Risk-Sensitive Optimization for Power Systems

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May 10, 2021

Expecting uncertainty in power systems

- Conventional power systems are composed of a grid connected to several large-scale dispatchable generators with predictable loads.

- This paradigm is shifting rapidly with
  - Increasing wind and solar plants that are intermittent, non-dispatchable, and uncertain.
  - Increasing adoption of distributed energy resources (rooftop solar or battery energy storage) and electric vehicles introducing further uncertainty and complexity.
  - Increasing frequency of extreme weather events posing greater risk of component availability and potential blackouts. For example, the 2021 Texas blackout.

- Modern models must more strongly account for uncertainty in component availability, available energy supply, and fluctations in demand.

What is risk-sensitive optimization?

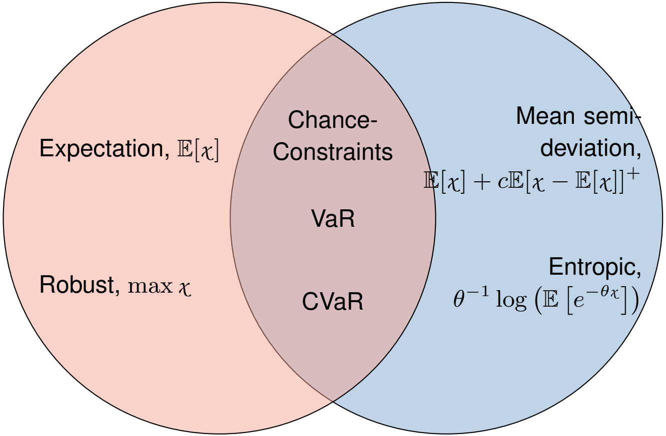
- For random functions, $f_\omega(\boldsymbol{x})$ and $\boldsymbol{g}_\omega(\boldsymbol{x})$, we want to solve the (ill-formed) problem

$$\underset{\boldsymbol{x}}{\text{minimize}} \quad f_\omega(\boldsymbol{x}), \qquad \text{subject to} \quad \boldsymbol{g}_\omega(\boldsymbol{x}) \le 0.$$

- We must extract some numerical property of these random functions to meaningfully optimize. These properties are referred to as *measures*.

"Easy" to optimize

...can we define data-driven single time-scale algorithms?

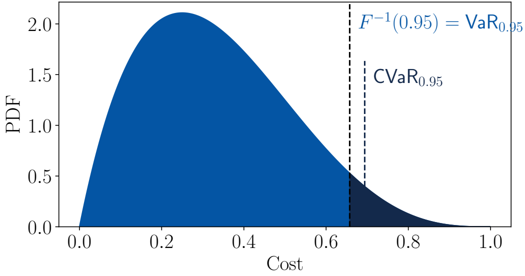


Expressive

...does it capture the cost-reliability tradeoff?

Understanding the conditional value at risk (CVaR) measure

- It measures the average over the $(1-\alpha)$ fraction of worst-case scenarios.



The CVaR measure has the properties:

- $\mathrm{CVaR}_0[\chi] \equiv \mathbb{E}[\chi]$.
- $\lim_{\alpha \uparrow 1} \mathrm{CVaR}_\alpha[\chi] = \operatorname{ess\,sup} \chi$.
- $\mathrm{CVaR}_\alpha[\chi] \geq \mathrm{VaR}_\alpha[\chi]$.
- CVaR limits severity of exceedance.

- For general distributions, CVaR is defined by the following variational form

$$\mathrm{CVaR}_\alpha[\chi] := \underset{z}{\text{minimize}}\ z + \frac{1}{1-\alpha}\mathbb{E}\left[\chi - z\right]^+$$

Rockafellar and Uryasev 2000; 2002

Summary of presentation

- **Why** do we need to model uncertainty in power system operation?
- **What** is risk-sensitive optimization and why does it address this concern?
- **How** do we formulate, optimize, and price risk-sensitive models?
  - Discrete uncertainty in a risk-sensitive security-constrained economic dispatch problem.
  - Continuous uncertainty in a risk-sensitive economic dispatch problem with uncertain wind.
  - Either uncertainty for voltage regulation in distribution networks.

