Shock Transmission and the Sources of Heterogeneous Expectations*

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Abstract

This paper studies how heterogeneity in expectation formation affects the transmission of macroeconomic shocks. In a general class of macroeconomic models, I first identify a novel channel of shock transmission that works through such heterogeneity. Agents forming expectations observe information about realized variables, and pass it through a model to map from that information to the expectation of interest. I show that shocks transmit through heterogeneous expectations whenever these two components are correlated across agents: when there are systematic relationships between agents' information and subjective models. This has broad implications, as many standard theories of bounded rationality generate such relationships if heterogeneity is permitted in both components of expectations. I then study this effect in a specific application to household beliefs around inflation. Using unique features of a UK survey, I document evidence of my novel channel in this context. In a model matching this data, transmission through expectations heterogeneity is substantial and time-varying. In particular, transitory inflation spikes may become 'baked in' to the expectations of certain households, with persistent effects on future shock transmission.
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1 Introduction

It has been well documented empirically that expectations of macroeconomic variables tend to be extremely heterogeneous. Even within groups of similar agents, expectations of inflation, unemployment, and other variables are extremely dispersed.[1] However, when policymakers study expectations data they typically ignore this, and focus on an average measure of the relevant expectation. Academic work frequently does the same.[2]

Where existing models do feature heterogeneous expectations, the heterogeneity is often a side-effect of underlying frictions, useful for distinguishing between different models or identifying parameters.[3] In this paper I propose an alternative reason to look beyond the first moment of the distribution of expectations: heterogeneous expectations are a *channel* through which shocks transmit to aggregate variables. I show that such effects can be large, and are missed by analysis relying on average expectations alone.

Critically, this transmission channel depends on the underlying *source* of the heterogeneity. To form an expectation an agent takes some information on the realizations of certain variables, and passes it through a model to map from their information to the expectation of interest. In workhorse macroeconomic models, for example, agents observe all variables realized up to the current period (full information), and map from that to expectations using each variable's equilibrium law of motion (rational expectations). Heterogeneity could therefore arise because agents have heterogeneous information, or because they use heterogeneous subjective models to interpret their information - or it could be both.

The first contribution of this paper is to show that in that last case, cross-sectional relationships between information and subjective models generate a novel *narrative heterogeneity* channel of aggregate shock transmission. I characterize this channel using a decomposition of the response of aggregate behavior to shocks in a general log-linear macroeconomic framework, with arbitrary expectation formation. While this is a general result, I then go on to show that the narrative heterogeneity channel has large and time-varying effects in a specific application to household beliefs about inflation. In particular, I find that temporary spikes in inflation may get 'baked in' to the expectations of certain households, with persistent consequences for future shock transmission.

[1]e.g. Carroll (2003), Mankiw et al. (2004), Dovern et al. (2012), Coibion and Gorodnichenko (2012), Andrade and Le Bihan (2013), Ma et al. (2021), Candia et al. (2022).

[2]Recent policy examples include Powell (2022), Mann (2022), Schnabel (2022). In academic work this is necessarily the case in models with a representative agent (e.g. Fuster et al., 2010; Bhandari et al., 2019; Caballero and Simsek, 2022; Gáti, 2022). It is also common when using expectations data in empirical work (e.g. Coibion and Gorodnichenko, 2015; Adam et al., 2022; Doh and Smith, 2022).

[3]e.g. Pfajfar and Santoro (2010); Coibion and Gorodnichenko (2012); Falck et al. (2021); Wang (2022).

The intuition for the narrative heterogeneity channel stems from the fact that an agent receiving possibly noisy information about a variable uses it for two purposes. First, they update expectations about that variable directly, depending on the precision of their information. Second, they update expectations of other variables, depending on how the variables are related in their subjective model of the economy. Information about a given shock therefore causes different reactions in agents with different subjective models. If that information is observed most precisely by agents with particular non-representative models, their subjective model has a disproportionate impact on aggregate expectations, and on aggregate behavior. Formally, the aggregate response to a shock depends on the cross-sectional covariance between the two rounds of updating: between information precision, and the perceived relationships between variables. I refer to this as the narrative heterogeneity channel because a simple definition of a narrative is that it consists of a state of the world (information) and a series of perceived consequences (subjective model) (Gibbons and Prusak, 2020).[4]

On top of this, the decomposition in the first part of the paper contains a second *response heterogeneity* channel. This may imply a further role for heterogeneous expectations, if there is heterogeneity in how agents respond to their own expectations. In this case, a shock is amplified if the resulting changes in expectations are largest among those who respond most strongly to those expectations. This extends the well-known effects of heterogeneity in marginal propensities to consume (Auclert, 2019; Bilbiie, 2019) to expectations. Evidence for such relationships with expectations is provided in Macaulay and Moberly (2022).

However, while examples of the response heterogeneity channel have appeared in some recent literature (Broer et al., 2020; Grimaud, 2021), existing theoretical models do not allow for the narrative heterogeneity channel. This is because they only allow heterogeneity in either information or subjective models, but not both.[5] Such approaches require minimal deviations from workhorse models with full information and rational expectations, but are at odds with growing evidence for heterogeneity across both information and subjective models in a variety of contexts.[6] Moreover, if this two-sided heterogeneity were permitted, many standard models of information frictions and subjective model formation would imply strong

[4]I use narratives here to mean stories an agent might use to form expectations, rather than narrative identification as in Romer and Romer (2004). See the related literature section below for how this paper links with other recent models of narratives in economics (e.g. Shiller, 2017; Eliaz and Spiegler, 2020).

[5]See for example Angeletos and Pavan (2009), Broer et al. (2020) for heterogeneous information, and Branch and Evans (2006), Malmendier and Nagel (2016) for heterogeneous subjective models. Models departing from both full information and rational expectations simultaneously (e.g. Angeletos et al., 2020; Bianchi et al., 2021) have so far abstracted from heterogeneity. See also the related literature section below.

[6]See for example Link et al. (2021) for information, Andre et al. (2022b) for subjective models, and Pfajfar and Santoro (2010), Beutel and Weber (2021), and Macaulay and Moberly (2022) for both.

systematic relationships between the two components of expectations, suggesting a powerful role for the narrative heterogeneity channel. For example, in models of rational inattention (Maćkowiak et al., 2020), different subjective models imply different incentives to acquire information. And if agents are learning (Evans and McGough, 2020), then observing different information will lead them to form different subjective models. The narrative heterogeneity channel is therefore relevant in a wide range of macroeconomic settings.

Having characterized the narrative heterogeneity channel in this general setting, I then show that it is present and powerful in a specific application, concerning household beliefs about inflation. Empirically, I document that information and subjective models are indeed systematically correlated across households. In a model that accounts for the specific patterns observed, the narrative heterogeneity channel has substantial time-varying effects on the transmission of inflationary shocks. The expectations of a representative agent are not therefore sufficient to understand aggregate dynamics in this context.

To show this, I first use unique features of the Bank of England's Inflation Attitudes Survey to separate information and subjective models about inflation at the household level. Respondents are asked about the information sources they used to arrive at their expectations, and how a hypothetical rise in inflation would affect the strength of the UK economy. The first of these questions concerns information without involving the conclusions drawn from it. The second concerns the respondent's subjective model of how inflation relates to the rest of the economy, without asking about information or expectations.

I document two key patterns in this data. First, households who believe inflation makes little difference to the strength of the economy use less information about inflation than households with other subjective models. Those who believe inflation has positive or negative effects use similar information sources. Crucially, this means there is a systematic relationship between information and subjective models, implying that the narrative heterogeneity channel will operate.

Second, information that inflation is high is associated with more negative subjective models of the effects of inflation, both in the cross-section and over time. A greater proportion of households report that inflation makes the economy weaker in periods with high realized inflation, and within a period, those who believe inflation is currently higher are more likely to hold such negative subjective models. The joint distribution of information and subjective models therefore varies over time, and is systematically related to the state of the economy.

In the final part of the paper I develop a model that is consistent with the empirical results, to evaluate the macroeconomic implications of the narrative heterogeneity channel.

The key ingredients required to match the data are that households face costs of acquiring information about inflation, and that they adjust their subjective models of how inflation affects real incomes when they observe the realizations of their chosen information.

Costly information acquisition, or rational inattention (Sims, 2003), generates the first cross-sectional result. Intuitively, if a household's subjective model implies that inflation is irrelevant for their choices, they perceive no benefit to information, and they will not pay for it. To match the remaining empirical observations it is then necessary to augment this with a belief-updating process, in which households with high perceptions of current inflation update their subjective model towards the view that inflation erodes real incomes.[7] When realized inflation rises, average inflation perceptions rise, and more households hold negative subjective models of the effects of inflation. In the cross-section, high perceived inflation is associated with more negative subjective models.

This two-way feedback between information and subjective models has several implications for aggregate dynamics. A selection effect weakens the aggregate effects of information frictions, as the households who precisely observe shocks to inflation are those intending to react strongly to such information. In addition, changes in inflation affect the joint distribution of information and subjective models, generating substantial state-dependent variation in the effects of inflationary shocks. Calibrating the model to the survey data, the narrative heterogeneity channel accounts for a third of the elasticity of aggregate consumption to inflation in steady state, and almost 40% of its variation.

Most importantly, the interaction between the two components of expectations can cause temporarily high inflation to become 'baked in' to the expectations of certain households, a concern for many economies in 2022 (Carstens, 2022). Households with subjective models in which inflation strengthens the real economy observe the higher inflation, and update their subjective model to a less positive view. As their long-run expectations also rise, they carry this more neutral model into the following periods, which means they place less value on any future inflation information. They pay less attention to inflation going forward. This in turn implies that they never adjust their expectations back down, even if inflation subsequently falls. Conversely, households with negative subjective models pay more attention, and so observe any future disinflation with great precision. The survey evidence is consistent with this mechanism. This selective baking in has persistent effects on the future dynamics of the economy, through a persistent change in the narrative heterogeneity channel. These effects would be missed in an analysis only considering average expectations.

[7] While the implications of the model are derived using a reduced-form version of this process, I offer a microfoundation in Appendix D.5.

Related literature. This paper principally contributes to the broad literatures on information frictions, subjective models, and heterogeneity in macroeconomics. In recent years, a large literature has documented an important role for heterogeneous household income and wealth in the transmission of macroeconomic shocks (see Kaplan and Violante, 2018, for a review). In particular, Auclert (2019) decomposes the channels of monetary policy transmission, highlighting those operating through heterogeneity in household asset positions. However, as this decomposition is done assuming perfect foresight, it cannot shed light on the heterogeneous macroeconomic expectations studied here.

Heterogeneous expectations are, however, common in models of limited information (see Coibion et al., 2018, for a review). Agents receive idiosyncratic signals (e.g. Sims, 2003), or update information sets in different periods (e.g. Reis, 2006). In addition, incentives to acquire information may differ across agents (Broer et al., 2020; Macaulay, 2021; Ciani et al., 2022). However, these models typically assume that agents know the true equilibrium model of the economy, so they abstract away from the narrative heterogeneity channel. Indeed, in models with no heterogeneity in other agent characteristics, so no response heterogeneity channel, the dispersion in expectations often plays no direct role in shock transmission, and is rather a byproduct of the sluggishness in average expectations that determines aggregate dynamics (e.g. Maćkowiak and Wiederholt, 2009; Coibion and Gorodnichenko, 2015).

Similarly, papers on learning (Evans and McGough, 2020), model uncertainty (Ilut and Schneider, 2022), imperfect common knowledge (Angeletos and Lian, 2018), level-k thinking (Farhi and Werning, 2019), and others study the effects of misperceptions of the true structural relationships in the economy, assuming that agents observe all variable realizations up to the current period (Molavi, 2019). Again, heterogeneous expectations feature frequently in this literature (see Hommes, 2021, for a review), for example because different cohorts use different life experiences to learn about laws of motion (Malmendier and Nagel, 2016). Similarly, heterogeneity in the way investors interpret data (i.e. heterogeneous subjective models) can explain a variety of phenomena in financial markets (Harris and Raviv, 1993; Scheinkman and Xiong, 2003; Banerjee and Kremer, 2010; Atmaz and Basak, 2018; Martin and Papadimitriou, 2022) and labor markets (Jäger et al., 2021; Braun and Figueiredo, 2022). With full information, however, this literature abstracts away from the narrative heterogeneity channel, and in many cases the average subjective model is sufficient to summarize aggregate shock transmission (e.g. Andrade et al., 2019).

Where existing literature does depart simultaneously from both full information and rational expectations, the focus is on settings with a representative agent (Ryngaert, 2018; Bordalo et al., 2018, 2020; Angeletos et al., 2020; Bianchi et al., 2021; Maxted, 2022). However, there is mounting evidence that in many contexts there is substantial heterogeneity in both information (Song and Stern, 2021; Link et al., 2021, 2022) and subjective models (Patton and Timmermann, 2010; Andrade et al., 2019; Laudenbach et al., 2021; Andre et al., 2022b). Pfajfar and Santoro (2010), Madeira and Zafar (2015), Beutel and Weber (2021), and Macaulay and Moberly (2022) find evidence for simultaneous heterogeneity along both dimensions. To my knowledge, this paper is the first to systematically study the transmission effects of simultaneous heterogeneity in these two components of expectation formation.

The empirical part of the paper also contributes to this literature, by separating information from subjective models around inflation in a survey with a long time series. This complements early work on household dislike of inflation (Shiller, 1997), and more recent evidence relating this to expectations of other variables and actions (Kamdar, 2019; Candia et al., 2020). Relatedly, Michelacci and Paciello (2020) and Dräger et al. (2020) document heterogeneity in household preferences over inflation and interest rates, which are plausibly linked to subjective models of how those variables affect other aspects of a household's environment. I extend this by documenting the correlation of those subjective models with household information, which drives the narrative heterogeneity channel.

Finally, while models of narratives have been developed in microeconomics and political economy (Bénabou et al., 2018; Akerlof et al., 2020; Eliaz and Spiegler, 2020), most work in macroeconomics has been concerned with empirically tracking particular narratives and their impacts (Shiller, 2017; Larsen et al., 2021; Goetzmann et al., 2022). Macaulay and Song (2022) in particular find that multiple distinct narratives often circulate about the same economic events: the framework in this paper is a step towards incorporating such heterogeneous narratives into macroeconomic models. Note that the Directed Acyclic Graphs increasingly used to define narratives in this literature (Eliaz and Spiegler, 2020; Andre et al., 2022a; Macaulay and Song, 2022) are nested in the subjective models considered here, as are the prior belief distortions in Flynn and Sastry (2022).

Outline. The rest of the paper is structured as follows. Section 2 derives the novel decomposition of aggregate responses to shocks in a general log-linear model with arbitrary information and subjective models. Section 3 explores information and subjective models about inflation in the data. Section 4 develops a model to match the empirical findings, and Sections 5 and 6 explore the implications of that model. Section 7 concludes.

2 General decomposition

I begin by presenting a decomposition of the effects of an arbitrary shock on the aggregate choices of a group of agents, in a general log-linear model. The decomposition highlights the roles played by information and subjective models, and their distribution across agents, in determining the strength of aggregate shock transmission. The aggregate response to a shock comes through three channels: the representative agent channel, the response heterogeneity channel, and the narrative heterogeneity channel.

2.1 The agent

Agent $i \in I$ chooses a $N_x \times 1$ vector of choice variables $\boldsymbol{X}_t^i$ in period t. Letting lower case letters be log-deviations of variables from some arbitrary point, a log-linear approximation of their policy function can be written:[8]

$$\boldsymbol{x}_t^i = \boldsymbol{\mu}_t^i \mathbb{E}_t^i \boldsymbol{z}_t^i \tag{1}$$

where $\boldsymbol{z}_t^i$ is a $N_z \times 1$ vector of variables taken as given by the agent,[9] and $\boldsymbol{\mu}_t^i$ is a $N_x \times N_z$ matrix of coefficients. This can be thought of as the log-linearized solution to some optimization problem, which has been left in the background.

The vector $\boldsymbol{z}_t^i$ may include both aggregate and idiosyncratic variables. Some elements of $\boldsymbol{z}_t^i$ may be known precisely by the agent; for the unknown elements, the agent-specific expectations operator $\mathbb{E}_t^i$ may or may not coincide with rational expectations. The elements of $\boldsymbol{z}_t^i$ may also be realized in any period: the indexation at time t simply reflects that they are the variables that matter for period t choices. This setup therefore encompasses a wide range of models, for choices made by households, firms, investors, and other types of agent. I show a particular example with a standard household consumption-saving problem in Section 2.2.

I now consider a shock ξ_t, which affects some or all of the variables in $\boldsymbol{z}_t^i$. The reaction of agent choices is determined by the effects of the shock on the expectation of each element

[8]This linearization need not be taken about a steady state, or about the same point for each agent. If two agents have different idiosyncratic state variables, they can therefore have different responses to aggregate variables and expectations, just as they would in a fully non-linear model. This is why the coefficients $\boldsymbol{\mu}_t^i$ are indexed by agent and by period, as the linearization could be taken about different points each period.

[9]This is without loss of generality, as any endogenous choice variable can also be expressed as a linear function of other elements of $\boldsymbol{z}_t^i$. Substituting out using that function, and repeating for any remaining endogenous variables, gives a policy function only in terms of variables exogenous to the agent.

of the policy function:

$$\frac{d\boldsymbol{x}_t^i}{d\xi_t} = \boldsymbol{\mu}_t^i \frac{d\mathbb{E}_t^i \boldsymbol{z}_t^i}{d\xi_t} \tag{2}$$

Applying the chain rule to the derivative of each element of $\mathbb{E}_t^i \boldsymbol{z}_t^i$ leads to a simple expression for the agent's response to the shock.

Proposition 1 *For any agent with policy function described by equation 1, the response to a shock ξ_t is given by:*

$$\frac{d\boldsymbol{x}_t^i}{d\xi_t} = \boldsymbol{\mu}_t^i (\boldsymbol{I} - \boldsymbol{\mathcal{M}}_t^i)^{-1} \boldsymbol{\delta}_t^i \tag{3}$$

where:

$$\boldsymbol{\mathcal{M}}_t^i = \begin{pmatrix} 0 & \mathcal{M}_{12,t}^i & \dots & \mathcal{M}_{1N_z,t}^i \\ \mathcal{M}_{21,t}^i & 0 & \dots & \mathcal{M}_{2N_z,t}^i \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{M}_{N_z1,t}^i & \mathcal{M}_{N_z2,t}^i & \dots & 0 \end{pmatrix}, \qquad \mathcal{M}_{jk,t}^i \equiv \frac{\partial \mathbb{E}_t^i z_{jt}^i}{\partial \mathbb{E}_t^i z_{kt}^i} \tag{4}$$

$$\boldsymbol{\delta}_t^i = \left(\left.\frac{d\mathbb{E}_t^i z_{1t}^i}{d\xi_t}\right|_{\mathbb{E}_t^i z_{m\neq 1,t}}, \left.\frac{d\mathbb{E}_t^i z_{2t}^i}{d\xi_t}\right|_{\mathbb{E}_t^i z_{m\neq 2,t}}, \dots, \left.\frac{d\mathbb{E}_t^i z_{N_zt}^i}{d\xi_t}\right|_{\mathbb{E}_t^i z_{m\neq N_z,t}} \right)'$$

Proof. Appendix A.1 ■

Equation 3 is useful because it distinctly highlights the separate roles played by the agent's information, subjective model, and policy function coefficients in determining the behavioral response to the shock. When the shock occurs, agent i first receives some direct information about how each of the variables in $\boldsymbol{z}_t^i$ have changed, and updates their expectations of each according to $\boldsymbol{\delta}_t^i$. Importantly, each element of $\boldsymbol{\delta}_t^i$ is defined as the update to that expectation in response to the shock, *holding constant* the expectations of all other variables. This first update does not therefore involve passing information about other variables through the agent's subjective model of how the variables relate to each other. $\boldsymbol{\delta}_t^i$ therefore captures a broad notion of the information observed about each variable, separately from that subjective model.[10] The j^{th} element of $\boldsymbol{\delta}_t^i$ will be zero for an agent who obtains no direct information about the corresponding z_{jt}^i, and will rise to the realized change $dz_{jt}^i/d\xi_t$ with perfect observation. If the agent is Bayesian, then between these extremes $\boldsymbol{\delta}_t^i$ reflects the signal-to-noise ratio of observed information (see the example in Section 2.2). Otherwise,

[10]Note that I am agnostic here about how the agents acquire this information, so this encompasses models of exogenous noisy information (Lucas, 1972), rational inattention (Sims, 2003), information avoidance (Golman et al., 2017), social learning (Mobius and Rosenblat, 2014), and others.

$\boldsymbol{\delta}_t^i$ simply reflects the agent's non-Bayesian use of direct information (e.g. De Filippis et al., 2022).

This, however, does not capture the entire response of expectations to the shock. After updating the expectation of each variable through the direct information effect, the agent engages in a second round of updating, where they use their newly updated expectations of each z_{jt} to inform their expectations of all other variables that they believe to be linked to z_{jt} through their subjective model. This secondary updating is reflected by $(\boldsymbol{I} - \boldsymbol{\mathcal{M}}_t^i)^{-1}$.[11] Once all expectations have been updated, the coefficients $\boldsymbol{\mu}_t^i$ determine the choice response.

Importantly, while the matrix $\boldsymbol{\mathcal{M}}_t^i$ reflects the direct effect of expectations of one variable on another, variables may also be linked indirectly. That is, an update to $\mathbb{E}_t^i z_{jt}^i$ may affect $\mathbb{E}_t^i z_{kt}^i$ directly, but also indirectly through its effect on the expectation of some other variable $\mathbb{E}_t^i z_{lt}^i$, which is linked in the household's subjective model to both z_{jt}^i and z_{kt}^i. The matrix $(\boldsymbol{I} - \boldsymbol{\mathcal{M}}_t^i)^{-1}$ captures all such direct and indirect links between variables. From here, it will be convenient to work directly with this, which I refer to as the cross-learning matrix:[12]

$$\boldsymbol{\chi}_t^i \equiv (\boldsymbol{I} - \boldsymbol{\mathcal{M}}_t^i)^{-1} \tag{5}$$

where the $(j,k)^{th}$ element of $\boldsymbol{\chi}_t^i$ will be denoted $\chi_{jk,t}^i$. It is these values that are measured in the empirical literature on cross-learning (e.g. Roth and Wohlfart, 2020). By allowing for direct and indirect perceived links across variables, this nests a wide range of possible subjective models, including those involving many variables (e.g. Crump et al., 2021).

Finally, having updated all of their expectations using their information, and then again using their subjective model, agent choices are determined by their reaction to each of those expectations, which is contained in the coefficient matrix $\boldsymbol{\mu}_t^i$. The information, subjective model, and response components of the agent's economic narrative are therefore represented by $\boldsymbol{\delta}_t^i$, $\boldsymbol{\chi}_t^i$, and $\boldsymbol{\mu}_t^i$ respectively.

Notice that full information rational expectations is nested in this framework, as the special case in which all variables realized up to period t are observed, and the subjective model coincides with the true model in equilibrium. This therefore differs from models in which narratives are represented by Directed Acyclic Graphs (DAGs) (Spiegler, 2020): while DAGs are also nested in the notion of subjective models in this section, most general equilibrium

[11] Since the variables held constant in the definition of $\boldsymbol{\delta}_t^i$ may include the actions of other agents, the effects of higher-order beliefs on the perceived optimal use of information also enter through this term.

[12] This has a parallel in the literature on production networks (Carvalho and Tahbaz-Salehi, 2019). The direct links in $\boldsymbol{\mathcal{M}}_t^i$ are analogous to the elements of the input-output matrix, and $\boldsymbol{\chi}_t^i$ is the corresponding Leontief inverse. As with production networks, this Leontief inverse regulates the transmission of shocks.

models do not have a recursive causal ordering of variables, so the true equilibrium laws of motion cannot be expressed as a DAG.

2.2 An example

Consider the textbook setup where infinitely lived households have CRRA utility over consumption, and can trade one-period risk-free bonds for intertemporal consumption smoothing. The consumption function of household i log-linearized about steady state is:

$$c_t^i = (1-\beta)\sum_{s=0}^{\infty}\beta^s \mathbb{E}_t^i y_{t+s} - \sigma\beta\sum_{s=0}^{\infty}\beta^s(\mathbb{E}_t^i r_{t+s} - \mathbb{E}_t^i \pi_{t+s+1}) \tag{6}$$

where y_t is real income in period t, r_t is the nominal interest rate, and π_t is gross inflation. The parameters β and σ are the discount factor and coefficient of relative risk aversion respectively. See Appendix A.2 for the derivation.

This is the familiar result that consumption depends on the expected present value of future income and all expected future real interest rates. Within the framework of equation 1, $\boldsymbol{z}_t^i$ contains all current and future realizations of y_t, r_t, and π_{t+1}. The coefficients $\boldsymbol{\mu}_t^i$ contain the relevant combinations of the preference parameters β and σ.

To see the interpretation of Proposition 1 in more detail, assume that households believe inflation and income are linked according to a simple subjective model:

$$\begin{aligned} y_t &= \alpha^i \pi_t + u_{yt}, \quad u_{yt} \sim N(0, \sigma_y^2) \\ \pi_t &= u_{\pi t}, \quad u_{\pi t} \sim N(0, \sigma_\pi^2) \end{aligned} \tag{7}$$

That is, inflation may have causal effects on real incomes, but there is believed to be no feedback from real incomes to inflation. For this example, assume that the household does not believe r_t is related to either y_t or π_t, so we can leave that out of the analysis.

