

**THE MILLIMETER-WAVELENGTH SULFUR DIOXIDE  
ABSORPTION SPECTRA MEASURED UNDER  
SIMULATED VENUS CONDITIONS**

A Thesis  
Presented to  
The Academic Faculty

by

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SIMULATED VENUS CONDITIONS**

Approved by:

Professor Paul G. Steffes, Advisor  
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Date Approved: TBD

*To myself,*

*Perry H. Disdainful,*

*the only person worthy of my company.*

## PREFACE

Theses have elements. Isn't that nice?

## ACKNOWLEDGEMENTS

I want to “thank” my committee, without whose ridiculous demands, I would have graduated so, so, very much faster.

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## SUMMARY

# CHAPTER I

## INTRODUCTION

Active and passive microwave remote sensing techniques have been extensively used in the study of our sister planet, Venus. Unlike Earth's atmosphere, the Venus atmosphere is mostly comprised of gaseous carbon dioxide ( $\text{CO}_2$ ).  $\text{CO}_2$  comprises 96.5% of the atmosphere along with gaseous nitrogen ( $\text{N}_2$ ) at about 3.5%. The Venus atmosphere has multiple trace constituents such as sulfur dioxide ( $\text{SO}_2$ ), carbon monoxide ( $\text{CO}$ ), water vapor ( $\text{H}_2\text{O}$ ), carbonyl sulfide ( $\text{OCS}$ ), and sulfuric acid vapor ( $\text{H}_2\text{SO}_4$ ) [?].

Two sulfur-bearing compounds dominate the millimeter-wave emission from Venus: sulfur dioxide ( $\text{SO}_2$ ) and gaseous sulfuric acid ( $\text{H}_2\text{SO}_4$ ). At higher pressures  $\text{H}_2\text{SO}_4$  thermally dissociates, forming  $\text{H}_2\text{O}$  and  $\text{SO}_2$ , both of which exhibit relatively small amounts of microwave absorption at the abundance levels present in the Venus atmosphere. Thus, in the deep atmosphere, only  $\text{SO}_2$  and  $\text{CO}_2$  have the potential to affect the observed microwave emission.

Utilizing the millimeter-wavelength system at the Planetary Atmospheres Laboratory at Georgia Institute of Technology it is possible to simulate the upper troposphere of Venus and take precision measurements of the millimeter-wavelength properties of sulfur dioxide. Using the measurements, a model can be created that accurately predicts the opacity of sulfur dioxide in the Venus atmosphere. This model will make it possible to determine the source of variations in the Venus millimeter-wavelength emission, such as were observed by Sagawa [?].

*1.1 Background and Motivation*

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## CHAPTER II

### THEORETICAL BASIS AND PREVIOUS WORK

*2.1 Physical Structure of  $SO_2$*

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## CHAPTER III

### EXPERIMENT DESIGN, THEORY, AND RESULTS

Verifying millimeter-wavelength absorption spectrum of SO<sub>2</sub> is important for the study of the atmosphere of Venus. Making measurements under simulated Venus conditions assures the accuracy of any model derived from such measurements. Described below is the theory, laboratory equipment, measurement procedure and derived uncertainties in the measurements of the millimeter-wavelength absorptivity of gaseous sulfur dioxide under simulated Venus conditions.

#### *3.1 Measurement Theory*

In this experimental program, quality factor (Q) of a resonant mode of a resonator is used to measure the absorption of a gas or gas mixture [?]. The quality factor of a resonance is given by [?]

$$Q = \frac{2\pi f_0 \times \text{Energy Stored}}{\text{Average Power Loss}} \quad (3.1)$$

where  $f_0$  is the resonant frequency. The Q of a resonance can be measured directly from  $f_0$  by dividing it by its half-power bandwidth (HPBW).

$$Q = \frac{f_0}{HPBW} \quad (3.2)$$

The Q of a lossy gas ( $\epsilon'/\epsilon''$ ) and its opacity are related by

$$\alpha \approx \frac{\epsilon''\pi}{\epsilon'\lambda} = \frac{1}{Q_{gas}} \frac{\pi}{\lambda} \quad (3.3)$$

where  $\epsilon'$  and  $\epsilon''$  are the real and imaginary permittivity of the gas,  $\lambda$  is the wavelength in km, and  $\alpha$  is the absorptivity of the gas in Nepers/km (1 Neper = 8.686 dB). Since

