# Homework 3: Mathematical Demography (Updated)

## 1. Stage Structured Population

In class we focused on age structured models, but similar principles can be used to model evolution in a "stage" structured population. Consider a tree for example, trees can be characterized as "seed", "seedlings", "sapling", and "adult".

Adult trees are have thick trunks and hence have relatively low mortality rates. Trees that reach adulthood often can live an average of 500 years (e.g., Douglas fir). Adult Douglas firs can produce up to 10,000 seeds per year.

Seeds have high mortality rates only 10% of seeds surviving to germinate each year. Of the surviving seeds only a fraction $g = 0.2$ germinate every year with the remainder staying in the seed bank.

Seedlings are germinated seeds. Small and climate sensitive seedlings have a high mortality rate. This stage lasts on average 2 years with a 90% mortality rate. Saplings are more robust with a 50% mortality rate, a stage lasting another 6 years. Neither seedlings or saplings produce seeds.

A. Use the information above to propose a matrix describing the population dynamics of Douglas fir. Note that if the average time spent in a stage is $T$ then the rate at which individuals leave that stage is $\frac{1}{T}$.

B. Use your matrix above to find the growth rate of the Douglas fir forest and its stable stage distribution.

C. Play around with the germination rate $g$ to find the germination rate for which the population is only slowly growing or slowly declining.

## 2. Canadian Demography

A. Statistics Canada has fantastic data on births and deaths in Canada. Use the following data to construct a Leslie matrix for Canada for the year 2020. Hint: use only females and consider 5 year time increments.

- https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=1310041601
- https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=1310071001&pickMembers[0]=1.1&pickMembers[1]=3.3&cubeTimeFrame.startYear=2016&cubeTimeFrame.endYear=2020&referencePeriods=20160101%2C20200101

B. Using your Leslie matrix from A, calculate the equilibrium age distribution in Canada.

C. Statistics Canada also has information on the current ages of residents. Compare the current age distribution to the equilibrium from B. Describe their differences/similarities and provide some possible explanations for deviations from equilibrium.

- https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=1710000501