The household observes a noisy signal about each variable of interest in period t:

$$\begin{aligned} s_{yt}^i &= y_t + \varepsilon_{yt}^i, \quad \varepsilon_{yt}^i \sim N(0, \sigma_{\varepsilon y}^2) \\ s_{\pi t}^i &= \pi_t + \varepsilon_{\pi t}^i, \quad \varepsilon_{\pi t}^i \sim N(0, \sigma_{\varepsilon \pi}^2) \end{aligned} \tag{8}$$

If the household follows Bayes' rule to incorporate these signals into their expectations of y_t and π_t, their posterior expectations of each are a linear combination of s_{yt}^i and $s_{\pi t}^i$, with the weights depending on the relative signal-to-noise ratios of each signal. Importantly, those ratios depend on α^i, as that determines how strongly the variables are believed to be linked, and therefore how informative s^i_{yt} is about π_t, and similarly how informative $s^i_{\pi t}$ is about y_t. Rearranging the resulting expressions for posterior expectations gives:

$$\begin{aligned}
\mathbb{E}^i_t y_t &= \frac{\sigma^2_y}{\sigma^2_y + \sigma^2_{\varepsilon y}} s^i_{yt} + \alpha^i \frac{\sigma^2_{\varepsilon y}}{\sigma^2_y + \sigma^2_{\varepsilon y}} \mathbb{E}^i_t \pi_t \\
\mathbb{E}^i_t \pi_t &= \frac{\sigma^2_\pi}{\sigma^2_\pi + \sigma^2_{\varepsilon\pi}\left(1 + \alpha^{i2}\frac{\sigma^2_y}{\sigma^2_\pi}\right)} s^i_{\pi t} + \alpha^i \frac{\sigma^2_{\varepsilon\pi}\frac{\sigma^2_y}{\sigma^2_\pi}}{\sigma^2_\pi + \sigma^2_{\varepsilon\pi}\left(1 + \alpha^{i2}\frac{\sigma^2_y}{\sigma^2_\pi}\right)} \mathbb{E}^i_t y_t
\end{aligned} \tag{9}$$

After a shock ξ_t that moves both y_t and π_t, these expectations change according to:

$$\begin{aligned}
\frac{d\mathbb{E}^i_t y_t}{d\xi_t} &= \frac{\sigma^2_y}{\sigma^2_y + \sigma^2_{\varepsilon y}} \frac{dy_t}{d\xi_t} + \alpha^i \frac{\sigma^2_{\varepsilon y}}{\sigma^2_y + \sigma^2_{\varepsilon y}} \frac{d\mathbb{E}^i_t \pi_t}{d\xi_t} \\
\frac{d\mathbb{E}^i_t \pi_t}{d\xi_t} &= \frac{\sigma^2_\pi}{\sigma^2_\pi + \sigma^2_{\varepsilon\pi}\left(1 + \alpha^{i2}\frac{\sigma^2_y}{\sigma^2_\pi}\right)} \frac{d\pi_t}{d\xi_t} + \alpha^i \frac{\sigma^2_{\varepsilon\pi}\frac{\sigma^2_y}{\sigma^2_\pi}}{\sigma^2_\pi + \sigma^2_{\varepsilon\pi}\left(1 + \alpha^{i2}\frac{\sigma^2_y}{\sigma^2_\pi}\right)} \frac{d\mathbb{E}^i_t y_t}{d\xi_t}
\end{aligned} \tag{10}$$

Combining these two equations to solve for each expectation change yields the form in equation 3. The first terms of each equation contain the elements of $\boldsymbol{\delta}^i_t$, and the coefficients in the second terms contain the elements of $\boldsymbol{\mathcal{M}}^i_t$.[13]

Consider first the change in $\mathbb{E}^i_t y_t$. The first term has two components: the signal-to-noise ratio of the income signal s^i_{yt}, and the underlying response of y_t to the shock. That is, if they precisely observe y_t, then $\mathbb{E}^i_t y_t$ responds to the shock in exactly the same way as the realized variable, regardless of changes in $\mathbb{E}^i_t \pi_t$. The noisier the household's direct information about y_t, the smaller that direct response. At the extreme with no direct information observed about y_t ($\sigma^2_{\varepsilon y} \to \infty$), the direct effect of the shock on expectations approaches 0 and the only way the household can update $\mathbb{E}^i_t y_t$ is through $\mathbb{E}^i_t \pi_t$.

The coefficient in the second term also has two components. First, a change in expected inflation only affects expected income if the household believes that the two are linked in their subjective model ($\alpha^i \neq 0$). The slope of the perceived relationship between them, α^i, therefore regulates the updating from $\mathbb{E}^i_t \pi_t$ to $\mathbb{E}^i_t y_t$. Second, this slope from the subjective model is scaled by a factor equal to one minus the signal-to-noise ratio. Intuitively, this scaling reflects how strongly the household weights the information in $\mathbb{E}^i_t \pi_t$ relative to the other information they have about y_t.

Now turn to the change in $\mathbb{E}^i_t \pi_t$. All of the effects described above are present, but there

[13]The equations have precisely the form of equation 66 used in the proof of Proposition 1.

is a further nuance. The weights on $d\pi_t/d\xi_t$ and $d\mathbb{E}_t^i y_t/d\xi_t$ are no longer determined by the simple signal-to-noise ratio in the relevant direct signal. This is because the first term of the $\mathbb{E}_t^i \pi_t$ updating equation reflects the extent of updating due to $s_{\pi t}^i$, holding $\mathbb{E}_t^i y_t$ constant. Since in the household's subjective model π_t is a direct cause of y_t, this conditioning involves assuming the structural shock u_{yt} offsets the perceived rise in π_t, effectively reducing the informativeness of $s_{\pi t}^i$ when it is used in this way. This distortion is smaller if income shocks are believed to be more volatile relative to inflation shocks, as then y_t is less strongly correlated with π_t in the subjective model.

The core insights, however, remain the same: the direct response varies between 0 (if $\sigma_{\varepsilon\pi}^2 \to \infty$) and the realized change in inflation (if $\sigma_{\varepsilon\pi}^2 = 0$), and the coefficient on $d\mathbb{E}_t^i y_t/d\xi_t$ is determined by the association between π_t and y_t in the subjective model (α^i), and how the household weights that information relative to the direct information.

2.3 Aggregate behavior

I now return to the general case. Consider a unit mass of the agents modeled in Section 2.1. Aggregate choices for each choice variable x_{st}^i are given by:

$$\bar{x}_{st} = \int_0^1 \omega_{st}^i x_{st}^i di \tag{11}$$

where ω_{st}^i denotes a weighting applied to agent i's choice x_{st}^i, such that:

$$\bar{x}_{st} = \mathbb{E}_I x_{st}^i \tag{12}$$

where $\mathbb{E}_I$ denotes the expected value across agents.

Again, consider a shock ξ_t that affects some or all of the variables in agent choice functions. Proposition 1 and the properties of covariances lead us to the following decomposition of the aggregate choice response:

Proposition 2 *The response of aggregate choice $\bar{x}_{st}$ to a shock ξ_t is given by:*

$$\frac{d\bar{x}_{st}}{d\xi_t} = \sum_{j=1}^{N_z} \sum_{k=1}^{N_z} \left[\bar{\mu}_{sj,t} \bar{\chi}_{jk,t} \bar{\delta}_{k,t} + Cov_I(\mu_{sj,t}^i, \chi_{jk,t}^i \delta_{k,t}^i) + \bar{\mu}_{sj,t} Cov_I(\chi_{jk,t}^i, \delta_{k,t}^i) \right] \tag{13}$$

where $\delta_{k,t}^i$ and $\mu_{sj,t}^i$ denote the k^{th} element of $\boldsymbol{\delta}_t^i$ and the $(s,j)^{th}$ element of $\boldsymbol{\mu}_t^i$ respectively, $\bar{\delta}_{k,t}$ and $\bar{\mu}_{sj,t}$ are their aggregate counterparts, and $\bar{\chi}_{jk,t}$ is the aggregate value of $\chi_{jk,t}^i$ across *agents i.*

Proof. Appendix A.1 ■

This decomposition shows that three groups of channels determine the aggregate response to shocks. The first term is the *representative agent channel*: the effects of the average coefficients, subjective model, and information about each variable. This summarizes all shock transmission channels in models with a representative agent, and indeed in many models with heterogeneity, where average expectation formation is sufficient to capture shock responses to first order. In Maćkowiak and Wiederholt (2009), for example, firms acquire idiosyncratic signals about aggregate shocks, but the dynamics of the price level are determined by the average level of inattention. Similarly, in Andrade et al. (2019) households differ in their interpretation of forward guidance announcements, but the aggregate effects of an announcement depend only on the average over this mix of beliefs. Heterogeneity might affect the average information or subjective model sustained in equilibrium, but unless one of the other two terms in the decomposition is non-zero, those averages alone drive aggregate shock transmission.

The second term is the *response heterogeneity channel*. Since $\boldsymbol{\chi}_t^i \boldsymbol{\delta}_t^i$ gives the total expectation response to the shock ($d\mathbb{E}_t^i \boldsymbol{z}_t^i / d\xi_t$), this reflects that shocks will be amplified if the agents whose expectations react the most to the shock are the agents whose actions are most sensitive to those expectations. Macaulay and Moberly (2022) provide evidence of one such correlation, between the behavior of inflation expectations and liquidity constraints among German households. This channel, for other expectations, is also behind the novel dynamics in Broer et al. (2020) and Macaulay (2021). In models with full information and rational expectations, the only way the expectations updates can be heterogeneous across agents is if they are differentially exposed to the shock. In that case the true response of idiosyncratic variables will differ across agents, and so observations of e.g. income will respond in heterogeneous ways to the shock. In this way the response heterogeneity channel nests the transmission effects of correlations between heterogeneous MPCs and shock exposure studied extensively in the heterogeneous-agent literature (Auclert, 2019; Bilbiie, 2019).

Finally, the third term is the *narrative heterogeneity channel*. Heterogeneous expectations can generate a channel of aggregate shock transmission even if every agent has the same policy function, if information ($\delta_{k,t}^i$) is correlated with subjective models ($\chi_{jk,t}^i$) across agents. Subjective models determine an agent's response to a given piece of information, so if information is concentrated among agents with particular non-representative subjective models, that distorts the aggregate response away from the representative agent effect.

This channel is novel to this paper. However, standard theories of information acquisition and subjective model formation will generate such correlations between information and subjective models, if heterogeneity is permitted across both. In models of rational inattention (Maćkowiak et al., 2020), agents with different subjective models will have different incentives to acquire information, leading to systematic relationships between the two. And different observed information will lead to agents forming different subjective models in, for example, models with recursive learning (Evans and McGough, 2020). Existing papers in these literatures only miss the narrative heterogeneity channel because they restrict heterogeneity to *either* information, *or* subjective models, but not both; the relevant covariance is always therefore forced to be zero.[14]

To further highlight the intuition for these channels, consider again the textbook consumption function in equation 6, and a shock that increases future inflation π_{t+1}. If all households believe that higher inflation is associated with lower real incomes, then the average $\chi^i_{y\pi,t}$ is negative, and the aggregate consumption response to the future inflation will be negative. This is the *representative agent channel*. If, however, this pessimistic subjective model of the effects of inflation only takes hold among hand-to-mouth households, then aggregate consumption will respond much more positively to the shock than the average would suggest, because the households who reduce their expected future real incomes are the ones who react the least to their expectations. This is the *response heterogeneity channel*. Finally, if all households are unconstrained, but the pessimistic model of inflation is concentrated among households who do not obtain any information about future inflation, then this again raises the aggregate consumption response. Those households who would update expected future incomes down and reduce consumption if they learned that inflation was about to rise are precisely the households who do not observe the shock, and so do not learn of the shock. This is the *narrative heterogeneity channel*.

It is important to be clear that this is a decomposition, not a solution for aggregate actions. $\boldsymbol{\delta}^i_t$ captures direct information received by agent i, but the information received depends on the true reaction of $\boldsymbol{z}^i_t$ to the shock, which I have taken as given so far. In Section 2.4 I extend this to general equilibrium, where the realized changes in $\boldsymbol{z}^i_t$ may depend on aggregate choices made by agents. Proposition 2, however, remains the main result of this section, as it gives the clearest expression of the channels through which heterogeneous expectation formation affects aggregate shock transmission. The general equilibrium effects explored below serve only to further amplify or dampen these existing channels.

[14] An exception is Berardi (2007), who studies a model where agents learn from heterogeneous information sets. However, he studies equilibrium convergence, and not aggregate shock transmission.

2.4 General Equilibrium

To extend this framework to general equilibrium, I make two further assumptions. I therefore lose some of the generality of Sections 2.1-2.3, while still nesting a range of common models.

Assumption 1 All elements of $\boldsymbol{z}_t^i$ are equal across agents i, and are such that:

$$A\boldsymbol{z}_t + B\bar{\boldsymbol{x}}_t + C\boldsymbol{\xi}_t = 0 \tag{14}$$

where $\boldsymbol{z}_t$ is the common value of $\boldsymbol{z}_t^i$, $\bar{\boldsymbol{x}}_t$ is the vector of aggregate agent choices $\bar{x}_{st}$, and $\boldsymbol{\xi}_t$ is a $N_\xi \times 1$ vector of exogenous shocks. A, B, C are coefficient matrices, with dimensions $N_z \times N_z$, $N_z \times N_x$, and $N_z \times N_\xi$ respectively.

As with the choice function (equation 1), equation 14 can be thought of as a log-linearization of N_z structural equations, in this case general equilibrium consistency requirements derived from resource constraints and/or the optimization of other agents beyond those choosing $\boldsymbol{x}_t^i$. For example, if $\bar{\boldsymbol{x}}_t$ are the aggregate choices of households, then equation 14 may contain conditions derived from firm optimization (e.g. a Phillips curve), policy rules, and market clearing conditions. Note that this may require extending the set of variables included in $\boldsymbol{z}_t$, if there are aggregate variables involved in the general equilibrium conditions which do not enter the choice functions for $\boldsymbol{x}_t^i$. For clarity I continue to refer to the agents choosing $\boldsymbol{x}_t^i$ as 'the agents' here.

The key restriction this places on the framework introduced in Sections 2.1-2.3 is that $\boldsymbol{z}_t$ may no longer contain idiosyncratic variables. However, with appropriate redefinitions of variables, this restriction is mild. For example, idiosyncratic income variation could be incorporated by specifying that the incomes of different groups of households are each included as separate variables within $\boldsymbol{z}_t$. Households from one group would simply have zeroes in their $\boldsymbol{\mu}_t^i$ coefficient matrices corresponding to the incomes of groups other than their own.

With this I define a *temporary equilibrium* (Grandmont, 1977; Woodford, 2013), which takes the agents' expectations as given, and defines all other variables such that, conditional on those expectations, agent decisions follow their choice functions and all general equilibrium conditions are satisfied.

Equilibrium definition. Given an exogenous shock vector $\boldsymbol{\xi}_t$ and agent expectations $\mathbb{E}_t^i \boldsymbol{z}_t$, a temporary equilibrium consists of values for aggregate variables $[\bar{\boldsymbol{x}}_t, \boldsymbol{z}_t]$ such that:

1. *Agents:* agents choose $\boldsymbol{x}_t$ according to their choice function (equation 1).
2. *Other Variables:* $\boldsymbol{z}_t$ is such that all general equilibrium conditions are satisfied (equation 14).

Note that for any process of expectation formation, existence of the temporary equilibrium is a necessary condition for existence of the full equilibrium in which expectations are formed endogenously. I restrict attention here to cases in which equilibrium exists, and is continuous in all elements of $\boldsymbol{\xi}_t$.

So far, agent expectations have been formed using general processes. To tractably solve for choice responses in general equilibrium, I now restrict the form of the information component of those expectations.

Assumption 2 Agent information is such that:

$$\boldsymbol{\delta}_t^i = \tilde{\boldsymbol{\delta}}_t^i \frac{d\boldsymbol{z}_t}{d\xi_t} \tag{15}$$

where ξ_t is an element of $\boldsymbol{\xi}_t$, and $\tilde{\boldsymbol{\delta}}_t^i$ is independent of $\boldsymbol{z}_t$.

That is, the direct update to expectations of each element of $\boldsymbol{z}_t$, given a shock ξ_t, is proportional to the realized change in that variable. $\tilde{\boldsymbol{\delta}}_t^i$ therefore reflects the strength of the direct updating of expectations through information, *relative* to the update that would be seen under full information. While this form does not cover all possible information structures, it is consistent with e.g. Bayesian updating under Gaussian uncertainty (see the example in Section 2.2).

With these two assumptions, the response of aggregate choices to a shock ξ_t, inclusive of general equilibrium effects, is given by Proposition 3.

Proposition 3 *Under Assumptions 1 and 2, the general equilibrium response of $\bar{\boldsymbol{x}}_t$ to a shock ξ_t is given by:*

$$\frac{d\bar{\boldsymbol{x}}_t}{d\xi_t} = -\mathbb{E}_I\left(\boldsymbol{\mu}_t^i \boldsymbol{\chi}_t^i \tilde{\boldsymbol{\delta}}_t^i\right)\left(A + B\mathbb{E}_I\left(\boldsymbol{\mu}_t^i \boldsymbol{\chi}_t^i \tilde{\boldsymbol{\delta}}_t^i\right)\right)^{-1} C e_\xi \tag{16}$$

where e_ξ is a $N_\xi \times 1$ vector with zero in every element, except for 1 in the element corresponding to the shocked element of $\boldsymbol{\xi}_t$.

Proof. Appendix A.1. ■

If $B = 0$, there is no general equilibrium feedback from agent choices to $\boldsymbol{z}_t$. This then reduces to the partial equilibrium result in Proposition 2. With $B \neq 0$ there are general equilibrium channels that affect agent choices. However, both the initial partial equilibrium response and the resulting feedback are determined by the product $\mathbb{E}_I(\boldsymbol{\mu}_t^i \boldsymbol{\chi}_t^i \tilde{\boldsymbol{\delta}}_t^i)$, which can still be decomposed into the three channels in Proposition 2.

This is not surprising: if the narrative heterogeneity channel amplifies the partial equilibrium response of agent choices to a particular shock, then it will also amplify the responses of other variables that depend on those choices. If those variables feed back into choices in general equilibrium, that will either amplify or dampen the choice response, depending on the role of that variable in agent choice functions. Whatever the form of this general equilibrium effect, it is still driven by the initial partial equilibrium channels.

In the remainder of the paper I go on to study the narrative heterogeneity channel in the particular case of household beliefs about inflation. The results in this section, however, are more general. In any situation with heterogeneity in how agents form expectations, understanding aggregate dynamics requires understanding the three channels presented in Proposition 2.

3 Survey evidence on information and subjective models of inflation

In this section I take the narrative heterogeneity channel to data, documenting three empirical results about the information and subjective models used by households. Specifically, the results refer to the information UK households obtain about inflation, and their subjective models of how inflation is related to aggregate economic performance. These results indicate the presence of a narrative heterogeneity channel, which varies over time. They will be used to inform the model in Section 4.

3.1 Data

To study the joint behavior of information and subjective models, we need data that is informative about each separately. This is a challenge, as most empirical papers on information frictions or subjective models use data on realized expectations, which combine both information and subjective models (as shown in Section 2), and so cannot be used to identify the narrative heterogeneity channel. I use data from the Bank of England Inflation Attitudes

Survey (IAS), which contains several unique questions which enable me to measure these components of expectation formation separately.

The IAS is a quarterly survey of a repeated cross-section of UK households, run since 2001 (annual until 2003). After weighting, the sample is representative of the UK adult population. I use the individual-level response data from 2001-2019, omitting the quarters conducted after the onset of the Covid-19 pandemic, as the implementation of the survey had to be changed substantially at this time (see Bank of England, 2020).

Alongside questions on expectations of inflation, interest rates, and other macroeconomic and personal variables, respondents are asked several questions which do not commonly appear in other household surveys. These questions are helpful in disentangling information and subjective models about inflation.

The first of these asks households about their subjective model of the relationship between inflation and the 'strength of the economy'.

Question 1 *If prices started to rise faster than they are now, do you think Britain's economy would end up stronger, or weaker, or would it make little difference?*

This differs from standard questions on expected future economic outcomes because it does not invoke the use of information about the state of the world. Similarly to the hypothetical vignettes used in Andre et al. (2022b), the answers to this question are informative about cross-learning, which is denoted $\chi^i_{jk,t}$ in Section 2 and summarizes the household's subjective model.[15] In the analysis below, I will refer to a respondent answering that inflation would make the economy stronger/little difference/weaker as having a positive/neutral/negative subjective model of inflation respectively.

There are two possible interpretations of this question. Households may view it as asking about the causal effects of inflation on the economy (as in the model of Spiegler, 2021). Alternatively, they could see it as asking about the most likely source of a rise in inflation, if they believe supply- and demand-driven inflation is associated with different real outcomes (Kamdar, 2019). For the purposes of this section, this distinction does not matter. In the decomposition of aggregate actions (Proposition 2), $\chi^i_{jk,t}$ is simply the degree to which households update their expectations of one variable when their expectation of another changes. In this case, it is the updating of expectations about the strength of the economy

[15] In Section 2.2 I noted that $\chi^i_{jk,t}$ comprised subjective models and any weighting the agent put on expectations of z^i_{kt}. Since these weights do not change the sign of $\chi^i_{jk,t}$, the qualitative responses to Question 1 still reflect the sign of the cross-learning from expected inflation to expectations of the state of the real economy, as long as no household perfectly observes the 'overall state of the economy'.

when expected inflation rises. The sign of this updating is captured by the question, whether it occurs because of a perceived direct causal link from inflation to the real economy, or a belief about the type of shocks hitting the economy.[16]

The next set of novel questions concern the information households use to form their inflation expectations, without asking what those expectations are. This allows us to learn about household information ($\delta^i_{k,t}$) without contamination from cross-learning ($\chi^i_{jk,t}$).

Question 2a *What were the most important factors in getting to your expectation for how prices in the shops would change over the next 12 months?*

Please select up to 4:

1. *How prices have changed in the shops recently, over the last 12 months*
2. *How prices have changed in the shops, on average, over the longer term i.e the last few years*
3. *Reports of current inflation in the media*
4. *Discussion of the prospects for inflation in the media*
5. *The level of interest rates*
6. *The inflation target set by the government*
7. *The current strength of the UK economy*
8. *Expectations about how economic conditions in the UK are likely to evolve*
9. *Other factors*
10. *None*

We can divide the possible answers into four categories. First, options 1 and 2 concern past experienced price rises. Options 3 and 4 are direct information about inflation. Options 5-8 concern other macroeconomic variables, either current or expected, and options 9 and 10 are extras. A rational household may well use the information sources in options 1,2 and 5-9 to forecast inflation, but in the decomposition in Proposition 2 this would represent cross-learning from information about other variables. To use the level of interest rates (5) to forecast inflation, for example, a household must employ a model of how interest rates relate to inflation. Similarly, to use past experienced price changes (1-2), households need a model of the persistence of inflation.[17] The only answers that represent the use of direct

[16] The distinction will matter when using a structural model to analyse counterfactual implications of this data. I therefore return to this issue in Section 4.

[17] Macaulay and Moberly (2022) find this perceived persistence is very heterogeneous across households. Note that strictly, option 3 also concerns past price changes, so the assumption here is that media reports of inflation tend to discuss both current and future inflation simultaneously. Appendix C.2 shows that the results below are robust to various small changes to this definition of the information indicator.

information about inflation are options 3 and 4.

Question 2a was only asked in 2016Q1, but very similar questions were asked at other times. In each, the respondent is asked about the information sources they used to arrive at their expected inflation, or that led them to change that expectation over the previous year. For each such question I construct a dummy variable equal to 1 if the respondent reports using direct information about inflation, and equal to 0 if they do not. Full details of these questions, and the options representing direct information, are in Appendix B. Combining these dummy variables gives an indicator for if the respondent used direct information on inflation in forming their expectations, that is whether $\delta^i_{\pi,t} > 0$. This indicator is observed for 8 separate quarters between 2009Q1-2019Q1. I confirm below that the key results of this section do not vary substantially with the changes in question wording over these periods.

In Appendix C.1 I confirm that these measures of information and subjective models correlate with questions on planned household consumption, and that the signs of these correlations are consistent with the measures picking up the desired elements of household beliefs. A further possible test of the information indicator would ask if households who obtain direct information about inflation make more accurate forecasts. However, if beliefs about the level of inflation affect subjective models, that may in turn change the incentives to acquire further information, complicating the predicted correlation between information and forecast accuracy. For this reason I leave discussion of this test for Section 6, after the model has been developed. The results are consistent with the model, adding further evidence that the information indicator reliably measures the object of interest.

The other questions used in this section are standard, asking households to give point estimates for "how prices have changed over the last twelve months" and "how much would you expect prices in the shops generally to change over the next twelve months". For each question, respondents choose from a list of ranges, and follow-up questions may then asked with more precise ranges, until the respondent has selected a 1 percentage-point bin between -5% and +10%, or end ranges $\leq -5\%, \geq 10\%$.

For the exercises in Section 3.4, I code perceptions and expectations at the midpoint of the selected bin, with the lowest and highest bins coded as -5.5% and 10.5% respectively. I refer to these answers as perceived and expected inflation respectively.

3.2 Information and subjective models in the cross-section

The first empirical result concerns the cross-sectional distribution of information and subjective models, the key relationship in the narrative heterogeneity channel. Table 1 shows the estimated average marginal effects from a probit regression of the information indicator defined in Section 3.1 on the respondent's subjective model of inflation, represented by their answer to Question 1. The first column shows this with quarter fixed effects only, while the second also includes a range of household controls.[18]

Table 1: Information correlates with subjective models

	(1)	(2)
end up stronger	-0.0102	-0.00827
	(0.0191)	(0.0192)
make little difference	-0.0356***	-0.0315**
	(0.0128)	(0.0129)
dont know	-0.0627***	-0.0605***
	(0.0172)	(0.0172)
Controls	None	All
Time FE	Yes	Yes
Observations	8270	8270

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the average marginal effects from estimating a probit regression of the information indicator on the responses to Question 1. The information indicator equals 1 if the household reports using a direct source of information about inflation when forming their expectations, as defined in Appendix B. The omitted category is the belief that inflation makes the economy weaker. All regressions are weighted using the survey weights provided in the IAS.

Those answering that inflation makes no difference to the aggregate economy, or who don't know the effect of inflation, are significantly less likely to use information about inflation than someone who believes inflation makes the economy weaker. There is no significant difference in the probability of using direct inflation information between those holding this view and those with positive subjective models of inflation. The probability of using direct inflation information is 3-3.5 percentage points lower for those with a neutral model of the effects of inflation than those who believe inflation weakens the economy. Over the whole population 23% of respondents use direct inflation information, so this difference is non-trivial. More important than the magnitude, however, is that this shows a systematic cross-sectional relationship between information and subjective models, indicating a role for

[18]These are gender, age, class, employment status, income, education, region, and home-ownership status. Age, class, income and education are all reported in bands, and included as categorical variables.

the narrative heterogeneity channel. Assessing the quantitative relevance of this requires a model, as developed in Sections 4-6 below.

Empirical Result 1 *Households who believe inflation makes no difference to the economy acquire less information about inflation on average than households who believe inflation does affect the economy (in either direction).*

The information indicator is composed of answers to several slightly different questions. In particular, some questions concern information used to arrive at the respondent's point expectation for inflation, while others concern the information they used in changing those expectations over the last year. Most questions concern expected inflation over the next 12 months, but a minority ask about a longer horizon. In Appendix C.2 I repeat the regressions of Table 1 on subsets of the questions, and find the results are robust to these alternatives. As some respondents do not answer the unique survey questions used here, I also account for the concern that there may be selection bias in whose answers are observed, using a selection model as in Heckman (1979). Again, the results are robust.

