics of aggregate consumption. In a standard consumption-saving model, the individual consumption response to inflation is convex in perceived inflation persistence. The same path of aggregate inflation expectations is therefore associated with much larger fluctuations in aggregate consumption if the individual-level subjective laws of motion are heterogeneous. The response of aggregate consumption to an inflation shock is consequently an order of magnitude larger when we calibrate the model using the heterogeneity observed in our data, relative to a representative-agent calibration based on average parameters. The persistence of aggregate consumption also rises by nearly one-half.

Moreover, elements of household expectation formation correlate systematically with each other, and with household characteristics, further distorting aggregate consumption away from the representative-agent case. In particular, hand-to-mouth households with little liquid wealth are more uncertain about current inflation relative to future inflation, and believe inflation is substantially more persistent. Since these households are likely to be less responsive to changes in their expectations, this dimension of heterogeneity further underlines the importance of investigating beyond average parameters of expectation formation for understanding macroeconomic behavior.

Related Literature: Angeletos et al. (2020) also study imperfect inflation expectations, directly estimating impulse responses of aggregate expectations to shocks. Our approach is complementary. While they can capture richer subjective models than the linear approximations we identify, our approach reveals heterogeneity in expectation formation, which cannot be observed with average forecasts.[4]

Several studies test noisy information models (e.g. Mankiw et al., 2004; Caplin and Dean, 2015; Coibion and Gorodnichenko, 2015a), often finding heterogeneity in information processing (Dovern and Hartmann, 2017; Link et al., 2021). We complement this literature by directly measuring the variance in perceptions and perceived persistence of inflation, which previous literature has typically inferred from assumptions on information and subjective laws of motion. To the best of our knowledge, there are no existing quantitative measures of the variance in perceptions.[5] Note that the US Survey of Consumer Expectations and the Survey of Professional Forecasters only elicit uncertainty about *future* rather than *current* inflation. We show that identifying these separately is important to understand the dynamics of aggregate expectations and consumption.

Perceived persistence has been measured in various contexts using laboratory experiments (Beshears et al., 2013; Afrouzi et al., 2020) and information treatments (Armona

[4] Ryngaert (2018) estimates perceived persistence and signal precision among professional forecasters, but similarly cannot uncover heterogeneity in these parameters.

[5] Armona et al. (2019) ask households to rate their uncertainty over past house-price growth on a 1-5 scale, but do not relate this quantitatively to the variance in expectations.

et al., 2019; Beutel and Weber, 2021; Laudenbach et al., 2021). Our method captures perceived inflation persistence even for households with very certain perceptions, who may not react to information treatments.[6] Moreover, we jointly measure perceived persistence and uncertainty over perceived and expected inflation, allowing us to observe the correlations between them.

Finally, we contribute to the literature on the role of expectations in business cycles. Like us, Branch and Evans (2006), Hommes and Lustenhouwer (2019), and Macaulay (2022) (among others) find that heterogeneity in the expectation formation process can substantially alter macroeconomic outcomes. We directly measure the relevant heterogeneity, and show that the resulting distribution of expectation processes amplifies the aggregate consumption response to inflation by an order of magnitude.

2 Expectations Framework

2.1 The Agent

Each agent i believes inflation follows the AR(1) process in equation 1. This subjective law of motion for inflation may or may not coincide with the true data generating process.

Each period, the agent receives a noisy signal $s_{i,t}$ about current inflation:

$$\begin{aligned} s_{i,t} &= \pi_t + q_{i,t} \\ q_{i,t} &\sim N(0, \sigma_q^2) \end{aligned} \tag{3}$$

Agents perceive the variance of $q_{i,t}$ to be $\tilde{\sigma}_{q,i}^2$, which is not necessarily equal to σ_q^2 (as in e.g. Daniel et al., 1998; Broer and Kohlhas, 2019). This allows the model to be consistent with any survey respondents who are simultaneously incorrect, but very certain, about their inflation perception.

After observing the signal, agents update their perception of inflation using the steady-

[6] An alternative method infers perceived inflation persistence from the ratio of expectations at different horizons (Reis, 2020; Andre et al., 2021). However, this assumes all agents share the same long-run expected inflation, and that there is no 'forward information' about future inflation innovations (Goldstein and Gorodnichenko, 2022). Methods using revisions to expectations in panel data may also be biased by survey tenure effects, which are large for inflation expectations (Kim and Binder, 2022).

state Kalman filter.[7] The evolution of the agent's one-period ahead inflation forecast is:

$$\tilde{E}_{i,t}\pi_{t+1} = (1-\chi_i)\tilde{\rho}_i\tilde{E}_{i,t-1}\pi_t + \tilde{\rho}_i\chi_i(\pi_t + q_{i,t}) \tag{4}$$

$$\chi_i = 1 - \frac{V_i^p}{V_i^f} \tag{5}$$

where χ_i is agent i's Kalman gain. V_i^p and V_i^f denote respectively the steady-state subjective variances of perceived inflation ($\tilde{Var}_{i,t}(\pi_t)$) and one-period ahead inflation ($\tilde{Var}_{i,t}(\pi_{t+1})$). These are such that $V_i^f \geq V_i^p$. All derivations for results here, and in Section 5, are in Appendix A.

The formula for χ_i is intuitive: the lower is V_i^p relative to V_i^f, the more informative is the signal received, and hence the larger the update the agent optimally makes to their inflation perceptions and forecasts. Note that the forecast variance in a given period, as measured in several household surveys, does not place any restrictions on the Kalman gain. This is why we require a novel question, not present in existing surveys, to identify V_i^p alongside V_i^f. One cannot identify V_i^p from the cross-sectional dispersion of inflation perceptions without assuming that agents all share the same $\tilde{\sigma}^2_{q,i}$, contrary to evidence of information heterogeneity among households (e.g. Link et al., 2021).

Using the law of motion for inflation, $\tilde{\sigma}^2_{\varepsilon,i}$ is given by:

$$\tilde{\sigma}^2_{\varepsilon,i} = V_i^f - \tilde{\rho}_i^2 V_i^p \tag{6}$$

The perceived noise in the signal is:

$$\tilde{\sigma}^2_{q,i} = \frac{V_i^f V_i^p}{V_i^f - V_i^p} \tag{7}$$

Our novel questions measuring V_i^p and $\tilde{\rho}_i$ therefore allow us to infer all parameters of the law of motion for household inflation expectations (equation 4), and the variances of both fundamental shocks and signal noise.

2.2 Expectations Impulse Responses

For simplicity, assume inflation is an exogenous AR(1) process, with autocorrelation ρ:

$$\pi_t = \rho\pi_{t-1} + \varepsilon_t \tag{8}$$

[7]The assumption of steady-state filtering is required to identify the agent's Kalman gain in the absence of panel data. The same assumption is commonly made in the rational inattention literature for tractability (Maćkowiak and Wiederholt, 2009).

We consider the impulse response of expectations to a one percentage point shock to inflation in time $t = 0$, with inflation and inflation expectations at steady-state (zero) before the shock. Abstracting from the effect of realized signal noise $q_{i,t}$,[8] the one-period ahead inflation forecast of agent i, t periods after the shock, is:

$$\tilde{E}_{i,t}\pi_{t+1} = \tilde{\rho}_i \chi_i \frac{\rho^{t+1} - (1-\chi_i)^{t+1}\tilde{\rho}_i^{t+1}}{\rho - (1-\chi_i)\tilde{\rho}_i} \tag{9}$$

Different combinations of χ_i and $\tilde{\rho}_i$ therefore imply very different impulse responses of expectations, even for the same V_i^f. On impact, the response of expectations is increasing in χ_i and $\tilde{\rho}_i$. The persistence of the expectation response increases in $\tilde{\rho}_i$, but decreases in χ_i. If $\tilde{\rho}_i$ is sufficiently large, and χ_i sufficiently small, then expectations display the hump-shaped impulse responses observed in previous empirical work (e.g. Angeletos et al., 2020). Equally, as in Angeletos et al. (2020), if $\tilde{\rho}_i > \rho$ then expectations overshoot, rising above realized inflation some periods after the shock. See Appendix A for derivations of these results.

2.3 The Role of Heterogeneity

If agents were homogeneous, equation 9 would also describe the impulse response of aggregate inflation expectations to the shock. With heterogeneity in $\tilde{\rho}_i$ and χ_i, however, the initial response of aggregate inflation expectations to the shock becomes:

$$\tilde{E}_0\pi_1 = E[\tilde{\rho}_i]E[\chi_i] + Cov(\tilde{\rho}_i, \chi_i) \tag{10}$$

A positive correlation between $\tilde{\rho}_i$ and χ_i therefore amplifies the initial effect of the shock on expectations, because those who extrapolate the most from perceived to expected inflation also update their perceptions the most in the period of the shock.

Heterogeneity continues to affect aggregate expectations in the periods after the shock. In Appendix A, we show that heterogeneity in $\tilde{\rho}_i$ increases the persistence of the response of aggregate expectations to the shock.

3 Data

We use the November 2021 wave of the Bundesbank-Online-Panel-Households survey, which is administered online to a representative sample of the German population. 4110 households were asked our questions.

[8] In effect, this measures the average expectation across many agents who share the same subjective law of motion and Kalman gain.

In the main survey, households give a point estimate of the inflation rate over the past 12 months, and give both point and density forecasts of inflation over the next 12 months. The density forecast involves households filling out the probabilities of inflation falling within various ranges.[9] Additionally, a range of household characteristics are collected. We report summary statistics in Appendix B.

We add two novel questions for the November 2021 survey wave, reproduced in Table 1 (see Appendix B for the German translations seen by respondents, and the point and density forecast questions).

Table 1: Novel questions added to the BOP-HH survey in November 2021

Question	Text	Sample
1	Now we would like to know how certain you are about your information on the inflation rate or deflation rate over the past 12 months ([Value of point estimate])%. In your opinion, how likely is it that the inflation rate has been between [Low inflation level]% and [High inflation level]% over the past twelve months?	All respondents
Respondents randomly shown one of three scenarios before Question 2		
General	Imagine the following hypothetical situation: Due to an unexpected economic event, the inflation rate increased by one percentage point in the past year.	Group A
Supply	Imagine the following hypothetical situation: Due to unexpected problems with local production technology in the Middle East, the price of crude oil rose in the past year, causing the inflation rate to rise by one percentage point.	Group B
Demand	Imagine the following hypothetical situation: Due to increased defense spending, government spending rose unexpectedly more than usual in the past year, causing the inflation rate to rise by one percentage point. The change is temporary and occurs even though the government's assessment of national security or economic conditions has not changed. In addition, taxes do not change in response to the spending program.	Group C
2	In this situation, would you adjust your inflation expectations for the next 12 months as stated in the first part of the questionnaire? If so, to what extent?	All respondents

[9]Similar questions appear in the FRBNY Survey of Consumer Expectations (Armantier et al., 2017) and the ECB's Consumer Expectations Survey (Coibion et al., 2021).

Question 1 elicits the uncertainty in the household's perceptions of current inflation.[10] The high and low inflation values seen by the respondent are determined by taking their point estimate of current inflation, and adding/subtracting 1 percentage point. If the respondent's point estimate of inflation is $\geq$5%, this range is widened to $\pm$2 percentage points, as uncertainty in inflation expectations is known to rise with point estimates (Ben-David et al., 2018). Answers are in percent, and must be within $[0, 100]$. Respondents also see a note giving further explanation of the question (see Appendix B).

To calculate the variance of perceived inflation V_i^p, we fit a symmetric triangular distribution using the respondent's answer and their point estimate. This is similar to the approach in Coibion et al. (2021), and in the Survey of Consumer Expectations when respondents only report positive probabilities in two bins of a density forecast question (Armantier et al., 2017). Note that this method is therefore different from that used to measure V_i^f, the uncertainty over future inflation, which uses all of the information in the density forecast for one-year ahead inflation.

In computing the Kalman gain for each respondent, we take the ratio of these two variances (equation 5). To confirm that the difference in measurement approaches does not bias our results, in Appendix C.2 we construct an alternative measure of V_i^f, which utilizes less of the available information from the density forecasts, but which corresponds closely to the measurement of V_i^p. All results below are robust to this alternative. This is consistent with the hypothesis in Kumar et al. (2022) that the discrepancy they find between their triangular and density variance measures is driven by a difference in the treatment of the end-points of each distribution, which is not present here.[11]

Question 2 elicits the respondent's perceived inflation persistence $\tilde{\rho}_i$. Following Andre et al. (2022), respondents are given a hypothetical scenario describing an exogenous shock, and asked how they would expect that to affect future inflation. Unlike Andre et al. (2022), in each scenario we tell the respondents that the shock caused current inflation to increase by 1 percentage point. Their answers on how that would change their inflation expectations consequently reflect their estimates of inflation persistence, not their predictions of the immediate impact of the shock, which Andre et al. (2022) find to be heterogeneous across their sample.

In Section 2 we did not distinguish between different types of shocks, and our main empirical analysis will do the same. However, households may associate different shocks

[10] We do not use a question with multiple bins, as for inflation expectations, as these questions are cognitively demanding. Too many can result in households dropping out of the survey.

