Advanced comprehensive examination in applied mathematics

Andrew MacFie

2017.Nov.2

Carleton University, Ottawa-Carleton Institute of Mathematics and Statistics

My background

Experience prior to PhD program

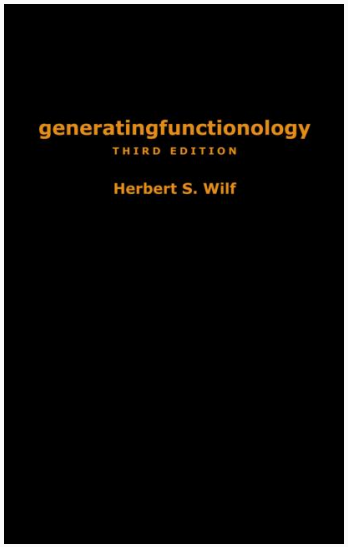
Counting words by number of occurrences of some patterns (with Zhicheng Gao and Daniel Panario)
Electronic Journal of Combinatorics, 2011

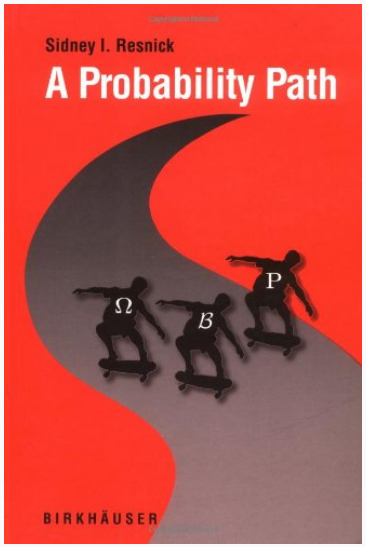
Random mappings with restricted preimages (with Daniel Panario)
LATINCRYPT 2012

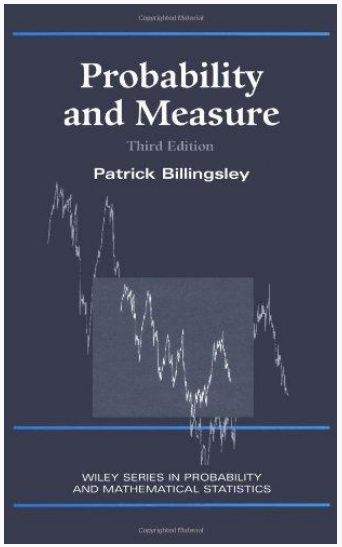
Software for enumerative and analytic combinatorics
Honours project (2013), supervised by Daniel Panario and Brett Stevens

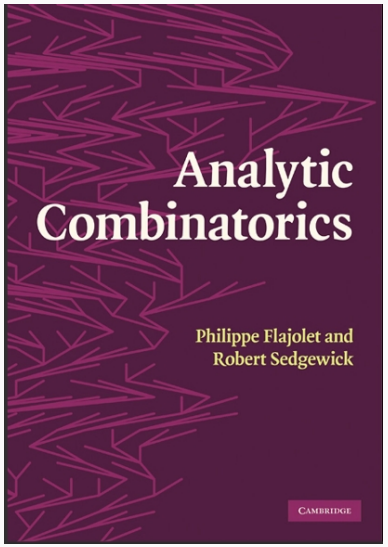
Software for combinatorial power series
MSc thesis (2014), supervised by Daniel Panario

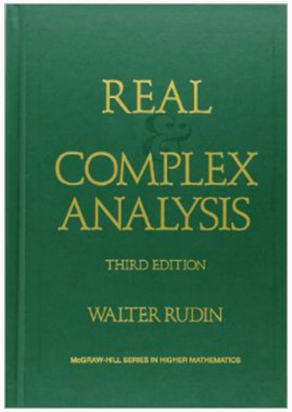
Training (during and prior to PhD program)

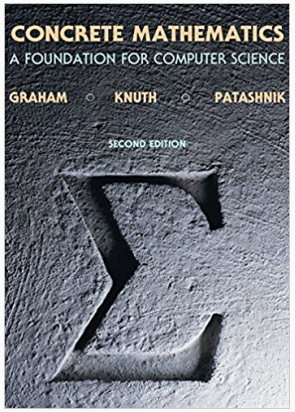


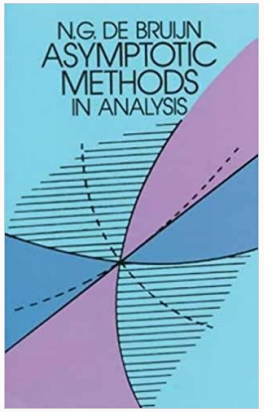












PhD program experience to date (research)

Analysis of algorithms and partial algorithms
AGI 2016

Plausibility and probability in deductive reasoning
public draft, 2017

Approximation algorithms and conditional expectation [working title] (with Boaz Barak)
public draft, 2017