The economic dispatch problem

- System operators make dispatch decisions at various intervals before the real time by solving an economic dispatch (ED) problem such as

$$\begin{aligned} \underset{\boldsymbol{g}}{\text{minimize}} \quad & \boldsymbol{c}^\top \boldsymbol{g}, \\ \text{subject to} \quad & \mathbb{1}^\top (\boldsymbol{g} - \boldsymbol{d}) = 0, \\ & \boldsymbol{H}(\boldsymbol{g} - \boldsymbol{d}) \leq \boldsymbol{f}, \\ & \underline{\boldsymbol{g}} \leq \boldsymbol{g} \leq \overline{\boldsymbol{g}}. \end{aligned}$$

- For simplicity, we adopt the DC approximations that reduce the nonlinear power flow equations into a linear map from the net power injections.
- This deterministic problem fails to account for:
    1. Potential component failures such as line outages.
    2. Uncertainty in available supply introduced by renewable sources.

A risk-sensitive security-constrained ED (R-SCED) problem

- For potential line failures, enumerated $k = 1, \ldots, K$, we augment the ED problem with additional constraints.

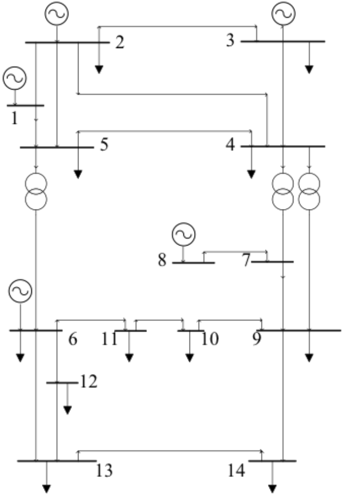
$$\begin{aligned}
\underset{\boldsymbol{g}, \boldsymbol{\delta g}, \boldsymbol{\delta d}}{\text{minimize}} \quad & \mathrm{CVaR}_\alpha \left[\boldsymbol{c}^\top \boldsymbol{g} + C(\boldsymbol{\delta g}, \boldsymbol{\delta d}) \right], \\
\text{subject to} \quad & \mathbb{1}^\top (\boldsymbol{g} - \boldsymbol{d}) = 0, \quad \boldsymbol{H}^\top (\boldsymbol{g} - \boldsymbol{d}) \le \boldsymbol{f}, \quad \boldsymbol{g} \in [\underline{\boldsymbol{g}}, \overline{\boldsymbol{g}}], \\
& \boldsymbol{H}_k^\top (\boldsymbol{g} - \boldsymbol{d}) \le \boldsymbol{f}_k^{\mathsf{DA}}, \\
& \boldsymbol{H}_k^\top (\boldsymbol{g} + \boldsymbol{\delta g}_k - \boldsymbol{d} + \boldsymbol{\delta d}_k) \le \boldsymbol{f}_k^{\mathsf{SE}}, \quad \boldsymbol{g} + \boldsymbol{\delta g}_k \in [\underline{\boldsymbol{g}}, \overline{\boldsymbol{g}}], \quad \boldsymbol{\delta g}_k \in \Delta_g, \\
& k = 1, \ldots, K.
\end{aligned}$$

where the random recourse cost $C(\boldsymbol{\delta g}, \boldsymbol{\delta d})$ takes values

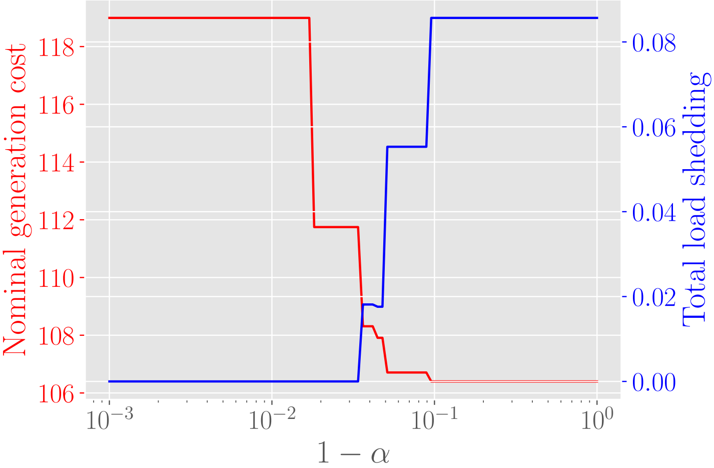
$$C_k(\boldsymbol{\delta g}_k, \boldsymbol{\delta d}_k) = \boldsymbol{c}_+^\top [\boldsymbol{\delta g}]^+ + \boldsymbol{c}_-^\top [\boldsymbol{\delta g}]^- + \boldsymbol{v}^\top \boldsymbol{\delta d} \qquad \text{w.p. } p_k.$$