[11] Specifically, Kumar et al. (2022) fit a triangular distribution using questions about firms' most optimistic, and most pessimistic, growth expectations. As they use these as the end-points of the distribution, they are assigned zero probability mass. In contrast, their density forecast question allows firms to place positive probability mass on these highest and lowest forecasts. Since our measure of V_i^p does not involve elicited end-points, we do not have this problem.

with different levels of persistence. To investigate this, we randomly split respondents into three groups. The first group are not told the nature of the shock, the second see a description of a supply shock (oil price), and the third see a demand shock (government spending). The specific hypothetical scenarios are adapted from Andre et al. (2022).

To answer, respondents see the following:

1) Yes, from [Value of point estimate]% to __%
2) No

and either input a number in the first line or select 'No'.

Using these questions we obtain $\tilde{\rho}_i$ and V_i^p for each respondent. We then infer the implied χ_i, $\tilde{\sigma}^2_{\varepsilon,i}$, and $\tilde{\sigma}^2_{q,i}$ using equations 5-7. Full details of the variable construction are in Appendix C.1.

4 Empirical Results

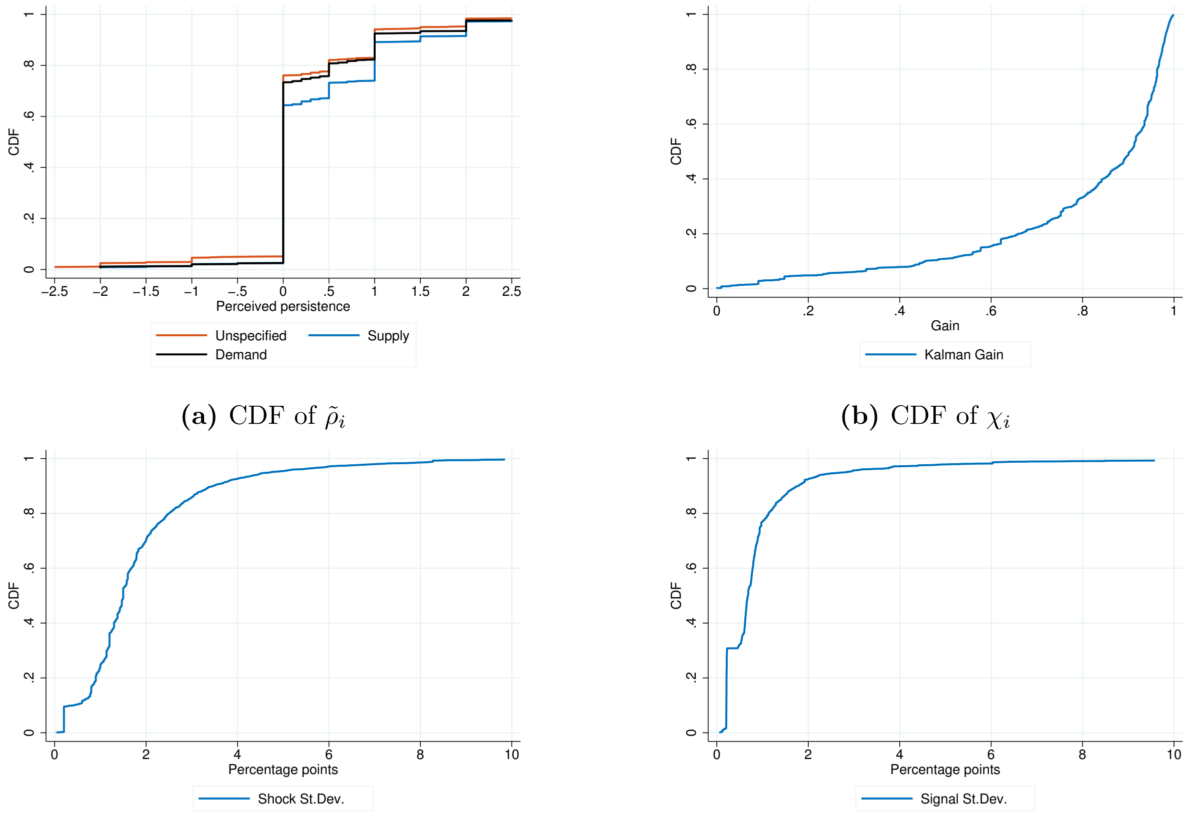
Figure 1a plots the CDF of $\tilde{\rho}_i$, truncated to remove the approximately 1% of responses outside the $[-5, 5]$ interval. Of the remaining responses, 89% report $\tilde{\rho}_i \in [0, 1]$, and 68% do not revise their expectations at all.

Result 1 *Conditional on perceiving persistence in* $[0, 1]$*, the average perceived persistence is broadly consistent with the data. However, the cross-sectional heterogeneity is large.*

Of those who give a response in $[0, 1]$, the mean $\tilde{\rho}_i$ is 0.18, close to the "correct" answer of 0.21 based on recent German data.[12] Including all responses in $[-5, 5]$, the mean $\tilde{\rho}_i$ is 0.29. The heterogeneity, however, is large. Restricting to responses in [0,1], the maximum possible standard deviation is 0.5: the standard deviation in our sample is 0.36.

This heterogeneity is not driven by households rounding to the nearest percentage point when reporting expectations (Binder, 2017). Excluding those whose reported inflation expectations are multiples of .5, *c*.60% of respondents do not revise their expectations, similar to the overall sample (see Appendix D). We continue with the full sample, as rounded expectations may still matter for consumption decisions. The large fraction of households choosing not to update their expectations is consistent with other information treatments in previous waves of this survey (Dräger et al., 2022), and with other hypothetical scenario surveys (Fuster et al., 2021).

[12] We obtain annual CPI data (2002-2021) from *www.destatis.de*, calculate inflation, and run a linear projection of this on its lagged value, finding a coefficient of 0.21.



(a) CDF of $\tilde{\rho}_i$

(b) CDF of χ_i

(c) CDF of $\tilde{\sigma}_{\varepsilon,i}$

(d) CDF of $\tilde{\sigma}_{q,i}$

Figure 1: CDFs of key parameters. Source: Bundesbank-Online-Panel-Households, November 2021 wave.

Among those shown the first scenario (unspecified shock), the mean $\tilde{\rho}_i$ for those within the $[0, 1]$ interval is 0.16. The corresponding figures are 0.22 and 0.16 for those shown the supply and demand shocks respectively. Supply shocks are therefore perceived to be slightly more persistent than demand shocks, consistent with evidence that supply shocks are particularly important for household inflation expectations (Coibion and Gorodnichenko, 2015b). While some of this difference across shock types could stem from the stronger reference to the temporary nature of the demand shock in our scenarios, note that the unspecified scenario is very similar to the supply shock in this respect; neither state explicitly that the shock is temporary. The fact that average $\tilde{\rho}_i$ is higher for supply shocks than *both* alternatives supports the interpretation that supply shocks are perceived to be more persistent.

Figure 1b shows the CDF of χ_i. The CDFs for V_i^p and V_i^f used to calculate this Kalman gain are in Appendix D.[13]

[13]For households who are completely certain that π_t is within the range shown in our question, we obtain a range of possible values for χ_i, because V_i^p cannot be exactly identified (see Appendix C.1 for details). Figure 1b uses the mid-point in these cases, for all respondents with a range of width $\leq$0.2. The CDFs using the upper and lower bounds on χ_i are shown in Appendix D. Figures 1c and 1d similarly

Result 2 *The average implied Kalman gain is high, at 0.8. There is considerable cross-sectional heterogeneity.*

The high average Kalman gain stems from most consumers being considerably more certain about their inflation perceptions than their expectations. There is, however, a long tail of very uncertain households, with low Kalman gains, leading to a substantial variance in χ_i.

The average χ_i exceeds values obtained from regressions on average forecast errors, which are typically close to 0.5 (e.g. Coibion and Gorodnichenko, 2015a). The discrepancy is unsurprising, since such regressions yield biased estimates if agents hold inaccurate beliefs about inflation persistence (Ryngaert, 2018). Moreover, at the time of the survey, CPI inflation in Germany exceeded 5%. It is plausible that consumers were particularly well-informed about inflation at this point because of media coverage.

Figure 1c shows the CDF of $\tilde{\sigma}_{\varepsilon,i}$. There is considerable heterogeneity; a tail of households believe future inflation is extremely volatile. This reinforces that much of the uncertainty in inflation expectations relates to future shocks, rather than uncertainty about current inflation.

Figure 1d shows the CDF of $\tilde{\sigma}_{q,i}$. Reflecting the high average χ_i, most households have little noise in their signals, though there is a long tail with very imprecise information.

4.1 Relationships between Expectation Components

Table 2 shows our next main result:

Result 3 *Households who are more uncertain about current inflation are also more uncertain about future inflation, believe that the inflation process is noisier, and have lower implied Kalman gains.*

This is consistent with noisy information models, in which greater uncertainty arises when households process less information. Importantly, this result is not imposed by our model assumptions. Our model allows more uncertainty about current inflation to be associated with higher or lower Kalman gains, depending on the relationships between V_i^p and V_i^f (equation 5). Indeed, noisy information can only partly explain the distribution of uncertainty in the data. More uncertain households also believe that inflation shocks are more volatile (higher $\tilde{\sigma}_{\varepsilon,i}$) than less uncertain households. This further highlights the importance of measuring the uncertainty in perceptions and expectations separately.

There are small positive correlations of $\tilde{\rho}_i$ with uncertainty about future inflation, and with Kalman gains. However, the results on $\tilde{\rho}_i$ are potentially complicated by two

use midpoints of implied ranges in these cases.

Table 2: Cross-sectional correlations of subjective law of motion elements.

	(1)				
	$SD_i(\pi_{t+1})$	$SD_i(\pi_t)$	$\tilde{\rho}_i$	$SD_i(\varepsilon_{t+1})$	χ_i
$SD_i(\pi_{t+1})$	1.000				
$SD_i(\pi_t)$	0.476***	1.000			
$\tilde{\rho}_i$	0.036*	-0.015	1.000		
$SD_i(\varepsilon_{t+1})$	0.988***	0.443***	-0.034	1.000	
χ_i	0.316***	-0.393***	0.038*	0.337***	1.000

Note: Bundesbank-Online-Panel-Households, November 2021 wave. For cases where χ_i is set-identified, respondents are excluded if the parameters are estimated very imprecisely (range> 0.2). For all remaining set-identified parameters, the mid-point of the range is used. Observations of $SD_i(\pi_{t+1})$, and $SD_i(\pi_t)$ below the 1st or above the 99th percentile of that variable's distribution are also excluded as outliers, as are observations of $\tilde{\rho}_i$ outside $[-5, 5]$ (c.1% of observations). * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

factors. First, households may believe that different types of shock have different levels of persistence. Second, households who believe inflation is non-stationary ($|\tilde{\rho}_i| \geq 1$) may be qualitatively different from households with $|\tilde{\rho}_i| < 1$. We explore these points in Appendix D. The variation across shock types is small, but there are noticeable differences between those with stationary and non-stationary subjective laws of motion. Within households who believe inflation is persistent and stationary, greater perceived persistence is associated with less uncertainty about current and future inflation, less perceived noise in the inflation process, and a greater Kalman gain. That is consistent with models of endogenous information acquisition; if inflation is more persistent, information about current inflation is more valuable.

4.2 Correlations with Household Characteristics

Table 3 shows results of regressing each component of expectation laws of motion on household characteristics. As our application in Section 5 is to consumption, we focus on characteristics known to relate to Marginal Propensities to Consume (MPCs). To account for the fact that households with $\tilde{\rho}_i = 0$ may differ qualitatively from those with $\tilde{\rho}_i \neq 0$, the final column shows the linear component of an estimated hurdle model (Cragg, 1971). It therefore shows the estimated associations conditional on the household revising their expectations in light of the hypothetical shock. Results from the selection stage, along with other ways of splitting the $\tilde{\rho}_i$ responses, are in Appendix D.

The key variables displayed are liquid wealth (bank deposits plus securities), illiquid wealth (property plus firm ownership), other wealth, debt, and household income. There is also an indicator for if the household is hand-to-mouth, defined here as having liquid wealth of less than €1250.

The first row of coefficients shows our next main result:

Result 4 *Hand-to-mouth households are more uncertain about current inflation, but no more uncertain about future inflation, than other households. They believe inflation is noisier and more persistent, and have higher implied Kalman gains.*

Table 3: Regressions of components of subjective laws of motion on household characteristics.

	(1) $\log(SD_i(\pi_{t+1}))$	(2) $\log(SD_i(\pi_t))$	(3) $\log(SD_i(\varepsilon_{t+1}))$	(4) $\log(\chi_i)$	(5) $\tilde{\rho}_i$
Hand-to-mouth	0.0285	0.1374**	0.1435**	0.0955**	0.4072**
	(0.0364)	(0.0541)	(0.0580)	(0.0397)	(0.2039)
Liquid wealth	0.0000	-0.0002**	0.0002**	0.0002***	-0.0013**
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0005)
Illiquid wealth	0.0000	-0.0000	-0.0000	0.0000	0.0002
	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0002)
Other wealth	-0.0001	-0.0001	0.0003	-0.0000	-0.0012
	(0.0002)	(0.0002)	(0.0003)	(0.0002)	(0.0007)
Debt	0.0000	0.0001	0.0001	0.0002*	-0.0009
	(0.0001)	(0.0002)	(0.0002)	(0.0001)	(0.0006)
log(income)	-0.0933***	-0.1247***	-0.1827***	-0.0038	0.0627
	(0.0235)	(0.0348)	(0.0380)	(0.0333)	(0.1830)
HH Controls	Yes	Yes	Yes	Yes	Yes
Hurdle model	No	No	No	No	Yes
Observations	4344	3161	2232	2014	3194
R^2	0.0484	0.0551	0.0645	0.0221	
Pseudo R^2					0.0238

Note: Bundesbank-Online-panel-Households, November 2021 wave. The units of the wealth and debt variables are €1000s. The household controls are age (in years up to a top bin of ≥ 80, coded as 80), age^2, gender, region (north/south/east/west), education, occupation category, and employment status (all categorical, for details see the full questionnaire at https://www.bundesbank.de/en/bundesbank/research/survey-on-consumer-expectations/questionnaires-850746). All controls except age and age^2 are treated as categorical. Robust standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Hand-to-mouth households also have a 9.5% larger average Kalman gain. Their perceived inflation persistence is higher by an average of 0.4, which coupled with the higher current uncertainty implies that they believe the inflation process is substantially noisier than other households.