Q can be affected by more than just the gas added, the Q of the gas-filled resonator is given by

$$\frac{1}{Q_{loaded}^m} = \frac{1}{Q_{gas}} + \frac{1}{Q_r} + \frac{1}{Q_{ext1}} + \frac{1}{Q_{ext2}} \quad (3.4)$$

where  $Q_{loaded}^m$  is the measured quality factor of a resonance in the presence of a test gas,  $Q_{gas}$  is the quality factor of the gas under test,  $Q_r$  is the quality factor of the resonator in the absense of coupling losses, and  $Q_{ext1}$  and  $Q_{ext2}$  are the external coupling losses. Since the resonator used is symmetric, it is safe to assume  $Q_{ext1} = Q_{ext2}$ . Coupling losses can be derived from the transmissivity  $t = 10^{-S/10}$ , where  $S$  is the measured insertion loss of the resonator in decibels (dB) at the frequency of a particular resonance using the following relationship

$$t = \left[ w \frac{Q^m}{Q_{ext}} \right]^2, \quad (3.5)$$

$$Q_{ext} = \frac{2Q^m}{\sqrt{t}} \quad (3.6)$$

$Q_r$  is related to the measured Q at a vacuum by

$$\frac{1}{Q_{vac}^m} = \frac{1}{Q_r} + \frac{1}{Q_{ext1}} + \frac{1}{Q_{ext2}} \quad (3.7)$$

where  $Q_{vac}^m$  is the measured Q at a vacuum. Substituting equation ?? into equations ?? and ?? gives

$$\frac{1}{Q_{gas}} = \frac{1 - \sqrt{t_{loaded}}}{Q_{loaded}^m} - \frac{1 - \sqrt{t_{vac}}}{Q_{vac}^m} \quad (3.8)$$

where  $t_{loaded}$  and  $t_{vac}$  are the transmissivity of the resonance taken in loaded and vacuum conditions respectively. When gas is added to the resonator there is a shift in the center frequency corresponding to the refractive index of the test gas. Since the quality factor is reliant on the center frequency this will affect the comparison

between the two measurements. This effect is called dielectric loading [?]. Dielectric matching can be achieved by performing additional measurements of the quality factor with a lossless gas present. Adding the lossless gas shifts the center frequency of the resonances, and by adding more or less gas the center frequency can be adjusted to be exactly the same as the lossy gas. These measurements are used in place of the vacuum measurements in equation ?? and by converting Nepers/km to dB/km we can rewrite equation ?? as

$$\alpha = 8.686 \frac{\pi}{\lambda} \left( \frac{1 - \sqrt{t_{loaded}}}{Q_{loaded}^m} - \frac{1 - \sqrt{t_{matched}}}{Q_{matched}^m} \right) dB/km \quad (3.9)$$

### ***3.2 Millimeter-Wavelength Measurement System***

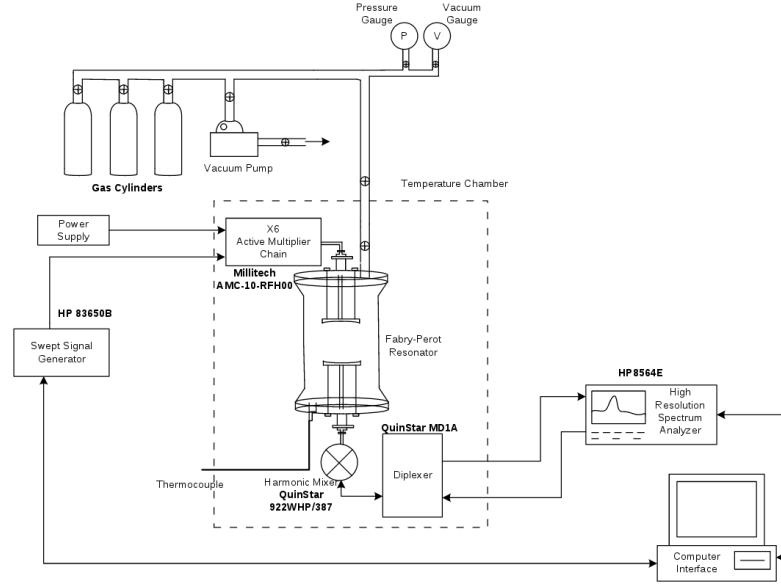
The high-sensitivity millimeter-wavelength system used for measuring the opacity of gaseous sulfur dioxide under Venus conditions is similar to the one used by Devaraj and Steffes [?] [?]. The system is comprised of two subsystems for measuring different bands of the millimeter-wavelength spectrum (W-band/F-band). The simulator consists of a glass pressure chamber capable of withstanding up to 3 bars of pressure along with a temperature chamber capable of operating up to 400K. The W-band subsystem is used for measurements in the 3-4 millimeter-wavelength range while the F-band system is used for the 2-3 millimeter-wavelength range. The following sections describe each subsystem and their components.

#### **3.2.1 W-band Subsystem**

The W-band measurement system is used to measure the 3-4 mm-wavelength properties of sulfur dioxide is shown in figure ??.

