Optimal Investment in Bitcoin Mining Farms: A Hedged Monte-Carlo Approach

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February 23, 2026

Abstract

The decision to invest in a Bitcoin mining facility involves navigating significant uncertainties, primarily driven by the high volatility of Bitcoin prices and fluctuating energy costs. This paper analyzes the optimal timing for such investments within a real options framework. We formulate the investment as an optimal stopping problem, considering both the option to defer the investment and the operational flexibility to temporarily suspend mining activities when unprofitable. To solve this complex valuation problem, we employ the Hedged Monte Carlo (HMC) method. HMC enhances the accuracy of the estimate by incorporating hedge sensitivities (Greeks) as control variables, thus reducing the variance of the Monte Carlo estimator. This approach allows for robust valuation under the real-world probability measure, accommodating complex dynamics for the underlying stochastic factors. We present a framework for determining the strategic value and optimal investment threshold for large-scale cryptocurrency mining projects

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HIGHLIGHTS

- Motivation: Evergroing Bitcoin cost due to energy consumption
- Determination of a suitable basis to tackle this problem (namely Margrabe type)
- Sustainability issues and energy consumptions
- Innovative usage of Real Options in the context of bit coin farming decisions
- Usage of the bitcoin index as a proxy to the actual bitcoin price.

IMPORTANT TOOL THE HASHPRICE `https://data.hashrateindex.com/network-data/bitcoin-hashprice-index`

Keywords: Real Options, Bitcoin Mining, Optimal Stopping, Hedged Monte Carlo, Investment under Uncertainty, Energy Finance.

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1. Introduction

The global expansion of Bitcoin mining operations represents a significant industrial development, characterized by substantial capital expenditures and high energy consumption. The economic viability of these ventures is inherently uncertain, being highly sensitive to the volatile market price of Bitcoin (BTC) and the cost of electricity. The scale of these investments is notable; for example, the establishment of a $ 2 billion crypto mining facility on Reem Island by the Phoenix group and the Abu Dhabi government underscores the strategic importance of robust valuation methods in this sector.

Given the irreversibility of the initial investment and the high uncertainty of future cash flows, traditional discounted cash flow (DCF) analysis is inadequate, as it fails to capture the value of managerial flexibility. Real Options Analysis (ROA) provides a superior framework, recognizing that investors possess critical flexibilities: the option to defer the investment (timing flexibility) and the option to suspend operations if the mining margin becomes negative (operational flexibility).

The main goal of this paper is to analyze the strategic value of investing in a Bitcoin mining farm under uncertainty. We model this decision as an optimal stopping problem with the objective of maximizing the expected net present value (NPV) by choosing the optimal time to execute the project.

Valuing these combined options (timing and switching) in the presence of multiple stochastic factors presents computational challenges. While traditional methods like binomial trees or finite difference methods exist, they often become intractable in higher dimensions or with complex dynamics. Monte Carlo simulation methods, particularly the Least-Squares Monte Carlo (LSM) approach [**?**], are frequently used, but can suffer from estimation biases and inefficiency.

To address these challenges, we utilize the Hedged Monte Carlo (HMC) method [**? ?**]. HMC is a variance reduction technique that leverages information about the sensitivities (deltas) of the option value to the underlying assets. By incorporating the hedging strategy directly into the simulation as a control variate, HMC provides a more stable and accurate estimation of the continuation value and the optimal exercise boundary. Furthermore, HMC can be easily applied under the real-world probability measure $\mathbb{P}$, using a subjective discount rate, which avoids the complexities of estimating the market price of risk for non-tradable assets.

This paper contributes to the literature by applying the HMC methodology to the specific context of Bitcoin mining investment decisions, offering a robust and flexible tool for strategic planning in this emerging industry.

2. Literature Review

Fast mean-reverting stochastic volatility. Asymptotic analysis of invest-or-wait problems under fast mean-reverting stochastic volatility shows that stochastic volatility raises the value of waiting and shifts the optimal investment boundary relative to constant-volatility benchmarks [7].

Mean-reverting project value and investment cost. When both the project value and the investment cost are mean-reverting, the optimal trigger becomes genuinely two-dimensional; simple value-to-cost ratio rules hold only in special cases, and the problem is addressed numerically [4].