- Other security-constrained ED (SCED) formulations include
  - Preventive-SCED: Alsac and Stott 1974; Capitanescu, Glavic, et al. 2007
  - Corrective-SCED: Capitanescu and Wehenkel 2007; 2008; Phan and Sun 2015 (without load shed), Francois Bouffard and Galiana 2008 (with load shed)

An illustrative example of R-SCED



IEEE 14-bus network



risk-averse ⟷ risk-neutral

The computational challenges of SCED at scale

- R-SCED and C-SCED are large linear programs which are *usually* considered easy.
- For the IEEE 2383-bus Polish network with 2,869 lines, 327 generators, and 1,817 loads

Formulation	**Variables**	**Constraints**
ED	327	6,447
C-SCED	947,319	28,949,071
R-SCED	6,212,249	28,954,865

"The resulting linear program is not solvable by traditional LP methods due to its large size"

-Liu et. al. 2015

- The following techniques have been used to make the problem more tractable:
  - Contingency filtering/pre-screening François Bouffard, Galiana, and Arroyo 2005; Capitanescu, Glavic, et al. 2007; Ejebe and Wollenberg 1979; Fliscounakis et al. 2013; Fu and A. Bose 1999
  - Exploiting decomposable structure Benders 2005; Gomez et al. 1991; Liu, Ferris, and Zhao 2015; Madavan, S. Bose, et al. 2019

A decomposition method for SCED problems

- The R-SCED (and C-SCED) formulation is an instance of the following with $\alpha' = (1-\alpha)^{-1}$

$$\begin{aligned} \underset{\boldsymbol{x}_0, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_K}{\text{minimize}} \quad & \boldsymbol{c}^\top \boldsymbol{x}_0 + \alpha' \sum_{k=1}^{K} p_k \boldsymbol{c}_k^\top \boldsymbol{x}_k, \\ \text{subject to} \quad & \boldsymbol{A}_0 \boldsymbol{x}_0 \leq \boldsymbol{b}_0, \\ & \boldsymbol{E}_k \boldsymbol{x}_0 + \boldsymbol{A}_k \boldsymbol{x}_k \leq \boldsymbol{b}_k, \quad \text{for each } k = 1, \ldots, K, \end{aligned}$$

- Making use of this structure, this can be written as

$$\underset{\boldsymbol{x}_0 \in \mathbb{X}_0}{\text{minimize}} \quad \boldsymbol{c}_0^\top \boldsymbol{x}_0 + \alpha' \sum_{k=1}^{K} J_k^*(\boldsymbol{x}_0),$$

where,

$$\begin{aligned} J_k^*(\boldsymbol{x}_0) := \underset{\boldsymbol{x}_k}{\text{minimize}} \quad & p_k \boldsymbol{c}_k^\top \boldsymbol{x}_k, \\ \text{subject to} \quad & \boldsymbol{E}_k \boldsymbol{x}_0 + \boldsymbol{A}_k \boldsymbol{x}_k \leq \boldsymbol{b}_k. \end{aligned}$$

The critical region exploration (CRE) algorithm

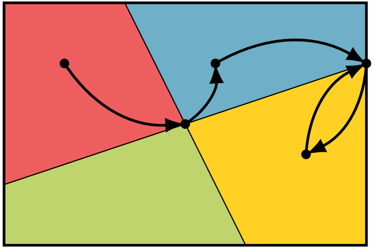
- The decomposed form of R-SCED and C-SCED is given by

$$\underset{\boldsymbol{x}_0 \in \mathbb{X}_0}{\text{minimize}} \quad \boldsymbol{c}_0^\top \boldsymbol{x}_0 + \alpha' \sum_{k=1}^{K} J_k^*(\boldsymbol{x}_0), \qquad J_k^*(\boldsymbol{x}_0) := \underset{\boldsymbol{x}_k}{\text{minimize}} \quad p_k \boldsymbol{c}_k^\top \boldsymbol{x}_k,$$

$$\text{subject to} \quad \boldsymbol{E}_k \boldsymbol{x}_0 + \boldsymbol{A}_k \boldsymbol{x}_k \leq \boldsymbol{b}_k.$$

- The critical region exploration algorithm uses the following:

  - J_k^* is a multi-parametric linear program (MPLP) linearly parameterized by $\boldsymbol{x}_0$.
  - J_k^* is piecewise affine and convex in $\boldsymbol{x}_0$.
  - The sets over which J_k^* is affine describe a polyhedral partition of $\mathbb{X}_0$.