Angeletos et al. (2020) find that aggregate inflation expectations display delayed overshooting, suggesting households perceive inflation to be more persistent than implied by the true data generating process. Our results suggest that overshooting may partly be driven by the expectations of hand-to-mouth households, who are less able to respond to expectations of future inflation by adjusting consumption and saving decisions. Their expectations may not therefore have much impact on the transmission of shocks. While this result is not consistent with simple models of rational inattention, the high Kalman gains for hand-to-mouth households could be driven by those who are close to leaving their borrowing constraints, who have highly non-linear policy functions and value information highly as a result (Broer et al., 2021).

Above the hand-to-mouth threshold, higher liquid wealth is associated with less uncertainty in current inflation and less perceived inflation persistence, though these effects are quantitatively small in comparison. Each further €1000 is associated with a 0.02% reduction in $SD_i(\pi_t)$ and a 0.001 fall in $\tilde{\rho}_i$. The lower current uncertainty relative to uncertainty over the future should imply that higher liquid wealth is associated with a lower Kalman gain, but in fact the reverse is true. This stems from some households giving answers to Question 1 such that we cannot precisely impute their χ_i, so the samples are not equal across these regressions. However, although statistically significant, the association of χ_i with wealth beyond hand-to-mouth status is small in magnitude. In Appendix D we show that the positive coefficient on liquid wealth in the χ_i regression is driven by a small number of very wealthy households.

The final row of coefficients gives our next main result:

Result 5 *Higher income is associated with less uncertainty about current and future inflation, and less perceived noise in the inflation process, but is not associated with differences in perceived inflation persistence or implied Kalman gains.*

As documented in other contexts (e.g. Ben-David et al., 2018), households with higher income are less uncertain about future inflation: a 10% rise in household income is associated with a 0.9% fall in $SD_i(\pi_{t+1})$. There is evidence that this partly results from high-income households acquiring more precise information, as they are even less uncertain about current inflation: the same 10% rise in income is associated with a 1.2% fall in $SD_i(\pi_t)$. However, this difference across current and future uncertainty is small, so there is no significant relationship between income and χ_i. Rather, the bulk of the lower uncertainty for high-income households is explained by them believing that the inflation process is less volatile. A 10% rise in income is associated with a 1.8% reduction in $SD_i(\varepsilon_{t+1})$. There is no significant correlation between income and perceived persistence.

5 Implications for Aggregate Consumption

5.1 Consumption-Saving Model

The model features both unconstrained and hand-to-mouth households.

Unconstrained households have an infinite horizon and face no borrowing constraint. They choose consumption each period to maximize the expected discounted sum of CRRA utility over consumption, and invest any unspent exogenous income $y_{i,t}$ in risk-free one-period bonds with gross nominal interest rate i_t. The household's log-linearized consumption function is:

$$\hat{c}_{i,t} = \sum_{h=0}^{\infty} \beta^h \left((1-\beta)\tilde{E}_{i,t} y_{i,t+h} - \beta\gamma^{-1}\tilde{E}_{i,t} i_{t+h} + \beta\gamma^{-1}\tilde{E}_{i,t}\pi_{t+h+1} \right) \tag{11}$$

where β is the discount factor and γ is the coefficient of relative risk aversion (see Gabaix (2020) Proposition 29). To isolate the effect of a shock to expected inflation, we hold expected $y_{i,t+h}$ and i_{t+h} constant in all exercises. We relax the latter assumption in Appendix E. Using equation 1, consumption is given by:[14]

$$\hat{c}_{i,t} = \frac{\beta\gamma^{-1}}{1-\beta\tilde{\rho}_i}\tilde{E}_{i,t}\pi_{t+1} \tag{12}$$

A higher $\tilde{\rho}_i$ therefore increases the responsiveness of consumption to those expectations, as it implies larger changes in longer-horizon expectations.

Using equation 9 and aggregating across households, the aggregate consumption response to a one percentage point inflation shock in $t=0$ is:

$$\hat{c}_0 = \beta\gamma^{-1}\left(E[\chi_i]E\left[\frac{\tilde{\rho}_i}{1-\beta\tilde{\rho}_i}\right] + Cov\left(\chi_i, \frac{\tilde{\rho}_i}{1-\beta\tilde{\rho}_i}\right)\right) \tag{13}$$

Heterogeneity in expectation laws of motion therefore affects aggregate consumption in two ways. First, heterogeneity in $\tilde{\rho}_i$ amplifies the initial aggregate consumption responses to inflation, because $\tilde{\rho}_i/(1-\beta\tilde{\rho}_i)$ is convex in $\tilde{\rho}_i$. If even a small fraction of agents believe that inflation is close to a unit root, they respond very strongly to current inflation, generating large aggregate consumption responses. Note that heterogeneity in $\tilde{\rho}_i$ increases the size of the consumption response *relative* to the response in inflation expectations. Similar aggregate impulse responses in inflation expectations may therefore correspond to very different impulse responses in consumption.

Second, any correlation between $\tilde{\rho}_i$ and χ_i will further distort the aggregate con-

[14] This result assumes $|\beta\tilde{\rho}_i| < 1$. When calibrating to the survey data we drop the minority of households for whom this is not true.

sumption response away from the representative-agent case. Intuitively, the response of aggregate consumption is amplified if the households who obtain precise information about the shock are also the ones who respond most strongly to that information. This is an example of the 'narrative heterogeneity channel' discussed in Macaulay (2022).

Constrained (hand-to-mouth) households, in contrast, do not respond to expectations. Since we are abstracting from indirect effects of nominal shocks through incomes, we set their $\hat{c}_{i,t} = 0$.

5.2 Model-implied Impulse Responses

We now generate IRFs of aggregate one-year ahead inflation expectations and consumption in three cases. First, we consider full information rational expectations (FIRE): all households know that $\rho = 0.21$, and observe π_t precisely. Second, we maintain homogeneity, and calibrate the model using the population averages for χ_i and $\tilde{\rho}_i$ in the survey data. Finally, we allow for heterogeneity, using the joint distribution of χ_i and $\tilde{\rho}_i$ from the data. In all cases we exclude observations with perceived persistence outside [0,1].[15]

Figure 2a plots the IRFs for aggregate one-year ahead inflation expectations. Expectations respond less on impact in both cases calibrated to the survey data than with FIRE, because $E[\chi_i] < 1$ and $E[\tilde{\rho}_i] \approx \rho$. On impact, expectations in these two cases are approximately the same. Aggregate inflation expectations are however somewhat more persistent under heterogeneity: a year after the shock, average $\tilde{E}_{i,t}\pi_{t+1}$ is c.35% greater than under homogeneity.

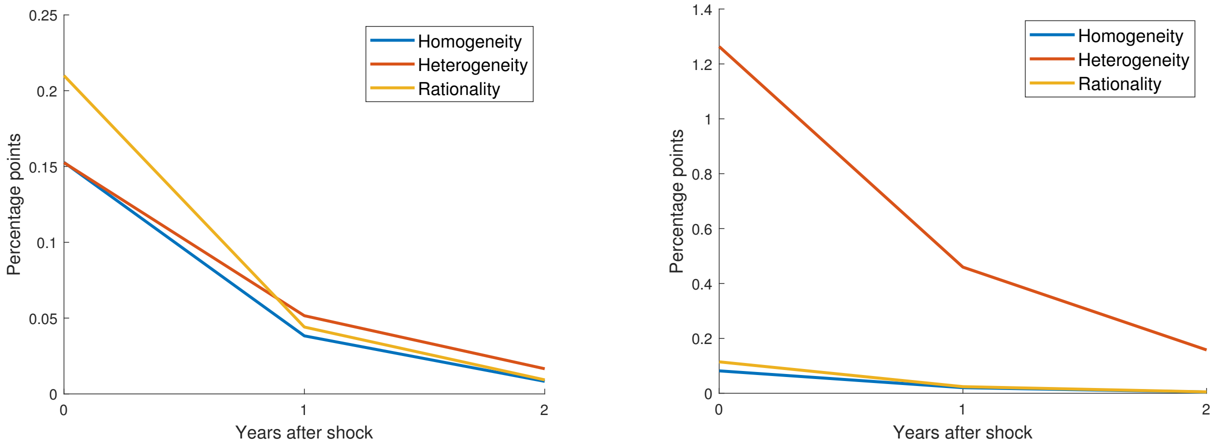
Figure 2b plots the IRFs for aggregate consumption. They differ considerably between cases:

Result 6 *The model-implied consumption response under heterogeneity is* $15.5\times$ *greater on impact than under homogeneity. The persistence of the consumption response under heterogeneity* ($\frac{\hat{c}_1}{\hat{c}_0}$) *is c.45% greater than under homogeneity.*

On impact, the FIRE response is small, at 0.11%. It has persistence of 0.21, so the deviation from steady-state in $t = 1$ is negligible. The homogeneous case has an even smaller initial consumption response (0.08%), and marginally greater persistence.

The heterogeneous case, however, has a vastly larger initial consumption response of 1.26%. The response is also more persistent, with a persistence of 0.36 between $t = 0$ and $t = 1$. Aggregate consumption therefore remains substantially above steady-state in the two years following the shock. In Appendix E, we show that these effects principally

[15] This is done to exclude outliers. The excluded households are disproportionately the hand-to-mouth, who are least able to respond to expectations (Result 4).



(a) Expectations

(b) Consumption

Figure 2: Implied IRFs of one-period ahead inflation expectations and consumption. Source: Bundesbank-Online-Panel-Households, November 2021 wave.

reflect the heterogeneity in perceived inflation persistence, which generates considerable amplification for any initial response of inflation expectations.

These figures demonstrate the challenges involved in inferring how expectations affect the response of macroeconomic variables to shocks using only aggregate data. The homogeneity and heterogeneity cases yield similar IRFs in aggregate inflation expectations, but entirely different IRFs in aggregate consumption.

Importantly, the results rely on the properties of the consumption function (equation 11), which assumes households behave according to an Euler equation. Dräger and Nghiem (2021) find evidence of this type of behavior amongst German households. Hanspal et al. (2021) also find that perceived persistence is important for consumption decisions in the context of Covid-19 expectations.

The amplification survives a number of robustness checks. In Appendix E, we repeat this exercise separately for each shock scenario posed in Question 2, and we exclude those who report rounded expectations. Amplification from heterogeneity remains large in all cases. We also allow for the possibility that households update their expectations of future nominal interest rates when their inflation expectations change. Again, under a wide range of assumptions about perceived interest rate rules, amplification from heterogeneity remains large.[16] If, for example, households believe the Taylor Principle will be satisfied, aggregate consumption falls after an inflation shock as expected real interest rates rise. Heterogeneity substantially amplifies this fall, just as it amplifies the rise in consumption studied here.

[16]Figures 2a and 2b can be interpreted as the results if households believe nominal interest rates are unresponsive to inflation. This is a reasonable baseline, since at the time of the survey, CPI inflation in Germany had risen sharply over the preceding year (from -0.3% to 5.2%), but the ECB was still a long way from raising nominal interest rates (Lagarde, 2021).

In general equilibrium, these effects will be amplified if the consumption increase leads to rising real incomes for hand-to-mouth households, and rising income expectations for unconstrained households. A further round of general equilibrium effects may then also occur through the Phillips curve, as the aggregate consumption response amplifies or dampens the inflation shock, depending on whether households believe that the Taylor principle will be satisfied. We leave exploration of these effects to future research.

6 Conclusion

Inflation expectations greatly affect aggregate dynamics in many theories of the business cycle. However, the quantities measured by existing surveys of expectations are consistent with multiple laws of motion for expectations, with contrasting aggregate implications. To distinguish between these models, we use novel survey data to elicit (i) consumers' uncertainty in their inflation perceptions, and (ii) how persistent they perceive inflation to be, at the individual level.

We find that, on average, consumers are relatively confident in their inflation perceptions and perceive little persistence in inflation. However, these averages mask considerable heterogeneity. Accounting for the observed heterogeneity in expectation laws of motion increases the aggregate consumption response to an inflation shock by an order of magnitude in an otherwise standard partial-equilibrium model. The persistence of consumption responses to shocks also increases substantially.

This effect occurs because individual consumption functions are highly non-linear in the components of expectation formation. Heterogeneity in those parameters, and correlations between them, can therefore have large effects on aggregate consumption, even for a given path of aggregate inflation expectations. The components of expectation formation are also correlated with household wealth and income, both of which correlate with consumption behavior (Kaplan et al., 2014; Kueng, 2018). Exploring the distribution of these components for expectations of other variables, and how the distributions change over time and states of the world, could be a fruitful avenue for future research.