Hedged Monte Carlo in incomplete markets. For settings driven by historical/simulation scenarios—common in commodities—Zubelli and coauthors propose a hedged Monte Carlo approach that prices the real option jointly with a dynamically learned hedge [2].

Energy application: LNG contracts. These ideas extend to energy, where robust, risk-aware valuation and management of LNG contracts with cancellation options are formulated and solved within an optimization framework [3].

3. Problem Formulation

We formalize the investment decision faced by a company considering the establishment of a Bitcoin mining farm.

3.1. Monthly Economic Baseline

We assume a project location in West Texas with an industrial electricity rate of $0.051/kWh. The facility faces fixed monthly operational expenditures (OpEx) of **$10,700**, covering rent, labor, insurance, and maintenance.

Under current market conditions (Hashprice $\approx$ $45/PH/day), the farm generates a monthly revenue of **$67,032**. After deducting electricity costs of $38,432 and fixed OpEx, the net monthly profit is **$17,900**.

This results in a static payback period of **82 months** (6.8 years).

3.2. Optimal Exit Analysis

To assess the impact of asset liquidity, we calculate the Net Present Value (NPV) of the project assuming a variable exit time T . This includes the discounted cash flows from operations up to month T and the resale value of the hardware and infrastructure at that time. We assume a monthly depreciation rate of 4% for the mining hardware.

As shown in Figure 1, the project value is maximized if the investor plans to exit at **Month 60**, yielding a total NPV of **$89,586**. This "concave" profile highlights the trade-off between accumulating operational profits and the rapid depreciation of ASIC hardware.

3.3. Investment Setup

Consider an investment horizon $[0,T]$. The company has the option to build a mining farm, requiring a fixed, irreversible sunk cost $K > 0$. If the company invests at time $\tau \in [0,T]$, the farm begins operations immediately and has a fixed operational lifetime T' . Consequently, the farm operates during the interval $[\tau, \tau + T']$.

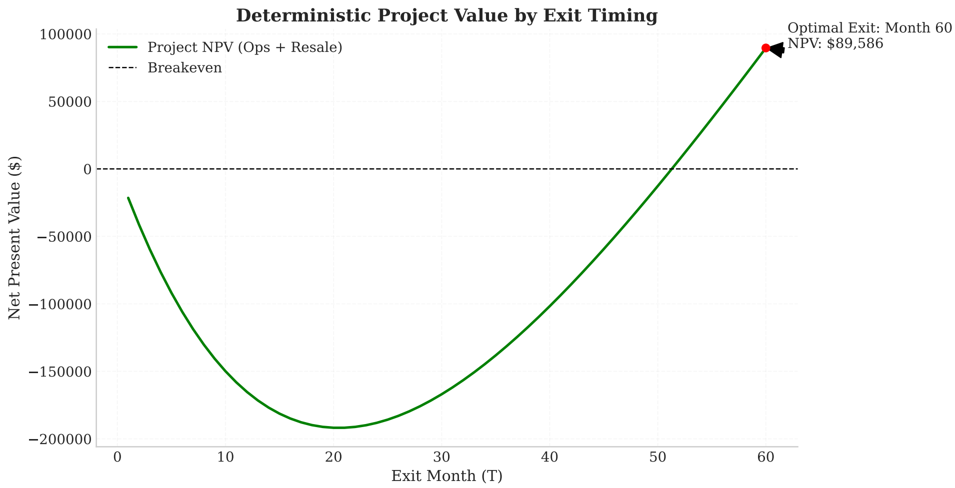

Figure 1: Project NPV as a function of the stopping time T, including asset resale value.

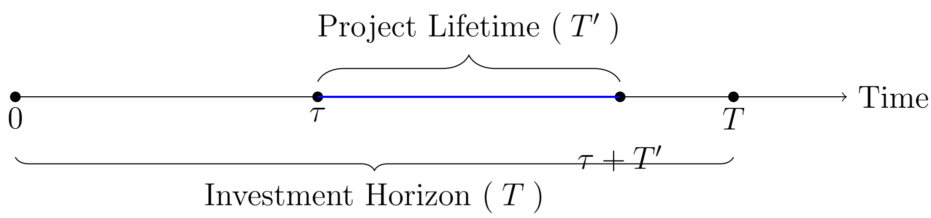

Figure 2: Timeline of the investment decision and project operation.