Theorem. CRE converges in finite iterations.

Preliminary results

- CRE and Benders' was implemented in C++ leveraging Eigen and SuiteSparse libraries. Available at: `https://github.com/amadavan/Stuka`.
- CRE is competitive with Benders' for small- and medium-sized problems but suffers from complexity issues for large-scale problems.
- Critical region defined by very large set of constraints, making main problem very large.
- Benders' requires several iterations to estimate global cost even if the initial point is close.
- Benders' solves all subproblems at each iteration while CRE solves a small subset.

Future work

- Design a composite algorithm making use of advantages of both approaches.
- Evaluate pricing schemes under the R-SCED formulation.

Avinash N. Madavan, Subhonmesh Bose, et al. (2019). "Risk-Sensitive Security-Constrained Economic Dispatch via Critical Region Exploration". In: *2019 IEEE Power Energy Society General Meeting (PESGM)*, pp. 1–5

Modeling uncertainty in wind in the ED problem

- Consider the ED problem augmented with some forecasted renewable supply

$$\mathcal{P}_{\text{det}}: \quad \underset{\boldsymbol{g}\in[\underline{\boldsymbol{g}},\overline{\boldsymbol{g}}]}{\text{minimize}} \quad \boldsymbol{c}^\top \boldsymbol{g},$$
$$\text{subject to} \quad \mathbb{1}^\top(\boldsymbol{g}-\boldsymbol{d}+\boldsymbol{\omega}) = 0,$$
$$\boldsymbol{H}^\top(\boldsymbol{g}-\boldsymbol{d}+\boldsymbol{\omega}) \leq \boldsymbol{f}.$$

- When the forecast is incorrect, the dispatch will change accordingly, i.e. $\boldsymbol{g} = \boldsymbol{g}(\omega)$. For simplicity, assume affine recourse policies, with $\Delta\boldsymbol{\omega} = \boldsymbol{\omega} - \mathbb{E}[\boldsymbol{\omega}]$,

$$\boldsymbol{g}(\omega) = \widehat{\boldsymbol{g}} + \boldsymbol{G}\Delta\boldsymbol{\omega}.$$

- The risk-sensitive problem is given by

$$\mathcal{P}_{\text{risk}}: \quad \underset{\widehat{\boldsymbol{g}},\boldsymbol{G}}{\text{minimize}} \quad \mathrm{CVaR}_\alpha\left[\boldsymbol{c}^\top(\widehat{\boldsymbol{g}}+\boldsymbol{G}\Delta\boldsymbol{\omega})\right],$$
$$\text{subject to} \quad \mathbb{1}^\top(\widehat{\boldsymbol{g}}+\boldsymbol{G}\Delta\boldsymbol{\omega}-\boldsymbol{d}+\boldsymbol{\omega}) = 0 \quad \text{a.s.},$$
$$\mathrm{CVaR}_{\beta^\ell}\left[\boldsymbol{H}^\top(\widehat{\boldsymbol{g}}+\boldsymbol{G}\Delta\boldsymbol{\omega}-\boldsymbol{d}+\boldsymbol{\omega})-\boldsymbol{f}\right] \leq 0,$$
$$\mathrm{CVaR}_{\beta^g}\left[\widehat{\boldsymbol{g}}+\boldsymbol{G}\Delta\boldsymbol{\omega}-\overline{\boldsymbol{g}}\right] \leq 0, \ \mathrm{CVaR}_{\beta^g}\left[\underline{\boldsymbol{g}}-\widehat{\boldsymbol{g}}-\boldsymbol{G}\Delta\boldsymbol{\omega}\right] \leq 0.$$