Counting restricted compositions over finite abelian groups (with Zhicheng Gao and Qiang Wang)
submitted (2017)

PhD program experience to date (external programs)

Summer fellowship, Machine Intelligence Research Institute, Berkeley, CA (2016)

Winter school on sum-of-squares algorithm, University of California, San Diego (2017)

Summer school on random graphs and probabilistic methods, Fields Institute (2017)

Thesis

Analytic combinatorics

combinatorial objects $\rightarrow$ generating functions $\rightarrow$ asymptotics

objects: words, integers compositions, integer partitions, mappings, trees, set partitions, permutations, graphs, . . .

counting: symbolic method, inclusion-exclusion, recurrences, refinements and decompositions, Pólya theory

asymptotics: Taylor and Puiseux expansions, Euler-Maclaurin summation, singularity analysis, saddle-point method, bootstrapping

multivariate asymptotics: quasi-powers theorem (basin of attraction around standard normal), discrete-to-discrete limits

Compositions avoiding subsequence patterns

Reduction of word: $red(22454) = 11232$. Subsequence pattern occurrence is subsequence w/ same reduction as pattern, e.g. pattern 122 in $\underline{47}34\underline{7}\underline{2}$.

Integer compositions

Previous: GF for avoiding 3-letter permutation patterns given in [SW06] based on alphabet vector GF in [ALW95]. Recurrences for 3-letter word pattern avoidance in [HM06]. Some partially-ordered-pattern (POP) avoidance in [HKM06].

Unsolved patterns include POP $1'1'' \cdots 1^{(p)}2^m$. (Equal letters with different number of primes are incomparable.)

Compositions over $\mathbb{Z}_k$

No previous work in subsequence pattern avoidance. Main idea: Restrict parts to $[0..k-1]$ and use multisection formula.

Transfer matrix simplification

Weak equivalence of DFAs: DFAs accept same number of strings of each length. Weak minimization algorithm (RE algorithm) given in [RE04].

Idea: Use RE algorithm to extract simpler recurrences from transfer matrices rather than to get faster computation. Apply as a technique for infinite families of transfer matrices. Much easier than computing symbolic power of matrix, even for non-symbolic matrices.

Examples: Locally Mullen compositions (compositions over $\mathbb{Z}_k$ s.t. local subsums are nonzero), subword patterns in compositions over $\mathbb{Z}_k$

Circular and cyclic locally-restricted words and compositions

Cyclic: local restriction wraps around from end to beginning

Includes "Research Direction 3.3" in [HM10], which refers only to Carlitz compositions. Main idea: count closed walks using modified transfer matrix method.

Circular: cyclic and counting only orbits under circular shift

To count orbits, we can use Moebius inversion. Main idea: if u satisfies cyclic local restriction, so does uu; this gives simple structure for orbits. Gotcha: must handle orbits under cyclic shifts smaller than width of one vertex.

Carlitz compositions (special case) done in [Had17].

Multivariate local limit quasi-powers theorem

Generalize Theorem IX.14 in [FS09] to multivariate generating functions beyond bivariate.

Sketch of above theorem: Let X_n be sequence of discrete RVs with PGFs $p_n(u)$, means μ_n, and variances σ_n. Assume

$$p_n(u) = A(u)B(u)^{\beta_n}\left(1 + O\left(\frac{1}{\kappa_n}\right)\right)$$

uniformly for u in a neighborhood $\Omega \subseteq \mathbb{C}$ of 1. Assume $|p_n(u)| \leq K^{-\beta_n}$ for some $K > 1$ and all $u \in D(0,1) \cap \mathbb{C} \setminus \Omega$. Then

$$\sigma_n p_{n,\lfloor \mu_n + x\sigma_n \rfloor} \to \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

uniformly for $x \in \mathbb{R}$.

Other problems 1

Count words avoiding pattern(s) with given longest run

Subword patterns dealt with in [BG16].

Circular and cyclic words and compositions avoiding subsequence patterns

No longer have property that if u avoids pattern so does uu.

Parameters of restricted mappings

For random finite function, $\boldsymbol{T}$ is lcm of cycle lengths (in functional graph). Find limiting distribution of (normalization of) $\log \boldsymbol{T}$ where mappings are restricted by a set of allowed indegrees. Indegree set $\{0, k(n)\}$ dealt with in [MPQS17].

Other problems 2

Largest part in compositional structure

Let $P(z)$ be a GF of *parts* and let $S(z)$ be a GF of *supports*. Then $S(P(z))$ is the GF of the relevant *compositional structures*. The composition $S(P(z))$ is *supercritical* if dominant singularity comes from S, *subcritical* if comes from P and *critical* if S and P are singular simultaneously.

Obtain distribution of largest part in critical and subcritical compositional classes. This is an open problem mentioned in [BG14] which deals with the supercritical case (which is probably more common than critical and subcritical).