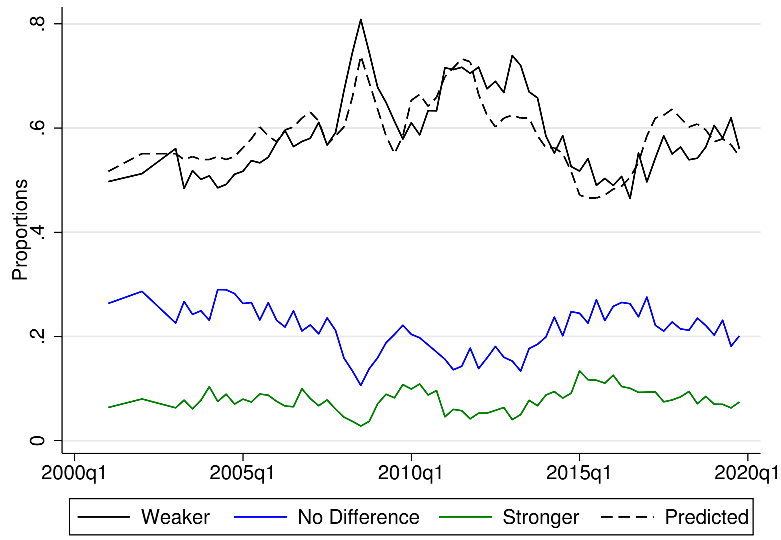
Result 1 is not consistent with models with exogenous information, as there would be no reason for information to be systematically correlated with household subjective models. It is, however, consistent with models of endogenous information acquisition, as the value of inflation information is lower for households who believe inflation makes little difference to other variables that matter for their decisions. The (broadly defined) strength of the aggregate economy is such a variable as long as households believe there is some relationship between the aggregate economy and their personal decisions, which is supported by evidence in Roth and Wohlfart (2020), among others. The implications of this link from subjective models to information acquisition are discussed further in Section 4.

3.3 Subjective models over time

I next turn to the time-series behavior of subjective models of the effects of inflation. Figure 1 shows the proportions answering Question 1 with each subjective model of inflation over time ('don't know' omitted for figure clarity).

The majority of households answer that inflation would make the economy weaker in all quarters, in keeping with the findings in Shiller (1997), Kamdar (2019), and Andre et al. (2022b). Combined with Empirical Result 1, this suggests that the covariance of information on inflation and cross-learning from inflation to the strength of the economy is negative. If

Figure 1: Proportions giving each answer to Question 1: "If prices started to rise faster than they are now, do you think Britain's economy would end up stronger, or weaker, or would it make little difference?"



Note: Proportions shown are calculated using the survey weights provided in the IAS. Proportion answering 'Don't know' is omitted for figure clarity. The dashed line is the predicted values from regressing the proportion reporting that inflation makes the economy weaker on annual CPI inflation: $\widehat{\Pr(\text{weaker})}_t = 0.057 \times \text{CPI inflation}_t + 0.466$. The coefficient on inflation is significant at the 1% level.

households consume more when they believe the economy is strong, the narrative heterogeneity channel will therefore reduce the consumption response to inflationary shocks.

The relatively long time series of the IAS also allows us to see that the distribution of answers varies substantially over time, and that much of this variation can be explained by recent inflation experiences. The correlation between annual CPI inflation and the proportion of respondents with negative models of inflation is extremely high, at 0.799. The dashed line in Figure 1 plots the predicted values from regressing this proportion on CPI inflation, showing that this correlation is strong across the whole sample. Tests in Appendix C.3 show that the correlation is robust to the addition of various macroeconomic controls, which themselves explain far less of the variation in the distribution of responses than realized inflation. The correlations are also robust to using inflation measures split by various household characteristics, to get closer to the rate of inflation in each household's own basket of goods. Finally, the proportions giving all other answers are also shown to be significantly negatively correlated with current inflation.

Empirical Result 2 *A greater proportion of households believe inflation weakens the economy when realized inflation is high.*

This is not what we would observe if households hold rational expectations. The question is about the effect of an aggregate variable (inflation) on the aggregate performance of the economy. Even if households are differentially exposed to the shock, if they all had model-consistent beliefs they would all give the same answer to this question. The fact that there is heterogeneity at all is evidence that at least some household subjective models are inconsistent with rational expectations.

The time-series patterns also suggest that the majority of households are not using New Keynesian-style models. In a textbook New Keynesian model, a rise in inflation causes the central bank to raise the nominal interest rate. If the Taylor Principle is satisfied, the real interest rate rises, so output falls. If it is not, the real rate falls, and output rises. If most households used this model, they should respond that inflation would make the economy weaker in the periods before interest rates reached the Zero Lower Bound, and they should switch to the view that inflation would make the economy stronger once we reach the ZLB in 2009. There is little evidence for this in Figure 1, and indeed statistical tests in Appendix C.3 find no evidence of such a shift.[19]

3.4 Inflation perceptions, expectations, and subjective models

Finally, I compare perceived and expected inflation across households with different subjective model beliefs. Figure 2 shows the time series of mean perceived and expected inflation by qualitative subjective model of inflation.

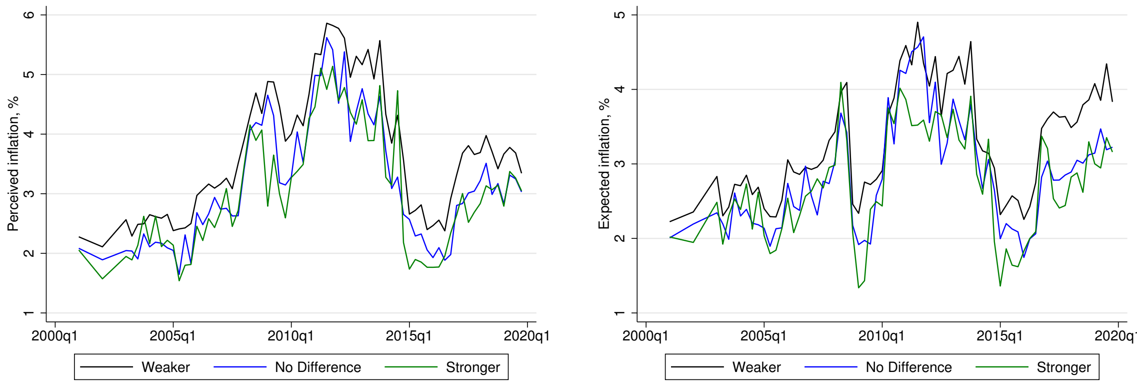
There are persistent differences between the perceptions and expectations of the different groups. Respondents who believe inflation weakens the economy systematically perceive that inflation has been higher, and expect it to be higher over the next year, than those who believe inflation makes no difference to the economy. They, in turn, perceive and expect higher inflation than those with positive subjective models of inflation.[20]

The differences are large: Table 2 shows that even after controlling for the full set of available household characteristics, those with a negative model of inflation perceive that

[19] This argument supposes that at least some households believe cost-push shocks are part of the drivers of inflation. While demand-driven inflation in a New-Keynesian model is associated with higher output whatever the monetary regime, we would still see some shifts in answer distributions at the ZLB if cost-push shocks are perceived to occur with positive probability. In principle, after 2009 a New Keynesian model would predict that a sufficiently large rise in inflation would lift the economy away from the ZLB, implying higher real interest rates and lower output. However, in 2013 the Bank of England began forward guidance committing to maintaining low interest rates, so it is unlikely that households were expecting them to contract in response to small rises in inflation at this time.

[20] Dräger et al. (2020) similarly find for German households that inflation expectations are higher among those reporting that they would prefer inflation to be lower.

Figure 2: Inflation perceptions and expectations by subjective model.



(a) Perception, past 12 months: $\mathbb{E}_t\pi_{t,t-12}$ (b) Expectation, next 12 months: $\mathbb{E}_t\pi_{t+12,t}$

Note: Perceived inflation refers to beliefs about what inflation has been over the past 12 months, and expected inflation refers to expectations for the next 12 months. Averages for each variable are calculated using the survey weights provided in the IAS. Average perceptions and expectations among respondents answering 'Don't know' to the subjective model question (Question 1) are omitted for figure clarity.

inflation has been 54 basis points higher than those with a neutral model, and 70 basis points higher than those with a positive model. The gaps are similarly large and strongly significant for expected inflation. Appendix C.4 shows that these results are not driven by selection bias from missing observations for inflation perceptions and expectations.

Empirical Result 3 *Households who believe inflation weakens the economy on average perceive higher current inflation, and expect higher future inflation, than those with less negative subjective models.*

This is not driven by the households using different kinds of information to arrive at their perceptions and expectations: Table 1 shows that the households with positive subjective models use similar information sources to those with negative models. It is, however, consistent with information about high inflation causing households to update their subjective models towards more negative views. Although the exercises here do not identify the direction of causation, such a mechanism can simultaneously account for Results 2 and 3. Within a period, those who receive signals that inflation is high shift to more negative subjective models, and when realized inflation rises more households receive such signals. This is explored in detail in Section 4.

Table 2: Perceived and expected inflation are higher for those with more negative subjective models.

	(1) Perceived inflation	(2) Expected inflation
end up stronger	-0.696***	-0.565***
	(0.0371)	(0.0353)
make little difference	-0.543***	-0.466***
	(0.0226)	(0.0207)
dont know	-0.462***	-0.413***
	(0.0315)	(0.0294)
Controls	Yes	Yes
Time FE	Yes	Yes
R-squared	0.179	0.113
Observations	85803	85201

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the results of regressing perceived and expected inflation on respondent subjective models (responses to Question 1) . The omitted category is the belief that inflation makes the economy weaker. All regressions are weighted using the survey weights provided in the IAS.

4 A consumption-savings model

In this section I present a heterogeneous-agent model that rationalizes the empirical findings documented above. The key elements needed to match the data are that households face costs of processing information about inflation, and that perceptions of recent inflation affect the perceived effect of inflation on real incomes. These features imply a two-way relationship between information and subjective models.

4.1 Model setup

Time is discrete, and the period is denoted by t. The economy is populated by a measure 1 of households. Each period, household i chooses consumption C_t^i to maximize expected discounted utility:

$$\tilde{\mathbb{E}}_0^i U_0^i = \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \frac{(C_t^i)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \tag{17}$$

subject to:

$$C_t^i + B_t^i = \frac{R_{t-1}}{\Pi_t} B_{t-1}^i + Y_t \tag{18}$$

where Y_t is the real income received by all households in period t, R_t is the gross nominal interest rate on one-period bonds B_t^i bought in period t, and Π_t is gross inflation between periods $t-1$ and t. $\beta \in (0,1)$ is the discount factor, and $\sigma > 0$ is the elasticity of intertemporal substitution. Income and prices are observed before the consumption choice in period t, but future income and prices are unknown. The operator $\tilde{\mathbb{E}}_t^i$ reflects the expectations of household i in period t, which may not coincide with rational expectations. However, given their subjective model for the evolution of R, Π, Y, the household uses their information optimally. Any non-rationality in expectations therefore comes only from misperceptions in these laws of motion.

While households observe the current price level when choosing consumption, I assume that they may not perfectly observe the current rate of inflation. This assumption is common in models where agents use a Kalman filter to update their inflation expectations (e.g. Coibion and Gorodnichenko, 2015), and is consistent with the evidence in Macaulay and Moberly (2022), who find substantial uncertainty about inflation over the past year.[21]

The first order condition is a standard consumption Euler equation:

$$(C_t^i)^{-\frac{1}{\sigma}} = \beta \tilde{\mathbb{E}}_t^i \frac{R_t}{\Pi_{t+1}} (C_{t+1}^i)^{-\frac{1}{\sigma}} \tag{19}$$

To proceed, I take a log-quadratic approximation to utility, as is common in the rational inattention literature (e.g. Maćkowiak and Wiederholt, 2009). The approximation is taken about a steady state with $\Pi = 1, R = \beta^{-1}$. The expected discounted utility loss relative to a household with full information about current inflation is then given by Lemma 1:

Lemma 1 *Let $\tilde{\mathbb{E}}_0^{i*} U_0^{i*}$ denote the expected utility of an otherwise identical household who observes Π_t precisely before choosing C_t^i. Furthermore, let $\hat{U}_0^{i*}$ and $\hat{U}_0^i$ denote the log-quadratic approximation to the discounted utility of the fully-informed and uninformed households respectively. The expected utility loss from imperfect information about Π_t is:*

$$\tilde{\mathbb{E}}_0^i (\hat{U}_0^{i*} - \hat{U}_0^i) = -\frac{(\bar{C}^i)^{1-\frac{1}{\sigma}}}{2\sigma} \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t (c_t^i - c_t^{i*})^2 \tag{20}$$

where lower-case letters are log-deviations of the corresponding variables from steady state, and c_t^{i*} denotes the period-t consumption of the fully-informed household.

[21] One way to microfound this is to assume that households consist of a forecaster, who forms expectations without observing current inflation, and a shopper who uses those forecasts (along with observed current prices) to make consumption decisions. A similar assumption is made in Pfäuti (2022).

Proof. Appendix D.1 ■

Note that the fully-informed household uses the same potentially non-rational expectations operator as the uninformed household. That is, they have the same subjective model, but different information. This will be helpful in solving for optimal information choices.

To focus on the feedback between subjective models and information choices, I take steady state assets $\bar{B}^i \to 0$. This implies that wealth plays no role in information choices, so abstracts away from wealth as an alternative source of information heterogeneity (as in e.g. Broer et al., 2020).[22] With this assumption, the problem of a fully-informed household is identical to that in Appendix A.2, and so the consumption function is:

$$c_t^{i*} = (1-\beta)\sum_{s=0}^{\infty}\beta^s \tilde{\mathbb{E}}_t^{i*} y_{t+s}^i - \sigma\beta\sum_{s=0}^{\infty}\beta^s(\tilde{\mathbb{E}}_t^{i*} r_{t+s} - \tilde{\mathbb{E}}_t^{i*}\pi_{t+s+1}) \tag{21}$$

Since utility losses from deviating from this are quadratic, a household with imperfect information sets $c_t^i = \tilde{\mathbb{E}}_t^i c_t^{i*}$.

The expectations of future real incomes, nominal interest rates, and inflation are therefore critical in determining consumption. Sections 4.2-4.4 describe how these expectations are formed when information processing is costly, and realizations of that information can affect the household's subjective model.

4.2 Expectations

Households form expectations of future variables by taking information on each variable and forecasting forward using their subjective models. Their information set in period t consists of the history up to period t of observations of y_t, r_t, and any signals acquired about π_t. These signals are specified in Section 4.3. The information set of the hypothetical fully-informed agent, used to aid in solving for optimal consumption and information acquisition, also includes the history to period t of realizations of π_t.

Subjective models may change within a period. Specifically, the timing is as follows: a household starts the period with an initial subjective model, which they use to make their information choices (Section 4.3). Once the household observes the realization of their chosen signals, they use that information to update the parameters in their subjective model (Section 4.4). The realized signals and updated subjective model are then combined to form

[22] Michelacci and Paciello (2020) show that with ambiguity aversion, wealth heterogeneity implies heterogeneity in subjective models. Combining this with endogenous information choices, wealth could therefore form an additional reason for a systematic relationship between information and subjective models. This is beyond the scope of this paper.

the expectations used to choose consumption. For now, the initial subjective models at the start of each period are fixed over time for each household. This aids the exposition of the core mechanisms, and is relaxed in Section 6.

I assume that both the initial and updated subjective models of all households take the simple form:

$$\pi_t = \rho^i_\pi \pi_{t-1} + u_{\pi t} \tag{22}$$

$$r_t = \phi^i \pi_t + u_{rt} \tag{23}$$

$$y_t = \alpha^i \pi_t + \lambda^i r_t + \rho^i_y y_{t-1} + u_{yt} \tag{24}$$

where $u_{xt} \sim N(0, \sigma^2_x)$ for $x \in \{\pi, r, y\}$, and $\rho^i_\pi, \rho^i_y \in (0,1)$. Note that the parameters of these subjective models may differ across households, so even if equations 22 - 24 nest the rational expectations solution to a general equilibrium model, it will not be the case that all households have rational expectations.

Unlike in Section 3, this specification of the subjective model does restrict the interpretation of Question 1 in the IAS. The only shock driving inflation is $u_{\pi t}$, so there is no room for disagreement about the source of inflation shocks. Heterogeneous cross-learning from inflation to the real economy can only therefore come from heterogeneous beliefs about the causal effects of inflation. This assumption aids tractability, but also reflects the fact that the distribution of survey answers is very consistent over time, in levels and in how it correlates with realized inflation. If the answers reflected beliefs about the type of shocks driving inflation, we would expect this distribution to change across time periods characterized by different types of shocks. Since the distribution of subjective models evolved in the same way with the run-up in inflation before the Great Recession and the currency devaluation-driven spike after the Brexit referendum, it does not appear that the source of inflation shocks plays a key role in the majority of survey answers. Finally, this formulation is also consistent with existing literature finding household inflation expectations are well-described by such simple forecasting rules (e.g. Adam, 2007).

With this subjective model, the expectations of a fully-informed household are (derivation in Appendix D.2):

$$\tilde{\mathbb{E}}^{i*}_t \pi_{t+s} = (\rho^i_\pi)^s \pi_t \tag{25}$$

$$\tilde{\mathbb{E}}^{i*}_t r_{t+s} = \phi^i (\rho^i_\pi)^s \pi_t \tag{26}$$

$$\tilde{\mathbb{E}}^{i*}_t y_{t+s} = \frac{(\alpha^i + \lambda^i \phi^i)\rho^i_\pi}{\rho^i_\pi - \rho^i_y}\big((\rho^i_\pi)^s - (\rho^i_y)^s\big)\pi_t + (\rho^i_y)^s y_t \tag{27}$$

Substituting these into the consumption function (21), the consumption function of the fully informed household is:

$$c_t^{i*} = \frac{1-\beta}{1-\beta\rho_y^i} y_t - \sigma\beta r_t + \frac{\beta\rho_\pi^i[(1-\beta)(\alpha^i+\lambda^i\phi^i) - \sigma(\phi^i\beta - 1)(1-\beta\rho_y^i)]}{(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)}\pi_t \tag{28}$$

The corresponding consumption function for an uninformed household is therefore:

$$c_t^{i} = \frac{1-\beta}{1-\beta\rho_y^i} y_t - \sigma\beta r_t + \frac{\beta\rho_\pi^i[(1-\beta)(\alpha^i+\lambda^i\phi^i) - \sigma(\phi^i\beta - 1)(1-\beta\rho_y^i)]}{(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)}\tilde{\mathbb{E}}_t^i\pi_t \tag{29}$$

where current inflation appears in expectation because the household may be imperfectly informed about current inflation. I refer to $\tilde{\mathbb{E}}_t^i\pi_t$ as perceived inflation below, in keeping with the evidence in Section 3.

4.3 Optimal information processing

Households choose the structure and precision of their inflation signals to maximize expected utility. Substituting the consumption functions of informed and uninformed households (equations 28 and 29) into the expected utility loss from imperfect information (equation 20) gives:

$$\tilde{\mathbb{E}}_0^i(\hat{U}_0^{i*} - \hat{U}_0^i) = \frac{(\bar{C}^i)^{1-\frac{1}{\sigma}}}{2\sigma}\left(\frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}\right)^2 \tilde{\mathbb{E}}_0^i\sum_{t=0}^{\infty}\beta^t(\pi_t - \tilde{\mathbb{E}}_t^i\pi_t)^2 \tag{30}$$

where:

$$\frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t} = \frac{\beta\rho_\pi^i[(1-\beta)(\alpha^i+\lambda^i\phi^i) - \sigma(\beta\phi^i - 1)(1-\beta\rho_y^i)]}{(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)} \tag{31}$$

is the elasticity of the household's consumption to perceived inflation, under the initial subjective model held at the start of the period.

That is, utility losses are proportional to the mean squared error in inflation perceptions, which will depend on the precision of the household's signals. Importantly, the parameters of the household's subjective model determine the expected utility loss from errors in perceived inflation, because they determine how those errors translate into errors in consumption. This is why subjective models affect household information choices.

Acquiring more precise information reduces these utility losses, but following the rational inattention literature I assume that increasing information precision is costly to the household. Specifically, the utility cost of a signal s_t^i is given by:

$$\mathcal{C}(\{s_t^i\}^t) = \psi \sum_{t=0}^{\infty} \beta^t I(\pi^t; s_t^i | \mathcal{I}_{t-1}^i) \tag{32}$$

where $\psi > 0$ is a positive constant and $I(\pi^t; s_t^i | \mathcal{I}_{t-1}^i)$ is the Shannon mutual information between priors and posteriors in period t. That is, the cost is proportional to the extra information provided by the signal s_t^i about the history of inflation to that point which was not contained in the previous period's information set. This cost function is common in the rational inattention literature (Maćkowiak et al., 2020).

To solve for optimal information processing, I make the simplifying assumption that the household chooses information as if they are certain about the parameters of their subjective model. Similarly, they ignore that they will update those parameters after receiving information. This is akin to the anticipated utility assumption in many models with least-squares learning, where agents do not consider that their perceived law of motion will change as they observe new periods of data in the future (see Bullard and Suda (2016) for a discussion of this in the learning literature).

I also assume that the household does not infer anything about π_t from the y_t and r_t that they observe each period. In principle, these are also noisy signals about π_t, but for simplicity I will not account for them in the information decision.[23]

The household information choice problem then has the same form as the firm's rational inattention problem in Maćkowiak and Wiederholt (2009). Under technical assumptions set out in Appendix D.3, Maćkowiak and Wiederholt (2009) show that the optimal signal is of the form:

$$s_t^i = \pi_t + \varepsilon_t^i, \quad \varepsilon_t^i \sim N(0, \sigma_{\varepsilon i}^2) \tag{33}$$

The household therefore uses a standard Kalman filter to form inflation perceptions:

$$\tilde{\mathbb{E}}_t^i \pi_t = K^i(\pi_t + \varepsilon_t^i) + (1 - K^i)\rho_\pi^i \tilde{\mathbb{E}}_{t-1}^i \pi_{t-1} \tag{34}$$

The information choice problem therefore reduces to choosing the variance of noise $\sigma_{\varepsilon i}^2$ in the signal s_t^i, which implies a particular Kalman gain K^i. The assumptions in Appendix D.3 ensure that the household uses the steady state K^i every period.

[23]Strictly, the rational inattention setup assumes that the agent chooses among all possible signals. So y_t and r_t are available signals, but the household chooses not to pay to process them when forming their inflation perception.

The optimal information choice is given by Proposition 4:

Proposition 4 *The utility-maximizing signal structure is as in equation 33, with $\sigma^2_{\varepsilon i}$ such that K^i satisfies:*

$$\begin{cases} K^i = 0 & \textit{if } \Gamma^i < \psi(1-(\rho^i_\pi)^2)^2 \\ \dfrac{1-K^i}{(1-(\rho^i_\pi)^2(1-K^i))^2} = \dfrac{\psi}{\Gamma^i} & \textit{if } \Gamma^i \geq \psi(1-(\rho^i_\pi)^2)^2 \end{cases} \tag{35}$$

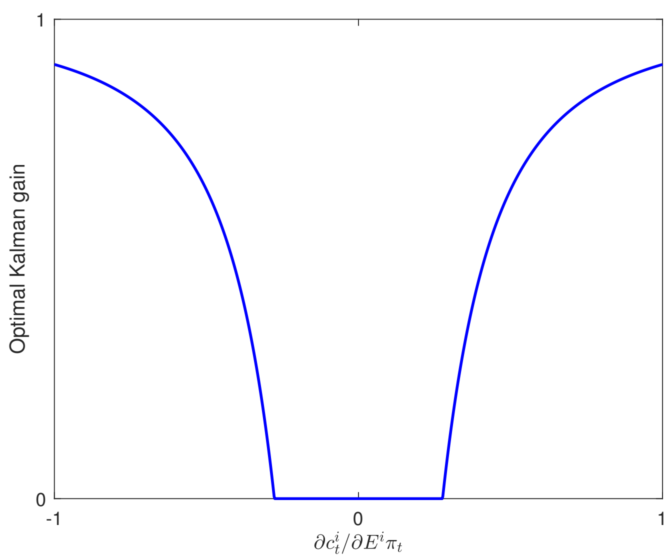
where:

$$\Gamma^i = \frac{(\bar{C}^i)^{1-\frac{1}{\sigma}}}{2\sigma}\sigma^2_\pi \ln(2) \cdot \left(\frac{\partial c^i_t}{\partial \tilde{\mathbb{E}}^i_t \pi_t}\right)^2 \tag{36}$$

Proof. Appendix D.3. ■

That is, if the elasticity of consumption to perceived inflation is close to 0, the household pays no attention to inflation. This occurs if the household's subjective model is such that the income and substitution effects of higher inflation come close to canceling out. Once $\partial c^i_t / \partial \tilde{\mathbb{E}}^i_t \pi_t$ is sufficiently positive or negative, perceived inflation affects decisions enough to warrant paying for some information, and $K^i > 0$. As the consumption elasticity to perceived inflation grows further, attention rises and K^i approaches 1, at which point perceived inflation is equal to realized inflation. These properties can be seen graphically in Figure 3.

Figure 3: Optimal K^i against the elasticity of consumption to perceived inflation. Calibration: Appendix E.



The no-attention region is wider if the perceived volatility of the inflation process $\sigma^2_\pi/(1-(\rho^i_\pi)^2)$ is lower, consistent with evidence in Cavallo et al. (2017) and Pfäuti (2022) that households pay less attention to inflation when it is less volatile. A higher information cost also implies a wider no-attention region. Similarly, outside of the no-attention region, attention is increasing in $\sigma_\pi^2/(1-(\rho_\pi^i)^2)$ and decreasing in ψ.

Information choices are therefore determined by the household's subjective model, and this naturally implies the model matches Empirical Result 1. A simple proxy for 'the strength of the economy' might be aggregate consumption. If households believe others hold beliefs similar to their own, then the households who report in the survey that inflation makes no difference to the economy are those with subjective models such that $\partial c_t^i/\partial\tilde{\mathbb{E}}_t^i\pi_t$ is close to zero.[24] They therefore process less information about inflation than those with stronger positive or negative $\partial c_t^i/\partial\tilde{\mathbb{E}}_t^i\pi_t$.

