DSA March 2022

Mathematics for Data Science: assignment

Due date : **13/03/2022**

Please try to send a **clean** version of your work with **clear** and **detailed** answers: no one wants to grade something that looks like a draft and remember that giving just calculations (or a "yes/no" answer) is, most of the time, not enough.

Also, remember this a personal work.

PART I Linear algebra (6.5 points)

a) Do you think that { $\begin{pmatrix}2\\3\end{pmatrix},\begin{pmatrix}-1\\0\end{pmatrix},\begin{pmatrix}0\\3\end{pmatrix}$ } is a basis of $\mathbb{R}^2$? **[0.5 point]**

b) What are the eigenvalues and eigenvectors of the matrix A = $\begin{pmatrix}3 & -2\\-2 & 3\end{pmatrix}$? **[2 points]**

c) Is the matrix A diagonalizable and why? If it is, what is a related diagonal matrix looks like? (reminder : there are several possible diagonal matrices if A is diagonalizable) **[2 points]**

d) Calculate A^3 ? **[1 point]**

e) Based on d), what is A^2 ? **[1 point]**

PART II Optimization (7 points)

a) Solve $max\, 3x - y$
s.t. $x^2 + y^2 = 5$ **[3.5 points]**

b) Do you think it is possible to find 2 functions f and g such as, for a specific point $(x_0, y_0) \in \mathbb{R}^2$ we have $\nabla f(x_0, y_0) = \begin{pmatrix}1\\2\end{pmatrix}$, $\nabla g(x_0, y_0) = \begin{pmatrix}2\\3\end{pmatrix}$ and (x_0, y_0) solution of the optimization problem

max f(x,y)
s.t. g(x,y) = c (with a specific $c \in \mathbb{R}$) ? **[3.5 point]**

PART III Probability and statistics (6.5 points)

A TV channel displays a football game and just after that, a show about mathematics.

We have the following information:
- 56% of the viewers watched the game
- one quarter of the viewers who have seen the game also have seen the math show
- 16.2% of viewers watched the show

We asked a few questions to a random viewer. We define 2 events:

- G: “the viewer watched the game”
- S: “the viewer watched the show”

a) What is the probability $P(G \cap S)$? **(1.5 points)**
b) If x is the probability that the viewer watched the show given the fact that he didn’t watch the game, show that P(S) = 0.44x + 0.14 **(2 points)**
c) Using b), find the value of x. **(1.5 points)**
d) It appears that the viewer hasn’t seen the show: what is the probability (up to 2 decimals) that he watched the game? **(1.5 points)**