Physically enforced

A stochastic approximation algorithm

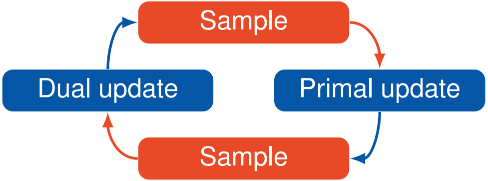
- The $\mathcal{P}_{\text{risk}}$ problem can be written more generally as

$$\mathcal{P}^{\text{CVaR}}: \quad \underset{\boldsymbol{x}}{\text{minimize}} \quad \mathrm{CVaR}_{\alpha}\left[f_{\omega}(\boldsymbol{x})\right], \\ \text{subject to} \quad \mathrm{CVaR}_{\beta}\left[g_{\omega}^{i}(\boldsymbol{x})\right] \leq 0, \quad i = 1, \ldots, m.$$

- With the variational form of CVaR and appropriately modified $\boldsymbol{x}$, $f_{\omega}(\boldsymbol{x})$, $\boldsymbol{g}_{\omega}(\boldsymbol{x})$, this can be

$$\mathcal{P}^{\text{E}}: \quad \underset{\boldsymbol{x}}{\text{minimize}} \quad \mathbb{E}\left[f_{\omega}(\boldsymbol{x})\right], \\ \text{subject to} \quad \mathbb{E}\left[g_{\omega}^{i}(\boldsymbol{x})\right] \leq 0, \quad i = 1, \ldots, m.$$

- We consider a primal-dual stochastic subgradient (PDSS) algorithm for the problem



$$\begin{aligned} \mathcal{L}_{\omega}(\boldsymbol{x}, \boldsymbol{z}) &= f_{\omega}(\boldsymbol{x}) + \boldsymbol{z}^{\top} \boldsymbol{g}_{\omega}(\boldsymbol{x}), \\ \boldsymbol{x}_{k+1} &= \boldsymbol{x}_{k} - \gamma \nabla_{\boldsymbol{x}} \mathcal{L}_{\omega_{k}}(\boldsymbol{x}_{k}, \boldsymbol{z}_{k}), \\ \boldsymbol{z}_{k+1} &= \left[\boldsymbol{z}_{k} + \gamma \nabla_{\boldsymbol{z}} \mathcal{L}_{\omega_{k+1/2}}(\boldsymbol{x}_{k+1}, \boldsymbol{z}_{k})\right]^{+}. \end{aligned}$$

Converges without knowing dual bound!

Theoretical guarantees of the PDSS algorithm

Theorem (Asymptotic convergence). Under certain assumptions and $\{\gamma_k\}_{k=1}^{\infty}$ non-summable square-summable nonnegative sequence, i.e., $\sum_{k=1}^{\infty} \gamma_k = \infty, \sum_{k=1}^{\infty} \gamma_k^2 < \infty$. Then, $(\boldsymbol{x}_k, \boldsymbol{z}_k)$ from PDSS remains bounded and $\lim_{k\to\infty} \mathcal{L}(\boldsymbol{x}_k, \boldsymbol{z}_\star) - \mathcal{L}(\boldsymbol{x}_\star, \boldsymbol{z}_k) = 0$

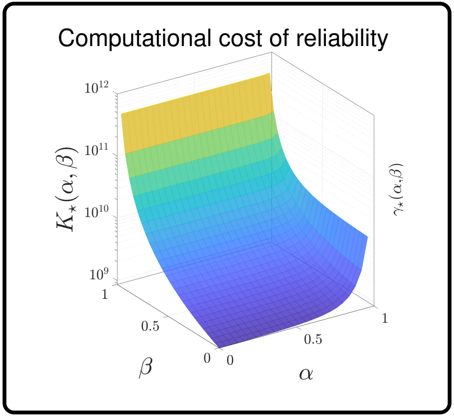
Theorem (Finite-time performance). Under certain assumptions, specified contants P_1, P_2, P_3, and positive sequence $\{\gamma_k\}_{k=1}^{K}$, with $P_3 \sum_{k=1}^{K} \gamma_k^2 < 1$, then the iterates satisfy

$$\mathbb{E}[F(\bar{\boldsymbol{x}}_{K+1})] - p_\star^{\mathrm{E}} \leq \frac{1}{4\sum_{k=1}^{K} \gamma_k} \left(\frac{P_1 + P_2 \sum_{k=1}^{K} \gamma_k^2}{1 - P_3 \sum_{k=1}^{K} \gamma_k^2} \right),$$