3.4. Stochastic Variables and Cash Flows

The profitability of the mining farm is driven by two primary stochastic processes:

- X_t : The price of 1 BTC in USD at time t .
- Y_t : The electricity price in USD/MWh at time t .

We assume the dynamics of X_t and Y_t are known or simulated under the real-world measure $\mathbb{P}$. Valuation is performed using a discount rate ρ , reflecting the company's cost of capital (WACC).

3.4.1. Net Present Value (NPV) of the Project

If the investment is made at time τ , the Net Present Value (NPV) of the project, evaluated at time τ , is given by:

$$G(x,y) = \mathbb{E}^{\mathbb{P}}_{x,y}\left[\int_{\tau}^{\tau+T'} e^{-\rho(t-\tau)}\left((X_t - \kappa Y_t)^+ - k\right) dt\right] - K \tag{3.1}$$

where $\mathbb{E}^{\mathbb{P}}_{x,y}[\cdot]$ denotes the expectation under $\mathbb{P}$ conditional on $X_\tau = x$ and $Y_\tau = y$. The parameters are:

- κ : The electricity consumption factor (MWh required to mine 1 BTC per unit time), capturing hardware efficiency and network difficulty.

- k : Fixed operating costs per unit time (e.g., maintenance, wages).
- K : The initial sunk cost.

We assume the firm has the operational flexibility to suspend mining (switch to an idle state) and resume operations without cost. The optimal strategy is to mine if $X_t > \kappa Y_t$, and stay idle otherwise. The instantaneous variable cash flow is thus $(X_t - \kappa Y_t)^+ := \max(X_t - \kappa Y_t, 0)$. The fixed cost k is incurred regardless of the operational state.

If the processes X_t and Y_t are time-homogeneous, the NPV function G depends only on the current values (x, y) .

3.4.2. Monte Carlo Estimation of NPV

The NPV function $G(x, y)$ must generally be estimated using Monte Carlo simulation. Discretizing the lifetime $[0, T']$ into J intervals of length $h = T'/J$, the MC estimator is:

$$\widehat{G}(x, y) \approx \frac{h}{N} \sum_{n=1}^{N} \sum_{i=1}^{J} e^{-\rho t_i} \left(\left(X_{t_i}^{(n)} - \kappa Y_{t_i}^{(n)} \right)^+ - k \right) - K \tag{3.2}$$

where N is the number of simulated paths starting from (x, y) . Note that evaluating $G(x, y)$ is computationally intensive, requiring a nested simulation whenever the option value is calculated.

4. The Hedged Monte Carlo Methodology

The objective is to find the optimal investment time τ to maximize the expected discounted value of the project. This is formulated as an optimal stopping problem:

$$V(x, y) = \operatorname{ess} \sup_{0 \le \tau \le T} \mathbb{E}_{x,y}^{\mathbb{P}} \left[e^{-\rho \tau} G(X_\tau, Y_\tau) \right] \tag{4.1}$$

where the supremum is taken over all stopping times τ adapted to the filtration generated by (X_t, Y_t) .

We apply the Hedged Monte Carlo (HMC) method to solve this problem. HMC utilizes dynamic programming and incorporates hedging information to improve the estimation of the continuation value.

4.1. Discretization and Backward Induction

We discretize the investment horizon $[0, T]$ into L intervals of length $\Delta = T/L$, with time points $t_j = j\Delta$. We simulate M independent trajectories of (X, Y) over this grid.

The algorithm proceeds via backward induction. At the terminal time $T = t_L$, the value is:

$$V(L, m) = G(X_T^{(m)}, Y_T^{(m)}) \tag{4.2}$$

for the m -th simulated path.

At time step t_j , the company compares the immediate investment value, $G(X_{t_j}, Y_{t_j})$, with the continuation value, $CV(j, X_{t_j}, Y_{t_j})$. We approximate the continuation value using a regression on a set of basis functions $\{C_a(x, y)\}_{a=1}^B$:

$$CV(j, x, y) \approx \sum_{a=1}^{B} \gamma_a^j C_a(x, y) \tag{4.3}$$

4.2. HMC Regression

In the HMC approach, the coefficients (γ_a^j) and the hedging coefficients (β_a^j, δ_a^j) are determined simultaneously by minimizing the squared replication error of the hedged portfolio. This minimization leverages the fact that the discounted option price process, when appropriately hedged, behaves like a martingale.

