# Homework 1: The Four Evolutionary Prcoesses

### 1. The WF Model with Mutation

In class we discussed the classic WF model of genetic drift. This WF model can be extended to include all the other evolutionary processes discussed. Suppose that when individuals reproduce their offspring may be mutants such that $A \to a$ with probability $u$ and $a \to A$ with probability $v$.

A. Derive the transition probabilities for the WF process with mutation. Hint: the probability an offspring carries a $A$ allele is the probability that their parent had a $A$ and did not mutate plus the probability their parent had a $a$ and did mutate.

B. For a population of size $100$, if the allele frequency of the $A$ allele is currently $p = 0.5$ what is the probability that the frequency is $p = 0.6$ in the next generation assuming $u = 0.001$ and $v = 0.003$? Compare and contrast the answer with the answer from class without mutation.

C. What is the stationary distribution of this WF+mutation process when $u = 0.001$ and $v = 0.003$ discuss your answer in light of the mutation only model presented in class.

### 2. Evolution of a recessive mutation

In class we discussed a model of directional selection for a dominant $A$ allele. An allele can however be under lots of different types of selection and exhibit lots of different forms of dominance. Suppose that the $A$ allele is recessive such that:

$$W_{aa} = 1, \ W_{Aa} = 1, \text{ and } W_{AA} = 1 + s$$

A. Derive a set of recursion equations for the evolution of this recessive mutation (assuming the same life-cycle used in class).

B. Find the equilibrium allele frequency and compare the answer to that for the dominant allele from class.

C. Assuming the allele frequency is initially $p(0) = 0.2$ plot the allele frequency dynamics for the dominant and recessive allele over time assuming that $s = 0.03$. Write a few sentences describing contrasting the two dynamics and describing why this difference makes sense.

### 3. Sequence of events in a life-cycle

In class we considered a case where individuals have the following life cycle: census>mutation>reproduction. Because reproduction results in Hardy-Weinberg genotype frequencies it didn't matter whether we followed genotype frequencies or allele frequencies in this model. I claimed that the order of events in this model was for mathematical convenience and has no effect on the outcome. Here we will show that this is true

A. Develop a set of recursion equations modelling with the following life cycle: census>reproduction>mutation. Show that the genotype frequencies at time time of population census are *not* in HW proportions.

B. Solve for the equilibrium genotype frequencies in this model. Show that at equilibrium the allele frequencies are the same as what we obtained in class.

C. Given the equilibrium genotype frequencies you found in B. Assume that reproduction occurs one more time and calculate the resulting HW genotype frequencies. Show that these are the same as the equilibrium genotype frequencies we found in class. Hence the only difference between the model here and the one we found in class is the arbitrary placement of the "census" in the life cycle.

### 4. Spatial differentiation and Fst

In class we showed that migration between two sub-populations leads to a blending of the genes between the two and erodes spatial differentiation. For there to be spacial differentiation then requires another one of the evolutionary processes to be at play. In other words, we need a process that creates spatial divergence. One such process is spatially heterogenous selection where, for example the $A$ allele is favoured in one location and the $a$ allele is favoured in the other. Let's build such a model.

Consider two populations connected by migration with a discrete-time life cycle of census>local selection>migration>random mating.

The fitness of the genotypes in each population are:

$$W_{AA,1} = 1 + 2s \quad W_{Aa,1} = 1 + s \quad W_{aa,1} = 1$$
$$W_{AA,2} = 1 \quad W_{Aa,2} = 1 + s \quad W_{aa,2} = 1 + 2s$$

where $s$ is the local selective advantage of the $A$ allele in population 1 and of the $a$ allele in population 2.

Suppose each generation a proportion $m$ of migrants are exchanged between the two populations.

A. Derive recursion equations describing the dynamics of the genotype frequencies in the two populations.

B. What is the equilibrium genotype frequencies? Are they in Hardy-Weinberg?

C. What is the equilibrium level of $F_{ST}$?

D. Here we have derived the dynamics in the population using genotypes. Could we have followed allele frequencies instead and gotten the same answer?