A synthesized swept signal generator (HP 83650B) is used to generate a signal in the 12.5-18.3 GHz range which is fed through a times-six active multiplier chain (AMC) via low-loss, high frequency coaxial cables. The radio frequency (RF) signal

from the output port of the Fabry-Perot resonator (FPR) is fed to a QuinStar Technology QMH series harmonic mixer. The local oscillator (LO) and the intermediate frequency (IF) are connected via an external diplexer. The harmonic mixer is locked to the 18th harmonic of the spectrum analyzer LO and is used in the “external mixer” mode with the spectrum analyzer (HP 8564E).



**Figure 3.1:** Block diagram of the W band measurement system. Solid lines represent the electrical connections and the arrows show the direction of the signal propagation. Valves controlling the flow of gasses are shown by small crossed circles.

### 3.2.2 F-band Subsystem

The F-band measurement system is used to measure the 2-3 mm-wavelength properties of sulfur dioxide and is shown in figure ??.

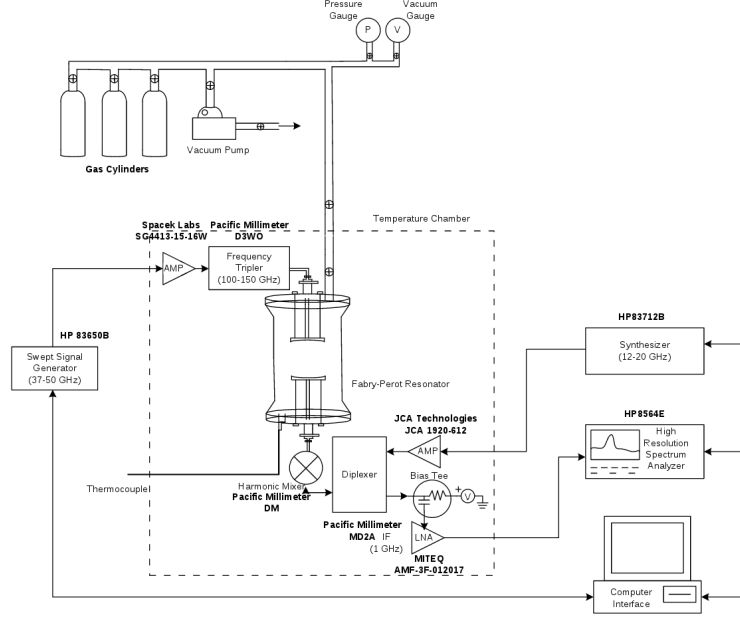
The swept signal generator (HP 83650B) is used to generate a signal in the 33-50 GHz range which is amplified and fed through a frequency tripler. The output of the tripler is fed to the input end of the FPR. The RF signal from the output port of the FPR is fed to a harmonic mixer which can operate with an LO frequency as high



as 18 GHz. An external diplexer is used to combine the IF and LO signals. For a particular RF and IF frequency, the LO frequency can be computed using

$$f_{LO} = \frac{f_{RF} - f_{IF}}{N_H} \quad (3.10)$$

where  $N_H$  is the lowest integer such that  $f_{lo} < 18GHz$ .



**Figure 3.2:** Block diagram of the F band measurement system. Solid lines represent the electrical connections and the arrows show the direction of the signal propagation. Valves controlling the flow of gasses are shown by small crossed circles.

### 3.3 Data Handling Subsystem

The data acquisition system consists of a computer connected to the spectrum analyzer (HP 8564E), swept signal generator (HP 83650B), and continuous wave (CW) signal generator (HP 83712B, the local oscillator for the F-Band system) via a general purpose interface bus (GPIB). The instruments are controlled via Matlab script and their appropriate programming language. The software used is similar to Devaraj and Steffes [?] [?] with modifications for equipment changes.

### ***3.4 Measurement Procedure***

The most important prerequisite for performing measurement of gas properties is ensuring a leak-proof system. This is done through two methods, the first is by drawing a vacuum inside the FPR and verifying the integrity of the vacuum over time. The second way is by adding a positive pressure of CO<sub>2</sub> to the system and making sure there are no leaks in any of the connectors and valves. Ensuring a leak-proof system allows for not only precise measurements but also ensures no toxic gases are released into the testing environment.