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A Proofs

Steady-State Kalman Filter

Before receiving the signal at time t, the agent's subjective distribution for π_t and the signal is:

$$\begin{pmatrix} \pi_t \\ \pi_t + q_{i,t} \end{pmatrix} \sim N\left(\begin{pmatrix} \tilde{\rho}_i \tilde{E}_{i,t-1}\pi_{t-1} \\ \tilde{\rho}_i \tilde{E}_{i,t-1}\pi_{t-1} \end{pmatrix}, \begin{pmatrix} V^f_{i,t-1} & V^f_{i,t-1} \\ V^f_{i,t-1} & V^f_{i,t-1} + \tilde{\sigma}^2_{q,i} \end{pmatrix} \right) \tag{14}$$

The conditional mean of π_t given the signal is then:

$$\tilde{E}_{i,t}\pi_t = (1-\chi_{i,t})\tilde{\rho}_i \tilde{E}_{i,t-1}\pi_{t-1} + \chi_{i,t}(\pi_t + q_{i,t}), \text{ where } \chi_{i,t} = \frac{V^f_{i,t-1}}{V^f_{i,t-1} + \tilde{\sigma}^2_{q,i}} \tag{15}$$

The conditional variance of π_t:

$$V^p_{i,t} = V^f_{i,t-1}\left(1 - \frac{(V^f_{i,t-1})^2}{V^f_{i,t-1}(V^f_{i,t-1} + \tilde{\sigma}^2_{q,i})}\right) = \frac{V^f_{i,t-1}\tilde{\sigma}^2_{q,i}}{V^f_{i,t-1} + \tilde{\sigma}^2_{q,i}}$$

In steady state, this variance is:

$$V^p_i = \frac{V^f_i \tilde{\sigma}^2_{q,i}}{V^f_i + \tilde{\sigma}^2_{q,i}} \tag{16}$$

The steady state Kalman gain is then:

$$\chi_i = \frac{V^f_i}{V^f_i + \tilde{\sigma}^2_{q,i}} = 1 - \frac{V^p_i}{V^f_i} \tag{17}$$

Response of Inflation Expectations to Shocks

Throughout, we assume that both inflation and the agent's inflation perception start in steady state in $t=-1$. That is, $\pi_{-1} = \tilde{E}_{i,-1}\pi_{-1} = 0$. The individual inflation perception is given by equation 15. Iterating backwards to time 0, we obtain:

$$\tilde{E}_{i,t}\pi_t = \chi_i \sum_{s=0}^{t} ((1-\chi_i)\tilde{\rho}_i)^s (\pi_{t-s} + q_{i,t-s}) \tag{18}$$

Abstracting from $q_{i,t}$, the h period ahead forecast is then:

$$\tilde{E}_{i,t}\pi_{t+h} = \chi_i \tilde{\rho}_i^h \sum_{s=0}^{t} ((1-\chi_i)\tilde{\rho}_i)^s \pi_{t-s} = \chi_i \tilde{\rho}_i^h \rho^t \sum_{s=0}^{t} ((1-\chi_i)\tilde{\rho}_i \rho^{-1})^s \varepsilon_0 \tag{19}$$

where the second equality uses that $\pi_{t-s} = \rho^{t-s}\varepsilon_0$.

Provided that $(1-\chi_i)\tilde{\rho}_i\rho^{-1} \neq 1$, then evaluating the summation and rearranging yields the result:

$$\tilde{E}_{i,t}\pi_{t+h} = \chi_i\tilde{\rho}_i^h \frac{\rho^{t+1} - ((1-\chi_i)\tilde{\rho}_i)^{t+1}}{\rho - (1-\chi_i)\tilde{\rho}_i}\varepsilon_0 \tag{20}$$

Setting $h = 1, \varepsilon_0 = 1$ yields equation 9.

Persistence of Expectations

From equation 20 we have:

$$\frac{\tilde{E}_{i,1}\pi_2}{\tilde{E}_{i,0}\pi_1} = \rho + (1-\chi_i)\tilde{\rho}_i \tag{21}$$

The persistence of the response of expectations to the shock is increasing in ρ and $\tilde{\rho}_i$, and decreasing in χ_i (assuming $\tilde{\rho}_i > 0$). If $\rho + (1-\chi_i)\tilde{\rho}_i > 1$, then the expectation rises between the period the shock hits and the period after, giving a hump-shaped response. This condition is both necessary and sufficient for a hump-shaped impulse response.

Under heterogeneity, the impact response of aggregate inflation expectations to the shock is given by equation 10. To see the role of heterogeneity in future periods, we consider the special case of $\rho = 0$, in which case:

$$\tilde{E}_t\pi_{t+1} = E[\tilde{\rho}_i^{t+1}]E[\chi_i(1-\chi_i)^t] + Cov(\tilde{\rho}_i^{t+1}, \chi_i(1-\chi_i)^t) \tag{22}$$

$\tilde{\rho}_i^{t+1}$ is strictly convex in $\tilde{\rho}_i$ for all $t \geq 1$, so heterogeneity in $\tilde{\rho}_i$ increases the time t response for $t \geq 1$. This convexity increases with t, so the persistence of the response of expectations also increases with heterogeneity in $\tilde{\rho}_i$. In addition, note that $\chi_i(1-\chi_i)^t$ is linear in χ_i for $t = 0$, concave in χ_i if $t = 1$, but becomes convex in χ_i as t becomes large. Heterogeneity in χ_i consequently decreases persistence for small t, but may increase persistence for large enough t.

To understand the role of the covariance term, consider the simple case where $\tilde{\rho}_i$ is monotonically increasing in χ_i. The covariance term is then positive for small t, but negative for large t. As such, a positive correlation between $\tilde{\rho}_i$ and χ_i tends to result in lower persistence of the response in inflation expectations, despite a larger initial response.

Formally, the persistence of aggregate expectations between $t = 0$ and $t = 1$ is:

$$\frac{\tilde{E}_1\pi_2}{\tilde{E}_0\pi_1} = \rho + \left(E[\tilde{\rho}_i] + \frac{Var(\tilde{\rho}_i)}{E[\tilde{\rho}_i]}\right)\left(1 - E[\chi_i] - \frac{Var[\chi_i]}{E(\chi_i)}\right) + \text{ covariance terms} \tag{23}$$

This highlights that heterogeneity in $\tilde{\rho}_i$ helps to generate hump-shaped IRFs in expectations, while heterogeneity in χ_i makes hump-shaped responses less likely to emerge.

Consumption responses

Consider an unconstrained agent facing an infinite-horizon consumption savings problem.

As in Gabaix (2020), take income as given. The consumption function is:

$$\hat{c}_{i,t} = \sum_{h\geq 0} \beta^h((1-\beta)\tilde{E}_{i,t}\hat{y}_{i,t+h} - \beta\gamma^{-1}\tilde{E}_{i,t}i_{t+h} + \beta\gamma^{-1}\tilde{E}_{i,t}\pi_{t+h+1}) \tag{24}$$

Since we hold expected income and nominal interest rates at steady state, we have $\tilde{E}_{i,t}\hat{y}_{i,t+h} = 0$ and $\tilde{E}_{i,t}i_{t+h} = 0$ for all t and h. The consumption function then reduces to:

$$\hat{c}_{i,t} = \beta\gamma^{-1}\sum_{h\geq 0}\beta^h\tilde{E}_{i,t}\pi_{t+h+1} = \beta\gamma^{-1}\frac{1}{1-\beta\tilde{\rho}_i}\tilde{E}_{i,t}\pi_{t+1} \tag{25}$$

To proceed, substitute in for the one period ahead expectation in time t using equation 20:

$$\hat{c}_{i,t} = \beta\gamma^{-1}\frac{1}{1-\beta\tilde{\rho}_i}\tilde{\rho}_i\chi_i\frac{\rho^{t+1}-(1-\chi_i)^{t+1}\tilde{\rho}_i^{t+1}}{\rho-(1-\chi_i)\tilde{\rho}_i} + d_{i,t} \tag{26}$$

Here, $d_{i,t}$ is an idiosyncratic noise term, which is a linear function of $q_{i,t}$, $q_{i,t-1}$, ...,$q_{i,0}$, and so has mean zero. Averaging across agents, one obtains:

$$\hat{c}_t = \beta\gamma^{-1}E\left[\frac{1}{1-\beta\tilde{\rho}_i}\tilde{\rho}_i\chi_i\frac{\rho^{t+1}-(1-\chi_i)^{t+1}\tilde{\rho}_i^{t+1}}{\rho-(1-\chi_i)\tilde{\rho}_i}\right] \tag{27}$$

Which in $t=0$ becomes:

$$\hat{c}_0 = \beta\gamma^{-1}E\left[\frac{\tilde{\rho}_i}{1-\beta\tilde{\rho}_i}\chi_i\right] \tag{28}$$

Applying the definition of a covariance then leads to equation 13.

B Survey details

B.1 Summary statistics

Table 4 shows summary statistics for the key variables used in our analysis, and several other household characteristics. The construction of $SD_i(\pi_t), SD_i(\pi_{t+1}), SD_i(\varepsilon_{t+1}), \chi_i, \tilde{\rho}_i$ is described in Appendix C.1.

Income and wealth variables are reported in bins. We take the mid-point of each bin. We code the lowest bin for income as if the lower bound is zero, and again take the midpoint (all wealth variables have a separate bin for zero). The top bin is coded as if it had the same width as the second-highest bin. Liquid wealth is (bank deposits + securities). Illiquid wealth is (property + firm ownership). Debt is (secured + unsecured debt). A respondent is classified as hand-to-mouth if their liquid wealth is < €1250.

Table 4: Summary statistics for expectations (point estimates and components), respondent characteristics, and income/wealth.

	Mean	Std Dev.	Min	Max
Panel A: Expectations				
$\tilde{E}_{i,t}\pi_t$	4.10	2.61	0	30
$\tilde{E}_{i,t}\pi_{t+1}$	4.90	4.99	-3	60
$SD_i(\pi_t)$	1.75	4.70	0.41	40.41
$SD_i(\pi_{t+1})$	1.72	1.35	0.30	8.80
$SD_i(\varepsilon_{t+1})$	1.87	1.52	0.04	12.12
χ_i	0.80	0.23	0	1
$\tilde{\rho}_i$	0.29	0.84	-5	5
Panel B: Demographics				
Age	56.87	14.66	16	80
Female	0.37	0.48	0	1
Higher Education	0.39	0.49	0	1
Is Working	0.55	0.50	0	1
Panel C: Income and Wealth				
Income	3.95	1.97	0.25	11
Liquid Wealth	90.49	154.90	0	1250
Illiquid Wealth	315.38	383.64	0	2375
Other Wealth	12.41	48.75	0	625
Debt	47.89	109.34	0	955
Owns Securities	0.62	0.48	0	1
Hand-to-mouth	0.14	0.34	0	1

Note: Bundesbank-Online-Panel-Households, November 2021 wave. For cases where χ_i is set-identified, respondents are excluded if the parameters are estimated very imprecisely (range$>$ 0.2). For all remaining set-identified parameters, the mid-point of the range is used. Observations of $\tilde{E}_{i,t}\pi_t, \tilde{E}_{i,t}\pi_{t+1}, SD_i(\pi_{t+1})$, and $SD_i(\pi_t)$ below the 1st or above the 99th percentile of that variable's distribution are also excluded as outliers, as are observations of $\tilde{\rho}_i$ outside $[-5, 5]$ (c.1% of observations). All income and wealth variables are in €1000s, and income refers to monthly net income of the household. Higher Education is an indicator for if the respondent has a bachelor's degree or higher, not including vocational training.

B.2 Survey questions (English and German)

Table 5 contains the existing questions in the Bundesbank survey eliciting point estimates of current and future inflation, and the density forecast of future inflation.

B.3 Novel questions for November 2021 (in German)

Table 6 contains the German text of our novel survey questions.