4.4 Subjective model updating

After processing their information, and forming a perception of current inflation, the households update their subjective model. Specifically, I assume that the only update is to the parameter α^i, the effect of inflation on real income. Denoting the parameter value used in making information choices at the start of the period as α_0^i, the updated parameter $\hat{\alpha}_t^i$ is given by:

$$\hat{\alpha}_t^i = \alpha_0^i + \alpha_1^i\tilde{\mathbb{E}}_t^i\pi_t \tag{37}$$

That is, each household takes the parameter from their subjective model at the start of the period, and distorts it up or down depending on the realization of perceived inflation. Specifically, to match Empirical Results 2 and 3 I assume that $\alpha_1^i < 0$, so when households perceive higher inflation they update their subjective model towards the view that inflation erodes real income.

The applications below are concerned with variation in perceived inflation, so the reduced-form specification here is sufficient. There are however several possible microfoundations for equation 37. For example, if households believe there is an optimal level of inflation, such that real income is increasing in inflation below that bliss point, but is decreasing beyond it, their subjective models would behave this way. Appendix D.5 provides an alternative formal microfoundation, in which households are ambiguity averse, and face Knightian uncertainty about α^i. In that environment households distort their subjective model towards the worst case, which varies with perceived inflation.[25]

[24] Indeed, Dräger et al. (2020) find that household beliefs about what is good for the economy overall and for them personally are highly correlated.

[25] This approach relates to that of Michelacci and Paciello (2020), who note that ambiguity aversion

Substituting the expression for $\hat{\alpha}^i_t$ (equation 37) into the consumption function (equation 29) yields:

$$\left.\frac{\partial c^i_t}{\partial \tilde{\mathbb{E}}^i_t \pi_t}\right|_{\hat{\alpha}^i_t} = \left.\frac{\partial c^i_t}{\partial \tilde{\mathbb{E}}^i_t \pi_t}\right|_{\alpha^i_0} - \Omega^i \tilde{\mathbb{E}}^i_t \pi_t \tag{38}$$

where Ω^i is a function of preference and subjective model parameters:

$$\Omega^i = -\frac{\beta(1-\beta)\rho^i_\pi \alpha^i_1}{(1-\beta\rho^i_\pi)(1-\beta\rho^i_y)} \tag{39}$$

Since $\alpha^i_1 < 0$, $\Omega^i > 0$ for all households. Higher perceived inflation is therefore associated with more negative consumption responses to perceived inflation. That consumption response reflects the household's beliefs about future aggregate variables, so this matches Empirical Result 3: households who believe that higher inflation would weaken the economy on average perceive higher recent inflation.

Proposition 5 shows that the updating process also implies that the model matches Empirical Result 2: more households hold negative models of the effects of inflation when realized inflation rises.

Proposition 5

$$\frac{\partial}{\partial \pi_t}\left[\Pr\left(\left.\frac{\partial c^i_t}{\partial \tilde{\mathbb{E}}^i_t \pi_t}\right|_{\hat{\alpha}^i_t} < X\right)\right] \geq 0 \tag{40}$$

for any threshold X. The inequality is strict if $K^i > 0$, and is an equality otherwise.

Proof. Appendix D.4 ■

That is, when inflation rises, households who acquire some information ($K^i > 0$) become more likely to hold negative subjective models of inflation. At higher levels of inflation, there are therefore more households with subjective models implying further inflation should be met with lower consumption. A greater proportion of households therefore believe that higher inflation would weaken the economy when inflation is high.

4.5 Closing the model

As the focus of this model is the behavior of households, I keep the production side of the model extremely simple. This allows for analytic solutions in the sections below.

naturally generates the negative correlation between preferences and expectations I observe for inflation. Similarly, in Ilut et al. (2020) firm worst-case beliefs depend on the direction of price changes.

There is a single consumption good, produced by a representative firm using labor as the only input to production. Each period, every household is employed, and supplies however much labor is required to meet aggregate consumption demand. By construction, output q_t therefore equals aggregate consumption $\bar{c}_t = \mathbb{E}_I(c_t^i)$, and the goods market clears. Household income is simply the revenues from the firm, which are distributed equally to households each period. Real income is therefore:

$$y_t = q_t = \bar{c}_t \tag{41}$$

Households, by assumption, do not take this into account when forming expectations of future real income. Rather, they use their subjective model (equations 22-24).

Note that this is not a full general equilibrium model, as inflation π_t and nominal interest rates r_t are left undetermined. I take these as exogenous, to explore the mechanisms involved in aggregate responses to inflation, and the role of the narrative heterogeneity channel, as clearly as possible.

Equilibrium definition. An equilibrium consists of values for aggregate variables $[\pi_t, r_t, y_t]$, and cross-household distributions of $[\hat{\alpha}_t^i, c_t^i, \tilde{\mathbb{E}}_t^i \pi_t, K^i]$, such that for all t:

1. *Driving forces:* (π_t, r_t) are given exogenously.
2. *Information:* households choose K^i optimally (equation 35). They use their information and Bayes rule to obtain $\tilde{\mathbb{E}}_t^i \pi_t$ (equation 34).
3. *Subjective models:* households update $\hat{\alpha}_t^i$ according to equation 37.
4. *Consumption choices:* given information and (updated) subjective models, households choose consumption c_t^i to maximize perceived household utility (equation 29).
5. *Market clearing:* real income y_t satisfies equation 41.

In general equilibrium, the narrative heterogeneity channel will have further dynamic implications beyond those derived in this paper, as the joint distribution of information and subjective models will affect, and be affected by, the equilibrium laws of motion of inflation and other variables. The exercises presented here should therefore be viewed as a first step in understanding the wide-ranging implications of this novel channel of shock transmission.

The other restrictive assumption made here is that information is only limited and heterogeneous about inflation, and the subjective model updating only occurs in a single parameter α^i. Of course, in reality these features are likely to be common to information on many different variables, and many aspects of subjective models. This means that narrative heterogeneity effects are potentially much more widespread than I allow for in this model.

Furthermore, updates to information and subjective models of other variables will in turn affect the information and subjective models around inflation. The full implications of narrative heterogeneity effects among households are therefore likely to be larger, and richer, than those derived here. I limit myself to these first-round effects, however, as the IAS data cannot discipline the behavior of information about other variables, or other aspects of subjective models.

5 Implications of narrative heterogeneity

In this section I show that the feedback between information and subjective models has important implications for macroeconomic dynamics, because it generates a large and time-varying narrative heterogeneity channel of shock transmission. Calibrating the model to the UK over the period of the survey data, the narrative heterogeneity channel accounts for 36% of the elasticity of aggregate consumption to inflation in steady state, and 39% of its volatility over the period.

5.1 Selection in attention

First, consider the effect of subjective models on information choice. To isolate this, assume for now that $\alpha_1^i = 0$, so the only heterogeneity in subjective models is that present at the start of each period, when households choose their information.

Consider a shock that increases inflation in period t, with no initial reaction of nominal interest rates. The effect of this on the consumption of household i on impact is given by:

$$\frac{\partial c_t^i}{\partial \pi_t} = \Theta^i \frac{\partial y_t}{\partial \pi_t} + \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \frac{\partial \tilde{\mathbb{E}}_t^i \pi_t}{\partial \pi_t} = \Theta^i \frac{\partial y_t}{\partial \pi_t} + \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} K^i \tag{42}$$

where the second equality follows from equation 34, and

$$\Theta^i = \frac{1-\beta}{1-\beta\rho_y^i} \in (0,1] \tag{43}$$

Denote the fraction of households who pay no attention ($K^i = 0$) as $1 - P_0$, and assume they are indexed by $i \in [P_0, 1]$. The response of aggregate consumption is therefore:

$$\frac{\partial \bar{c}_t}{\partial \pi_t} = (1-\bar{\Theta})^{-1} \int_0^{P_0} \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} K^i di \tag{44}$$

where ω^i is a weight on household i as in equation 11, $\bar{\Theta}$ is the (similarly-weighted) average Θ^i across households, and I have used that real income y_t always equals aggregate consumption $\bar{c}_t$ in equilibrium (equation 41).

To see how the relationship between information and subjective models affects aggregate outcomes, compare this to a model in which all households have the same Kalman gain $\bar{K}$, equal to the average K^i from the baseline model:

$$\bar{K} = \mathbb{E}_I(K^i) = \mathbb{E}_I(K^i | K^i > 0) \cdot P_0 \tag{45}$$

This, for example, could reflect an economist calibrating a model with homogeneous information frictions to micro-level evidence on household information. In such a homogeneous-K model the aggregate response of consumption to the inflation shock can be decomposed into two integrals:

$$\begin{aligned} \left.\frac{\partial \bar{c}_t}{\partial \pi_t}\right|_{K^i=\bar{K}} &= (1-\bar{\Theta})^{-1} \int_0^1 \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \bar{K} di \\ &= (1-\bar{\Theta})^{-1} \left(\int_0^{P_0} \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} K^i \frac{\bar{K}}{K^i} di + \int_{P_0}^1 \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \bar{K} di \right) \end{aligned} \tag{46}$$

The first term is identical to the expression for $\partial \bar{c}_t / \partial \pi_t$ in the baseline model with endogenous attention (equation 44), except that each household's response is weighted by $\bar{K}/K^i$. Relative to the baseline model, the consumption responses of more attentive households receive a lower weight, while less attentive households are over-weighted.

The second integral concerns the consumption responses of inattentive households. In the baseline model, their response to perceived inflation is irrelevant, because their inflation perceptions are unaffected by the shock. Here, however, their perceptions react to the shock with elasticity $\bar{K}$. The least attentive households are also therefore over-weighted in the homogeneous-K model.

This leads to systematic differences in aggregate consumption responses, because the most attentive households in the baseline model have high K^i precisely because they respond strongly to perceived inflation. Formally, the difference between the aggregate consumption responses in the endogenous-K^i baseline and the homogeneous-K model is:

$$\begin{aligned} \frac{\partial \bar{c}_t}{\partial \pi_t} - \frac{\partial \bar{c}_t}{\partial \pi_t}\bigg|_{K^i=\bar{K}} &= (1-\bar{\Theta})^{-1}\mathbb{E}_I\left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}(K^i - \bar{K})\right) \\ &= (1-\bar{\Theta})^{-1} Cov_I\left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}, K^i\right) \end{aligned} \tag{47}$$

The difference therefore depends on the covariance of information and subjective models: by making attention exogenous, the homogeneous-K model omits the narrative heterogeneity channel of shock transmission discussed in Section 2.[26] This covariance depends on the distribution of subjective models, as K^i is increasing in the absolute value of $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$. Among households with $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t > 0$, the covariance of consumption responses and K^i is positive, but among those with $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t < 0$ it is negative.

This implies that, for most distributions of subjective models, the narrative heterogeneity channel amplifies the aggregate consumption response to the shock, relative to the homogeneous-K model. If most households increase consumption when perceived inflation rises, then the baseline aggregate consumption response to a π_t increase is positive. At the same time, the narrative heterogeneity channel in expression 47 is positive. Conversely, if most households have strong negative subjective models of inflation, the baseline aggregate response is negative, as is the narrative heterogeneity channel in expression 47.[27]

Figure 4 shows this effect graphically. It plots the consumption response of an individual household to a shock to π_t, holding real income fixed, against the same household's response to an increase in perceived inflation $\tilde{\mathbb{E}}_t^i \pi_t$. If households observed inflation precisely, this would simply be the 45° line (red dashed line).

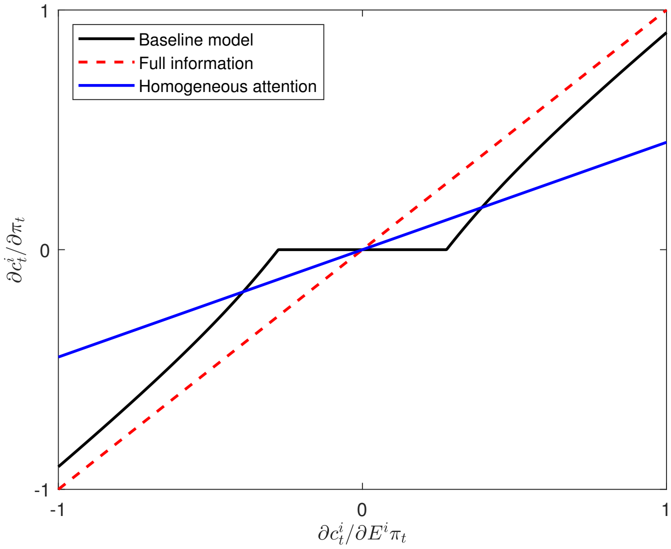
The black solid line shows this relationship in the baseline model with endogenous K^i. Households with $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$ close to zero pay no attention to current inflation, and so their perceptions of inflation do not change when the shock hits. They therefore do not react. Households with greater $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$ pay more attention, so their perceptions are more sensitive to the shock, and their elasticity of consumption to π_t is closer to the 45° line.

If the endogenous K^i is replaced by a fixed $\bar{K}$ for all households, the elasticity of c_t^i to π_t is instead given by the blue solid line. Relative to the baseline model, consumption

[26] Note there is no response heterogeneity channel because all households have the same policy functions. All heterogeneity in $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$ therefore comes from cross-learning from current inflation to expectations of other variables.

[27] While this intuition dominates for most subjective model distributions, it is possible to construct cases in which the narrative heterogeneity effect instead attenuates aggregate transmission relative to the homogeneous-K^i case. These are discussed in Appendix D.6.

Figure 4: Consumption response to a change in actual inflation against response to perceived inflation. Parameters listed in Appendix E.



responses are drawn closer to the full-information line for all households with $\partial c_t^i/\partial \tilde{\mathbb{E}}_t^i \pi_t$ such that $K^i < \bar{K}$ in the baseline model. Conversely, consumption responses are reduced towards zero for all those who are more attentive than average in the baseline model. Since the less attentive households are the ones who would react the least under full information, removing the narrative heterogeneity channel in this way weakens the effect of the shock. The equilibrium response of real income only amplifies this effect, as a smaller partial-equilibrium consumption response implies a smaller change in real income, further weakening consumption responses.[28]

This is analogous to the selection effect in menu cost models of price setting (Caplin and Spulber, 1987; Golosov and Lucas, 2007). In those models, the aggregate price level is less sticky than the average of firm-level stickiness, because price adjustments are disproportionately drawn from firms desiring large price changes. Here, households obtaining information about inflation are disproportionately drawn from those who would react strongly to that information.[29] Just as the price level in a menu cost model is more flexible than the average firm-level flexibility, this implies that aggregate consumption is typically more responsive to inflation than is implied by micro-level estimates of household attention. The narrative heterogeneity channel can therefore explain why representative-agent models typically require

[28]Note this amplification is small unless average ρ_y^i is very close to 1, as households in this model have a small MPC out of transitory income shocks. With a more realistic MPC the amplification would be larger.

[29]Afrouzi and Yang (2021) study a similar mechanism, in which firms pay attention to aggregate variables only when they need to change prices.

only small information frictions to match aggregate data (Maćkowiak and Wiederholt, 2015), while micro-level studies find very large degrees of inattention (Link et al., 2021).

A further implication concerns identification in information treatment experiments aimed at estimating the causal effects of expectations (see Candia et al., 2020, for a review). The standard approach in these studies is to regress an outcome variable on the expectation of interest, instrumented using an indicator for whether the respondent was in the treatment or control group.[30] The estimate is therefore consistent for the local average treatment effect on those who update their expectations as a result of the information provision, and is most influenced by those who update the furthest. The selection effect studied here suggests that those compliers will disproportionately be those with the smallest responses to information: they start out with the most uncertain beliefs due to their lack of attention, and so they update expectations the most when shown publicly available information. However, when a shock hits the economy, these are not the households whose expectations matter. Rather, it is the attentive households who observe the shock precisely, and react most strongly.[31]

5.2 State-dependent shock transmission

I now return to the two-way feedback between information and subjective models. Restoring subjective model updating ($\alpha_1^i < 0$), the interaction between the two components of expectations implies that the transmission of inflation shocks to aggregate consumption depends on the size and recent history of realized inflation deviations from steady state.

To explore these effects, I begin by showing how the aggregate consumption response to an inflation shock depends on the distribution of inflation perceptions, before showing how that distribution varies with the size of inflation shocks and recent inflation history.

The distribution of $\tilde{\mathbb{E}}_t^i \pi_t$. Using equation 38, we can decompose the aggregate consumption response to inflation (equation 44) as follows:

$$\begin{aligned}\frac{\partial \bar{c}_t}{\partial \pi_t} &= (1-\bar{\Theta})^{-1}\left[\int_0^1 \omega^i K^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\bigg|_{\alpha_0^i} di - \int_0^1 \omega^i K^i \Omega^i \tilde{\mathbb{E}}_t^i \pi_t di\right] \\ &= (1-\bar{\Theta})^{-1}\left[\mathbb{E}_I\left(K^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\bigg|_{\alpha_0^i}\right) - \mathbb{E}_I(K^i)\mathbb{E}_I(\Omega^i \tilde{\mathbb{E}}_t^i \pi_t) - Cov_I(K^i, \Omega^i \tilde{\mathbb{E}}_t^i \pi_t)\right]\end{aligned} \tag{48}$$

[30] It is also common to use a second instrument, the interaction of the treatment indicator with the agent's prior expectation (e.g. Coibion et al., 2019). This does not substantially change the intuition discussed here.

[31] In some settings the response of inattentive households is precisely the object of interest, such as in the literature on central bank communication with the general public (Haldane et al., 2021; Coibion et al., 2022).

The first term of the aggregate elasticity to inflation is a function of underlying parameters only. Since the initial subjective models held by households at the start of each period are assumed to be fixed here, this is unaffected by realized shocks.

The second term, however, shows that the average subjective model will adjust towards lower values of $\hat{\alpha}_t^i$ as perceived inflation rises. This more negative average subjective model will reduce the aggregate consumption elasticity to inflation. The third term shows that such a rise in perceived inflation will have more of an effect if it occurs in households who process a lot of information about inflation. These are the time-varying components of the representative agent and narrative heterogeneity channels identified in Section 2.

Size dependence. Differentiating equation 48 with respect to current inflation, and using the Kalman filtering equation (34) to extract the response of perceived inflation, we obtain:

$$\frac{d}{d\pi_t}\left(\frac{\partial \bar{c}_t}{\partial \pi_t}\right) = -(1-\bar{\Theta})^{-1}\Big[\mathbb{E}_I(K^i)\mathbb{E}_I(\Omega^i K^i) + Cov_I(K^i, \Omega^i K^i)\Big] \tag{49}$$

The effects on each of the terms is especially clear if we further assume that all households share the same α_1^i, ρ_π^i, and ρ_y^i, and so the same Ω^i. In that case equation 49 becomes:

$$\frac{d}{d\pi_t}\left(\frac{\partial \bar{c}_t}{\partial \pi_t}\right) = -(1-\bar{\Theta})^{-1}\Omega\Big[\big(\mathbb{E}_I(K^i)\big)^2 + Var_I(K^i)\Big] \tag{50}$$

The elasticity of aggregate consumption to inflation therefore falls for two reasons as the inflationary shock gets larger. First, the average inflation perception rises, so the average subjective model becomes more negative about inflation. This matches up with the survey data: the large 0.9% point rise in annual CPI inflation from August to November 2021 in the UK coincided with a 9% point increase in the share of households responding that inflation weakens the economy in the IAS.

Second, the narrative heterogeneity channel also contributes to a fall in $\partial \bar{c}_t / \partial \pi_t$. As the shock size increases, the difference between the inflation perceptions of attentive (high K^i) and less attentive (low K^i) households grows. The most attentive households therefore adjust their subjective models more towards lower $\hat{\alpha}_t^i$ relative to inattentive households, which makes the covariance of K^i and $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$ more negative. Intuitively, as the most attentive households adjust their perceptions by the most, larger shocks lead to a greater concentration of very negative subjective models among the most attentive households. This effect is particularly strong if information choices are very heterogeneous across households, as suggested by the empirical evidence in Link et al. (2021).

History dependence. If households believe inflation is persistent, recent inflation history will also affect the distribution of inflation perceptions. Differentiating equation 48 with respect to realized inflation in period $t-1$ gives:

$$\frac{d}{d\pi_{t-1}}\left(\frac{\partial \bar{c}_t}{\partial \pi_t}\right) = -(1-\bar{\Theta})^{-1}\Big[\mathbb{E}_I(K^i)\mathbb{E}_I(\Omega^i K^i(1-K^i)\rho_\pi^i) + Cov_I(K^i, \Omega^i K^i(1-K^i)\rho_\pi^i)\Big] \quad (51)$$

The first effect is as with the size dependence: high inflation in period $t-1$ implies high average inflation perceptions in period t (through higher prior beliefs), which lowers $\partial \bar{c}_t / \partial \pi_t$.

The narrative heterogeneity effect is more subtle. Again assuming that households all share the same Ω^i, equation 51 becomes:

$$\frac{d}{d\pi_{t-1}}\left(\frac{\partial \bar{c}_t}{\partial \pi_t}\right) = -(1-\bar{\Theta})^{-1}\Omega\rho_\pi\Big[\mathbb{E}_I(K^i)\mathbb{E}_I(K^i(1-K^i)) + Cov_I(K^i, K^i(1-K^i))\Big] \quad (52)$$

The second term may be positive or negative, because there are two opposing effects: on the one hand, as for the size dependence, any inflation shock has the greatest effects within the period on the perceptions of the most attentive households. This acts to reduce $\partial \bar{c}_t / \partial \pi_t$. However, on the other hand, the most attentive households are the least reliant on their prior beliefs when forming perceptions of π_t, and so are least affected by their past inflation perceptions. If average K^i is sufficiently large, this second effect dominates and high past inflation increases the covariance of information and $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$, and so increases $\partial \bar{c}_t / \partial \pi_t$.

5.3 Quantifying the narrative heterogeneity channel

To understand the relative sizes of the effects derived above, I now take the model to data on the UK economy from 2001-2019, the sample period of the IAS data in Section 3. The narrative heterogeneity channel accounts for substantial fractions of the steady state aggregate consumption elasticity to inflation, and of its variation over time.

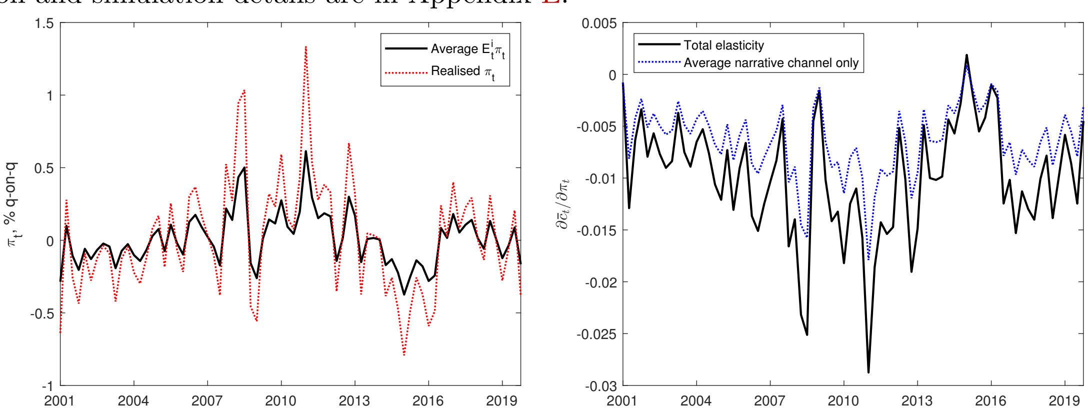
To calibrate the model, I first set some parameters to standard values from the literature. Second, I assume that subjective models are identical across households, with the exception of α_0^i. I obtain the common subjective model parameters from a naïve OLS estimation of equations 22 - 24 with the relevant UK macroeconomic time series. I assume that α_0^i is normally distributed across households, and set the mean equal to the estimated α from the regression of equation 24. Finally, I choose the variance of α_0^i, the updating parameter α_1, and the cost of information ψ to target three key moments from the IAS data: the average proportion of households who believe inflation makes the economy weaker, the elasticity of this proportion to increases in inflation, and an estimate of the average Kalman gain in inflation perceptions. Full details of the calibration are in Appendix E.

Note that the estimation of equations 22 - 24 used in this calibration, while naïve from the point of view of modern empirical macroeconomics, are not naïve from the households' point of view. If their subjective model has the correct structure, then these regressions uncover the intended underlying parameters. In this it is important that y_t does not appear in the law of motion for π_t, as in that case π_t would be endogenous in equation 24.

I obtain a stationary distribution of inflation perceptions by assuming that $\pi_t = 0$ for many periods, so the only variation in $\tilde{\mathbb{E}}_t^i \pi_t$ comes from idiosyncratic noise in household signals. In the steady state with $\tilde{\mathbb{E}}_t^i \pi_t$ drawn from this distribution, $\partial \bar{c}_t / \partial \pi_t$ is negative. This is because the majority of households believe inflation weakens the economy in the survey, so most have negative $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$. As observed in the IAS data, there is a negative correlation between information K^i and $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$, so the narrative heterogeneity channel is also negative, accounting for 36% of the steady state $\partial \bar{c}_t / \partial \pi_t$.

I next simulate the model for 1000000 households, feeding in the path of de-meaned quarterly CPI inflation observed in the UK over the sample period as realizations of π_t. Figure 5 shows the paths of average perceived inflation and $\partial \bar{c}_t / \partial \pi_t$. Compared to realized inflation, average perceived inflation is relatively smooth, despite the selection effect discussed in Section 5.1. However, this still implies substantial volatility in $\partial \bar{c}_t / \partial \pi_t$. For example, when inflation spiked in 2016Q3 after the Brexit referendum, the elasticity of aggregate consumption to inflation was 5.5x larger than in the previous quarter.

Figure 5: Simulated inflation perceptions and aggregate consumption elasticity to inflation. Calibration and simulation details are in Appendix E.



(a) Average $\tilde{\mathbb{E}}_t^i \pi_t$ and realized π_t (b) $\partial \bar{c}_t / \partial \pi_t$

The transmission of inflation shocks therefore varies a great deal over time due to the interaction of information and subjective models. Using the decomposition from Section 2, we can further split that variation into the representative agent and narrative heterogeneity channels. The blue line in Figure 5b shows $\partial\bar{c}_t/\partial\pi_t$ without the narrative heterogeneity channel. It is substantially less volatile: fluctuations in the covariance of information and subjective models account for 39% of the standard deviation of $\partial\bar{c}_t/\partial\pi_t$. As discussed in Section 5.2, when inflation rises the narrative heterogeneity channel becomes more negative, widening the gap between the total $\partial\bar{c}_t/\partial\pi_t$ and that implied by representative agent effects alone. The average perceptions in Figure 5a are therefore not sufficient to capture a large part of the transmission of inflationary shocks.