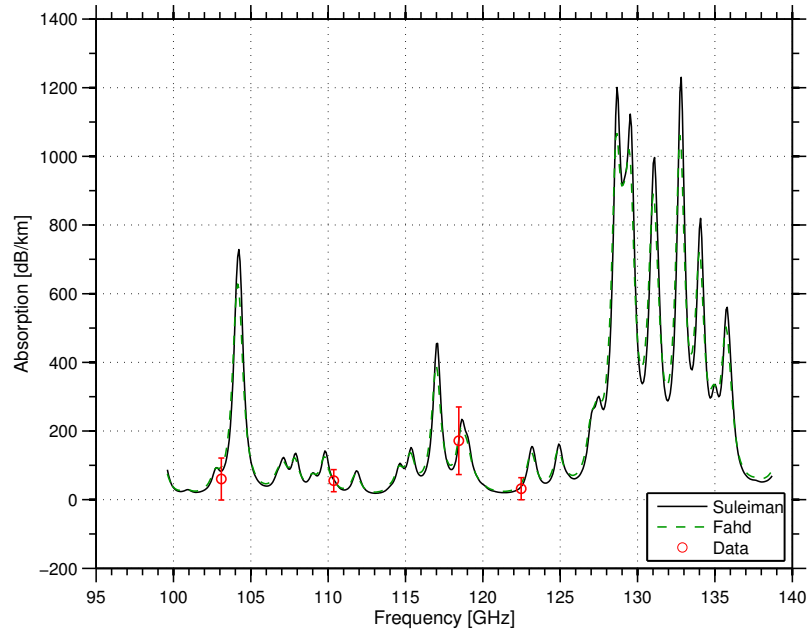
After the system is ensured to be leak-proof and at a stable temperature, a vacuum is drawn and a measurement is taken using the appropriate subsystem (W-band for 3-4 mm-wavelengths, F-band for 2-3 mm-wavelengths). This allows for a base line measurement of the FPR's resonances and the Quality factor. Once this baseline is established the gas under test is added to the system.

Once the gas temperature has stabilized, another set of tests measuring the resonant frequencies along with the quality factors is taken. More gas is added and the procedure is repeated until all suitable pressures are taken. A vacuum is drawn once again but this time it is pumped overnight due to the gas being tested (SO<sub>2</sub>) and its properties of "sticking" (or adsorbing) to metal. Another vacuum measurement is taken to measure any possible system error.

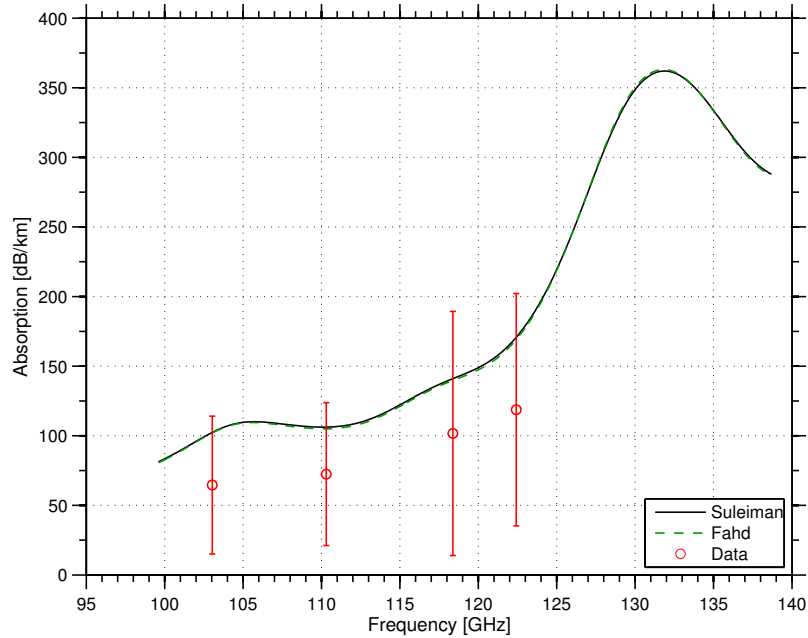
Once the second vacuum measurements are taken CO<sub>2</sub> is then added to the chamber until the resonances are matched to the same frequency of our test gas. Again measurements are taken and this is repeated for every pressure of the test gas. Once this is finished a vacuum is again drawn and another test is taken.

Lastly the system is set up for a transmissivity test where we measure  $t$  (equation ??) for each given resonant frequency. The system is then set back up and is ready for a new test.