Table 5: Existing questions in the BOP-HH survey

Label	Text
Inflation Development	
Question	What do you think the rate of inflation or deflation in Germany was over the past twelve months?
Note	If you assume there was deflation, please enter a negative value. Values may have one decimal place.
Question	Wass denken Sie, wie hoch war die Inflationsrate oder Deflationsrate in den letzten zwölf Monaten in Deutschland?
Note	Im Falle einer angenommenen Deflationsrate tippen Sie bitte einen negativen Wert ein. Die Eingabe maximal einer Nachkommastelle ist möglich. Bitte geben Sie einen Wert hier ein.
Input Field	Percent
Inflation Expectations Qualitative	
Question	Do you think inflation or deflation is more likely over the next twelve months?
Note	Inflation is the percentage increase in the general price level. It is mostly measured using the consumer price index. A decrease in the price level is generally described as “deflation”.
Question	Was denken Sie, ist in den kommenden zwölf Monaten eher mit einer Inflation oder einer Deflation zu rechnen?
Note	Inflation ist der prozentuale Anstieg des allgemeinen Preisniveaus. Sie wird meist über den Verbraucher-preisindex gemessen. Ein Rückgang des Preisniveaus wird gemeinhin als “Deflation” bezeichnet.
Input Field	Select one answer
Inflation Expectations Quantitative	
Question	What do you think the rate of inflation/deflation will roughly be over the next twelve months? (select based on answer to *Inflation Expectations Qualitative*)
Note	Inflation is the percentage increase in the general price level. It is mostly measured using the consumer price index. A decrease in the price level is generally described as “deflation”.
Question	Was denken Sie, wie hoch wird die Inflationsrate/Deflationsrate in den kommenden zwölf Monaten in etwa sein? (select based on answer to *Inflation Expectations Qualitative*)

Note	Inflation ist der prozentuale Anstieg des allgemeinen Preisniveaus. Sie wird meist über den Verbraucherpreisindex gemessen. Ein Rückgang des Preisniveaus wird gemeinhin als “Deflation” bezeichnet. Bitte tippen Sie einen Wert in das Zahlenfeld ein (eine Nachkommastelle möglich).
Input Field	Percent

Inflation Expectations Probabilistic

Question	In your opinion, how likely is it that the rate of inflation will change as follows over the next twelve months?
Note	The aim of this question is to determine how likely you think it is that something specific will happen in the future. You can rate the likelihood on a scale from 0 to 100, with 0 meaning that an event is completely unlikely and 100 meaning that you are absolutely certain it will happen. Use values between the two extremes to moderate the strength of your opinion. Please note that your answers to the categories have to add up to 100.
Input Field	The rate of deflation (opposite of inflation) will be 12% or higher. __ The rate of deflation (opposite of inflation) will be between 8% and less than 12%. __ The rate of deflation (opposite of inflation) will be between 4% and less than 8%. __ The rate of deflation (opposite of inflation) will be between 2% and less than 4%. __ The rate of deflation (opposite of inflation) will be between 0% and less than 2%. __ The rate of inflation will be between 0% and less than 2%. __ The rate of inflation will be between 2% and less than 4%. __ The rate of inflation will be between 4% and less than 8%. __ The rate of inflation will be between 8% and less than 12%. __ The rate of inflation will be 12% or higher. __
Question	Für wie wahrscheinlich halten Sie es, dass sich die Inflationsrate in den kommenden zwölf Monaten wie folgt entwickelt?
Note	Bei dieser Frage geht es darum, wie Sie die Wahrscheinlichkeit einschätzen, dass ein bestimmter Sachverhalt in der Zukunft eintritt. Ihre Antworten können in einer Spanne zwischen 0 und 100 liegen, wobei 0 absolut unwahrscheinlich bedeutet und 100 absolut sicher. Mit Werten dazwischen können Sie Ihre Einschätzung abstufen. Bitte beachten Sie, dass sich die Angaben über alle Kategorien auf 100 summieren müssen.
Input Field	die Deflationsrate (Gegenteil von Inflation) wird 12% oder höher sein. __ die Deflationsrate (Gegenteil von Inflation) wird zwischen 8% und 12% liegen. __ die Deflationsrate (Gegenteil von Inflation) wird zwischen 4% und 8% liegen. __ die Deflationsrate (Gegenteil von Inflation) wird zwischen 2% und 4% liegen. __ die Deflationsrate (Gegenteil von Inflation) wird zwischen 0% und 2% liegen. __ die Inflationsrate wird zwischen 0% und 2% liegen. __ die Inflationsrate wird zwischen 2% und 4% liegen. __ die Inflationsrate wird zwischen 4% und 8% liegen. __ die Inflationsrate wird zwischen 8% und 12% liegen. __ die Inflationsrate wird 12% oder höher sein. __

Table 6: Novel questions added to the BOP-HH survey in November 2021

Question	Text
1	Nun möchten wir wissen, wie sicher Sie sich über Ihre Angabe zur Inflationsrate oder Deflationsrate in den letzten 12 Monaten sind ([Value of point estimate])%. Wie wahrscheinlich ist es Ihrer Meinung nach, dass die Inflationsrate in den letzten zwölf Monaten zwischen [Low inflation level]% und [High inflation level]% lag?
Hinweis	Bei dieser Frage geht es darum, wie Sie die Wahrscheinlichkeit einschätzen, dass die von Ihnen angegebene Inflationsrate oder Deflationsrate in den letzten 12 Monaten tatsächlich ungefähr diesen Wert angenommen hat. Ihre Antworten können zwischen 0 und 100 liegen, wobei 100 bedeutet, dass Sie absolut sicher sind. Kleinere Zahlen bedeuten, dass Sie sich weniger sicher sind.
Input Field	Prozent
Respondents randomly shown one of three scenarios before Question 2	
Group A	Stellen Sie sich die folgende hypothetische Situation vor: Aufgrund eines unerwarteten wirtschaftlichen Ereignisses hat sich die Inflationsrate im vergangenen Jahr um einen Prozentpunkt erhöht.
Group B	Stellen Sie sich die folgende hypothetische Situation vor: Aufgrund von uner-warteten Problemen mit der lokalen Produktionstechnologie im Nahen Osten ist der Rohölpreis im vergangenen Jahr gestiegen, was zu einem Anstieg der Inflationsrate um einen Prozentpunkt geführt hat.
Group C	Stellen Sie sich die folgende hypothetische Situation vor: Aufgrund gestiegener Verteidigungsausgaben sind die Staatsausgaben im vergangenen Jahr unerwartet stärker als üblich gestiegen, was zu einem Anstieg der Inflationsrate um einen Prozentpunkt geführt hat. Die Änderung ist vorübergehend und tritt ein, obwohl sich die Einschätzung der Regierung zur nationalen Sicherheit oder den wirtschaftlichen Bedingungen nicht geändert hat. Darüber hinaus ändern sich die Steuern nicht als Reaktion auf das Ausgabenprogramm.
2	Würden Sie in dieser Situation Ihre im vorderen Teil des Fragebogens genannten Inflationserwartungen für die nächsten 12 Monate anpassen? Wenn ja, inwiefern?
Input Field	1) Ja, von [Value of point estimate]Prozent auf __Prozent 2) Nein

C Variable construction

C.1 Main variables

To obtain V_i^p, we fit a symmetric triangular distribution to household i's answers:

$$V_i^p = \begin{cases} \frac{1}{6}\left(1 - \sqrt{1 - \frac{x_{1i}}{100}}\right)^{-2} & \text{if } \tilde{E}_i(\pi_t) \in (-5, 5) \\ \frac{2}{3}\left(1 - \sqrt{1 - \frac{x_{1i}}{100}}\right)^{-2} & \text{if } \tilde{E}_i(\pi_t) \notin (-5, 5) \end{cases} \tag{29}$$

where x_{1i} is respondent i's response to Question 1. Note that for households who report $x_i = 0$, this method provides an upper bound on their $Var_i(\pi_t)$.

To obtain $\tilde{\rho}_i$, we set $\tilde{\rho}_i = 0$ for households who select 'No' in answer to Question 2. For all others, we set:

$$\tilde{\rho}_i = x_{2i} - \tilde{E}_i(\pi_{t+1}) \tag{30}$$

where x_{2i} is respondent i's response to Question 2.

We then calculate V_i^f. For agents who are certain future inflation will lie within one specific bin, we calculate an upper bound on the variance using the symmetric triangular distribution, just as for the perception. The lower bound on V_i^f is given by zero.

For the remaining agents, we calculate V_i^f by taking the midpoints of each of the bins in the probability distribution. Denote these midpoints as z_j for the bins $j = 1, ..., n$. Denote the probability assigned to each bin as p_j. We then calculate the mean:

$$\bar{z}_i = \sum_{j=1}^{n} p_{i,j} z_j \tag{31}$$

The variance is then:

$$V_i^f = \sum_{j=1}^{n} p_{i,j}(z_j - \bar{z}_i) \tag{32}$$

The calculation of the Kalman gain is complicated by the fact that that for some respondents we have ranges of possible V_i^p or V_i^f, in which case we can only find ranges for χ_i and the other key parameters. We now describe how we calculate these parameters for each such case.

Case (i): V_i^p and V_i^f both point-identified

Calculate χ_i using:

$$\chi_i = 1 - \frac{V_i^p}{V_f^i} \tag{33}$$

Back out $\tilde{\sigma}^2_{\varepsilon,i}$ and $\tilde{\sigma}^2_{q,i}$ using:

$$\tilde{\sigma}^2_{\varepsilon,i} = V_i^f - \tilde{\rho}_i^2 V_i^p \tag{34}$$

$$\tilde{\sigma}^2_{q,i} = \frac{V_i^f V_i^p}{V_i^f - V_i^p} \tag{35}$$

Datapoints are inconsistent with Kalman filtering (and so are dropped) if $\chi_i < 0$ or $\tilde{\sigma}^2_{\varepsilon,i} < 0$.

Case (ii): V_i^f point-identified, V_i^p set-identified

This occurs if the respondent is certain that π_t lies within the specified interval, but places strictly positive probability in multiple intervals in the expectation question. V_i^p is then bounded below by zero, and the upper bound is calculated using the symmetric triangular distribution as above.

Denote the upper bound on V_i^p by a, so that $V_i^p \in [0, a]$. Under steady state Kalman filtering, it must be that $V_i^p \leq V_i^f$ and $V_i^p \leq \tilde{\rho}_i^{-2} V_i^f$. The latter is more restrictive if $|\tilde{\rho}_i| > 1$. This may shrink the upper bound on V_i^p, and hence raise the lower bound on the Kalman filter. As such, $V_i^p \in [0, \tilde{a}]$, where $\tilde{a}$ is given by:

$$\tilde{a} = \min(V_i^f, \tilde{\rho}_i^{-2} V_i^f, a) \tag{36}$$

Then we have the following ranges for the key parameters:

$$\chi_i \in \left[1 - \frac{\tilde{a}}{V_i^f}, 1\right], \; \tilde{\sigma}^2_{q,i} \in \left[0, \frac{V_i^f \tilde{a}}{V_i^f - \tilde{a}}\right], \; \tilde{\sigma}^2_{\varepsilon,i} \in \left[V_i^f - \tilde{\rho}_i^2 \tilde{a}, V_i^f\right] \tag{37}$$

Case (iii): V_i^f set-identified, V_i^p point-identified

In this case, the consumer is not certain that current inflation lies within the specified interval, but is certain that future inflation lies within one specific interval. As such, V_i^p is known, but $V_i^f \in [0, b]$, where b is given by the symmetric triangular distribution.

Under steady state Kalman filtering, it must be the case that $V_i^f \geq V_i^p$ and $V_i^f \geq \tilde{\rho}_i^2 V_i^p$. Hence $V_i^f \in [\tilde{b}, b]$, where:

$$\tilde{b} = \max(V_i^p, \tilde{\rho}_i^2 V_i^p) \tag{38}$$

Note that if $\tilde{b} > b$, then the observations must be dropped as they are inconsistent with steady state Kalman filtering. Using the equation for the Kalman gain, we then have:

$$\chi_i \in \left[1 - \frac{V_i^p}{\tilde{b}}, 1 - \frac{V_i^p}{b}\right] \tag{39}$$

The variance of the signal then lies in the interval:

$$\tilde{\sigma}_{q,i}^2 \in \left[\frac{bV_i^p}{b - V_i^p}, \frac{\tilde{b}V_i^p}{\tilde{b} - V_i^p}\right] \tag{40}$$

Note that if $\tilde{b} = V_i^p$, then the upper end of this interval is infinite, implying the signal may be infinitely noisy (i.e. contains no information).

Finally, the perceived variance of the shock lies in the range:

$$\tilde{\sigma}_{\varepsilon,i}^2 \in [\tilde{b} - \tilde{\rho}_i^2 V_i^p, b - \tilde{\rho}_i^2 V_i^p] \tag{41}$$

Case (iv): V_i^f and V_i^p both set-identified

In this case, the consumer is certain that current inflation lies within the specified interval, and certain that future inflation will lie within one specific interval. Hence, we have $V_i^p \in [0, a]$ and $V_i^f \in [0, b]$. If $|\tilde{\rho}_i| \leq 1$, then χ_i is unrestricted within the interval $[0, 1]$. If $|\tilde{\rho}_i| > 1$, then χ_i is bounded below as described above. Hence $\chi_i \in [0, 1]$ if $|\tilde{\rho}_i| \leq 1$, and $\chi_i \in [1 - \tilde{\rho}_i^{-2}, 1]$ if $|\tilde{\rho}_i| > 1$.

We then know that:

$$\tilde{\sigma}_{q,i}^2 = \frac{V_i^f V_i^p}{V_i^f - V_i^p} \tag{42}$$

If $|\tilde{\rho}_i| < 1$, this can take any value. It could be infinite large if $V_i^p = V_i^f$, and could be zero if $V_i^p = 0$ but $V_i^f > 0$. If $|\tilde{\rho}_i| > 1$, then $V_i^f \geq \tilde{\rho}_i^2 V_i^p$. In that case, $\tilde{\sigma}_{q,i}^2$ could still be zero, but the maximum value it can now take is:

$$\tilde{\sigma}_{q,i}^2 = \frac{V_i^f V_i^p}{V_i^f - V_i^p} \leq \frac{V_i^f V_i^p}{\tilde{\rho}_i^2 V_i^p - V_i^p} \tag{43}$$

$$= \frac{V_i^f}{\tilde{\rho}_i^2 - 1} \leq \frac{b}{\tilde{\rho}_i^2 - 1} \tag{44}$$

To summarize, then, $\tilde{\sigma}_{q,i}^2 \in [0, \infty)$ if $|\tilde{\rho}_i| \leq 1$, and $\tilde{\sigma}_{q,i}^2 \in [0, \frac{b}{\tilde{\rho}_i^2 - 1}]$ if $|\tilde{\rho}_i| > 1$.