6 Endogenous long-run expectations

So far in this analysis, information about inflation has mostly affected expectations about aggregate variables in the near future, as all variables are perceived to be stationary. Policymakers, however, are often also concerned about longer-term expectations (e.g. Powell, 2021). In this section I extend the model to allow households to use current information to update their expectations of long-run inflation. Inflationary shocks may become 'baked in' to expectations after an inflationary shock, but only among households who held positive subjective models of the effects of inflation before the shock. This in turn has persistent effects on the transmission of inflationary shocks, with the majority of the long-run effect driven by the narrative heterogeneity channel.

Suppose that household i's subjective model for inflation includes a long-run mean of inflation $\bar{\pi}_t$ which is not necessarily equal to 0:

$$\pi_t = \rho_\pi^i \pi_{t-1} + (1 - \rho_\pi^i)\bar{\pi}_t + u_{\pi t} \tag{53}$$

To begin with, assume that households treat the long-run mean of inflation as a parameter of their subjective model, rather than as a time-varying variable. Following the anticipated utility assumption used above, they therefore make information choices expecting $\bar{\pi}_t$ to remain constant at their current estimate for certain. This assumption greatly simplifies the analysis and allows for analytic results, but is not critical for the mechanisms. I relax it in Appendix F, and the qualitative results below continue to hold numerically.

Re-deriving the consumption function with this new subjective model for inflation gives (derivation in Appendix D.7):

$$c_t^i = \frac{1-\beta}{1-\beta\rho_y^i} y_t - \sigma\beta r_t + \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \left(\tilde{\mathbb{E}}_t^i \pi_t + \frac{1-\rho_\pi^i}{\rho_\pi^i(1-\beta)} \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t \right) \tag{54}$$

where $\tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t$ is household i's estimate of $\bar{\pi}_t$ before information processing in period t. This consumption function is as in equation 29, except for the additional term in $\bar{\pi}_t$.

In the previous sections, household information choices were determined by the constant subjective model parameter α_0^i. However, as the household may now expect inflation to deviate from 0 in the long term, I allow the perceived long-run mean of inflation to affect that initial model:

$$\alpha_t^{i,prior} = \alpha_0^i + \alpha_1^i \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t \tag{55}$$

In this way the model allows us to understand the consequences of a rise in long-term inflation expectations for both information and subjective models.

These assumptions imply that the expected utility loss from imperfect information is given by:

$$\tilde{\mathbb{E}}_0^i(\hat{U}_0^{i*} - \hat{U}_0^i) = \frac{(\bar{C}^i)^{1-\frac{1}{\sigma}}}{2\sigma} \left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \bigg|_{\alpha_t^{i,prior}} \right)^2 \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \big((\pi_t - \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t) - (\tilde{\mathbb{E}}_t^i \pi_t - \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t) \big)^2 \tag{56}$$

Rewriting equation 53 with the assumption that $\bar{\pi}_t$ will remain at $\tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t$ for all t gives:

$$(\pi_t - \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t) = \rho_\pi^i (\pi_{t-1} - \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t) + u_{\pi t} \tag{57}$$

The information choice problem is therefore isomorphic to that in Section 4.3, with π_t replaced with $\pi_t - \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t$ and the constant in the objective function adjusted for $\alpha_t^{i,prior}$. The optimal signal is therefore:

$$s_t^i = \pi_t - \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t + \varepsilon_t^i, \quad \varepsilon_t^i \sim N(0, \sigma_{\varepsilon it}^2) \tag{58}$$

and the optimal $\sigma_{\varepsilon it}^2$ is as in Proposition 4, with the coefficient Γ^i computed using $\alpha_t^{i,prior}$.

The household then uses this signal to update their beliefs about current inflation, and also their beliefs about the long-run mean $\bar{\pi}_t$. For that updating they therefore acknowledge that $\bar{\pi}_t$ may in fact change over time. Specifically, they assume that $\bar{\pi}_t$ follows a random walk (as in e.g. Cogley and Sbordone, 2008; Fisher et al., 2021):

$$\bar{\pi}_t = \bar{\pi}_{t-1} + v_t, \quad v_t \sim N(0, \sigma_v^2) \tag{59}$$

With this assumption, we can write the household's forecasting problem in state-space form:

$$\xi_t = F^i \xi_{t-1} + e_t^i \tag{60}$$

$$(s_t^i + \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t) = C' \xi_t + \varepsilon_t^i \tag{61}$$

where:

$$\xi_t = \begin{pmatrix} \pi_t \\ \bar{\pi}_t \end{pmatrix}, \quad F^i = \begin{pmatrix} \rho_\pi^i & 1 - \rho_\pi^i \\ 0 & 1 \end{pmatrix}, \quad e_t^i = \begin{pmatrix} u_{\pi t} + (1 - \rho_\pi^i) v_t \\ v_t \end{pmatrix}, \quad C = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{62}$$

It therefore remains optimal for households to incorporate signals into their perceptions of π_t and $\bar{\pi}_t$ using the Kalman filter:

$$\tilde{\mathbb{E}}_t^i \xi_t = (I - K_t^i C') F^i \tilde{\mathbb{E}}_{t-1}^i \xi_{t-1} + K_t^i s_t^i \tag{63}$$

where K_t^i is a 2×1 vector of gain parameters.

This means that households do not use their signals in the way they expected when they made their information decisions, as they did not anticipate the update to beliefs about $\bar{\pi}_t$. This is a direct consequence of the anticipated utility assumption, relaxed in Appendix F. To avoid $K_t^i = 0$ becoming an absorbing state, I further add that each household has a small probability of resetting to $\tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t = 0$ each period. As with other fluctuations in $\bar{\pi}_t$ beliefs, households do not take this reset shock into account when making information choices.

With these assumptions, Proposition 6 shows how perceived long-run inflation affects optimal attention and expectation updating.

Proposition 6 *Let $\sigma_{\varepsilon it}^{2*}$ denote the optimally chosen noise variance in s_t^i. Then, for $\sigma_{\varepsilon it}^{2*} < \infty$:*

$$\frac{\partial \sigma_{\varepsilon it}^{2*}}{\partial \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t} < 0 \quad \textit{if and only if} \quad \left. \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \right|_{\alpha_t^{i,prior}} < 0 \tag{64}$$

$$\frac{\partial K_t^i}{\partial \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t} > 0 \quad \textit{if and only if} \quad \left. \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \right|_{\alpha_t^{i,prior}} < 0 \tag{65}$$

Proof. Appendix D.8. ■

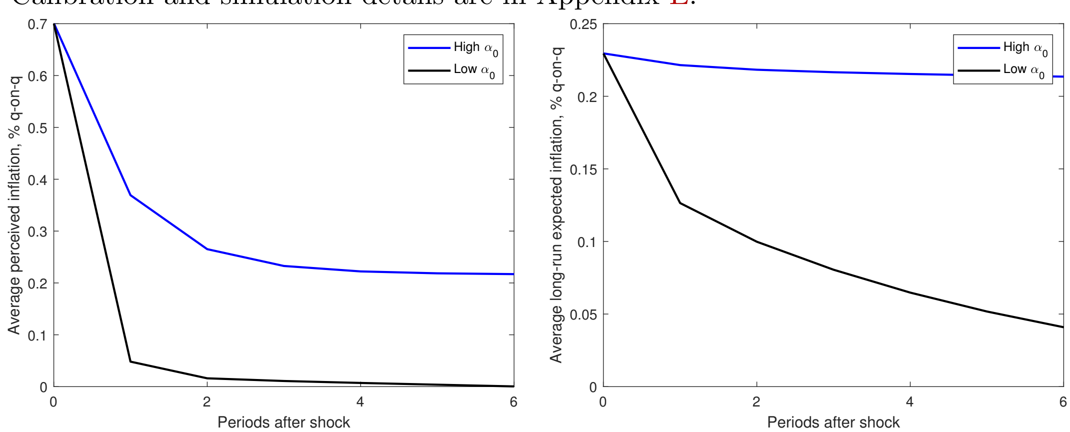
That is, if a household starts the period with a negative subjective model, such that they reduce consumption when perceived inflation rises, then higher long-run inflation expectations cause them to acquire more precise signals about inflation. Higher expected $\bar{\pi}_t$ causes them to update their subjective model towards inflation eroding real incomes even more strongly (equation 55). This increases the magnitude of their consumption response to inflation, so information gets more valuable, and they pay to acquire more of it. Their perceptions of π_t and $\bar{\pi}_t$ become more responsive to realized π_t as a result.

The reverse is true for a household with $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t > 0$ under $\alpha_t^{i,prior}$. Higher long-run expected inflation similarly reduces their $\alpha_t^{i,prior}$, but that shifts the consumption response to inflation towards zero. Inflation is believed to matter less on balance for decisions, which reduces the value of inflation information. Perceived current and long-run inflation get less responsive to realized π_t.

Information about π_t therefore not only affects the subjective model used that period, but also the subjective model used to make information choices in the next period, through perceptions of $\bar{\pi}_t$. These interdependencies imply that the expectations of different households may follow very different paths after a shock. To show this, Figure 6 plots the average perceived π_t and $\bar{\pi}_t$ for two groups of households after a 1 percentage point i.i.d. inflation shock. Within a group, all households share the same subjective model parameters, but obtain idiosyncratic signals.

The figure is drawn assuming all households have $\tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t = 0$ when the shock hits, and prior beliefs in the period of the shock are drawn from the stationary distribution obtained in the absence of aggregate shocks.

Figure 6: Simulated average $\tilde{\mathbb{E}}_t^i \pi_t$ and $\tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t$ for two household groups after an i.i.d. inflation shock. Calibration and simulation details are in Appendix E.



(a) Average $\tilde{\mathbb{E}}_t^i \pi_t$

(b) Average $\tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t$

The first group of households, shown in black, begin the shock period with low α_0^i, so they have $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t < 0$ and $K_t^i > 0$. Since they process some information, both perceived current and long-run inflation rise when the shock hits. However, as this leads them to increase their information processing, they observe that inflation has fallen in the periods after the shock, and their perceptions quickly return to 0.

The second group of households, shown in blue, are identical to the first except that they have a higher α_0^i, such that $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t > 0$. Their α_0^i has been chosen such that both groups have the same K_t^i in the period of the shock, so average inflation perceptions initially rise by the same amount. However, the rise in perceived $\bar{\pi}_t$ causes this second group to pay *less* attention to inflation, as their subjective models shift towards inflation making less difference for their consumption. This slows down the return of long-run expectations, and perceived current inflation, to steady state among this group, as they do not precisely observe the fall in inflation after the shock. In turn, this means their attention remains low.

High inflation can therefore become 'baked in' to expectations, but only among households who start out believing inflation strengthens the economy, and who subsequently reduce their attention after an inflationary shock. This is a novel effect from the interaction of the two components of expectations: if households had limited information but knew the true equilibrium law of motion for inflation, they would know that the shock is transitory, and would not update their long-run expectations. If households didn't know the true model but had full information, they would all observe inflation returning to 0 after the shock.

Empirical evidence. As the IAS does not contain a panel dimension, we cannot track individual households over time to test this 'baking in' mechanism directly. However, we can test the underlying process by studying the relationship between inflation perceptions and information. Proposition 6 implies that among those with negative subjective models of inflation, higher perceived inflation encourages more information processing, so there should be a positive correlation between $\tilde{\mathbb{E}}_t^i \pi_t$ and information.[32] Among those with positive models, that correlation should be reversed. I test this in the survey data in Appendix D.9, and find evidence of the relevant correlations, lending support to the mechanism in the model.

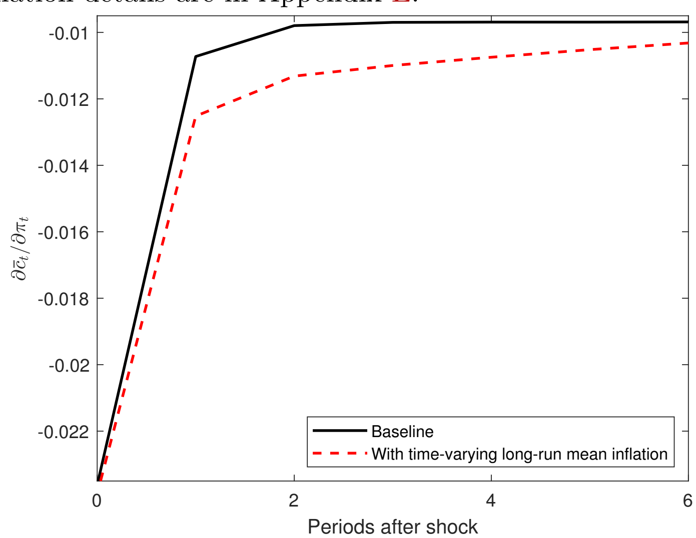
This rationalizes a key result in Pfajfar and Santoro (2010): in the Michigan Survey of Consumers, they estimate that higher inflation is principally associated with more frequent information acquisition among those with higher than average expected inflation. In the model developed here, those households mostly hold negative subjective models of the ef-

[32] As in other surveys, households overestimate inflation on average (Carroll, 2003; Kumar et al., 2015), so this implies households with more information make larger forecast errors.

fects of inflation, and so Proposition 6 generates the result. Similarly, Link et al. (2022) find that greater information acquisition about inflation is associated with higher expected inflation on average, even though this implies greater average forecast errors. Again this is explained by Proposition 6, combined with the observation that most households in the data believe inflation weakens the economy.

Implication for aggregate dynamics. The fact that this 'baking in' is correlated with subjective models implies that it has a persistent effect on the aggregate transmission of inflationary shocks. Figure 7 shows $\partial \bar{c}_t / \partial \pi_t$ in the calibrated model (Section 5.3) after the one-off inflationary shock from Figure 6, with and without time-varying long-run perceptions.

Figure 7: Simulated aggregate consumption elasticity to inflation after an i.i.d. inflation shock. Calibration and simulation details are in Appendix E.



The aggregate consumption elasticity to inflation returns quickly to its pre-shock level when the long-run inflation mean is known to be constant at 0, because the perceived persistence of inflation in the calibration is low. However, with time-varying perceived long-run inflation, $\partial \bar{c}_t / \partial \pi_t$ remains depressed persistently after the shock, because of the households whose expectations have become 'baked in' at a high level. Their subjective models of the effects of inflation are persistently less positive than before the shock.

This has an effect on the average subjective model, but also importantly on the covariance of information and subjective models. A group of well-informed households who believed in positive effects of inflation move to being uninformed, which persistently lowers the narrative heterogeneity channel. Intuitively, inflation information becomes more concentrated among those who react to it in the most negative way. Decomposing the changes in $\partial \bar{c}_t / \partial \pi_t$ reveals that the narrative heterogeneity channel accounts for 70% of the difference between $\partial\bar{c}_t/\partial\pi_t$ and its pre-shock value after 6 quarters.

7 Conclusion

This paper studies the transmission of macroeconomic shocks through heterogeneity in expectation formation. Importantly, it allows for interactions between the information and subjective models involved in forming expectations, which previous literature has tended to treat separately.

In a general log-linear model, shocks pass through to aggregate actions along three channels. The first is the transmission that would be seen in a representative agent model. The second comes from heterogeneity in the parameters of policy functions, extending well-known results from the literature on heterogeneous-agent macroeconomics. The third channel is novel. The narrative heterogeneity channel operates when information and subjective models covary systematically across agents. Heterogeneous subjective models imply heterogeneous responses to information, so systematic patterns in the distribution of information across agents with different subjective models distort the aggregate response to shocks.

I use unique features of the Bank of England Inflation Attitudes Survey to document that subjective models and information about inflation do indeed covary systematically with each other, and with inflation perceptions and expectations. The distribution of subjective models also varies systematically with realized inflation. A model with rational inattention and time-varying subjective models accounts for these empirical results. The model generates a selection effect on information, size- and history-dependent shock transmission, and the possibility that temporarily high inflation may become 'baked in' to expectations, but only among certain households.

When tracking if high inflation is becoming 'baked in' to expectations, not all households are therefore of equal concern. The households who believed before the shock that more inflation would make the economy stronger pose the greatest risk, because they reduce their attention to inflation as perceived inflation rises. If their expectations increase substantially, reducing realized inflation will not be sufficient to bring their expectations back down. From August 2021 to February 2022, the perceived inflation of households in this positive group in the IAS rose by just 40 basis points, substantially less than the average rise in perceived inflation across all households in the survey (180 b.p.). This suggests that the cat was not yet out of the bag in UK inflation expectations at the start of 2022.

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A Log-linear model proofs and derivations

A.1 Proofs of Propositions 1 - 3

Proposition 1. The derivative of the expectation of each element z^i_{jt} of $\boldsymbol{z}^i_t$ can be decomposed using the chain rule:

$$\frac{d\mathbb{E}^i_t z^i_{jt}}{d\xi_t} = \left.\frac{d\mathbb{E}^i_t z^i_{jt}}{d\xi_t}\right|_{\mathbb{E}^i_t z_{k\neq j,t}} + \sum_{k\neq j}^{N_z} \frac{\partial\mathbb{E}^i_t z^i_{jt}}{\partial\mathbb{E}^i_t z^i_{kt}} \frac{d\mathbb{E}^i_t z^i_{kt}}{d\xi_t} \tag{66}$$

Stacking this expression over all elements of $\boldsymbol{z}^i_t$ and rearranging gives:

$$\frac{d\mathbb{E}^i_t \boldsymbol{z}^i_t}{d\xi_t} = (\boldsymbol{I} - \boldsymbol{\mathcal{M}}^i_t)^{-1}\boldsymbol{\delta}^i_t \tag{67}$$

which substituted into equation 2 gives the result.

Proposition 2. From the definition of $\bar{x}_{st}$ (equation 12), we have:

$$\frac{d\bar{x}_{st}}{d\xi_t} = \mathbb{E}_I \frac{dx^i_{st}}{d\xi_t} \tag{68}$$

The s^{th} row of equation 3 can be written as:

$$\frac{dx^i_{st}}{d\xi_t} = \sum_{j=1}^{N_z}\sum_{k=1}^{N_z} \mu^i_{sj,t}\chi^i_{jk,t}\delta^i_{k,t} \tag{69}$$

Substituting this into equation 68 gives:

$$\frac{d\bar{x}_{st}}{d\xi_t} = \sum_{j=1}^{N_z}\sum_{k=1}^{N_z} \mathbb{E}_I \mu^i_{sj,t}\chi^i_{jk,t}\delta^i_{k,t} \tag{70}$$

From the definition of covariance, $\mathbb{E}(XY) = \mathbb{E}(X)\mathbb{E}(Y) + Cov(X,Y)$ for any X, Y. Applying this to equation 70 implies:

$$\frac{d\bar{x}_{st}}{d\xi_t} = \sum_{j=1}^{N_z}\sum_{k=1}^{N_z} \left[\bar{\mu}_{sj,t}\mathbb{E}_I(\chi^i_{jk,t}\delta^i_{k,t}) + Cov_I(\mu^i_{sj,t}, \chi^i_{jk,t}\delta^i_{k,t})\right] \tag{71}$$

Applying the covariance formula again to the first term inside the sum in equation 71 implies equation 13.

Proposition 3. Differentiating equation 14 with respect to ξ_t we have:

$$A\frac{d\boldsymbol{z}_t}{d\xi_t} + B\frac{d\bar{\boldsymbol{x}}_t}{d\xi_t} + Ce_\xi = 0 \tag{72}$$

From Proposition 1 and Assumption 2 we have:

$$\frac{d\bar{\boldsymbol{x}}_t}{d\xi_t} = \mathbb{E}_I\left(\boldsymbol{\mu}^i_t\boldsymbol{\chi}^i_t\tilde{\boldsymbol{\delta}}^i_t\right)\frac{d\boldsymbol{z}_t}{d\xi_t} \tag{73}$$

Substituting equation 73 into equation 72 and rearranging:

$$\frac{d\boldsymbol{z}_t}{d\xi_t} = -\left(A + B\mathbb{E}_I\left(\boldsymbol{\mu}^i_t\boldsymbol{\chi}^i_t\tilde{\boldsymbol{\delta}}^i_t\right)\right)^{-1} Ce_\xi \tag{74}$$

Substituting equation 74 into equation 73 yields equation 16.

A.2 Consumption function in a standard household problem

Household i maximizes:[33]

$$\mathbb{E}^i_t\sum_{s=0}^{\infty}\beta^s\frac{(C^i_{t+s})^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \text{ s.t. } C^i_{t+s} + B^i_{t+s} = \tilde{R}_{t+s-1}B^i_{t+s-1} + Y_{t+s} \tag{75}$$

[33]This derivation closely follows that in Bilbiie (2019) appendix A, and is also similar to consumption functions derived in Farhi and Werning (2019) and others.

where C_t^i is consumption, σ is the intertemporal elasticity of substitution, B_t^i are real one-period bonds bought in period t, $\tilde{R}_t$ is the gross real interest rate on such a bond, and Y_t is real income (assumed equal across households). The first order condition is the standard Euler equation. Log-linearizing about steady state and substituting forward we obtain:

$$c_t^i = \mathbb{E}_t^i c_{t+s}^i - \sigma \sum_{k=0}^{s-1} \mathbb{E}_t^i \tilde{r}_{t+k} \tag{76}$$

where lower-case letters denote log-deviations from steady state. Assuming that $b_t^i = 0$ (as it is in equilibrium in a standard representative-agent or two-agent New Keynesian model), the log-linearized present value budget constraint is:

$$\sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^i (c_{t+s}^i - \sum_{k=0}^{s-1} \tilde{r}_{t+k}) = \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^i (y_{t+s} - \sum_{k=0}^{s-1} \tilde{r}_{t+k}) \tag{77}$$

Use the Euler equation to substitute out for $\mathbb{E}_t^i c_{t+s}^i$ to obtain:

$$\sum_{s=0}^{\infty} \beta^s (c_t^i - (1-\sigma) \mathbb{E}_t^i \sum_{k=0}^{s-1} \tilde{r}_{t+k}) = \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^i (y_{t+s} - \sum_{k=0}^{s-1} \tilde{r}_{t+k}) \tag{78}$$

Rearranging:

$$\frac{1}{1-\beta} c_t^i = \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^i y_{t+s} - \frac{\sigma\beta}{1-\beta} \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^i \tilde{r}_{t+s} \tag{79}$$

Multiplying through by $1-\beta$, and applying the Fisher equation $\mathbb{E}_t^i \tilde{r}_t = \mathbb{E}_t^i (r_t - \pi_{t+1})$ (where r_t is the nominal interest rate), we obtain equation 6.

B Defining the direct information indicator in the IAS

The full set of questions used to construct the information dummy is set out below, along with the dates at which each was asked and how the answers are mapped into the information indicator used above. Note that my question numbering differs from the labels in the IAS microdata, to aid the logical organization of the paper. All of the questions were only asked in the first quarter of the year(s) indicated. In the main exercises I exclude questions 2e and 2g from the total information variable, to ensure that there are no periods in which two questions are asked. I remove these rather than the short run questions in those periods to keep the majority of questions as short run expectations. The results are robust to including these extra questions. See Appendix C.2 for this, and robustness checks with other variations in the definition of the information indicator.

Question 2b *What were the most important factors that led you [to change (insert their response to how expectation has changed)] your expectation of prices in the shops over the next 12 months?*

Please select up to 4:

1. *How prices have changed in the shops recently, over the last 12 months*
2. *How prices have changed in the shops, on average, over the longer term i.e the last few years*
3. *Reports of current inflation in the media*
4. *Discussion of the prospects for inflation in the media*
5. *The level of interest rates*
6. *The inflation target set by the government*
7. *The current strength of the UK economy*
8. *Expectations about how economic conditions in the UK are likely to evolve*
9. *The level of the exchange rate (the value of sterling)*
10. *Other factors*
11. *None*

Asked: 2017
Information indicator: =1 if items 3 or 4 selected, =0 otherwise.

Question 2c *What were the most important factors that led you to change/not change your expectation of prices in the shops in the longer term?*

1. *How prices have changed in the shops recently, over the last 12 months*
2. *How prices have changed in the shops, on average, over the longer term i.e the last few years*
3. *Reports of current inflation in the media*
4. *Discussion of the prospects for inflation in the media*
5. *The level of interest rates*
6. *The inflation target set by the government*
7. *The current strength of the UK economy*
8. *Expectations about how economic conditions in the UK are likely to evolve*
9. *The level of the exchange rate (the value of sterling)*
10. *Other factors*

11. *None*

Asked: 2018, 2019
Information indicator: =1 if items 3 or 4 selected, =0 otherwise.

Question 2d *When you said prices would go up in the next 12 months, how important were the following things in getting to that answer?*

For each option, possible answers are:

- *Very important*
- *Fairly important*
- *Not very important*
- *Not at all important*
- *Don't know*
- *Refused*

Options:

1. *How prices have changed in the shops in your most recent visits (i.e. the last 1 to 6 months).*
2. *How prices have changed in the shops over the longer term (i.e. the last 12 months or more)*
3. *The current level of interest rates.*
4. *The current strength of the British Economy.*
5. *The inflation target set by the government.*
6. *Reports on inflation outlook in the media.*
7. *Reports of VAT changes in the media.*
8. *Other factor(s).*

Asked: 2009, 2010, 2011, 2013
Information indicator: =1 if 'very important' selected for option 6, =0 otherwise.

Question 2e *And which, if any, of the same factors were important in getting to your expectation of how prices will change over the longer term (say in 5 years time)?*

1. *How prices have changed in the shops in your most recent visits (i.e. the last 1 to 6 months).*
2. *How prices have changed in the shops over the longer term (i.e. the last 12 months or more)*
3. *The current level of interest rates.*
4. *The current strength of the British Economy.*

5. *The inflation target set by the government.*
6. *Reports on inflation outlook in the media.*
7. *Reports of VAT changes in the media.*
8. *Other factor(s).*

Asked: 2011, immediately after Question 2d
Information indicator: =1 if item 6 selected, =0 otherwise.

Question 2f *What were the most important factors in getting to your expectation for how prices in the shops would change over the next 12 months?*

Please select up to 4:

1. *How prices have changed in the shops recently, over the last 12 months*
2. *How prices have changed in the shops, on average, over the longer term i.e the last few years*
3. *Reports of current inflation in the media*
4. *Discussion of the prospects for inflation in the media*
5. *The level of interest rates*
6. *The inflation target set by the government*
7. *The current strength of the UK economy*
8. *Expectations about how economic conditions in the UK are likely to evolve*
9. *Other factors*
10. *None*

Asked: 2016
Information indicator: =1 if items 3 or 4 selected, =0 otherwise.