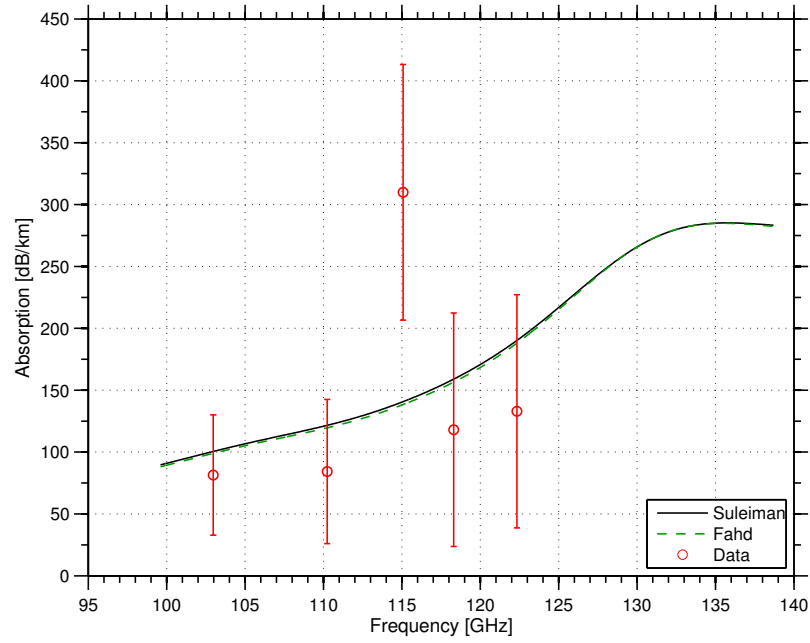
### ***3.5 Experimental Results***



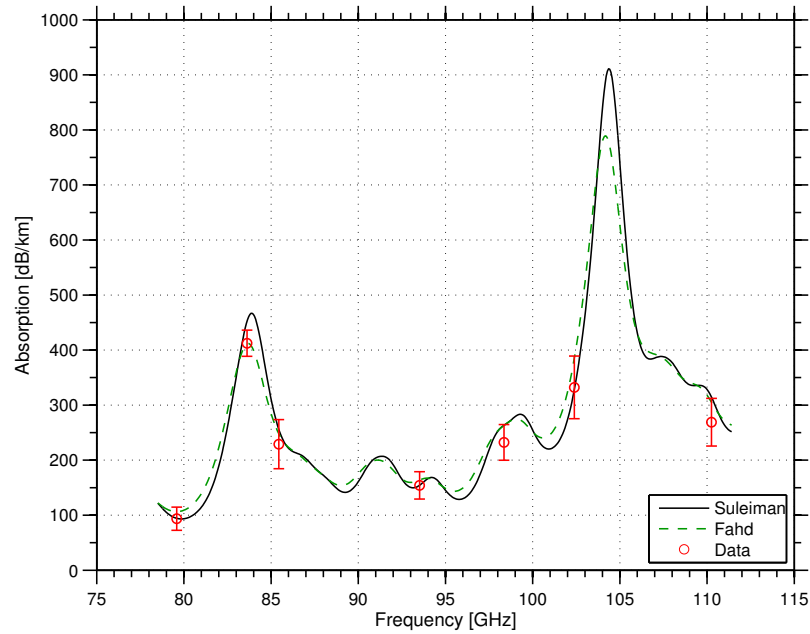
**Figure 3.3:** Opacity data using the 2-3 mm-wavelength system for a mixture of  $\text{SO}_2 = 84.7\%$  ,  $\text{CO}_2 = 0\%$  at a pressure of 0.030 bar and a temperature of 308.8 K compared to various models