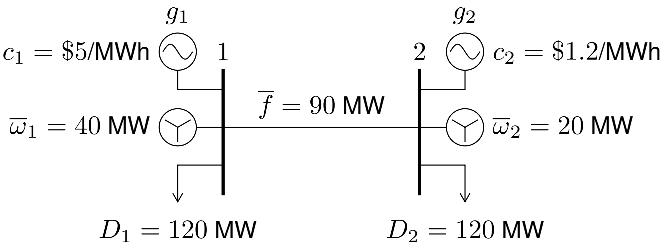
$$\mathbb{E}[G^i(\bar{\boldsymbol{x}}_{K+1})] \leq \frac{1}{4\sum_{k=1}^{K} \gamma_k} \left(\frac{P_1 + P_2 \sum_{k=1}^{K} \gamma_k^2}{1 - P_3 \sum_{k=1}^{K} \gamma_k^2} \right)$$

for each $i = 1, \ldots, m$, where $\bar{\boldsymbol{x}}_{K+1} := \frac{\sum_{k=1}^{K} \gamma_k \boldsymbol{x}_{k+1}}{\sum_{k=1}^{K} \gamma_k}$.

Illustrating PDSS with examples



A two-bus network example with wind sample pairs (ω_1, ω_2) taken from NREL's synthetic dataset over 5 min intervals between 2008-2011.



Aversion	α	β	γ	Highest Cost (\$)	Worst-Case Constraint Violation Line (MW)	Worst-Case Constraint Violation Generation (MW)	Iterations $(\times 10^9)$	Runtime (s)
None	0	0	0	400	25	24	2.9	3767
Line capacity	0	0.6	0.2	402	0	50	9.2	21400
Dispatch cost	0.6	0.2	0.2	352	13	0	4.6	9660

Pricing for the risk-sensitive ED problem

- Assume $\alpha = 0$, and replace the almost sure constraint using

$$\mathbb{1}^\top(\widehat{\boldsymbol{g}} + \boldsymbol{G}\Delta\boldsymbol{\omega} - \boldsymbol{d} + \boldsymbol{\omega}) = 0 \; a.s. \quad \Longleftarrow \quad \mathbb{1}^\top(\widehat{\boldsymbol{g}} - \boldsymbol{d} + \mathbb{E}[\boldsymbol{\omega}]) = 0, \; \boldsymbol{G}^\top\mathbb{1} = \mathbb{1}.$$

- Applying the variational form, the risk-sensitive ED problem can be written as

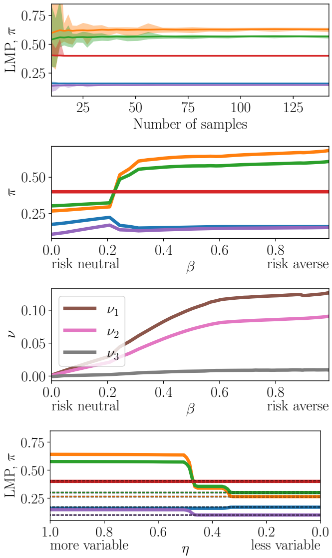
$$\begin{aligned}
\underset{\boldsymbol{g}_0, \boldsymbol{G}, \boldsymbol{u}, \underline{\boldsymbol{v}}, \overline{\boldsymbol{v}}}{\text{minimize}} \quad & \boldsymbol{c}^\top \boldsymbol{g}_0, \\
\text{subject to} \quad & \mathbb{1}^\top(\boldsymbol{g}_0 + \mathbb{E}[\boldsymbol{\xi}] - \boldsymbol{d}) = 0, \quad \boldsymbol{G}^\top\mathbb{1} = \mathbb{1} && : \lambda, \; \boldsymbol{\nu} \\
& \boldsymbol{u} + \frac{1}{\beta}\mathbb{E}\left[(\boldsymbol{H}(\boldsymbol{g}_0 - \boldsymbol{G}\Delta\boldsymbol{\xi} + \boldsymbol{\xi} - \boldsymbol{d}) - \boldsymbol{f} - \boldsymbol{u})^+\right] \le 0, && : \boldsymbol{\mu} \\
& \underline{\boldsymbol{v}} + \frac{1}{\gamma}\mathbb{E}\left[(-\boldsymbol{g}_0 + \boldsymbol{G}\Delta\boldsymbol{\xi} - \underline{\boldsymbol{v}})^+\right] \le 0, \\
& \overline{\boldsymbol{v}} + \frac{1}{\gamma}\mathbb{E}\left[(\boldsymbol{g}_0 - \boldsymbol{G}\Delta\boldsymbol{\xi} - \overline{\boldsymbol{g}} - \overline{\boldsymbol{v}})^+\right] \le 0.
\end{aligned}$$