Question 2g *And what were the most important factors in getting to your expectation for how prices in the shops would change over the longer term (say in 5 years' time)?*

Please select up to 4:

1. *How prices have changed in the shops recently, over the last 12 months*
2. *How prices have changed in the shops, on average, over the longer term i.e the last few years*
3. *Reports of current inflation in the media*
4. *Discussion of the prospects for inflation in the media*
5. *The level of interest rates*
6. *The inflation target set by the government*

7. *The current strength of the UK economy*
8. *Expectations about how economic conditions in the UK are likely to evolve*
9. *Other factors*
10. *None*

Asked: 2016
Information indicator: =1 if items 3 or 4 selected, =0 otherwise.

C Further empirical results

C.1 The relationship of planned consumption with measured information and subjective models

To confirm that the survey measures of information and subjective models uncover meaningful aspects of household beliefs, I consider how they correlate with planned consumption behavior. To this end, I use the following survey question:

Question 3 *Which, if any, of the following actions are you taking, or planning to take, in the light of your expectations of price changes over the next twelve months?*

- *Cut back spending and save more.*

Crucially, this asks about consumption choices which are explicitly driven by expected inflation.[34] A household answering 'yes' to this question, and who reports elsewhere in the survey that they expect prices to rise in the next year, is therefore indicating that $dc_t^i/d\mathbb{E}_t^i p_{t+1} < 0$. A question that only asked about consumption or consumption changes, without reference to the cause of the behavior, would conflate this with reactions to expectations of other variables, which might also be influenced by the same shocks as expected inflation, either directly or through cross-learning. Question 3 is therefore informative about the sign of $\frac{dc_t^i}{d\mathbb{E}_t^i p_{t+1}}$. If current prices are taken as given by the household, then this is the same as the sign of $\frac{dc_t^i}{d\mathbb{E}_t^i \pi_{t+1}}$.

[34]Another question in the survey asks if the respondent will "bring forward major purchases such as furniture or electrical goods" as a result of expected inflation. I do not use this for two reasons. First, as Nunes and Park (2020) note, the question refers specifically to durable goods, which may not respond to prices in the same way as aggregate consumption, the object of interest. Second, it is very rarely chosen: just 6% of respondents said they would bring forward major purchases. In contrast, 40% report that they will cut back spending and save more. Any estimation on this variable will therefore be heavily influenced by a small subset of agents.

The vast majority of respondents (98%) expect positive inflation over the next 12 months.[35] For these households, yes and no responses to Question 3 respectively indicate that:

$$\frac{dc_t^i}{d\mathbb{E}_t^i p_{t+1}} \begin{cases} < 0 & \text{if answer yes} \\ \geq 0 & \text{if answer no} \end{cases} \tag{80}$$

For the minority who expect deflation, these inequalities are reversed: responding with 'yes' indicates consumption is being cut because of an expected fall in prices. I therefore define the following indicator:

$$\widetilde{\frac{dc_t^i}{d\mathbb{E}_t^i p_{t+1}}} = \begin{cases} 1 & \text{if Q3='no' and } \mathbb{E}_t^i \pi_{t+1} > 0 \\ 0 & \text{if Q3='yes' and } \mathbb{E}_t^i \pi_{t+1} > 0 \\ 1 & \text{if Q3='yes' and } \mathbb{E}_t^i \pi_{t+1} < 0 \\ 0 & \text{if Q3='no' and } \mathbb{E}_t^i \pi_{t+1} < 0 \end{cases} \tag{81}$$

For the large majority who expect inflation, this is equal to 1 if $\frac{dc_t^i}{d\mathbb{E}_t^i p_{t+1}} \geq 0$, and equal to 0 if the reaction to expected price rises is strictly negative. The same is true of the minority who expect deflation, except that any household with $\frac{dc_t^i}{d\mathbb{E}_t^i p_{t+1}} = 0$ would respond 'no' to Question 3, and so is counted as if their response to expected price rises is strictly negative. The mislabeling is not a large issue, as less than 1% of respondents to Question 3 both expect deflation and answer 'no'. The results below are robust to removing the few households who expect deflation (see Table 3 column 2).

Table 3 shows how this is related to the information indicator and the subjective models (responses to Question 1). Column 1 shows the results from estimating a probit regression of $\frac{\widetilde{dc_t^i}}{d\mathbb{E}_{+1}}$ on the information indicator interacted with subjective models (Question 1), plus the standard household controls and time fixed effects used above. The coefficient on information is significantly negative for those with negative subjective models of inflation, despite the fact that substitution effects imply $\frac{dc_t^i}{d\mathbb{E}_t^i p_{t+1}} \geq 0$ in many standard models. Being informed is therefore associated with a *lower* probability of responding positively to expected inflation for these households.

[35] The analysis in this section excludes any households who report expecting zero inflation over the next 12 months, or who do not answer the inflation expectation question, as Question 3 is difficult to interpret for these households. I discuss the appropriate counterfactual implicit in the question below. Including these people, 79% of respondents to Question 3 expect positive inflation, 7% expect zero inflation, 2% expect deflation, and 12% do not answer.

Table 3: Consumption response to inflation correlates with information, by subjective model

	(1)	(2)
	c response to Eπ	c response to Eπ
information indicator=1	-0.213***	-0.224***
	(0.0611)	(0.0613)
end up stronger	0.0108	0.0392
	(0.0891)	(0.0906)
information indicator=1 × end up stronger	0.348*	0.313*
	(0.185)	(0.186)
make little difference	0.130**	0.157***
	(0.0594)	(0.0600)
information indicator=1 × make little difference	0.0240	-0.0149
	(0.126)	(0.128)
dont know	0.0958	0.0978
	(0.0833)	(0.0846)
information indicator=1 × dont know	-0.0158	-0.0342
	(0.186)	(0.187)
Expected Inflation	All	Exclude Deflation
Controls	All	All
Time FE	Yes	Yes
Observations	4940	4871

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the results of probit regressions of the $\frac{\widetilde{dc_t^i}}{d\mathbb{E}_t^i \pi_{t+1}}$ indicator on the information indicator, interacted with responses to Question 1. The omitted category is a household with information indicator=0 who holds the belief that inflation makes the economy weaker. All regressions are weighted using the survey weights provided in the IAS.

However, for those who believe inflation makes the economy stronger, being informed is associated with a significantly higher $\Pr(\frac{dc_t^i}{d\mathbb{E}_t^i \pi_{t+1}} \geq 0)$. For those who believe inflation makes no difference, the average value of $\Pr(\frac{dc_t^i}{d\mathbb{E}_t^i \pi_{t+1}} \geq 0)$ with and without information, which is also consistent with the interpretation of these variables as $\frac{\widetilde{dc_t^i}}{d\mathbb{E}_{+1}} = 1$ includes the case where $\frac{dc_t^i}{d\mathbb{E}_t^i \pi_{t+1}} = 0$.

This is consistent with individuals filtering information through their subjective models of the economy. If a household who believes inflation weakens the economy gets more information about future positive inflation, their subjective model implies that they should cut consumption, because bad times lie ahead. If instead a household believes inflation strengthens the economy, then they will react in the opposite way to the same inflation. The overall correlation of information and consumption response is negative because the majority of households believe inflation makes the economy weaker. This therefore supports the claim that the information indicator and answers to Question 1 reflect the information and subjective models used by households in making their consumption decisions.

The analysis here assumes that when asked whether they will cut back consumption and save more, households are comparing their actions to a counterfactual in which there are no price rises over the next 12 months. An alternative possibility is that they are comparing with a consumption plan made in the past, in which case the relevant counterfactual is where expected inflation is unchanged from the level expected when the plan was made. I consider this in two ways, and find that the qualitative patterns in reported consumption responses to inflation are the same for households expecting inflation to increase or decrease relative to the previous year. It does not therefore appear that past inflation is the relevant counterfactual for most respondents.

First, column 2 of Table 3 re-runs the regression in column 1, excluding any respondent who reports expecting prices to fall over the next year. All results are qualitatively the same as over the full sample, showing that the few respondents expecting deflation are not driving the results.

Second, I split the sample by the sign of the respondent's expected change in inflation, computed as the sign of the difference between 12-month ahead inflation forecast and their perception of inflation over the previous 12 months. The results are in Table 4. The sample sizes in each group are substantially smaller than over the full sample, so some significance is lost, but importantly the signs of the key coefficients remain the same. In each group, households who believe inflation makes the economy weaker are less likely to have $\frac{dc_t^i}{d\mathbb{E}_t^i\pi_{t+1}} \geq 0$ when they get inflation information. For households who believe inflation makes the economy stronger, this effect is reversed. The similarity of these patterns suggests that most respondents use 'no price change' as the counterfactual when answering Question 3, not 'no inflation change'. If the latter was used, we would expect to see changes of sign across the columns in Table 4, as a household expecting a fall in inflation would be reporting $-1 \times \frac{dc_t^i}{d\mathbb{E}_t^i\pi_{t+1}}$, while one expecting a rise in inflation would report $\frac{dc_t^i}{d\mathbb{E}_t^i\pi_{t+1}}$.

Table 4: Consumption response to inflation correlates with information, by subjective model and sign of perceived Eπ change.

	(1)	(2)	(3)
	E$\Delta\pi < 0$	E$\Delta\pi = 0$	E$\Delta\pi > 0$
Dc_Dpi			
Information=1	-0.140	-0.305***	-0.257**
	(0.116)	(0.101)	(0.107)
end up stronger	0.0668	-0.178	0.195
	(0.164)	(0.151)	(0.165)
Information=1	0.586	0.349	0.397
× end up stronger	(0.441)	(0.293)	(0.307)
make little	0.165	0.136	0.181
difference	(0.111)	(0.0957)	(0.112)
Information=1	0.129	-0.300	0.113
× make little difference	(0.241)	(0.211)	(0.216)
dont know	0.156	0.0293	0.0264
	(0.176)	(0.128)	(0.167)
Information=1	-0.141	0.469	0.117
× dont know	(0.354)	(0.359)	(0.325)
Controls	All	All	All
Time FE	Yes	Yes	Yes
Observations	1384	1876	1463

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the results of probit regressions of the $\frac{\widetilde{dc_t^i}}{d\mathbb{E}_t^i \pi_{t+1}}$ indicator on the information indicator, interacted with responses to Question 1, split by the sign of the respondent's inflation expectations. The omitted category in all cases is a household with information indicator=0 who holds the belief that inflation makes the economy weaker. All regressions are weighted using the survey weights provided in the IAS.

C.2 Cross-sectional patterns in information on inflation

The first three columns of Table 5 show the results of probit regressions of the information indicator on subjective models, controls, and period fixed-effects, for three subsamples. The first only uses questions about the information used to arrive at the respondent's *change* in expected inflation, and the second uses only questions about information used to form point forecasts. The third column excludes questions relating to forecast horizons longer than 12 months. The signs of the marginal effects are the same as in the main exercise in Table 1, though they are not significant in the case of the revisions questions, as the sample size is small.

Table 5: Information correlates with subjective models, split by information question type

	(1) Revision	(2) Point	(3) Short horz.	(4) Extra Qs	(5) Q2d wider	(6) +Other
end up stronger	0.0575	-0.0335	-0.0123	0.00114	-0.00126	-0.00779
	(0.0380)	(0.0218)	(0.0206)	(0.0196)	(0.0196)	(0.0205)
make little difference	-0.0191	-0.0331**	-0.0392***	-0.0310**	-0.0312**	-0.0429***
	(0.0233)	(0.0155)	(0.0141)	(0.0132)	(0.0131)	(0.0139)
dont know	-0.0408	-0.0715***	-0.0622***	-0.0663***	-0.0472***	-0.0663***
	(0.0297)	(0.0206)	(0.0192)	(0.0174)	(0.0180)	(0.0191)
Controls	All	All	All	All	All	All
Time FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2364	5906	6848	8306	8270	8270

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the average marginal effects from estimating probit regressions of the information indicators constructed from subsets of the questions listed in Appendix B on the responses to Question 1. The omitted category is the belief that inflation makes the economy weaker. All regressions are weighted using the survey weights provided in the IAS.

The remaining columns of Table 5 repeat the regression for broader definitions of the information dummy than that used in Table 1. In the fourth column, the information indicator includes Questions 2e and 2g. In the fifth column, I extend the criteria for setting the information indicator equal to 1 in Question 2d to account for the fact that some people may be unwilling to select the highest importance box for any information source. I therefore set the information indicator to 1 if in answer to Question 2d, the respondent selects 'very important' for direct inflation information (as before), or if they do not select 'very important' for any option, but do respond that four or fewer options were 'fairly important', and direct inflation information is among them. In the final column, I set the information indicator =1 if the household chooses a direct information source or 'Other', in case this includes direct information sources (e.g. checking the Bank of England published forecasts). In all of these, the results are robust.

To account for possible selection bias from missing observations, I estimate a version of Table 1 amended for selection as in Heckman (1979). As in Michelacci and Paciello (2020),

Table 6: Information correlates with subjective models, with selection correction

	(1) Information	(2) Information
end up stronger	-0.0178	-0.0183
	(0.0172)	(0.0172)
make little difference	-0.0306**	-0.0315***
	(0.0120)	(0.0121)
dont know	-0.0575***	-0.0579***
	(0.0172)	(0.0173)
Inverse Mills ratio	-0.282***	-0.0696**
	(0.0820)	(0.0355)
Selection stage		
Economic Literacy	0.205***	
	(0.0226)	
HH does not know past π		-0.876***
		(0.0365)
r affects π in 1-2 months		0.0334
		(0.0236)
r affects π in 1-2 yrs		0.0882***
		(0.0231)
Pseudo-R^2 (selection)	0.103	0.127
Controls	All	All
Time FE	Yes	Yes
Observations	18026	18026

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the coefficients from estimating a linear regression of the information indicator defined in Section 3.1 on the responses to Question 1, augmented with the inverse Mills ratio from a first-stage probit regression of whether the information indicator is observed on measures of economic literacy defined above. The omitted category is the belief that inflation makes the economy weaker. The selection stage is only run for quarters in which the information questions were asked. The model is run using the 2-step limited information method in Heckman (1979). Time fixed effects and controls as in footnote 18 are included in both stages.

I predict observing the relevant survey response using a measure of economic literacy. Here the relevant response is only the information indicator, as there are no missing values for Question 1. Following Michelacci and Paciello (2020), economic literacy is measured with three indicators: the household reports a value for perceived current inflation, and they answer 'agree' or 'strongly agree' to the statements "a rise in interest rates makes prices in the high street rise more slowly in the short term (say a month or two)" and "a rise in interest rates makes prices rise more slowly in the medium term (say a year or two)". I estimate versions of the model with this as an aggregate index (=1 if and only if the household scores on all components), and with the components disaggregated. The results are in Table 6. The predictors used in the first stage are strongly significant. Qualitatively the second-stage results are unchanged from Table 1, and the quantitative differences are small.

C.3 Time series patterns in subjective models of inflation

Bhandari et al. (2019) also study the time series of responses to Question 1, and conclude that households are more pessimistic about inflation when output growth is low. To explore this, I regress the proportion of households responding 'end up weaker' on realized annual CPI inflation and quarterly GDP growth. The results are in column 2 of Table 7. Consistent with Bhandari et al. (2019), the coefficient on GDP growth is significantly negative. However, the R^2 is only slightly higher than that of a regression on inflation only (column 1), so GDP growth does not account for much of the variation in survey answers. Indeed, GDP growth does not have any significant relationship with the proportion of households with a negative view of inflation outside of the four worst months of the Great Recession (column 3).

Table 7: Regressions of the proportion of households answering weaker to Question 1 on aggregate variables.

	(1) Proportion weaker	(2) Proportion weaker	(3) Proportion weaker
Inflation	0.0568***	0.0517***	0.0501***
	(0.00489)	(0.00479)	(0.00469)
GDP growth		-0.0261***	-0.0110
		(0.00869)	(0.0180)
Constant	0.466***	0.487***	0.482***
	(0.0109)	(0.0123)	(0.0152)
Omitted quarters	None	None	2008Q2-2009Q1
R-squared	0.615	0.647	0.554
Observations	70	70	66

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the results of regressing the proportion of households answering Question 1 that inflation makes the economy weaker on annual CPI inflation and quarter-on-quarter real GDP growth. Proportions are computed using survey weights.

To explore which measure of inflation affects subjective models, Table 8 reports the results of regressing an indicator variable for if the respondent reports a negative subjective model on a variety of inflation measures. The first column uses CPI inflation, so is very similar to the time-series regression in Table 7. Columns 2-4 use more granular measures of the inflation rate experienced by different households, split by whether they are above retirement age (65), above median income, and by their housing tenure. Inflation rates split by these characteristics are provided by the ONS.[36] Column 5 uses perceived current inflation. The different realized inflation measures are strongly correlated, so cannot be included jointly. Although the coefficient sizes vary as the different inflation rates have different levels of volatility, in all cases higher inflation is associated with a significantly greater probability of reporting a negative subjective model. The R^2 is highest for perceived inflation, supporting the choice of modeling assumption in Section 4.4.

Table 8: Probability of reporting negative subjective model by experienced and perceived inflation

	(1) Weaker	(2) Weaker	(3) Weaker	(4) Weaker	(5) Weaker
Inflation	0.0510***	0.0463***	0.0457***	0.0292***	0.0254***
	(0.00177)	(0.00170)	(0.00165)	(0.00137)	(0.000720)
Inflation measure	CPI	by retirement	by income	by housing	perceived
Controls	All	All	All	All	All
R-squared	0.0303	0.0286	0.0292	0.0237	0.0371
Observations	68269	68269	68269	68269	68269

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the results of estimating a linear probability model of whether a respondent reports that inflation makes the economy weaker in response to Question 1 on various measures of inflation. These are annual CPI inflation, inflation split by whether the respondent is of retirement age, split by whether the respondent has above or below median income, split by the respondent's housing tenure, and finally the respondent's perceived current inflation. Sample begins in 2006 Q1, as this is when the ONS sub-group inflation data is available from. Households not reporting a perceived rate of inflation are dropped in all regressions. All regressions are weighted using the survey weights provided in the IAS.

Similar patterns in reverse are observed for the other answers. Table 9 repeats the regressions of Table 7, replacing the dependent variable with the proportion of respondents choosing each of the other answers to Question 1. In all cases, inflation accounts for a large share of the variation in survey answers, and higher inflation is associated with significantly

[36]Finer decompositions of inflation by household characteristics are not reliable, given the data available for the UK (see e.g. Dawber et al., 2022).

lower proportions giving each answer. Higher GDP growth is associated with higher proportions on these other answers, but that relationship is not significantly different from zero for any answer when excluding the worst of the Great Recession.

Table 9: Regressions of the proportion of households giving each answer to Question 1 on aggregate variables.

	(1) Proportion	(2) Proportion	(3) Proportion
Stronger			
Inflation	-0.0123***	-0.0116***	-0.0108***
	(0.00193)	(0.00215)	(0.00221)
GDP growth		0.00346	-0.00392
		(0.00363)	(0.00646)
Constant	0.104***	0.102***	0.104***
	(0.00431)	(0.00550)	(0.00638)
No difference			
Inflation	-0.0292***	-0.0262***	-0.0257***
	(0.00303)	(0.00313)	(0.00314)
GDP growth		0.0150***	0.0106
		(0.00473)	(0.0107)
Constant	0.277***	0.264***	0.266***
	(0.00772)	(0.00883)	(0.0103)
Don't know			
Inflation	-0.0154***	-0.0139***	-0.0135***
	(0.00249)	(0.00262)	(0.00267)
GDP growth		0.00762*	0.00428
		(0.00423)	(0.00987)
Constant	0.153***	0.147***	0.148***
	(0.00687)	(0.00757)	(0.00884)
Omitted quarters	None	None	2008Q2-2009Q1
Observations	70	70	66

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the results of regressing the proportion of households giving each answer to Question 1 (except 'weaker') on annual CPI inflation and quarter-on-quarter real GDP growth. Proportions are computed using survey weights. The R^2 of the core regressions in column 1 are 0.388 (stronger), 0.534 (no difference), and 0.355 (don't know).

To test if the distribution of beliefs about inflation shifts when the economy reaches the Zero Lower Bound, I estimate an ordered probit regression of subjective models of inflation in the zero lower bound period, and a variety of controls.[37] A response that inflation makes the economy stronger is coded as the highest value, and inflation makes the economy weaker is the lowest value (I exclude the 'don't know' answers). A positive coefficient on the zero lower bound period would therefore imply a shift towards believing inflation makes the economy stronger, as we would expect if households follow a standard New Keynesian model. This is not what the results in Table 10 show: there is no significant shift towards a positive view of inflation in the ZLB period.

Table 10: Ordered probit regressions of subjective models of inflation on whether the economy is at the zero lower bound on nominal interest rates.

	(1) Subjective model	(2) Subjective model	(3) Subjective model
Subjective model			
ZLB	-0.00801	-0.00785	-0.00513
	(0.00937)	(0.00962)	(0.00972)
Controls	None	Household	Household + macro
Observations	83526	83526	83526

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the results of an ordered probit regression of answers to Question 1 on an indicator for whether the UK economy was at the zero lower bound, defined as the period from 2009Q2 to the end of 2019 (end of the sample). The ordering is: "stronger", "no difference", "weaker". Those answering "no idea" are omitted. All regressions are weighted using the survey weights provided in the IAS.

C.4 Perceived and expected inflation across households

To account for possible selection bias from missing observations, I estimate a version of Table 2 amended for selection as in Heckman (1979). As in Appendix C.2, I predict observing perceived and expected inflation using the components of the economic literacy indicator in Michelacci and Paciello (2020), this time removing the component concerning whether perceived inflation is reported as this is closely related to the dependent variables of the regressions. The results are in Table 11. The predictors used in the first stage are strongly significant. Qualitatively the second-stage results are unchanged from Table 2, and the quantitative differences are small.

[37]The first column of Table 10 has no controls, the second includes the set of household-level covariates used throughout the paper, and the third adds inflation and GDP growth.

Table 11: Perceived and expected inflation are higher for those with more negative subjective models, with selection correction

	(1) Perceived inflation	(2) Expected inflation
end up stronger	-0.724***	-0.607***
	(0.0319)	(0.0301)
make little difference	-0.548***	-0.478***
	(0.0213)	(0.0201)
dont know	-0.452***	-0.407***
	(0.0292)	(0.0280)
Inverse Mills ratio	0.714***	0.168
	(0.248)	(0.191)
Selection stage		
r affects π in 1-2 months	0.133***	0.192***
	(0.0209)	(0.0209)
r affects π in 1-2 yrs	0.278***	0.310***
	(0.0205)	(0.0205)
Pseudo-R^2 (selection)	0.0477	0.0536
Controls	All	All
Time FE	Yes	Yes
Observations	95339	95339

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the coefficients from estimating a linear regression of perceived and expected inflation on the responses to Question 1, augmented with the inverse Mills ratio from a first-stage probit regression of whether the dependent variable is observed on measures of economic literacy. The omitted category is the belief that inflation makes the economy weaker. The model is run using the 2-step limited information method in Heckman (1979). Time fixed effects and controls as in footnote 18 are included in both stages.

D Dynamic model: derivations and proofs

D.1 Proof of Lemma 1

The proof is an adaptation of the derivation of expression (34) in Maćkowiak and Wiederholt (2015). First, substitute the budget constraint (18) into the utility function (17) to obtain:

$$\tilde{\mathbb{E}}_0^i U_0^i = \tilde{\mathbb{E}}_0^i \beta^t \frac{1}{1-\frac{1}{\sigma}} \left(\frac{R_{t-1}}{\Pi_t} B_{t-1}^i + Y_t - B_t^i \right)^{1-\frac{1}{\sigma}} \tag{82}$$

Write this in log-deviations from steady state:

$$\tilde{\mathbb{E}}_0^i U_0^i = \tilde{\mathbb{E}}_0^i \beta^t \frac{1}{1-\frac{1}{\sigma}} \left(\bar{R}\bar{B}^i \exp(r_{t-1} - \pi_t + b_{t-1}^i) + \bar{Y}\exp(y_t) - \bar{B}^i \exp(b_t^i) \right)^{1-\frac{1}{\sigma}} \tag{83}$$

where $\bar{X}$ denotes the steady state value of the corresponding variable X_t, and $x_t \equiv \log(X_t/\bar{X})$ is the corresponding log-deviation.

We then take a quadratic approximation of this with respect to each variable in log-deviation, about the steady state. For this, define $z_t = (r_{t-1}, \pi_t, y_t)'$ as the vector of exogenous variables taken as given by the household in period t. The past asset choice b_{t-1}^i is also taken as given in period t, and b_t^i is the only choice variable. After the quadratic approximation, expected discounted utility is given by:

$$\begin{aligned}
\tilde{\mathbb{E}}_0^i U_0^i \approx \tilde{\mathbb{E}}_0^i \hat{U}_0^i = \bar{U}^i + \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \Bigg[& h_b b_t^i + h_z z_t + \frac{1}{2} H_{bb,-1} b_t^i b_{t-1}^i + \frac{1}{2} H_{bb,0} (b_t^i)^2 + \frac{1}{2} H_{bb,1} b_{t+1}^i b_t^i \\
& + \frac{1}{2} b_t^i H_{bz,0} z_t + \frac{1}{2} b_t^i H_{bz,1} z_{t+1} + \frac{1}{2} z_t' H_{zz,0} z_t + \frac{1}{2} z_t' H_{zb,-1} b_{t-1}^i + \frac{1}{2} z_t' H_{zb,0} b_t^i \Bigg] \\
& + \beta^{-1} \left(h_{-1} b_{-1}^i + \frac{1}{2} H_{-1} (b_{-1}^i)^2 + \frac{1}{2} H_{bb,1} b_{-1}^i b_0^i + \frac{1}{2} b_{-1}^i H_{bz,1} z_0 \right)
\end{aligned} \tag{84}$$

where $\beta^t h_b$ denotes the first derivative of U_0^i with respect to b_t^i, evaluated at the steady state. Similarly, h_z denotes the vector of first derivatives of U_0^i with respect to z_t, evaluated at steady state. The matrices $\beta^t H_{jk,\tau}$ denote the second derivatives of U_0^i with respect to j_t and $k_{t+\tau}$, for $j_t, k_t \in \{b_t^i, z_t\}$, evaluated at steady state. Finally, $\beta^{-1} h_{-1}$ and $\beta^{-1} H_{-1}$ are the first and second derivatives of U_0^i with respect to initial wealth b_{-1}^i, evaluated at steady state.