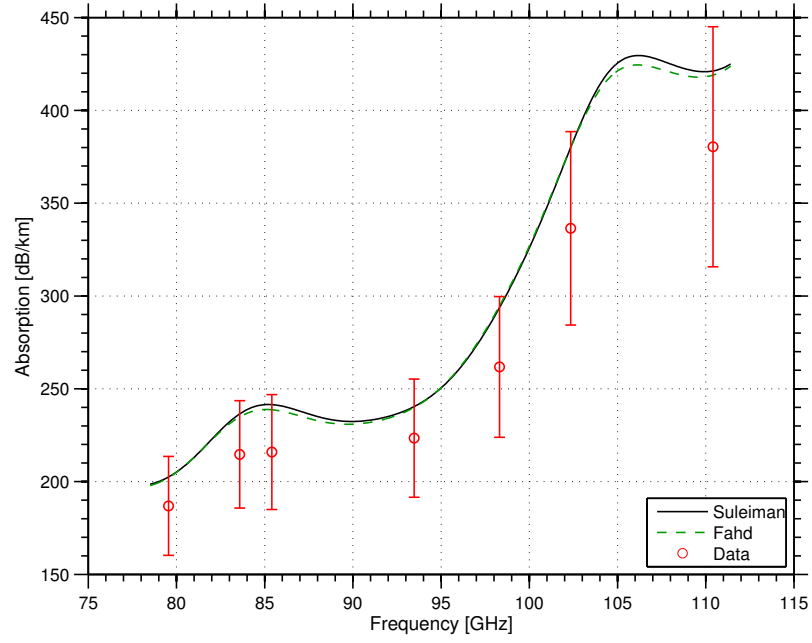
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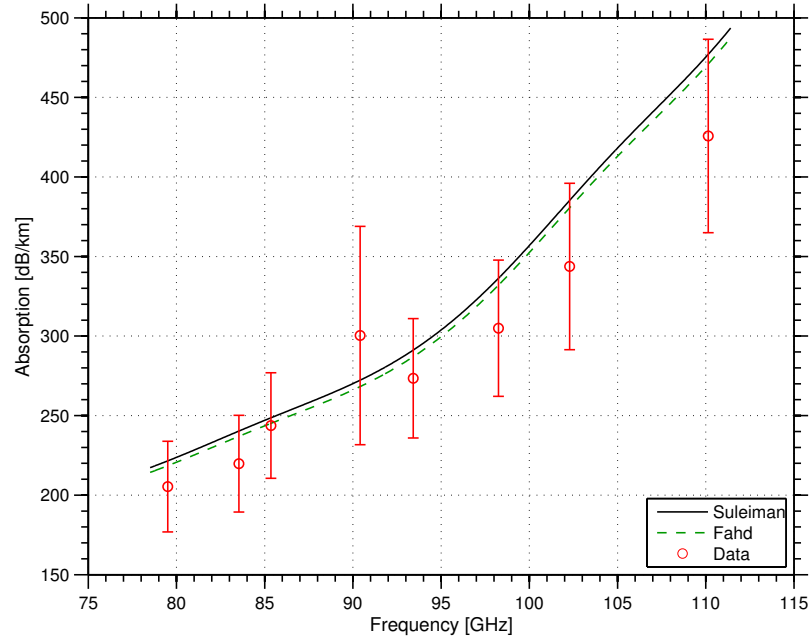
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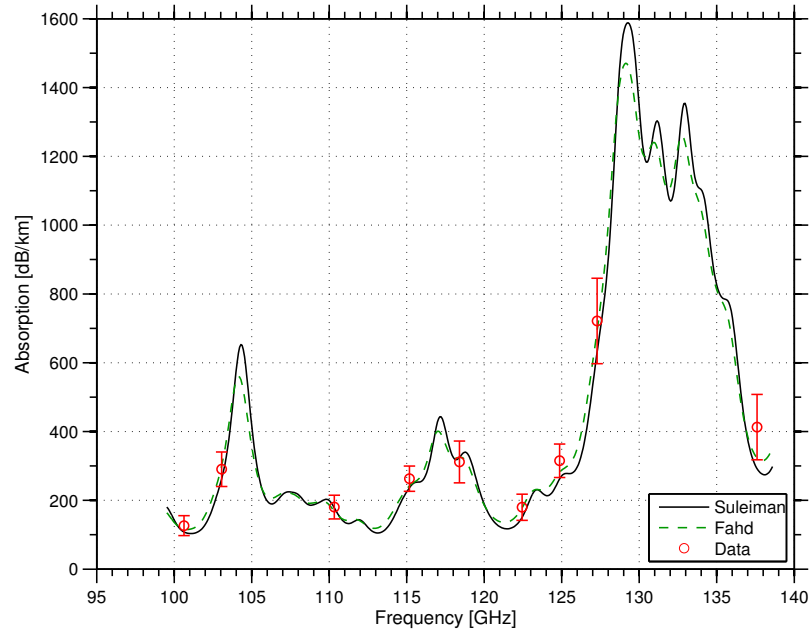
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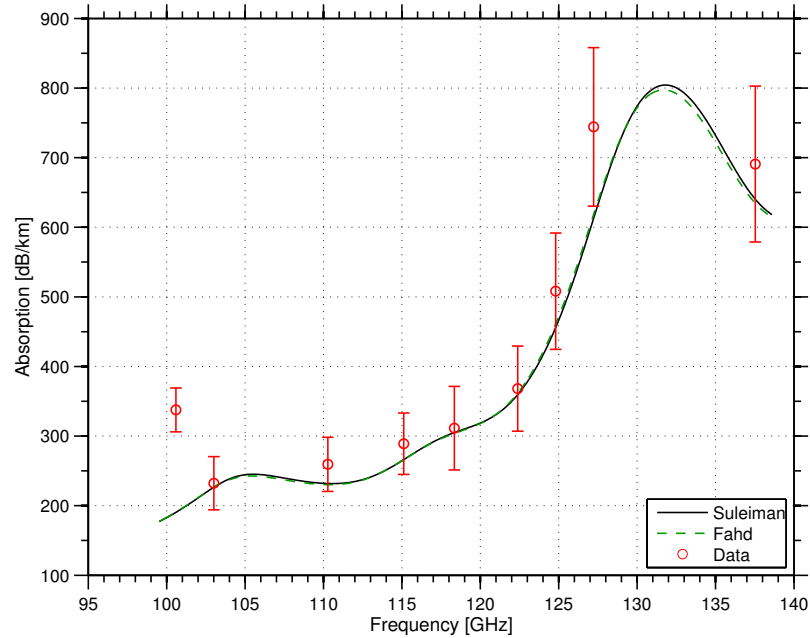
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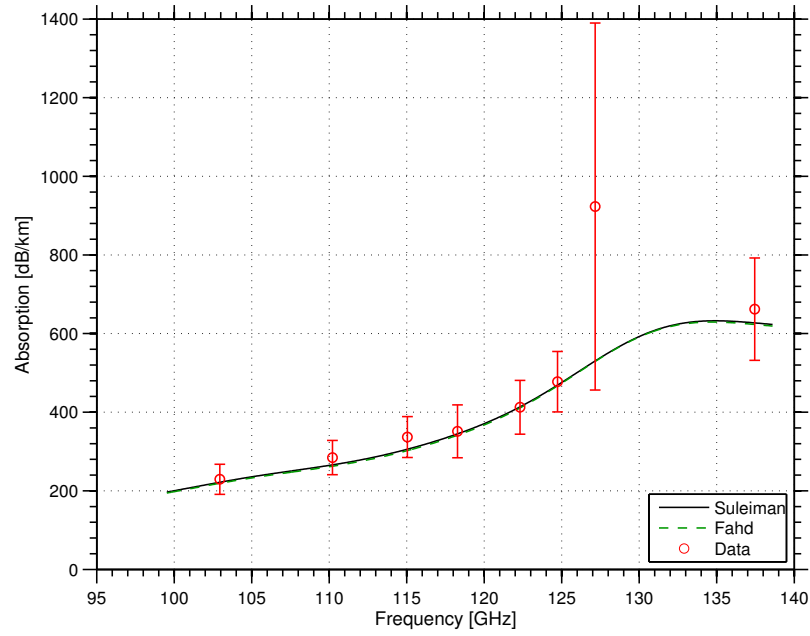