The objective function to minimize is:

$$\min_{\gamma,\beta,\delta} \sum_{m=1}^{M} \left(e^{-\rho\Delta} V \left(j+1, X_{t_{j+1}}^{(m)}, Y_{t_{j+1}}^{(m)} \right) - \sum_{a=1}^{B} \gamma_a^j C_a \left(X_{t_j}^{(m)}, Y_{t_j}^{(m)} \right) \right.$$
$$+ \sum_{a=1}^{B} \beta_a^j \frac{\partial C_a}{\partial x} \left(X_{t_j}^{(m)}, Y_{t_j}^{(m)} \right) \left[X_{t_j}^{(m)} - e^{-\rho\Delta} X_{t_{j+1}}^{(m)} \right]$$
$$\left. + \sum_{a=1}^{B} \delta_a^j \frac{\partial C_a}{\partial y} \left(X_{t_j}^{(m)}, Y_{t_j}^{(m)} \right) \left[Y_{t_j}^{(m)} - e^{-\rho\Delta} Y_{t_{j+1}}^{(m)} \right] \right)^2 \tag{4.4}$$

The terms involving β_a^j (hedging X) and δ_a^j (hedging Y) act as control variates. They represent the P&L of delta-hedge portfolios constructed using the known sensitivities of the basis functions. By incorporating these ”perfect hedges” for the basis functions, HMC significantly reduces the variance in the estimation of the continuation value.

4.3. Option Value Update

Once the coefficients $\hat{\gamma}_a^j$ are estimated, the continuation value is calculated. The project value at time t_j for path m is then:

$$V(j,m) = \max \left(G(X_{t_j}^{(m)}, Y_{t_j}^{(m)}), \sum_{a=1}^{B} \hat{\gamma}_a^j C_a \left(X_{t_j}^{(m)}, Y_{t_j}^{(m)} \right) \right) \tag{4.5}$$

This process is repeated backwards until t_0 , yielding the value of the investment option.

5. Numerical Implementation and Case Study

5.1. Strategic Baseline: The Hybrid Hosting Model

To evaluate the investment, we adopt a “Hybrid” or “Colocation” investment strategy, which represents the most common and practical entry point for mid-scale mining operations today. In this model, the investor retains ownership of the mining hardware (capital expenditure) but outsources the infrastructure and energy procurement (operational expenditure).

- **Capital Investment:** The firm purchases the mining hardware at an adjusted secondary-market price of $2,500/unit. For our base case of 350 units, the initial sunk cost (K) totals **$875,000**. This structure avoids the high interest premiums associated with full hardware leasing.

- **Operational Structure:** The hardware is deployed in a third-party hosting facility at an all-inclusive rate of $0.065/kWh. This rate removes the need for capital investment in transformers, land development, and maintenance staff. The facility’s total daily energy conversion factor (κ) is calculated as **25,284 kWh/day**.

Assuming current market conditions remain completely static (deterministic), the project generates a positive monthly net profit of **$17,092**. This static scenario results in a traditional payback period of **4.3 years**.

5.2. Deterministic Project Value and Optimal Exit

Before applying stochastic volatility, we establish a strict deterministic floor by calculating the optimal exit timing. This analysis evaluates the Net Present Value (NPV) of the project assuming a variable stopping time T. The NPV incorporates both the discounted operational cash flows up to month T and the salvage value of the hardware, which we assume depreciates at an aggressive rate of **4%** per month.

Assuming constant hashprices, the deterministic project value is maximized if the investor liquidates the operation at **Month 60**, yielding a maximum theoretical NPV of **$89,586**. This establishes the baseline value of the project ignoring all crypto-market volatility.

6. Results and Discussion

6.1. Scenario 2: The "Hosting" Case (Stochastic Revenue)

In our first stochastic scenario, we isolate the value of *Revenue Volatility.* Because the firm pays a fixed electricity hosting rate ($0.065/kWh), energy price risk is eliminated. However, the project remains fully exposed to the extreme volatility of Bitcoin prices and Hashprice.