The risk-sensitive locational marginal prices (risk-LMPs): $\boldsymbol{\pi} = \lambda\mathbb{1} - \boldsymbol{H}^\top\boldsymbol{\mu}$.

Properties of the risk-sensitive ED prices

- The risk-sensitive locational marginal prices (risk-LMPs), $\boldsymbol{\pi} = \lambda \mathbb{1} - \boldsymbol{H}^\top \boldsymbol{\mu}$, satisfy
  - Equal to marginal sensitivity of optimal cost to nodal demands.
  - Varies with congestion through $\boldsymbol{\mu}$.
  - Uniform under no congestion.
  - Equal to classical LMP for deterministic wind.
- The payment scheme for players at bus i is
  - Demander pays $\pi_i d_i$.
  - Wind producer paid $\pi_i \mathbb{E}[\omega_i] - \nu_i$.
  - Dispatchable generator paid $\pi_i \bar{g}_i + \sum_{k=1}^{n} G_{ik} \nu_k$.
- Merchandising surplus $\geq \boldsymbol{\mu}^\top \boldsymbol{f} +$ risk-dependent term.

Risk-LMPs generalize traditional LMPs to risk-sensitive case.



Preliminary Results

- Created a performant, open-source C++ implementation of PDSS algorithm. Available at: `https://github.com/amadavan/Stuka`.
- Sample-average approximation approach can solve risk-sensitive problem with good accuracy while PDSS is slow but has strong theoretical guarantees.

Future work

- Leverage power systems problem structure to improve runtime of the PDSS algorithm.
- Evaluate pricing under model with uncertain wind and unit commitment.
- Consider a distributed variant and extend the i.i.d. sampling model of PDSS to the Markov case.

Avinash N. Madavan and Subhonmesh Bose (2021). *A Stochastic Primal-Dual Method for Optimization with Conditional Value at Risk Constraints*. arXiv: 1908.01086 [math.OC]

Avinash N. Madavan and Subhonmesh Bose (2019). "Risk-Sensitive Energy Procurement with Uncertain Wind". In: *2019 IEEE Global Conference on Signal and Information Processing (GlobalSIP)*, pp. 1–5

Mariola Ndrio, Avinash N. Madavan, and Subhonmesh Bose (2021). "Conditional-Value-at-Risk-Sensitive Locational Marginal Pricing for Electricity Markets". In: *2021 IEEE Power & Energy Society General Meeting (PESGM)*. IEEE

Dispatching for distribution networks

- Volt-VAR control (VVC) seeks to determine optimal control actions for voltage regulating and VAR control devices.
- Traditionally, VVC uses models of the network. However, network models are notoriously unreliable for distribution networks due to size and poor record-keeping.
- Model-free methods, such as reinforcement learning (RL), offer a solution for this problem.
- There are two important considerations when applying RL to power systems decision-making:
  - The applied policy must satisfy the constraints of the grid, such as line flow capacity limits, and resource limits.
  - The policy will be trained on a faulty or simulated model and must be able to provide strong out-of-sample guarantees.