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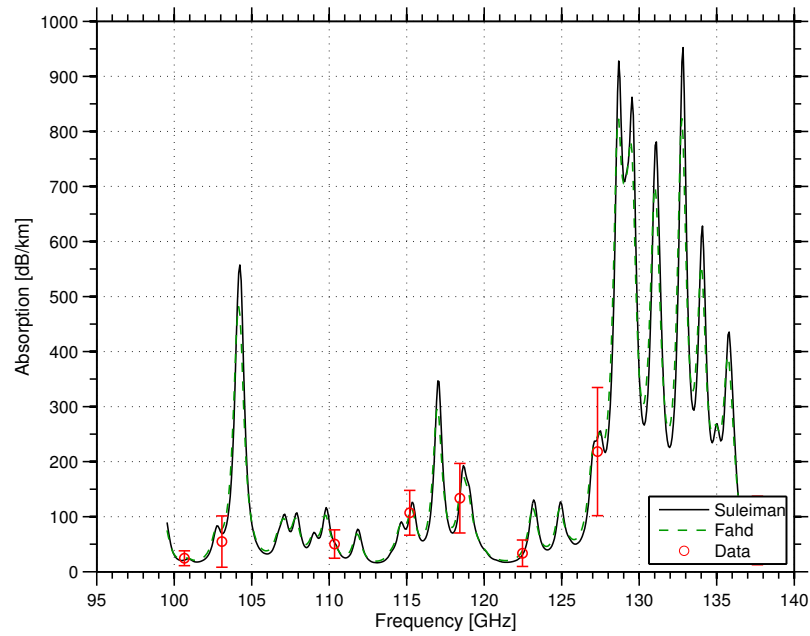
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**Figure 3.10:** Opacity data using the 2-3 mm-wavelength system for a mixture of  $\text{SO}_2 = 8.3\%$  ,  $\text{CO}_2 = 90.2\%$  at a pressure of 0.923 bar and a temperature of 344.6 K compared to various models