Turning to $\tilde{\sigma}_{\varepsilon,i}^2$, this could always be zero in this case. The maximum it could be is b if $V_i^f = b$ and $V_i^p = 0$. Hence $\tilde{\sigma}_{\varepsilon,i}^2 \in [0, b]$.

C.2 Alternative measurement for V_i^f

We consider two alternative measures for V_i^f. Like the measurement of V_i^p, both make use of just two pieces of information per respondent: their point estimate for inflation in the following year, and the probability that inflation will be within a particular range around that point estimate. We use these two pieces of information to fit a symmetric triangular distribution to beliefs about future inflation, and infer the variance from that.

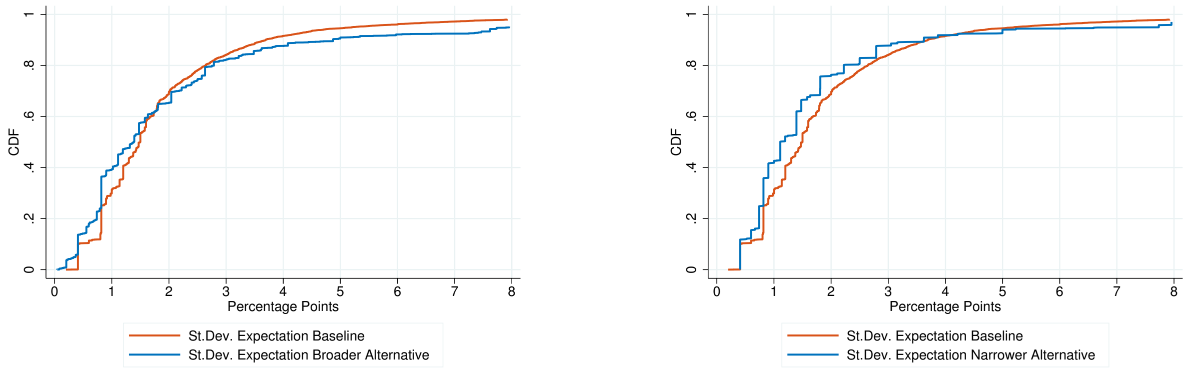
For both of these measures, we take the point estimate for future inflation from the existing question in the survey (see Table 5). For the first (broader) measure, we then consider the density forecast question, and focus just on the inflation rate bin containing the respondent's point estimate. The probability assigned to this inflation range gives us the second piece of information. That is, we observe:

1. $\tilde{E}_{i,t}\pi_{t+1}$
2. $\Pr(lb < \pi_{t+1} \leq ub)$

for lb, ub defined by the edges of the relevant bin in the density forecast. We then fit the symmetric triangular distribution as described in Appendix C.1. If the point estimate is on the boundary between two bins in the density forecast, we combine the bins to form one wider inflation range, and take the sum of the probabilities given. This disregards some information contained within the future inflation density forecasts, and indeed requires dropping a small number of observations where the point estimate is completely inconsistent with the density forecast (i.e. the density forecast assigns 0 probability to the bin containing the point estimate). The sample size therefore shrinks somewhat, to c.93% of the original sample size. It is however much closer to the measurement of V_i^p: the only differences are that the respondent has been simultaneously asked about the probabilities of inflation being in several ranges rather than just one, and that the bin we use is not necessarily symmetric about their point estimate.

In the second (narrower) alternative measure for V_i^f, we go further and remove the second of these points of difference. That is, we restrict the sample to only respondents whose point estimate is at the mid-point of one of the bins in the density forecast question, then apply the same method described above. This substantially reduces the number of observations, but does leave us with a measure of V_i^f computed in the same way as V_i^p. The only assumption required to make them exactly comparable is an independence of irrelevant alternatives: the fact that respondents are also asked about the probability of inflation being in other ranges far away from their point estimate does not affect their answer for the range around their point estimate.

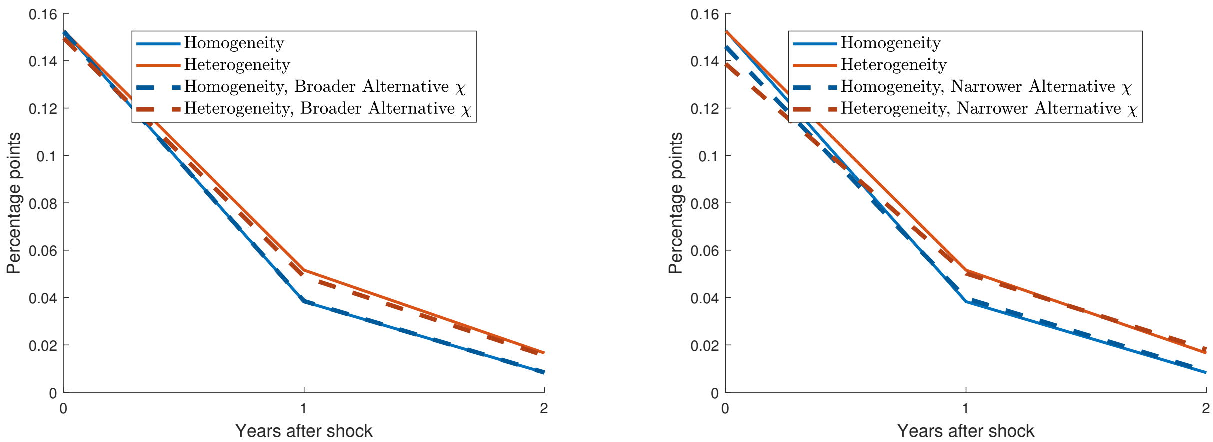
Figure 3 shows the distributions of V_i^f computed using our baseline measure using all of the information in density forecasts, and our two alternative triangular measures. They are all extremely similar. Moreover, the ranking of individuals within these distributions is strongly correlated. The Spearman's rank correlations of the first and second alternative measures with the baseline measure across individuals is 0.76 and 0.84 respectively.



(a) Broader Alternative Measure **(b)** Narrower Alternative Measure

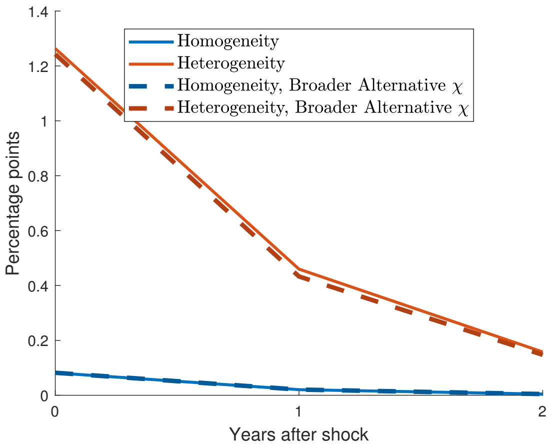
Figure 3: CDF of $St.Dev_i(\pi_{t+1})$ under the narrower and broader alternative calculation measures for V_i^f. Source: Bundesbank-Online-Panel-Households, November 2021 wave.

Unsurprisingly, the key results are therefore robust to these alternative variance measures. The mean Kalman gains in the two cases are 0.79 and 0.72, similar to to the 0.80 we find using our baseline measure. The different ways of calculating the Kalman gain correlate strongly across individuals, giving a Spearman's rank correlation of our baseline measure of χ_i with the first (broader) alternative of 0.69, and with the second (narrower) measure of 0.75. The impulse responses to an inflation shock in the model calibrated using the alternative measures are extremely close to those using the baseline measure (figures 4 and 5).

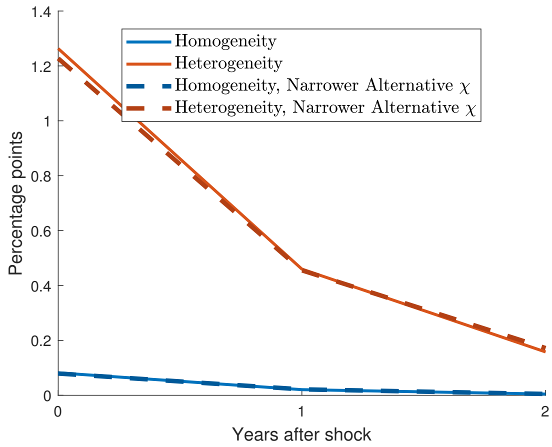


(a) Broader Alternative Measure **(b)** Narrower Alternative Measure

Figure 4: IRF of $\tilde{E}_t\pi_{t+1}$ under the narrower and broader alternative calculation measures for V_i^f. Source: Bundesbank-Online-Panel-Households, November 2021 wave.



(a) Broader Alternative Measure



(b) Narrower Alternative Measure

Figure 5: IRF of $\hat{c}_t$ under the narrower and broader alternative calculation measures for V_i^f. Source: Bundesbank-Online-Panel-Households, November 2021 wave.

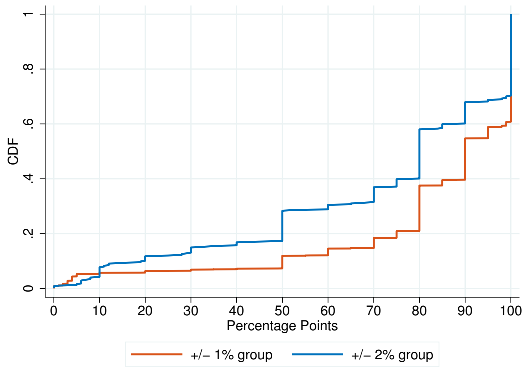
D Additional empirical results

D.1 Additional parameter distributions

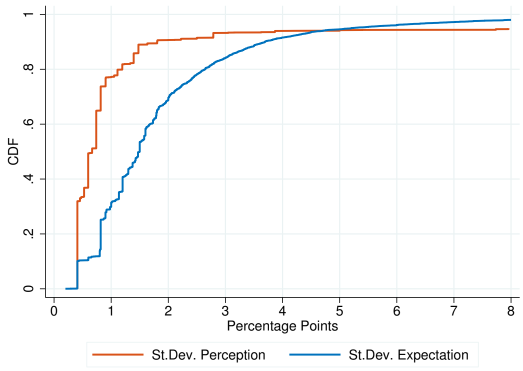
Figure 6a shows the CDF of the raw responses to question 1: respondents' assessment of the probability that current inflation lies within the specified range around their point estimate. We split the data between those with $\tilde{E}_{i,t}\pi_t \in (-5, 5)$, who were shown a $\pm 1\%$ interval, and those with $\tilde{E}_{i,t}\pi_t$ outside of this range, who were shown a $\pm 2\%$ interval. In both distributions, the majority believe there is at least an 80% chance that inflation lies within that range. Note that the $\pm 1\%$ group are more confident, despite seeing a smaller range, consistent with the notion that those who perceive lower rates of inflation or deflation are more certain in their perceptions. Figure 6b plots the CDFs of V_i^p and V_i^f. In cases where these are only set-identified, this plots the upper bound from fitting a symmetric triangular distribution. The lower bound in all such cases is 0. On average households are less uncertain about current inflation than about future inflation.

For respondents where we can only identify ranges for V_i^p and V_i^f, we can similarly only identify bounds for $\chi_i, \tilde{\sigma}^2_{q,i}, \tilde{\sigma}^2_{\varepsilon,i}$. Figure 7 shows the distributions of these parameters if we take the upper or lower bounds of the parameter ranges for those households respectively.

Figure 8 plots the distribution of $\tilde{\rho}_i$ when we exclude respondents whose response to the initial inflation expectations question ends in .0 or .5. Even excluding these households with the strongest tendency to round their answers, there is a large mass with $\tilde{\rho}_i = 0$, and substantial heterogeneity.

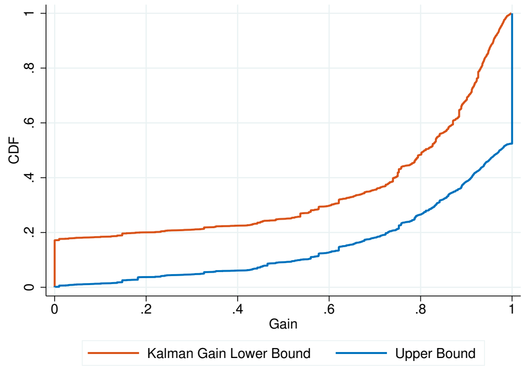


(a) CDFs of responses to question 1.

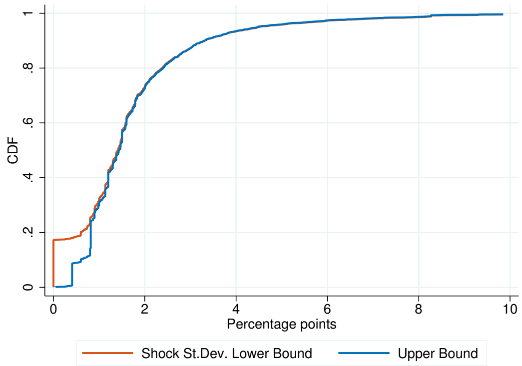


(b) CDFs of V_i^p and V_i^f.