As in Maćkowiak and Wiederholt (2015), note that there are no cross-products of b_t and z_{t-1}, because from equation 83 the first derivative of U_0^i with respect to b_t^i does not depend on any elements of z_{t-1}. Similarly, there are no terms in the interaction of z_t and z_{t-1} or z_{t+1}.

We now simplify this, using several properties of the coefficient vectors and matrices.

First, we have that $z_t' H_{zb,0} b_t^i = b_t^i H_{bz,0} z_t$. Second:

$$\begin{aligned}\tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{2} H_{bb,-1} b_t^i b_{t-1}^i &= \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \beta^{-1} H_{bb,1} b_t^i b_{t-1}^i \\ &= \frac{1}{2} \beta^{-1} H_{bb,1} b_{-1}^i b_0^i + \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \beta^{-1} H_{bb,1} b_t^i b_{t+1}^i\end{aligned} \tag{85}$$

Similarly:

$$\begin{aligned}\tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{2} z_t' H_{zb,-1} b_{t-1}^i &= \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \beta^{-1} b_{t-1}^i H_{bz,1} z_t \\ &= \frac{1}{2} \beta^{-1} b_{-1}^i H_{zb,1} z_0 + \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{2} b_t^i H_{zb,1} z_{t+1}\end{aligned} \tag{86}$$

Using these, and the fact that $h_b = 0$, the log-quadratic approximation to utility becomes:

$$\begin{aligned}\tilde{\mathbb{E}}_0^i \hat{U}_0^i = \bar{U}^i + \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \Bigg[& h_z z_t + \frac{1}{2} H_{bb,0} (b_t^i)^2 + H_{bb,1} b_{t+1}^i b_t^i \\ & + b_t^i H_{bz,0} z_t + b_t^i H_{bz,1} z_{t+1} + \frac{1}{2} z_t' H_{zz,0} z_t \Bigg] \\ & + \beta^{-1} \left(h_{-1} b_{-1}^i + \frac{1}{2} H_{-1} (b_{-1}^i)^2 + H_{bb,1} b_{-1}^i b_0^i + b_{-1}^i H_{bz,1} z_0 \right)\end{aligned} \tag{87}$$

Next, we find b_t^{i*}, the optimal asset holdings chosen each period by a fully-informed household. This satisfies the first order condition:

$$\tilde{\mathbb{E}}_0^{i*} \left[H_{bb,0} b_t^{i*} + H_{bb,1} b_{t+1}^{i*} + \beta^{-1} H_{bb,1} b_{t-1}^{i*} \right] = -\tilde{\mathbb{E}}_0^{i*} \left[H_{bz,0} z_t + H_{bz,1} z_{t+1} \right] \tag{88}$$

Define the expected utility of a fully-informed household, $\tilde{\mathbb{E}}_0^{i*} \hat{U}_0^{i*}$, as the expected discounted utility if the household chooses this optimal saving behavior. The expected utility loss from deviating from this rule is:

$$\begin{aligned}\tilde{\mathbb{E}}_0^i (\hat{U}_0^{i*} - \hat{U}_0^i) &= \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} H_{bb,0} (b_t^{i*})^2 + H_{bb,1} b_{t+1}^{i*} b_t^{i*} - \frac{1}{2} H_{bb,0} (b_t^i)^2 - H_{bb,1} b_{t+1}^i b_t^i \right] \\ & + \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t (b_t^{i*} - b_t^i)(H_{bz,0} z_t + H_{bz,1} z_{t+1}) + \tilde{\mathbb{E}}_0^i \beta^{-1} \left(H_{bb,1} b_{-1}^i b_0^{i*} - H_{bb,1} b_{-1}^i b_0^i \right)\end{aligned} \tag{89}$$

where I have used that $b^{i*}_{-1} = b^i_{-1}$.

Substituting in equation 88 we have:

$$\tilde{\mathbb{E}}^i_0(\hat{U}^{i*}_0 - \hat{U}^i_0) = \tilde{\mathbb{E}}^i_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} H_{bb,0}(b^{i*}_t)^2 + H_{bb,1} b^{i*}_{t+1} b^{i*}_t - \frac{1}{2} H_{bb,0}(b^i_t)^2 - H_{bb,1} b^i_{t+1} b^i_t\right]$$
$$-\tilde{\mathbb{E}}^i_0 \sum_{t=0}^{\infty} \beta^t (b^{i*}_t - b^i_t)(H_{bb,0} b^{i*}_t + H_{bb,1} b^{i*}_{t+1} + \beta^{-1} H_{bb,1} b^{i*}_{t-1}) + \tilde{\mathbb{E}}^i_0 \beta^{-1}\Big(H_{bb,1} b^i_{-1} b^{i*}_0 - H_{bb,1} b^i_{-1} b^i_0\Big) \tag{90}$$

Collecting terms and rearranging:

$$\begin{aligned}\tilde{\mathbb{E}}^i_0(\hat{U}^{i*}_0 - \hat{U}^i_0) = \tilde{\mathbb{E}}^i_0 \sum_{t=0}^{\infty} \beta^t \bigg[&-\frac{1}{2} H_{bb,0}(b^{i*}_t)^2 - \frac{1}{2} H_{bb,0}(b^i_t)^2 + H_{bb,0} b^i_t b^{i*}_t + H_{bb,1} b^{i*}_{t+1} b^i_t \\ &- H_{bb,1} b^i_{t+1} b^i_t \bigg] + \tilde{\mathbb{E}}^i_0 \sum_{t=0}^{\infty} \beta^t \beta^{-1} H_{bb,1} b^{i*}_{t-1}(b^i_t - b^{i*}_t) + \tilde{\mathbb{E}}^i_0 \beta^{-1} H_{bb,1} b^i_{-1}(b^{i*}_0 - b^i_0)\end{aligned} \tag{91}$$

The second summation can be written as:

$$\begin{aligned}\tilde{\mathbb{E}}^i_0 \sum_{t=0}^{\infty} \beta^t \beta^{-1} H_{bb,1} b^{i*}_{t-1}(b^i_t - b^{i*}_t) = \beta^{-1} H_{bb,1} b^i_{-1} b^i_0 - \tilde{\mathbb{E}}^i_0 \beta^{-1} H_{bb,1} b^i_{-1} b^{i*}_0 \\ + \tilde{\mathbb{E}}^i_0 \sum_{t=0}^{\infty} \beta^t H_{bb,1} b^{i*}_t (b^i_{t+1} - b^{i*}_{t+1})\end{aligned} \tag{92}$$

where I have again used $b^{i*}_{-1} = b^i_{-1}$.

Substituting this into the expected utility loss and collecting terms:

$$\tilde{\mathbb{E}}^i_0(\hat{U}^{i*}_0 - \hat{U}^i_0) = \tilde{\mathbb{E}}^i_0 \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2} H_{bb,0}(b^i_t - b^{i*}_t)^2 - H_{bb,1}(b^i_t - b^{i*}_t)(b^i_{t+1} - b^{i*}_{t+1})\right] \tag{93}$$

Differentiating the instantaneous utility function $U_{p,t}$ twice gives:

$$H_{bb,0} = \frac{\partial^2 U^i_{p,t}}{\partial (b^i_t)^2}\bigg|_{ss} = -\frac{1}{\sigma}(\bar{C}^i)^{-\frac{1}{\sigma}-1}(\bar{B}^i)^2(1+\beta^{-1}), \quad H_{bb,1} = \frac{\partial^2 U^i_{p,t}}{\partial b^i_t \partial b^i_{t+1}}\bigg|_{ss} = \frac{1}{\sigma}(\bar{C}^i)^{-\frac{1}{\sigma}-1}(\bar{B}^i)^2 \tag{94}$$

Therefore:

$$\tilde{\mathbb{E}}^i_0(\hat{U}^{i*}_0 - \hat{U}^i_0) = -\frac{1}{\sigma}(\bar{C}^i)^{-\frac{1}{\sigma}-1}(\bar{B}^i)^2 \tilde{\mathbb{E}}^i_0 \sum_{t=0}^{\infty} \beta^t \left[-\frac{1+\beta^{-1}}{2}(b^i_t - b^{i*}_t)^2 + (b^i_t - b^{i*}_t)(b^i_{t+1} - b^{i*}_{t+1})\right] \tag{95}$$

Next, we transform this into an equation involving consumption choices, rather than asset choices. Log-linearizing the budget constraint (18) gives:

$$\bar{C}^i c_t^i = \beta^{-1}\bar{B}^i(r_{t-1} - \pi_t + b_{t-1}^i) - \bar{B}^i b_t^i + \bar{Y} y_t \tag{96}$$

Subtracting the equivalent for the fully-informed household:

$$\bar{C}^i(c_t^i - c_t^{i*}) = \beta^{-1}\bar{B}^i(b_{t-1}^i - b_{t-1}^{i*}) - \bar{B}^i(b_t^i - b_t^{i*}) \tag{97}$$

We substitute this into equation 95 and rearrange. To see how the rearrangement works, define $\Delta_t^i = \bar{B}^i/\bar{C}^i \cdot (b_t^i - b_t^{i*})$, so that equation 97 becomes:

$$\Delta_t^i = \beta^{-1}\Delta_{t-1}^i - (c_t^i - c_t^{i*}) \tag{98}$$

Substituting out for $(b_t^i - b_t^{i*})$ and $(b_t^i - b_t^{i*})$ in equation 95 using the definition of Δ_t^i gives:

$$\tilde{\mathbb{E}}_0^i(\hat{U}_0^{i*} - \hat{U}_0^i) = -\frac{1}{\sigma}(\bar{C}^i)^{1-\frac{1}{\sigma}}\tilde{\mathbb{E}}_0^i\sum_{t=0}^{\infty}\beta^t\left[-\frac{1+\beta^{-1}}{2}(\Delta_t^i)^2 + \Delta_t^i\Delta_{t+1}^i\right] \tag{99}$$

The terms inside the square brackets can be rearranged to:

$$\begin{aligned}
-\frac{1}{2}(\Delta_t^i)^2 - \frac{1}{2\beta}(\Delta_t^i)^2 + \Delta_t^i\Delta_{t+1}^i &= -\frac{1}{2}\frac{1}{\beta^2}(\Delta_{t-1}^i)^2 + \frac{1}{\beta}\Delta_{t-1}^i(c_t^i - c_t^{i*}) - \frac{1}{2}(c_t^i - c_t^{i*})^2 \\
&\qquad - \frac{1}{2\beta}(\Delta_t^i)^2 + \Delta_t^i\Delta_{t+1}^i \\
= -\frac{1}{2}\frac{1}{\beta^2}(\Delta_{t-1}^i)^2 + \frac{1}{\beta}\Delta_{t-1}^i(c_t^i - c_t^{i*}) &- \frac{1}{2}(c_t^i - c_t^{i*})^2 - \frac{1}{2\beta}(\Delta_t^i)^2 + \Delta_t^i(\beta^{-1}\Delta_t^i - (c_{t+1}^i - c_{t+1}^{i*})) \\
&= -\frac{1}{2}(c_t^i - c_t^{i*})^2 + \frac{1}{2\beta}\big((\Delta_t^i)^2 - \frac{1}{\beta}(\Delta_{t-1}^i)^2\big) - \big(\Delta_t^i(c_{t+1}^i - c_{t+1}^{i*}) - \frac{1}{\beta}\Delta_{t-1}^i(c_t^i - c_t^{i*})\big)
\end{aligned} \tag{100}$$

where the first and second equalities involve substituting out using equation 98.

Substituting this into equation 99, canceling terms when they appear from multiple periods, and noting that $\Delta_{-1}^i = 0$, we obtain:

$$\tilde{\mathbb{E}}_0^i(\hat{U}_0^{i*} - \hat{U}_0^i) = -\frac{1}{2\sigma}(\bar{C}^i)^{1-\frac{1}{\sigma}}\tilde{\mathbb{E}}_0^i\sum_{t=0}^{\infty}\beta^t(c_t^i - c_t^{i*})^2 \tag{101}$$

D.2 Forecasts using the subjective model

The subjective model represented by equations 22 - 24 can be written in VAR form as:

$$Y_t = A^i Y_{t-1} + B^i U_t \tag{102}$$

where:

$$\begin{aligned}
Y_t &= (\pi_t, y_t, i_t)' \\
A &= \begin{bmatrix} \rho_\pi^i & 0 & 0 \\ (\alpha^i + \lambda^i \phi^i)\rho_\pi^i & \rho_y^i & 0 \\ \phi \rho_\pi & 0 & 0 \end{bmatrix} \\
U_t &= (u_{\pi t}, u_{yt}, u_{it})' \\
B &= \begin{bmatrix} 1 & 0 & 0 \\ \alpha^i + \lambda^i \phi^i & 1 & \lambda^i \\ \phi^i & 0 & 1 \end{bmatrix}
\end{aligned} \tag{103}$$

To form a forecast of future variables, the fully-informed agent uses:

$$\tilde{\mathbb{E}}_t^{i*} Y_{t+s} = (A^i)^s Y_t \tag{104}$$

That is, their forecasts are optimal given their subjective model.

To find $(A^i)^s$, first find diagonal matrix D^i and matrix P^i such that:

$$A^i = P^i D^i (P^i)^{-1} \tag{105}$$

This is satisfied with:

$$\begin{aligned}
P &= \begin{bmatrix} 0 & (\phi^i)^{-1} & 0 \\ 0 & \dfrac{(\alpha^i + \lambda^i \phi^i)\rho_\pi^i}{\phi^i(\rho_\pi^i - \rho_y^i)} & 1 \\ 1 & 1 & 0 \end{bmatrix} \\
D^i &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & \rho_\pi^i & 0 \\ 0 & 0 & \rho_y^i \end{bmatrix}
\end{aligned} \tag{106}$$

We then have that:

$$
\begin{aligned}
(A^i)^s = P^i(D^i)^s(P^i)^{-1} &= P^i \cdot \begin{bmatrix} 0^s & 0 & 0 \\ 0 & (\rho_\pi^i)^s & 0 \\ 0 & 0 & (\rho_y^i)^s \end{bmatrix} \cdot (P^i)^{-1} \\
&= \begin{bmatrix} (\rho_\pi^i)^s & 0 & 0 \\ \dfrac{(\alpha^i + \lambda^i\phi^i)\rho_\pi^i}{\rho_\pi^i - \rho_y^i}((\rho_\pi^i)^s - (\rho_y^i)^s) & (\rho_y^i)^s & 0 \\ \phi^i(\rho_\pi^i)^s & 0 & 0 \end{bmatrix}
\end{aligned} \tag{107}
$$

This implies equations 25 - 27.

D.3 Proposition 4

As in Maćkowiak and Wiederholt (2009), I solve the rational inattention problem under three further assumptions, as is standard in the rational inattention literature.

Assumption 1: (π_t, s_t^i) has a stationary Gaussian distribution.

Assumption 2: When the household decides on their information strategy in period 0, they receive a long sequence of signals of their chosen form. This implies that $\tilde{\mathbb{E}}_t^i(\pi_t^2|\mathcal{I}_t^i)$ is constant over time.

Assumption 3: In period t, households can only process information about variables realized up to period t. They cannot process any information about realizations of inflation in future periods.[38]

With these assumptions, Maćkowiak and Wiederholt (2009) show that the optimal signal is of the form in equation 33.

Using that signal structure, the utility cost of period-t signal s_t^i is given by:

$$
\begin{aligned}
I(\pi^t; s_t^i|\mathcal{I}_{t-1}^i) \equiv H(\pi_t|s^{t-1,i}) - H(\pi_t|s^{t,i}) &= \frac{1}{2}\log_2\left(\frac{Var(\pi_t|\mathcal{I}_{t-1}^i)}{Var(\pi_t|\mathcal{I}_t^i)}\right) \\
&= \frac{1}{2}\log_2\left(\frac{1}{1-K^i}\right)
\end{aligned} \tag{108}
$$

where the final equality uses standard properties of the Kalman filter.

Assumption 2 further implies that $\tilde{\mathbb{E}}_0^i(\pi_t - \mathbb{E}_t\pi_t)^2$ is constant over time. From the prop-

[38] This ensures that as the cost of information approaches zero the household information set in period t contains realized values of all period t variables, but not realizations of variables in future periods, as in standard full-information models. See Jurado (2021) for a detailed discussion of this assumption.

erties of the Kalman filter:

$$\tilde{\mathbb{E}}_0^i(\pi_t - \mathbb{E}_t\pi_t)^2 = \frac{(1-K^i)\sigma_\pi^2}{1-(\rho_\pi^i)^2(1-K^i)} \tag{109}$$

Using these results, and evaluating the resulting geometric series in the utility losses and costs of information, the household information choice problem reduces to:

$$\min_K \frac{(\bar{C}^i)^{1-\frac{1}{\sigma}}}{2\sigma}\left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right)^2 \frac{(1-K^i)\sigma_\pi^2}{1-(\rho_\pi^i)^2(1-K^i)} + \frac{\psi}{2}\log_2\left(\frac{1}{1-K^i}\right) \tag{110}$$

subject to $K^i \in [0,1]$.

The first order condition for an interior solution is:

$$\frac{1-K^i}{(1-(\rho_\pi^i)^2(1-K^i))^2} = \frac{\psi}{\Gamma^i} \tag{111}$$

where Γ^i is as defined in Proposition 4:

$$\Gamma^i = \frac{(\bar{C}^i)^{1-\frac{1}{\sigma}}}{2\sigma}\sigma_\pi^2 \ln(2)\cdot\left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right)^2 \tag{112}$$

Since $\psi/\Gamma^i > 0$, the K^i implied by this first order condition is always strictly less than 1. However, it remains to find the region where the constraint $K^i \geq 0$ binds. First, note that the interior solution implies $K^i = 0$ when:

$$\Gamma^i = \psi(1-(\rho_\pi^i)^2)^2 \tag{113}$$

Differentiating the left hand side of equation 111 with respect to K^i gives:

$$\frac{\partial}{\partial K^i}\left(\frac{1-K^i}{(1-(\rho_\pi^i)^2(1-K^i))^2}\right) = -\frac{1+(\rho_\pi^i)^2(1-K^i)}{(1-(\rho_\pi^i)^2(1-K^i))^3} \tag{114}$$

The left hand side of equation 111 is therefore strictly decreasing in K^i. The constraint therefore binds, and optimal $K^i = 0$, whenever the right hand side is sufficiently large, that is when:

$$\Gamma^i < \psi(1-(\rho_\pi^i)^2)^2 \tag{115}$$

D.4 Proposition 5

Equation 38 implies:

$$\begin{aligned} &\left.\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right|_{\hat{\alpha}_t^i} < X \\ \iff &\tilde{\mathbb{E}}_t^i \pi_t > \frac{(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)}{\beta(1-\beta)\rho_\pi^i \alpha_1^i}\left(X - \left.\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right|_{\alpha_0^i}\right) \end{aligned} \tag{116}$$

where X is an arbitrary threshold.

That is, household i's consumption response to inflation, after updating their subjective model, is below any threshold X if their perceived inflation is sufficiently high. Using equation 34, and holding priors and realized inflation fixed, the household's perceived inflation is distributed according to:

$$\tilde{\mathbb{E}}_t^i \pi_t \sim N(K^i \pi_t + (1-K^i)\rho_\pi^i \tilde{\mathbb{E}}_{t-1}^i \pi_{t-1}, (K^i)^2 \sigma_{\varepsilon i}^2) \tag{117}$$

Since $K^i \in [0,1]$, this is sufficient to prove the proposition.

D.5 Microfounding subjective model updating

This section provides one way to microfound the process described in Section 4.4, in which households update their subjective models towards the view that inflation erodes real incomes when $\tilde{\mathbb{E}}_t^i \pi_t$ is high. The household faces Knightian uncertainty about the α^i parameter in their subjective model. After observing the realization of s_t^i, the household updates their subjective model to reflect this: following the literature on ambiguity aversion they make decisions using worst-case beliefs (Hansen and Sargent, 2008).

This leads to distorted subjective models in which α^i falls as perceived inflation rises. Intuitively, when perceived inflation is high, the worst case is that high inflation is associated with low real incomes. However, when perceived inflation is lower, the reverse is true. The worst case is then that inflation supports real incomes, and so the ambiguity averse household distorts their subjective model in that direction, with a positive $\hat{\alpha}_t^i$.

Formally, the household selects beliefs and actions as if they are playing a game with an 'evil agent', who distorts α^i to minimize expected utility, while the household simultaneously chooses c_t^i to maximize expected utility. The maximization problem is solved by the consumption function in equation 29 with the updated α^i. To solve the evil agent problem, we then need to find the indirect expected utility when households follow this consumption function.

To find this indirect utility, begin with the expected utility of a household who is fully-informed about inflation each period. To simplify the problem, here I assume that $\sigma \to 1$, so the instantaneous utility from consumption C_t^i is $\log(C_t^i)$. As this is already log-linear, a log-quadratic approximation to this instantaneous utility simply yields $\log(\bar{C}^i) + c_t^i$. The log-quadratic approximation of expected discounted utility, substituting in the consumption function of the informed household (equation 28), is therefore:

$$\tilde{\mathbb{E}}_0^{i*}\hat{U}_0^{i*} = \tilde{\mathbb{E}}_0^{i*}\sum_{t=0}^{\infty}\beta^t\Bigg(\frac{1-\beta}{1-\beta\rho_y^i}y_t - \sigma\beta r_t + \frac{\beta\rho_\pi^i[(1-\beta)(\alpha^i+\lambda^i\phi^i) - \sigma(\phi^i\beta-1)(1-\beta\rho_y^i)]}{(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)}\pi_t\Bigg) \tag{118}$$

Substituting out for expected future inflation, interest rates, and real income using the subjective model (equations 25 - 27) gives indirect utility as a function of current observables and subjective model parameters:

$$\tilde{\mathbb{E}}_0^{i*}\hat{U}_0^{i*} = \frac{1-\beta}{(1-\beta\rho_y^i)^2}y_0 - \sigma\beta r_0 + \frac{1}{1-\beta\rho_\pi^i}\left(\frac{\beta\rho_\pi^i(\alpha^i+\lambda^i\phi^i)}{1-\beta\rho_y^i} - \sigma\beta^2\phi^i\rho_\pi^i + \frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}\right)\pi_0 \tag{119}$$

Finally, use the expression for the expected utility loss from limited information (equation 30) to find the expected indirect utility of the potentially uninformed household:

$$\tilde{\mathbb{E}}_0^{i}\hat{U}_0^{i} = \frac{1-\beta}{(1-\beta\rho_y^i)^2}y_0 - \sigma\beta r_0 + \frac{1}{1-\beta\rho_\pi^i}\left(\frac{\beta\rho_\pi^i(\alpha^i+\lambda^i\phi^i)}{1-\beta\rho_y^i} - \sigma\beta^2\phi^i\rho_\pi^i + \frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}\right)\tilde{\mathbb{E}}_0^i\pi_0 - \frac{\log(\bar{C}^i)}{2(1-\beta)}\left(\frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}\right)^2\frac{(1-K^i)\sigma_\pi^2}{1-(\rho_\pi^i)^2(1-K^i)} \tag{120}$$

where I have used that the expected variance of inflation perception gaps is constant at:

$$\tilde{\mathbb{E}}_0^i(\pi_t - \mathbb{E}_t\pi_t)^2 = \frac{(1-K^i)\sigma_\pi^2}{1-(\rho_\pi^i)^2(1-K^i)} \tag{121}$$

Differentiating the expected indirect utility with respect to α^i gives:

$$\frac{\partial\tilde{\mathbb{E}}_0^i\hat{U}_0^i}{\partial\alpha^i} = \frac{\beta(2-\beta)\rho_\pi^i}{(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)}\tilde{\mathbb{E}}_0^i\pi_0 - \frac{\beta\rho_\pi^i\log(\bar{C}^i)(1-K^i)\sigma_\pi^2}{(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)(1-(\rho_\pi^i)^2(1-K^i))}\cdot\frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t} \tag{122}$$

Expected indirect utility is therefore increasing in α^i if:

$$\tilde{\mathbb{E}}_0^i \pi_0 > \frac{\log(\bar{C}^i)(1-K^i)\sigma_\pi^2}{(2-\beta)(1-(\rho_\pi^i)^2(1-K^i))} \cdot \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \tag{123}$$

That is, expected indirect utility is increasing in α^i if and only if perceived inflation is sufficiently high, and otherwise it is decreasing.[39] Whether the worst-case distortions to α^i are restricted to a set $\hat{\alpha}_t^i \in [\alpha_0^i - \alpha^*, \alpha_0^i + \alpha^*]$, or the evil agent can pay a convex cost to distorting $\hat{\alpha}_t^i$ away from α_0^i, the solution will therefore have the distorted $\hat{\alpha}_t^i$ falling as $\tilde{\mathbb{E}}_t^i \pi_t$ rises, as in equation 37.

D.6 Selection and amplification

It is shown in Section 5.1 that the difference between the elasticity of aggregate consumption to inflation with heterogeneous and homogeneous information is given by:

$$\frac{\partial \bar{c}_t}{\partial \pi_t} - \left.\frac{\partial \bar{c}_t}{\partial \pi_t}\right|_{K^i=\bar{K}} = (1-\bar{\Theta})^{-1} Cov_H\left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}, K^i\right) \tag{124}$$

The selection effect therefore amplifies aggregate transmission of inflation shocks if this covariance has the same sign as $\partial \bar{c}_t / \partial \pi_t$, and is less than twice its magnitude. While this will be true for most distributions of subjective models, as discussed above, it is possible to construct counter-examples where the selection effect weakens the transmission of inflation shocks, so gives a larger role to information frictions in aggregate outcomes.