Applying a simulator-trained model to the real world

- Safe RL studies methods to avoid risky rewards or satisfy constraints.
  - Risky policy avoidance: Gaskett 2003; Heger 1994 (worst-case), Basu, Bhattacharyya, and Borkar 2008; Borkar 2001; 2002; Howard and Matheson 1972; Kashima 2007 (risk-sensitive)
  - Constraint satisfaction: Di Castro, Tamar, and Mannor 2012; Kadota, Kurano, and Yasuda 2006; Moldovan and Abbeel 2012
- Evaluation of the optimal policy typically requires exploration over adverse states, the effect of which can be mitigated through a simulator.
- Robust RL seeks to establish strong out-of-sample guarantees when the transition kernel is not reflective of the true system (ex. from a simulator). Iyengar 2005; Nilim and El Ghaoui 2005; Wiesemann, Kuhn, and Rustem 2013
- Robust RL, distributionally robust optimization (DRO) and CVaR are closely related.

$$\text{Robust RL} \longrightarrow \text{DRO} \longleftarrow \mathrm{CVaR}\text{-optimization}$$

- Based on the literature behind safe RL and robust RL, we expect that these notions can be combined using primal-dual analysis and DRO.

Summary of presentation

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- **How** do we formulate, optimize, and price risk-sensitive models?
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Thank you!

amadavan.github.io

Appendix: CRE algorithm stated in full

Algorithm 1: A critical region exploration algorithm for R-SCED.

Initialization: Choose $\boldsymbol{x}_0 \in \mathbb{X}_0$, $J^* = \infty$, $\mathbb{D} = \emptyset$ and a small positive ϵ.

1 **while** $\boldsymbol{v}^* \neq 0$ **do**
2   Given $\boldsymbol{x}^0$, compute $\boldsymbol{\rho}_k, \eta_k, \mathbb{C}_k$ for $k = 1, \ldots, K$.
3   Solve main problem over current critical region.
4     $\left[\boldsymbol{x}^0\right]^{\text{opt}} \leftarrow$ lexicographically smallest minimizer of step 3
5     $J^{\text{opt}} \leftarrow$ optimal cost of step 3
6   **if** $J^{opt} < J^*$ **then**
7         $\left[\boldsymbol{x}^0\right]^* \leftarrow \left[\boldsymbol{x}^0\right]^{\text{opt}}, J^* \leftarrow J^{\text{opt}}, \mathbb{D} \leftarrow \{\boldsymbol{c}\}$
8   **else**
9         $\mathbb{D} \leftarrow \mathbb{D} \cup \{\boldsymbol{c}^0 + \alpha' \sum_{k=1}^{K} \boldsymbol{\rho}_k\}$
10     $\boldsymbol{v}^* \leftarrow \operatorname{argmin}_{\boldsymbol{v} \in \operatorname{conv}(\mathbb{D}) + N_{\mathbb{X}^0}\left([\boldsymbol{x}^0]^*\right)} \|\boldsymbol{v}\|^2$
11     $\boldsymbol{x}^0 \leftarrow \left[\boldsymbol{x}^0\right]^{\text{opt}} - \epsilon \boldsymbol{v}^*$

Appendix: PDSS algorithm stated in full

Algorithm 2: Primal-dual stochastic subgradient method for $\mathcal{P}^{\mathrm{E}}$.

Initialization: Choose $\boldsymbol{x}_1 \in \mathbb{X}$, $\boldsymbol{z}_1 = 0$, and a positive sequence γ.

1 **for** $k \geq 1$ **do**

2 Sample $\omega_k \in \Omega$. Update $\boldsymbol{x}$ as

$$\boldsymbol{x}_{k+1} \leftarrow \underset{\boldsymbol{x}\in\mathbb{X}}{\operatorname{argmin}} \left\langle \nabla f_{\omega_k}(\boldsymbol{x}_k) + \sum_{i=1}^{m} z_k^i \nabla g_{\omega_k}^i(\boldsymbol{x}_k), \boldsymbol{x} - \boldsymbol{x}_k \right\rangle + \frac{1}{2\gamma_k} \|\boldsymbol{x} - \boldsymbol{x}_k\|^2 . \quad (1)$$

3 Sample $\omega_{k+1/2} \in \Omega$. Update $\boldsymbol{z}$ as

$$\boldsymbol{z}_{k+1} \leftarrow \underset{\boldsymbol{z}\in\mathbb{R}_+^m}{\operatorname{argmax}} \left\langle \boldsymbol{g}_{\omega_{k+1/2}}(\boldsymbol{x}_{k+1}), \boldsymbol{z} - \boldsymbol{z}_k \right\rangle - \frac{1}{2\gamma_k} \|\boldsymbol{z} - \boldsymbol{z}_k\|^2. \quad (2)$$