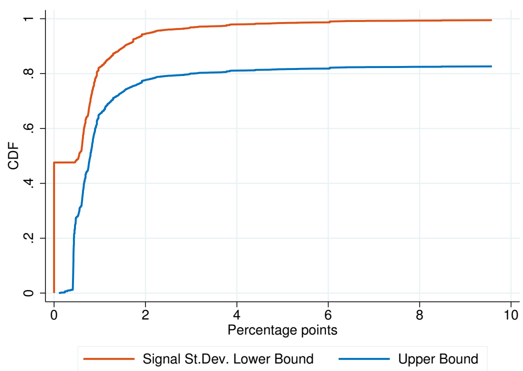
Figure 6: CDF of raw responses to question 1, for both the group shown a ±1% range and the group show a ±2% range, and the implied CDFs of V_i^p and V_i^f. Source: Bundesbank-Online-Panel-Households, November 2021 wave.



(a) χ_i



(b) $\tilde{\sigma}_{\varepsilon,i}$



(c) $\tilde{\sigma}_{q,i}$

Figure 7: CDFs of upper and lower bounds for inferred parameters. Source: Bundesbank-Online-Panel-Households, November 2021 wave.

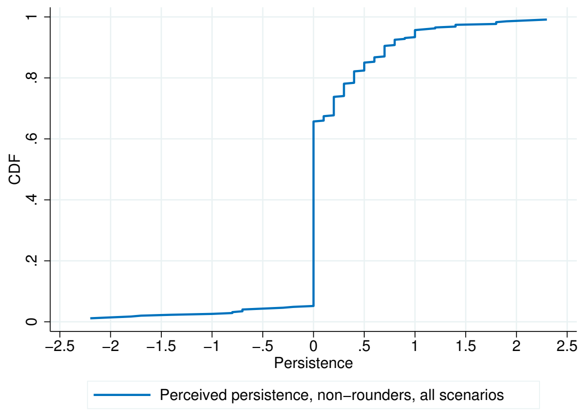


Figure 8: CDF of perceived persistence, only respondents whose response to initial inflation expectations question does not end in .0 or .5. Source: Bundesbank-Online-Panel-Households, November 2021 wave.

D.2 Relationship between components of expectation formation and point estimates

Table 7 shows the means of the elements of the expectation laws of motion, broken down by the respondent's inflation perception ($\tilde{E}_{i,t}\pi_t$). Those with an inflation perception far away from the actual value (which was approximately 5% at the time of the survey) tend to be the least certain in their perceptions. Those with the highest perceptions are also the least certain in their expectations and perceive the noise in the inflation process to be the highest. Those with perceptions that are either very high or very low also tend to have very low perceived persistence on average, and lower Kalman gains.

D.3 Relationship between components of expectation formation: further details

Table 8 breaks down the different elements of the expectation law of motion according to $\tilde{\rho}_i$, divided into five categories; $\tilde{\rho}_i < 0$ $\tilde{\rho}_i = 0$, $\tilde{\rho}_i \in (0,1)$, $\tilde{\rho}_i = 1$, and $\tilde{\rho}_i > 1$. This follows the classification for stock return beliefs in Dominitz and Manski (2011). Table 8 includes all respondents, with the most extreme 1% of responses for each of the standard deviation variables excluded as outliers.[17]

[17]Note that the first two columns include those for whom $V_i^p > V_i^f$, whose responses are inconsistent with steady-state Kalman filtering.

Table 7: Means of elements of expectation laws of motion, by inflation perception

	$SD_i(\pi_t)$	$SD_i(\pi_{t+1})$	$SD_i(\varepsilon_{t+1})$	χ_i	$\tilde{\rho}_i$
$\tilde{E}_{i,t}\pi_t < 0$	1.07	2.60	2.53	0.67	-0.13
	(0.27)	(0.52)	(0.48)	(0.13)	(0.30)
$\tilde{E}_{i,t}\pi_t \in [0, 2)$	0.77	1.85	1.80	0.70	0.21
	(0.04)	(0.16)	(0.17)	(0.04)	(0.08)
$\tilde{E}_{i,t}\pi_t \in [2, 4)$	0.67	1.84	1.78	0.78	0.24
	(0.01)	(0.03)	(0.03)	(0.01)	(0.02)
$\tilde{E}_{i,t}\pi_t \in [4, 6)$	0.71	2.13	2.07	0.81	0.22
	(0.01)	(0.04)	(0.04)	(0.01)	(0.02)
$\tilde{E}_{i,t}\pi_t \in [6, 8)$	1.44	3.35	3.22	0.74	0.22
	(0.10)	(0.19)	(0.19)	(0.03)	(0.09)
$\tilde{E}_{i,t}\pi_t \in [8, 10)$	1.44	3.95	3.89	0.81	0.17
	(0.17)	(0.44)	(0.41)	(0.07)	(0.17)
$\tilde{E}_{i,t}\pi_t \geq 10$	2.11	4.83	4.73	0.72	0.04
	(0.18)	(0.29)	(0.30)	(0.04)	(0.07)

Note: Bundesbank-Online-panel-Households, November 2021 wave. Standard errors in parentheses. For cases where χ_i is set-identified, respondents are excluded if the parameters are estimated very imprecisely (range> 0.2). For all remaining set-identified parameters, the mid-point of the range is used. Observations of $\tilde{E}_{i,t}\pi_t$, $\tilde{E}_{i,t}\pi_{t+1}$, $SD_i(\pi_{t+1})$, and $SD_i(\pi_t)$ below the 1st or above the 99th percentile of that variable's distribution are also excluded as outliers, as are observations of $\tilde{\rho}_i$ outside $[-5, 5]$ (c.1% of observations).

Table 8: Means of elements of expectation laws of motion, by persistence type

	$SD_i(\pi_t)$	$SD_i(\pi_{t+1})$	$SD_i(\varepsilon_{t+1})$	χ_i	HTM	Owns Stocks
$\tilde{\rho}_i < 0$	1.58	1.79	1.65	0.85	0.09	0.65
	(0.39)	(0.11)	(0.14)	(0.02)	(0.03)	(0.04)
$\tilde{\rho}_i = 0$	1.80	1.72	1.96	0.79	0.14	0.56
	(0.09)	(0.03)	(0.04)	(0.01)	(0.01)	(0.01)
$\tilde{\rho}_i \in (0, 1)$	0.90	1.59	1.68	0.80	0.09	0.67
	(0.15)	(0.07)	(0.08)	(0.02)	(0.02)	(0.03)
$\tilde{\rho}_i = 1$	1.59	1.56	1.51	0.79	0.10	0.59
	(0.20)	(0.05)	(0.06)	(0.01)	(0.01)	(0.02)
$\tilde{\rho}_i > 1$	2.33	2.01	2.21	0.92	0.19	0.47
	(0.35)	(0.09)	(0.20)	(0.01)	(0.02)	(0.03)

Note: Bundesbank-Online-panel-Households, November 2021 wave. Standard errors in parentheses. For cases where χ_i is set-identified, respondents are excluded if the parameters are estimated very imprecisely (range> 0.2). For all remaining set-identified parameters, the mid-point of the range is used. Observations of $\tilde{E}_{i,t}\pi_t, \tilde{E}_{i,t}\pi_{t+1}, SD_i(\pi_{t+1})$, and $SD_i(\pi_t)$ below the 1st or above the 99th percentile of that variable's distribution are also excluded as outliers, as are observations of $\tilde{\rho}_i$ outside $[-5, 5]$ (c.1% of observations). Note that the first two columns include those for whom $V_i^p > V_i^f$, whose responses are inconsistent with steady-state Kalman filtering.

There is a highly non-linear relationship between the standard deviation of the perception and the persistence type. In particular, those who perceive that inflation is persistent but mean reverting are the most confident in their inflation perceptions. Those who believe that $\tilde{\rho}_i > 1$ have the lowest confidence in their perceptions. This fits with the notion that those who track inflation most closely are also those who have the best knowledge of its dynamic properties. Those who believe inflation has zero persistence and those who believe it is explosive tend to also believe that the noise in the inflation process is highest.

As noted in Result 4, those who believe that $\tilde{\rho}_i > 1$ are more likely to be hand-to-mouth than any of the other persistence types, over twice as likely if one only includes those whose responses are consistent with Kalman filtering. They are also much less likely to own securities.

Finally, note that those who believe that $\tilde{\rho}_i > 1$ have the highest χ_i on average. However, this is partly mechanical, since if $|\tilde{\rho}_i| > 1$ then that places a lower bound on the values of χ_i that are consistent with steady-state Kalman filtering. Between the groups with $\tilde{\rho}_i \in [0, 1]$, the average Kalman filter varies little.

Table 9 shows regressions of each component of the expectation laws of motion on $\tilde{\rho}_i$, split in two ways. The first panel splits respondents according to which hypothetical scenario they were shown before Question 2, to explore the role of different shock types. That is, each dependent variable is regressed on $\tilde{\rho}_i$ interacted with a categorical variable reflecting which shock scenario the respondent saw.

The second panel splits households into some of the persistence categories outlined above, specifically those who believe the price level is mean-reverting ($\tilde{\rho}_i < 0$), those who believe inflation is persistent but stationary ($\tilde{\rho}_i \in (0, 1)$), and those who believe inflation is non-stationary with positive persistence ($\tilde{\rho}_i \geq 1$). The final panel shows the results of regressing each dependent variable on an indicator equal to 1 if the household does no updating of expectations at all when faced with the hypothetical shock ($\tilde{\rho}_i = 0$).

In the first panel, there are some significant differences between shock types in the relationships of $\tilde{\rho}_i$ with other elements of expectation laws of motion. However, the magnitudes are generally small. For that reason we pool households across shock types for the analysis in Section 4.2.

The differences are much larger, however, across persistence types. Panel 2 shows that within households who believe inflation is persistent and stationary, greater perceived persistence is associated with less uncertainty about current and future inflation, less perceived noise in the inflation process, and a greater implied Kalman gain. This is consistent with models of endogenous information acquisition, as with a more persistent inflation process information about the current rate of inflation is more valuable.

Although there are only weak relationships between $\tilde{\rho}_i$ and uncertainty over current and future inflation across the whole sample, the second panel reveals that this is driven by weak relationships among those who believe in inflation processes that are qualitatively different from the data. Among those who believe that inflation is persistent but stationary, the relationships between $\tilde{\rho}_i$ and uncertainty are very strong. Since an AR(1) process estimated on German CPI inflation over the previous 20 years implies a persistence of

$\rho = 0.21$, this suggests that the group of households most aware of the time-series properties of inflation behave as predicted by models of rational inattention (e.g. Sims, 2003). However outside of this group, households behave less in line with those predictions.

Table 9: Breakdown of $\tilde{\rho}_i$ relationships with other expectation law of motion components by shock type and persistence category.

	(1) $SD_i(\pi_{t+1})$	(2) $SD_i(\pi_t)$	(3) $SD_i(\varepsilon_{t+1})$	(4) χ_i
Panel A: Shock type				
Shock unspecified $\times\ \tilde{\rho}_i$	-0.00278	0.0103	-0.162*	-0.00969
	(0.105)	(0.0306)	(0.0913)	(0.0149)
Supply $\times\ \tilde{\rho}_i$	0.0956	0.00437	-0.118	0.0104
	(0.105)	(0.0246)	(0.102)	(0.00983)
Demand $\times\ \tilde{\rho}_i$	0.145	-0.0546***	0.0278	0.0418***
	(0.118)	(0.0165)	(0.116)	(0.00872)
Constant	2.077***	0.751***	2.055***	0.789***
	(0.0308)	(0.0115)	(0.0307)	(0.00526)
Panel B: Persistence type				
$\tilde{\rho}_i < 0 \times \tilde{\rho}_i$	-0.150	0.0268	0.132	-0.0390***
	(0.117)	(0.0306)	(0.123)	(0.0142)
$\tilde{\rho}_i \in (0,1) \times \tilde{\rho}_i$	-0.459***	-0.321***	-0.532***	0.0497*
	(0.161)	(0.0351)	(0.162)	(0.0301)
$\tilde{\rho}_i \geq 1 \times \tilde{\rho}_i$	0.164**	-0.00462	-0.0968	0.0246***
	(0.0795)	(0.0178)	(0.0788)	(0.00727)
Constant	2.082***	0.765***	2.087***	0.784***
	(0.0340)	(0.0130)	(0.0340)	(0.00574)
Panel C: Updating indicator				
$\tilde{\rho}_i \neq 0$	-0.106*	-0.0744***	-0.306***	0.0132
	(0.0594)	(0.0196)	(0.0585)	(0.0102)
Constant	2.129***	0.770***	2.128***	0.789***
	(0.0343)	(0.0136)	(0.0343)	(0.00596)
Observations	2317	2317	2317	2317

Note: Bundesbank-Online-panel-Households, November 2021 wave.
Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

D.4 Correlations of expectation components with household characteristics: further details

Table 10 repeats the analysis in Table 3, allowing the coefficient on liquid wealth to differ for those with high liquid wealth (top 10% of respondents). The positive association between χ_i and liquid wealth found in Table 3 is only present for wealthy households.

Table 10: Regressions of components of subjective laws of motion on household characteristics, interacting liquid wealth with an indicator for having liquid wealth above the 90th percentile.