For example, consider the case where the distributions of $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$ with positive and negative reactions to realized inflation are exact mirror images of one another. Denoting $\phi(\cdot)$ as the PDF of $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$:

$$\phi\left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right) = \phi\left(-\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right) \quad \text{for all } \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \text{ such that } K^i > 0 \tag{125}$$

That is, among the households paying positive amounts of attention to inflation, the distributions of $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$ above and below 0 cancel each other out. This means that the

[39] Note that since K^i is decided before any distortion to α^i, it is also not a function of expected inflation. Everything on the right hand side of condition 123 is a function of underlying parameters and the parameters of the subjective model only.

partial-equilibrium response of aggregate consumption is 0:

$$\begin{aligned}\frac{\partial \bar{c}_t}{\partial \pi_t} &= (1-\bar{\Theta})^{-1}\left[\int_0^{P_0/2} \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} K^i di + \int_{P_0/2}^{P_0} \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} K^i di\right] \\ &= (1-\bar{\Theta})^{-1}\left[\int_0^{P_0/2} \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} K^i di - \int_0^{P_0/2} \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} K^i di\right] = 0\end{aligned} \tag{126}$$

The average Kalman gain K^i is however positive, and so the response of aggregate consumption in the homogeneous-K^i model is:

$$\begin{aligned}\left.\frac{\partial \bar{c}_t}{\partial \pi_t}\right|_{K^i=\bar{K}} &= (1-\bar{\Theta})^{-1}\bar{K}\left[\int_0^{P_0} \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} di + \int_{P_0}^{1} \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} di\right] \\ &= (1-\bar{\Theta})^{-1}\bar{K}\int_{P_0}^{1} \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} di\end{aligned} \tag{127}$$

and the full-information response is:

$$\begin{aligned}\left.\frac{\partial \bar{c}_t}{\partial \pi_t}\right|_{K^i=1} &= (1-\bar{\Theta})^{-1}\left[\int_0^{P_0} \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} di + \int_{P_0}^{1} \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} di\right] \\ &= (1-\bar{\Theta})^{-1}\int_{P_0}^{1} \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} di\end{aligned} \tag{128}$$

If the average $\partial c_t^i/\partial \tilde{\mathbb{E}}_t^i \pi_t$ among the households paying no attention is non-zero, then the aggregate consumption response with a homogeneous-K^i will be non-zero, and closer to the full-information benchmark. This is because the link between the sign of $\partial c_t^i/\partial \tilde{\mathbb{E}}_t^i \pi_t$ and the covariance in equation 124 breaks down at $K^i = 0$.

D.7 Consumption function with time-varying long-run inflation

Changing the subjective model of inflation does not change anything about the model before the initial consumption function of a fully informed household (equation 21).[40]

However, the change in subjective model to include long-run inflation $\bar{\pi}_t$ does affect how we evaluate the expectation terms. Specifically, the subjective model in VAR(1) form is now:

$$Y_t = A^i Y_{t-1} + B^i U_t \tag{129}$$

[40] Note we assume this fully-informed household observes $\bar{\pi}_t$ as well as π_t.

where:

$$
\begin{aligned}
Y_t &= (\pi_t, y_t, i_t, \bar{\pi}_t)' \\
A &= \begin{bmatrix} \rho_\pi^i & 0 & 0 & 1-\rho_\pi^i \\ (\alpha^i+\lambda^i\phi^i)\rho_\pi^i & \rho_y^i & 0 & (\alpha^i+\lambda^i\phi^i)(1-\rho_\pi^i) \\ \phi\rho_\pi & 0 & 0 & \phi^i(1-\rho_\pi^i) \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
U_t &= (u_{\pi t}, u_{yt}, u_{it}, v_t)' \\
B &= \begin{bmatrix} 1 & 0 & 0 & 1-\rho_\pi^i \\ \alpha^i+\lambda^i\phi^i & 1 & \lambda^i & (\alpha^i+\lambda^i\phi^i)(1-\rho_\pi^i) \\ \phi^i & 0 & 1 & \phi^i(1-\rho_\pi^i) \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
\tag{130}
$$

This is the same for the case where $\bar{\pi}_t$ is a random walk and where it is assumed to be constant. In the latter case, simply set $\sigma_v^2 = 0$.

To form a forecast of future variables, the fully-informed agent uses:

$$\tilde{\mathbb{E}}_t^{i*} Y_{t+s} = (A^i)^s Y_t \tag{131}$$

To find $(A^i)^s$, first find diagonal matrix D^i and matrix P^i such that:

$$A^i = P^i D^i (P^i)^{-1} \tag{132}$$

This is satisfied with:

$$
\begin{aligned}
P &= \begin{bmatrix} 1 & 0 & (\phi^i)^{-1} & 0 \\ \dfrac{\alpha^i+\lambda^i\phi^i}{1-\rho_y^i} & 0 & \dfrac{(\alpha^i+\lambda^i\phi^i)\rho_\pi^i}{\phi^i(\rho_\pi^i-\rho_y^i)} & 1 \\ \phi^i & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
D^i &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_\pi^i & 0 \\ 0 & 0 & 0 & \rho_y^i \end{bmatrix}
\end{aligned}
\tag{133}
$$

We then have that:

$$(A^i)^s = \begin{bmatrix} (\rho^i_\pi)^s & 0 & 0 & 1-(\rho^i_\pi)^s \\ \dfrac{(\alpha^i+\lambda^i\phi^i)\rho^i_\pi}{\rho^i_\pi-\rho^i_y}((\rho^i_\pi)^s-(\rho^i_y)^s) & (\rho^i_y)^s & 0 & \Lambda^i(s) \\ \phi^i(\rho^i_\pi)^s & 0 & 0 & \phi^i(1-(\rho^i_\pi)^s) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{134}$$

where

$$\Lambda^i(s) = \frac{(\alpha^i+\lambda^i\phi^i)\big(\rho^i_\pi-\rho^i_y-(\rho^i_\pi)^{s+1}(1-\rho^i_y)+(\rho^i_y)^{s+1}(1-\rho^i_\pi)\big)}{(\rho^i_\pi-\rho^i_y)(1-\rho^i_y)} \tag{135}$$

Using this to evaluate the infinite sums in the consumption function (21) gives:

$$\begin{aligned} c^{i*}_t = \frac{1-\beta}{1-\beta\rho^i_y}y_t - \sigma\beta r_t + \frac{\beta\rho^i_\pi[(1-\beta)(\alpha^i+\lambda^i\phi^i)-\sigma(\phi^i\beta-1)(1-\beta\rho^i_y)]}{(1-\beta\rho^i_\pi)(1-\beta\rho^i_y)}\pi_t \\ + \frac{\beta(1-\rho^i_\pi)[(1-\beta)(\alpha^i+\lambda^i\phi^i)-\sigma(\phi^i\beta-1)(1-\beta\rho^i_y)]}{(1-\beta)(1-\beta\rho^i_\pi)(1-\beta\rho^i_y)}\bar{\pi}_t \end{aligned} \tag{136}$$

The consumption function of an uninformed household, who believes $\bar{\pi}_t = \tilde{\mathbb{E}}^i_{t-1}\bar{\pi}_t$ for certain, is therefore:

$$\begin{aligned} c^{i}_t = \frac{1-\beta}{1-\beta\rho^i_y}y_t - \sigma\beta r_t + \frac{\beta\rho^i_\pi[(1-\beta)(\alpha^i+\lambda^i\phi^i)-\sigma(\phi^i\beta-1)(1-\beta\rho^i_y)]}{(1-\beta\rho^i_\pi)(1-\beta\rho^i_y)}\tilde{\mathbb{E}}^i_t\pi_t \\ + \frac{\beta(1-\rho^i_\pi)[(1-\beta)(\alpha^i+\lambda^i\phi^i)-\sigma(\phi^i\beta-1)(1-\beta\rho^i_y)]}{(1-\beta)(1-\beta\rho^i_\pi)(1-\beta\rho^i_y)}\tilde{\mathbb{E}}^i_{t-1}\bar{\pi}_t \end{aligned} \tag{137}$$

Simplifying the final two terms, we obtain equation 54.

D.8 Proof of Proposition 6

First, define $\tilde{K}^i_t$ as the Kalman gain the household expects to use when they make their information decision (that is, assuming no updating of $\bar{\pi}_t$ beliefs). From Proposition 4 we have:

$$\begin{cases} \tilde{K}^i_t = 0 & \text{if } \Gamma^i_t < \psi(1-(\rho^i_\pi)^2)^2 \\ \dfrac{1-\tilde{K}^i_t}{(1-(\rho^i_\pi)^2(1-\tilde{K}^i_t))^2} = \dfrac{\psi}{\Gamma^i_t} & \text{if } \Gamma^i_t \geq \psi(1-(\rho^i_\pi)^2)^2 \end{cases} \tag{138}$$

where:

$$\Gamma_t^i = \frac{(\bar{C}^i)^{1-\frac{1}{\sigma}}}{2\sigma}\sigma_\pi^2 \ln(2) \cdot \left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\bigg|_{\alpha_t^{i,prior}} \right)^2$$
$$= \frac{(\bar{C}^i)^{1-\frac{1}{\sigma}}}{2\sigma}\sigma_\pi^2 \ln(2) \cdot \left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\bigg|_{\alpha_0^i} - \Omega^i \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t \right)^2 \tag{139}$$

Among those with $\Gamma_t^i \geq \psi(1-(\rho_\pi^i)^2)^2$, we have that:

$$\frac{\partial \tilde{K}_t^i}{\partial \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t} = \frac{\psi(1-(\rho_\pi^i)^2(1-\tilde{K}_t^i))^3}{(\Gamma_t^i)^2(1+(\rho_\pi^i)^2(1-\tilde{K}_t^i))} \frac{\partial \Gamma_t^i}{\partial \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t} \tag{140}$$

where:

$$\frac{\partial \Gamma_t^i}{\partial \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t} = -\frac{\Omega^i(\bar{C}^i)^{1-\frac{1}{\sigma}}}{\sigma}\sigma_\pi^2 \ln(2) \cdot \left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\bigg|_{\alpha_t^{i,prior}} \right)$$
$$> 0 \text{ if and only if } \left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\bigg|_{\alpha_t^{i,prior}} \right) < 0 \tag{141}$$

Since ρ_π^i and $\tilde{K}_t^i$ are both $\in [0,1]$, the coefficient in front of $\partial\Gamma_t^i/\partial\tilde{\mathbb{E}}_{t-1}^i\bar{\pi}_t$ in equation 140 is always positive. This proves that, for households with $\tilde{K}_t^i > 0$, and so $\sigma_{\varepsilon it}^{2*} < \infty$, $\tilde{K}_t^i$ strictly increases in $\tilde{\mathbb{E}}_{t-1}^i\bar{\pi}_t$ if and only if $\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\Big|_{\alpha_t^{i,prior}} < 0$. The statement in equation 64 then follows from the inverse relationship between $\sigma_{\varepsilon it}^{2*}$ and $\tilde{K}_t^i$, from the standard properties of the steady state Kalman filter. Those with $\Gamma_t^i \geq \psi(1-(\rho_\pi^i)^2)^2 < 0$ do not change attention with marginal changes in $\tilde{\mathbb{E}}_{t-1}^i\bar{\pi}_t$.

Second, we turn to the actual Kalman gains employed by the household. Define Σ as the steady state variance-covariance matrix of ξ_t conditional on the information set in period $t-1$. From the standard properties of the steady state Kalman filter:

$$\Sigma = F(\Sigma - \Sigma C(C'\Sigma C + \sigma_{\varepsilon it}^2)^{-1}C'\Sigma)F' + Q \tag{142}$$

The Kalman gain vector is then given by:

$$K_t^i = \Sigma C(C'\Sigma C + \sigma_{\varepsilon it}^2)^{-1} \tag{143}$$

The statement in equation 65 then follows from equation 64 and the fact that the elements of the Kalman gain vector grow as signal precision improves ($\partial K_t^i/\partial\sigma_{\varepsilon it}^2 < 0$).

D.9 Empirical test of Proposition 6

Proposition 6 states that information processing is increasing in perceived long-run inflation if and only if the household's subjective model has $\alpha_t^{i,prior} < 0$: that is if they start the period believing that inflation erodes real income. Since higher perceived current π_t imply higher perceived $\bar{\pi}_t$, information processing should be increasing in perceived inflation among this group. For households starting with positive models of inflation ($\alpha_t^{i,prior} > 0$), higher perceived π_t implies higher perceived $\bar{\pi}_t$, which implies less information processing. Within this group lower perceived inflation should therefore be associated with more information.

To test this, I regress perceived and expected inflation on the information indicator described in Section 3.1. For each dependent variable, I first run the regression for the households who report negative subjective models of inflation in response to Question 1, corresponding to those with $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t < 0$. I then repeat the regression for those reporting non-negative subjective models.[41]

Table 12: Information is associated with higher perceived and expected inflation among those with negative subjective models.

	(1) Perceived	(2) Perceived	(3) Expected	(4) Expected
Information	0.226**	-0.122	0.311***	-0.0109
	(0.102)	(0.138)	(0.0990)	(0.119)
Subjective Model	Negative	Non-negative	Negative	Non-negative
Controls	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
R-squared	0.111	0.127	0.111	0.115
Observations	5114	2787	5298	2923

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the results of regressing perceived and expected inflation on the information indicator, split by responses to Question 1. The first and third columns are the results using those who answer that inflation would make the economy weaker, and the second and fourth columns use all other respondents. All regressions are weighted using the survey weights provided in the IAS.

The results are in Table 12. Within the group with negative subjective models of inflation, both perceived and expected inflation are significantly higher among those obtaining direct information about inflation. This relationship turns negative among those with other

[41] As the information indicator is not observed every quarter there are too few observations to draw conclusions from regressions on each non-negative subjective model option individually. This is also the reason for not using the longer-horizon expectations in the IAS: the sample giving answers to both this and the information questions is small.

subjective models, though this is not significant. These results are therefore in line with Proposition 6, and the model in Section 6.

E Parameters for figures in Sections 4 - 6

Figures 3 and 4:

All parameters as in Table 13, except α_0^i distributed such that $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$ is in the range $[-1, 1]$, and ψ set at 0.2×10^{-3}. This scales all attention so that the change in attention with subjective models is clear in the figures.

Figure 5:

The calibrated parameters are set out in Table 13. β and σ are set to standard values, and ϕ is set such that the Taylor principle is just satisfied, as found on average for the UK by Lee et al. (2013). For the remaining parameters of subjective models, including the mean of the α_0^i distribution, I estimate equations 22-24 using OLS on UK data from 1993-2019. The longer sample than the survey data is to allow for more precise estimation of model parameters. It is not extended further back because of the structural break in many UK macroeconomic time series at the end of 1992 identified by Benati (2006).

For the inflation data, I take the log first difference of quarterly CPI (ONS series MM23). I de-mean and remove seasonal variation by regressing the series on quarter-of-the-year dummies, and taking the residual as my quarterly inflation series. As well as being used in the calibration, this series is used to generate the simulated paths for perceived inflation and the aggregate consumption elasticity to inflation.

The interest rate data is 3-month money market rates, taken from the OECD Main Economic Indicators. To be consistent with the model equations, I transform this annualized rate into a gross quarterly interest rate, then take logs and de-mean. Following Harrison and Oomen (2010), I allow the mean interest rate to vary with changes in the broad regime of UK monetary policy, which I take to occur in 2009Q1 as interest rates hit the ZLB.

I proxy real income with real wages, since the model is approximated around a steady state with no saving. I begin by summing ONS series ROYH, ROYK, and ROYJ to obtain a measure of total nominal wages. I then divide this by total hours (ONS series YBUS) and working age population (ONS series MGSL) to obtain nominal wages per worker per hour. Finally, I divide by the level of CPI (including the seasonal adjustment carried out in the computation of inflation) to obtain real wages. I then take logs and hp-filter the series to obtain the cyclical component. This is estimated to be reasonably persistent, ($\rho_w = 0.731$), but still this implies a very small amplification from real income changes: $(1 - \bar{\Theta})^{-1} = 1.04$.

For $(\sigma(\alpha_0^i), \alpha_1^i, \psi)$ I target three moments from the IAS data. The first is the average ratio of 'weaker' to 'stronger' answers in response to Question 1. The raw proportions are inappropriate since we do not know how far either side of a true $dc_t^i/d\tilde{\mathbb{E}}_t^i\pi_t = 0$ is considered 'little difference' by the respondents, but the ratio still gives the balance between negative and positive models of the economy. That ratio is on average 7.533.

The second target is the estimated elasticity of the proportion with negative models to inflation, that is the coefficient from regressing Pr('weaker') on current inflation and a constant. That elasticity is 0.090.

Finally, the third target is an estimate of the average Kalman gain across the population, which helps to identify the information cost parameter ψ. For this, take Equation 34 and average across households to give:

$$\mathbb{E}_H\big(\tilde{\mathbb{E}}_t^i\pi_t\big) = \mathbb{E}_H\big(K^i\big)\pi_t + \big(1 - \mathbb{E}_H\big(K^i\big)\big)\rho_\pi\mathbb{E}_H\big(\tilde{\mathbb{E}}_{t-1}^i\pi_{t-1}\big) \tag{144}$$

where I have used the fact that all households are calibrated to have the same ρ_π, and in the model information, and so K^i, is decided before the households update their subjective models, and so is independent of perceived inflation. Denoting $\bar{\mathbb{E}}_t\pi_t$ as the average perceived inflation in time t, I therefore estimate:

$$\bar{\mathbb{E}}_t\pi_t = \gamma_1\pi_t + \gamma_2\bar{\mathbb{E}}_{t-1}\pi_{t-1} \tag{145}$$

by OLS, restricting $\gamma_2 = \rho_\pi(1-\gamma_1)$, where ρ_π is as in Table 13. The estimated γ_1 therefore gives an estimate of the average Kalman gain across the population. This target is 0.448.

Table 13: Calibration

Parameter	Value	Source	Parameter	Value	Source
β	0.99	standard	σ_π	0.003	estimated model
σ	1	standard	σ_i	0.004	estimated model
ϕ	β^{-1}	Lee et al. (2013)	σ_y	0.008	estimated model
$\mathbb{E}_H\alpha_0^i$	-0.732	estimated model	$\sigma(\alpha_0^i)$	0.613	targets
λ	-0.037	estimated model	α_1^i	-234	targets
ρ_π	0.329	estimated model	ψ	0.787×10^{-9}	targets
ρ_y	0.731	estimated model			

Figures 6 and 7:

All shared parameters are as in Table 13, except for ψ, which is set to 0.453×10^{-9} to ensure that average K^i_{1t} remains equal to the target level from the survey (0.448) in the period before the shock. For Figure 6, the high-α group have $\alpha^i_0 = 0.997$, while the low-α group have $\alpha^i_0 = -0.923$. These are chosen such that both households have the same initial $K^i_{1t} = 0.7$. The variance of v_t in equation 59 is set at $\sigma^2_v = \sigma^2_\pi/10$, and the reset shock probability is set at 0.005.

To simulate these figures, optimal attention is derived using equation 138. The variance of noise in the signals is then given by:

$$\sigma^2_{\varepsilon it} = \frac{\sigma^2_\pi(1-\tilde{K}^i_t)}{\tilde{K}^i_t(1-(\rho^i_\pi)^2(1-\tilde{K}^i_t))} \tag{146}$$

Plugging this into equations 142 and 143 for each household each period gives the Kalman gain vector, to be used to simulate the path of each household's expectations.

F Relaxing anticipated utility in Section 6

In this section I relax the assumption that households make information choices assuming $\bar{\pi}_t$ will remain constant at $\partial\tilde{\mathbb{E}}^i_{t-1}\bar{\pi}_t$ for certain. Instead, they know that $\bar{\pi}_t$ follows the persistent process:

$$\bar{\pi}_t = \bar{\rho}\bar{\pi}_{t-1} + v_t, \quad v_t \sim N(0, \sigma^2_v) \tag{147}$$

where $\bar{\rho}$ is close to but strictly less than 1. That is, I now assume that $\bar{\pi}_t$ is very persistent but stationary. This ensures that it is possible for households to pay no attention to inflation, without their utility losses from inattention becoming infinite. Note that this also implies that zero attention is no longer an absorbing state, so there is no need for the reset shocks used in Section 6.

Repeating the steps in Appendix D.7, the consumption function becomes:

$$c^i_t = \frac{1-\beta}{1-\beta\rho^i_y} y_t - \sigma\beta r_t + \frac{\partial c^i_t}{\partial\tilde{\mathbb{E}}^i_t\pi_t}\left(\tilde{\mathbb{E}}^i_t\pi_t + \frac{(1-\rho^i_\pi)\bar{\rho}}{\rho^i_\pi(1-\beta\bar{\rho})}\tilde{\mathbb{E}}^i_t\bar{\pi}_t\right) \tag{148}$$

For simplicity, I restrict households to obtaining signals of the same form as in the model without $\bar{\pi}_t$:

$$s^i_t = \pi_t + \varepsilon^i_t \quad \varepsilon^i_t \sim N(0, \sigma^2_{\varepsilon it}) \tag{149}$$

In Sections 4 and 5, this was the optimal signal structure chosen endogenously by households. This is no longer the case here. First, without this restriction households would also acquire information about $\bar{\pi}_t$ directly. Forcing households to estimate long-run inflation from realized inflation is in line with the approach taken by the literature on inflation forecasting (Stock and Watson, 2007). As $\bar{\pi}_t$ is a latent variable that cannot be observed directly in the data, it is plausible that households cannot obtain direct signals about it, but must infer it from observing other variables.

Second, π_t no longer follows an AR(1) process, so unrestricted households would not choose the simple Gaussian signal over current π_t only.[42] Restricting households to the simple signal form in equation 149 is a common way to simplify rational inattention problems (e.g. Lei, 2019).

In state-space form, the subjective model is:

$$\xi_t = F^i \xi_{t-1} + e_t^i \tag{150}$$

$$s_t^i = C' \xi_t + \varepsilon_t^i \tag{151}$$

where:

$$\xi_t = \begin{pmatrix} \pi_t \\ \bar{\pi}_t \end{pmatrix}, \quad F^i = \begin{pmatrix} \rho_\pi^i & (1-\rho_\pi^i)\bar{\rho} \\ 0 & \bar{\rho} \end{pmatrix}, \quad e_t^i = \begin{pmatrix} u_{\pi t} + (1-\rho_\pi^i) v_t \\ v_t \end{pmatrix}, \quad C = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{152}$$

It therefore remains optimal for households to incorporate signals into their perceptions of π_t and $\bar{\pi}_t$ using the Kalman filter:

$$\tilde{\mathbb{E}}_t^i \xi_t = (I - K_t^i C') F^i \tilde{\mathbb{E}}_{t-1}^i \xi_{t-1} + K_t^i s_t^i \tag{153}$$

where K_t^i is a 2×1 vector of gain parameters.

The household's attention problem is to choose the noise in their signals $\sigma_{\varepsilon it}^2$ to minimize expected utility losses from limited information plus information costs, as in Section 4.3. Formally, define Σ_0 and Σ_1 as the steady state variance-covariance matrices of ξ_t conditional on the information sets in period t and $t-1$ respectively.

The per-period expected utility loss from limited information in steady state is given by:

$$\frac{(\bar{C}^i)^{1-\frac{1}{\sigma}}}{2\sigma} \left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \right)^2 (\zeta' \Sigma_0 \zeta) \tag{154}$$

[42] It can be shown that π_t follows an ARMA(2,1) process. Even without the incentives to forecast $\bar{\pi}_t$ accurately, the optimal signal in period t would therefore also contain information on π_{t-1} and the current shock realization, as these help to forecast π_{t+1} (Maćkowiak et al., 2018).

where:

$$\zeta = \left(1, \frac{(1-\rho_\pi^i)\bar{\rho}}{\rho_\pi^i(1-\beta\bar{\rho})}\right)' \tag{155}$$

Following Maćkowiak et al. (2018), the attention problem can therefore be written:

$$\min_{\sigma^2_{\varepsilon it}} \frac{(\bar{C}^i)^{1-\frac{1}{\sigma}}}{2\sigma}\left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right)^2 \zeta'\Sigma_0\zeta + \frac{\psi}{2}\log_2\left(\frac{C'\Sigma_1 C}{\sigma^2_{\varepsilon it}} + 1\right) \tag{156}$$

Where in the steady state Kalman filter, Σ_1 and Σ_0 are defined by:

$$\Sigma_1 = F(\Sigma_1 - \Sigma_1 C(C'\Sigma_1 C + \sigma^2_{\varepsilon it})^{-1}C'\Sigma_1)F' + Q \tag{157}$$

$$\Sigma_0 = \Sigma_1 - \Sigma_1 C(C'\Sigma_1 C + \sigma^2_{\varepsilon it})^{-1}C'\Sigma_1 \tag{158}$$

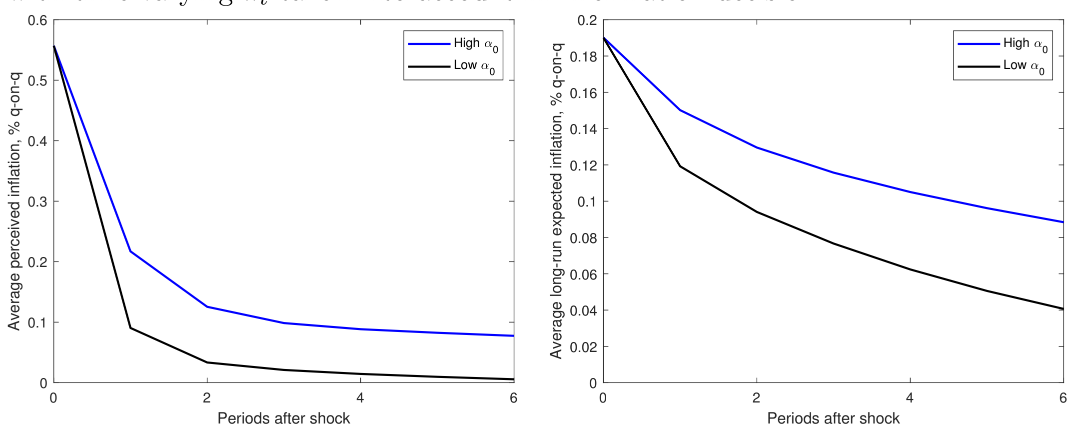
And the Kalman gain vector is:

$$K_t^i = \Sigma_1 C(C'\Sigma_1 C + \sigma^2_{\varepsilon it})^{-1} \tag{159}$$

Note that I am maintaining the assumption here that households immediately use the steady state Kalman filter each period, even though their attention is potentially changing. This is an approximation to maintain tractability, and is related to a remaining aspect of the anticipated utility assumption. Households do not expect their subjective model to change in the future, so do not expect their information processing decisions to change, even though they account for changing $\bar{\pi}_t$ in their decisions.

In Figure 8 I repeat the exercise of Figure 6 above, using $\bar{\rho} = 0.99$, and adjusting ψ to 0.485×10^{-5} to ensure average $K_{1t}^i = 0.448$ before the shock hits. All other parameters are the same. The core mechanism from Section 6 remains: after the shock, low-α_0 households increase attention, and so quickly learn that inflation has fallen. High-α_0 households reduce attention, and so their perceived current and long-run inflation fall much more slowly.

Figure 8: Simulated average $\tilde{\mathbb{E}}_t^i \pi_t$ and $\tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t$ for two household groups after an i.i.d. inflation shock, with time-varying $\bar{\pi}_t$ taken into account in information decision.



(a) Average $\tilde{\mathbb{E}}_t^i \pi_t$ (b) Average $\tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t$