Crucially, the firm possesses an operational *Switching Option*: management can temporarily suspend mining operations if the Hashprice drops below the hosting break-even threshold, preventing negative cash flows, and resume operations when profitability returns.

We evaluate this flexibility using the Hedged Monte Carlo (HMC) algorithm. When evaluated purely intrinsically (forcing immediate exercise without the flexibility to wait or switch), the traditional NPV under simulated paths drops to **$0**.

However, when properly valuing the operational flexibility to weather "crypto winters" and capitalize on "bull markets", the Real Option Value rises significantly to **$165,027**. The difference between these two metrics yields an option premium of **$165,027**.

This result highlights a fundamental principle in highly volatile industries: traditional DCF and static NPV severely underprice assets because they fail to account for management's ability to asymmetricaly capture upside volatility while truncating downside risk through operational suspension.

7. Conclusion

This paper presents a framework for valuing investments in Bitcoin mining facilities using real options analysis. By employing the Hedged Monte Carlo method, we address the complexities arising from stochastic Bitcoin prices and electricity costs, the option to defer investment, and the operational flexibility inherent in mining operations. The HMC approach provides a robust and variance-reduced method for determining the optimal investment timing under uncertainty.

A. Appendix: Overview of Bitcoin Mining

Bitcoin mining is the decentralized consensus mechanism by which transactions are validated and added to the blockchain, and new bitcoins are issued. It relies on the **Proof-of-Work (PoW)** protocol, requiring participants (miners) to expend significant computational effort to solve complex cryptographic puzzles [6].

A.1. Process and Rewards

Miners aggregate transactions into a candidate block and repeatedly hash the block header until they find a value below a network-defined target (the difficulty). This requires specialized hardware, predominantly **Application-Specific Integrated Circuits (ASICs)**.

The successful miner broadcasts the block, and upon validation by the network, receives a block reward. This reward consists of the block subsidy (newly minted bitcoins) and transaction fees [5, 1]. The network difficulty adjusts approximately every two weeks to maintain an average block discovery time of ten minutes. The block subsidy halves roughly every four years; as of the 2024 halving, the subsidy is 3.125 BTC.

A.2. Mining Strategies and Time to Reward

The time until a mining farm generates revenue depends on its strategy:

- **Pool Mining:** Miners combine their hash rate in a pool and share rewards proportionally. This reduces the variance of returns, allowing for consistent, frequent payouts (hours or days).
- **Solo Mining:** The farm keeps the entire block reward if it successfully mines a block. However, the time to discover a block is stochastic and can be extremely long, following an exponential distribution.

A.3. Illustrative Numerical Example

This example calculates the expected revenue for a hypothetical mining farm.

Assumptions (Illustrative):

- Global Network Hash Rate (H_{Global}): 965 EH/s (1 EH/s = 10^{18} H/s).
- Block Reward (R): 3.125 BTC .
- Average Blocks per Day (B_{Day}): 144 .
- Farm Hash Rate (H_{Farm}): 20 PH/s (0.020 EH/s).

Calculations: The farm's share of the global hash rate (α):

$$\alpha = \frac{H_{Farm}}{H_{Global}} = \frac{20 \text{ PH/s}}{965{,}000 \text{ PH/s}} \approx 2.07 \times 10^{-5} \quad (0.00207\%).$$

Expected blocks mined per day by the farm ($E[B_{Farm}]$):

$$E[B_{Farm}] = B_{Day} \times \alpha = 144 \times 2.07 \times 10^{-5} \approx 0.00298 \text{ blocks/day}.$$

Expected time to mine one block (solo mining, mean of exponential distribution):

$$\frac{1}{E[B_{Farm}]} \approx \frac{1}{0.00298} \approx 335 \text{ days}.$$

Median time-to-first block (solo mining):

$$\frac{\ln(2)}{E[B_{Farm}]} \approx \frac{0.693}{0.00298} \approx 232 \text{ days}.$$

Expected daily revenue in BTC (assuming pool mining and ignoring fees):

$$E[\text{Revenue}_{BTC}] = E[B_{Farm}] \times R = 0.00298 \times 3.125 \approx 0.00933 \text{ BTC/day}.$$

Interpretation: For this hypothetical farm, solo mining presents high variance, with an average wait of 335 days to find a block. Pool mining provides a much more stable expected revenue stream.

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