	(1) $\log(SD_i(\pi_{t+1}))$	(2) $\log(SD_i(\pi_t))$	(3) $\log(SD_i(\varepsilon_{t+1}))$	(4) $\log(\chi_i)$	(5) $\tilde{\rho}_i$
Hand-to-mouth	0.0138	0.1147**	0.1113*	0.0803*	0.4183*
	(0.0376)	(0.0556)	(0.0598)	(0.0421)	(0.2141)
High liquid wealth=0	-0.0004*	-0.0008**	-0.0005	-0.0002	-0.0010
× Liquid wealth	(0.0002)	(0.0003)	(0.0004)	(0.0003)	(0.0016)
High liquid wealth=1	0.0000	-0.0002**	0.0002**	0.0002**	-0.0013**
× Liquid wealth	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0005)
Illiquid wealth	0.0000	-0.0000	-0.0000	0.0000	0.0002
	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0002)
Other wealth	-0.0001	-0.0001	0.0003	-0.0000	-0.0012
	(0.0002)	(0.0002)	(0.0003)	(0.0002)	(0.0008)
Debt	0.0000	0.0001	0.0001	0.0002	-0.0009
	(0.0001)	(0.0002)	(0.0002)	(0.0001)	(0.0006)
log(income)	-0.0905***	-0.1212***	-0.1777***	-0.0016	0.0605
	(0.0235)	(0.0349)	(0.0380)	(0.0334)	(0.1840)
HH Controls	Yes	Yes	Yes	Yes	Yes
Hurdle model	No	No	No	No	Yes
Observations	4344	3161	2232	2014	3194
R^2	0.0490	0.0560	0.0669	0.0228	
Pseudo R^2					0.0239

Note: Bundesbank-Online-panel-Households, November 2021 wave. The household controls are age, age^2, gender, region, education, occupation category, and employment status, coded as described in Table 3. Robust standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 11 column 1 shows the full results of the hurdle model regressing $\tilde{\rho}_i$ on household characteristics. The upper panel is as in Table 3 column 4, and the lower panel is the selection step. The remaining columns of Table 11 repeat this exercise, splitting by shock type. The association between $\tilde{\rho}_i$ and being hand-to-mouth is absent for those who saw the demand shock scenario, though this is imprecisely estimated.

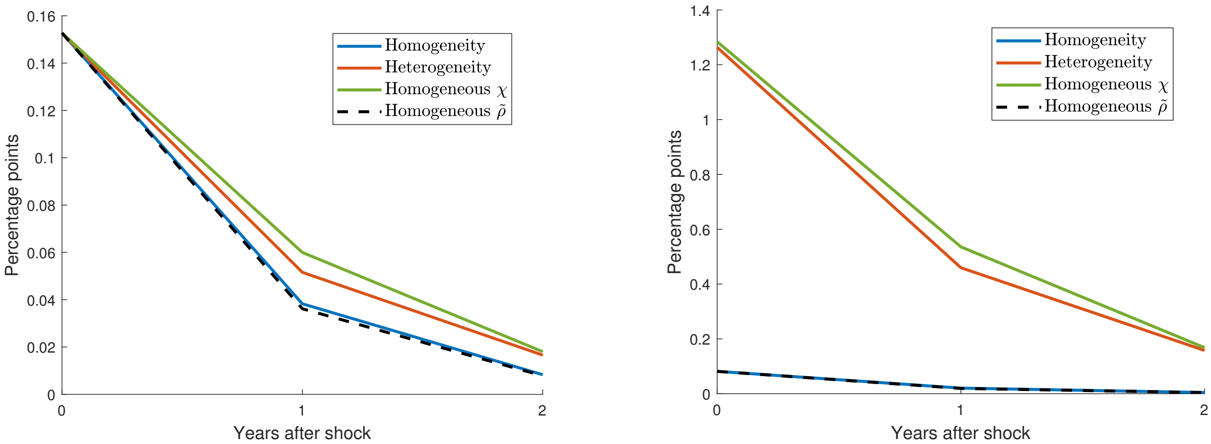
Table 11: Regressions of $\tilde{\rho}_i$ on household characteristics, split by shock type.

	(1)	(2)	(3)	(4)
	$\tilde{\rho}_i$	$\tilde{\rho}_i$	$\tilde{\rho}_i$	$\tilde{\rho}_i$
$\tilde{\rho}_i$				
Hand-to-mouth	0.4072**	0.4296	0.3946	-0.0898
	(0.2039)	(0.3519)	(0.2736)	(0.3867)
Liquid wealth	-0.0013**	-0.0006	-0.0016**	-0.0016
	(0.0005)	(0.0006)	(0.0007)	(0.0014)
Illiquid wealth	0.0002	-0.0003	0.0004	0.0006
	(0.0002)	(0.0002)	(0.0003)	(0.0004)
Other wealth	-0.0012	-0.0002	-0.0019	-0.0035*
	(0.0007)	(0.0009)	(0.0016)	(0.0018)
Debt	-0.0009	-0.0007	-0.0003	-0.0014
	(0.0006)	(0.0008)	(0.0009)	(0.0011)
log(income)	0.0627	0.2877	0.1716	-0.5092
	(0.1830)	(0.2258)	(0.2256)	(0.3664)
selection ($\tilde{\rho}_i \neq 0$)				
Hand-to-mouth	-0.0882	-0.0859	-0.2268*	0.0726
	(0.0774)	(0.1418)	(0.1346)	(0.1368)
Liquid wealth	0.0000	0.0002	0.0002	-0.0001
	(0.0002)	(0.0003)	(0.0003)	(0.0003)
Illiquid wealth	0.0001	0.0002	0.0000	-0.0001
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Other wealth	0.0004	0.0011	-0.0005	0.0012
	(0.0005)	(0.0009)	(0.0009)	(0.0008)
Debt	0.0000	0.0002	-0.0007	0.0007
	(0.0003)	(0.0004)	(0.0005)	(0.0004)
log(income)	-0.0363	-0.1734	0.0130	0.0122
	(0.0582)	(0.1083)	(0.1060)	(0.1001)
HH Controls	Yes	Yes	Yes	Yes
Hurdle model	Yes	Yes	Yes	Yes
Shock type	All	Unspecified	Supply	Demand
Observations	3194	1053	1057	1084
Pseudo R^2	0.0238	0.0593	0.0469	0.0520

Note: Bundesbank-Online-panel-Households, November 2021 wave. The household controls are age, age^2, gender, region, education, occupation category, and employment status, coded as described in Table 3. Robust standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

E Further impulse response exercises

In Figure 9b, we break down the amplification from heterogeneous expectation formation into its components. The impulse response with heterogeneity in $\tilde{\rho}_i$ only, but homogeneous χ_i, is close to that with full heterogeneity. This is therefore the main driver of the amplification we find. Note however that the difference between the IRFs with full heterogeneity and with homogeneous χ_i is small relative to the response with full heterogeneity, it remains large relative to the consumption responses with homogeneous expectation formation, and with homogeneous $\tilde{\rho}_i$. The covariance between $\tilde{\rho}_i$ and χ_i, though small in the data, does still play a non-trivial role in aggregate consumption dynamics.

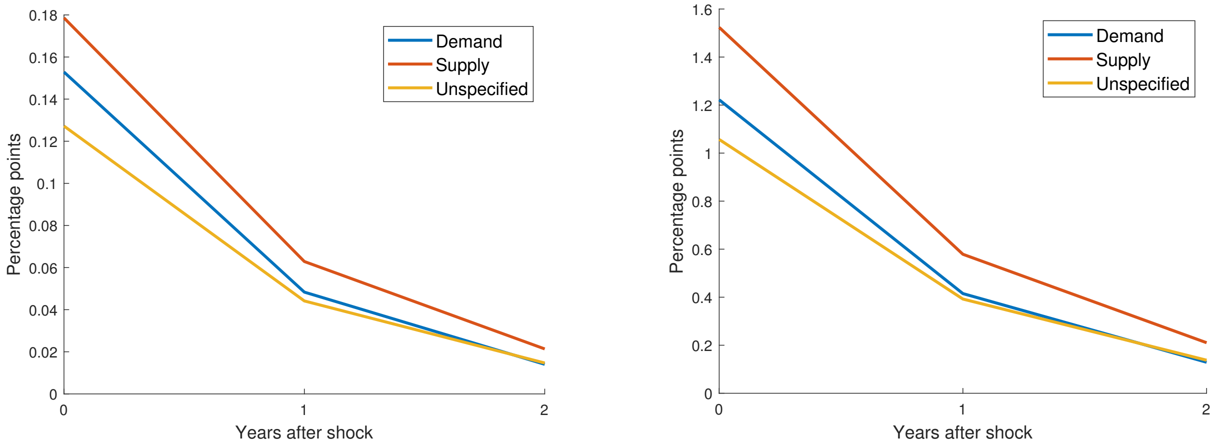


(a) Expectations **(b)** Consumption

Figure 9: Implied IRF of aggregate expectations and aggregate consumption. The homogeneity and heterogeneity cases are as described in Section 5.2. The remaining cases set χ_i and $\tilde{\rho}_i$ respectively to their average values for all households. Source: Bundesbank-Online-Panel-Households, November 2021 wave.

Using the survey responses to the different hypothetical scenarios in Question 2, we can further compare the effects of heterogeneous expectation laws of motion for different types of shock. We find somewhat greater amplification and persistence in consumption responses to supply shocks than other types of shock. A comparison of the IRFs between the three cases is shown in Figures 10a and 10b. This result is consistent with the higher average perceived persistence of supply shocks discussed in Section 4.

As in Section 4, we also repeat these exercises with the distributions of subjective models after excluding those whose answers are rounded to a multiple of 0.5. The model with heterogeneity does deliver smaller initial consumption responses in this case, but it is still 4.4× larger than under the homogeneity case calibrated using the average $\tilde{\rho}_i$ and χ_i across all respondents. This rises to 5.5× if one compares to a representative agent model calibrated using the average $\tilde{\rho}_i$ and χ_i for the population of non-rounders (as they have slightly smaller perceived persistence on average). As such, the result



(a) Expectations

(b) Consumption

Figure 10: Implied IRFs of one-period ahead inflation expectations and consumption by shock. Source: Bundesbank-Online-Panel-Households, November 2021 wave.

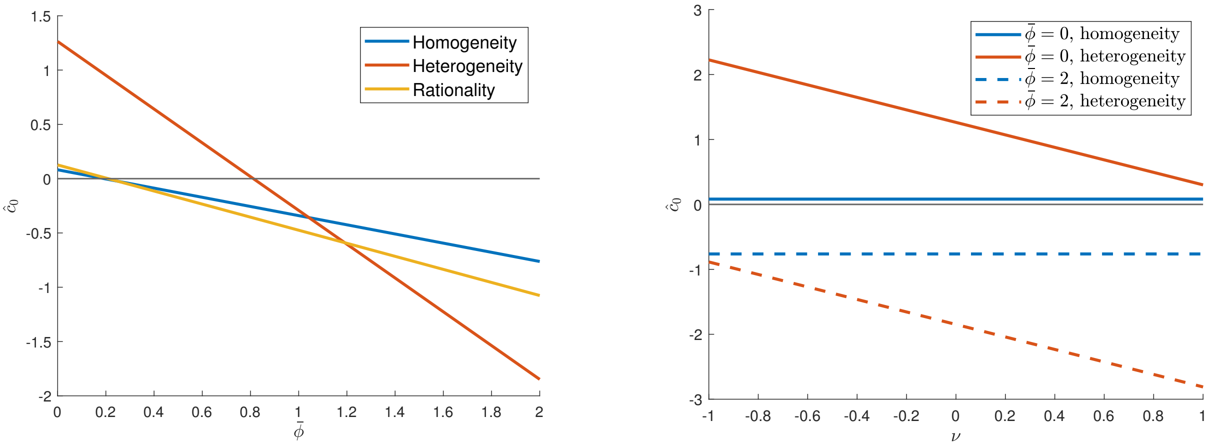
that heterogeneity generates very significant amplification of the transmission of inflation shocks to consumption still holds.

Finally, we allow for households to expect nominal interest rates to respond to inflation. Specifically, we assume that:

$$\tilde{E}_{i,t} i_{t+h} = \tilde{\phi}_i \tilde{E}_{i,t} \pi_{t+h} \tag{45}$$

Households are assumed to not observe i_t precisely when they choose $c_{i,t}$, consistent with them not observing π_t. They therefore infer i_t from their perceived current inflation, just as they do for expectations of future periods.

Figure 11 shows the initial consumption response $\hat{c}_0$ under homogeneity, heterogeneity, and rationality for a range of assumptions about $\tilde{\phi}_i$. In panel (a), we assume that all households share a common $\tilde{\phi}_i = \bar{\phi}$. When the Taylor principle is expected to be satisfied, higher inflation leads households to expect higher real interest rates, and so to reduce consumption. Aside from a small region around $\bar{\phi} = 1$, in which inflation shocks are not expected to affect real interest rates at all, heterogeneity provides substantial amplification of aggregate consumption, and indeed heterogeneity also increases the sensitivity of aggregate consumption to the perceived interest rate rule. In panel (b), we show that our results also remain robust even if $\tilde{\phi}_i$ is allowed to covary with $\tilde{\rho}_i$. Such covariances do make some quantitative difference to the results, so future research could consider ways to measure these relationships.



(a) Varying $\bar{\phi}$

(b) Varying ν

Figure 11: Implied $\hat{c}_0$ under different assumptions on $\tilde{\phi}_i$. In panel (a) all households form interest rate expectations using the same perceived Taylor Rule parameter $\bar{\phi}$. In panel (b) this parameter varies across households according to $\tilde{\phi}_i = \bar{\phi} + \nu(\tilde{\rho}_i - E[\tilde{\rho}_i])$. Source: Bundesbank-Online-Panel-Households, November 2021 wave.