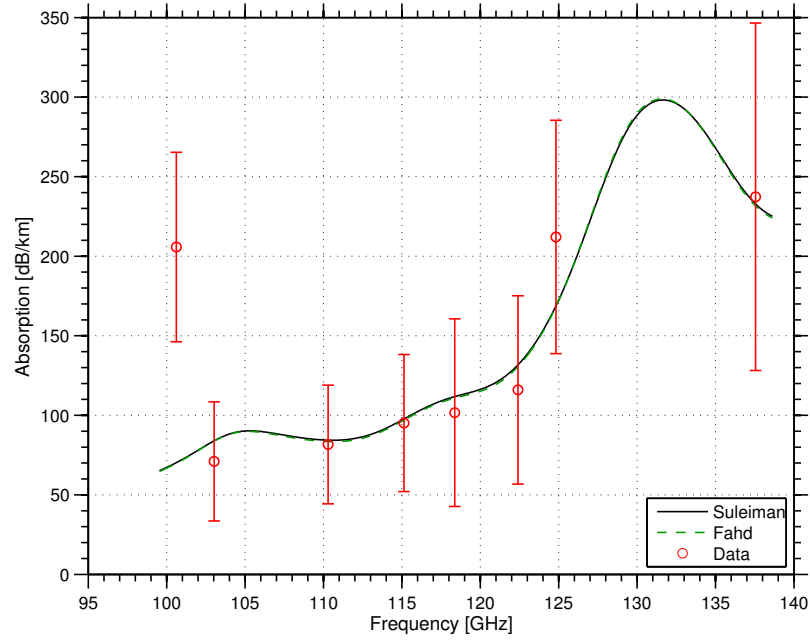


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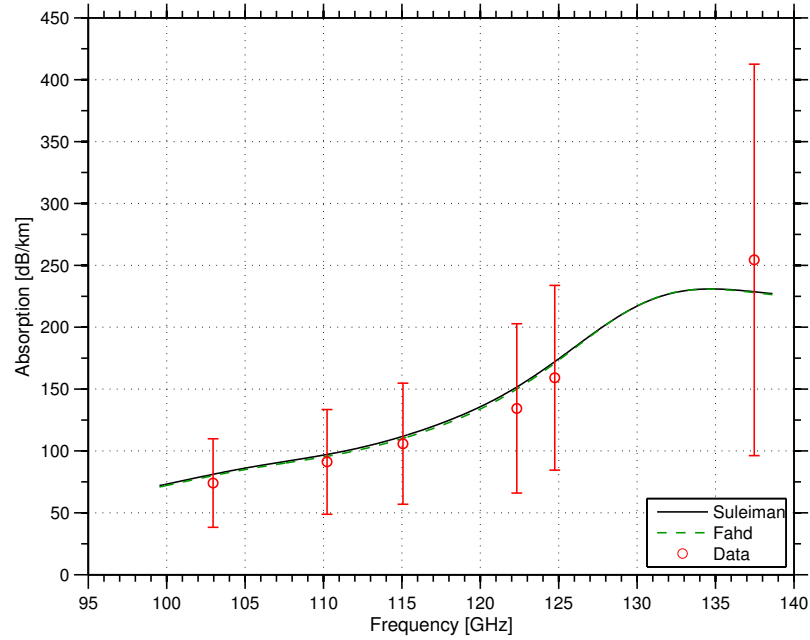


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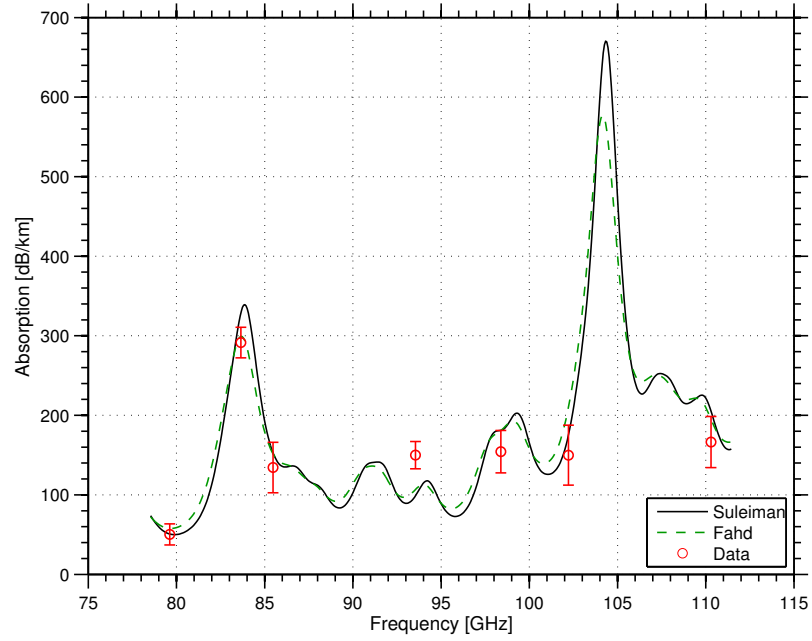




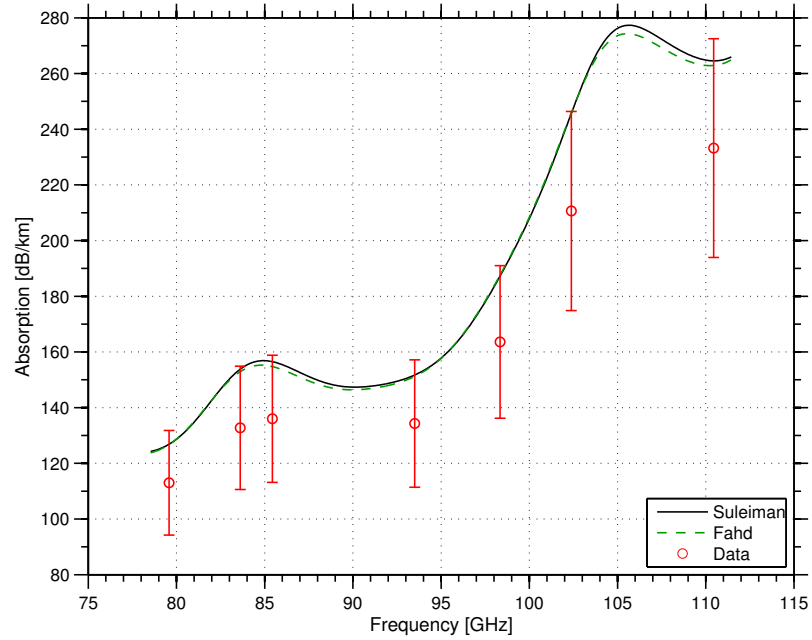
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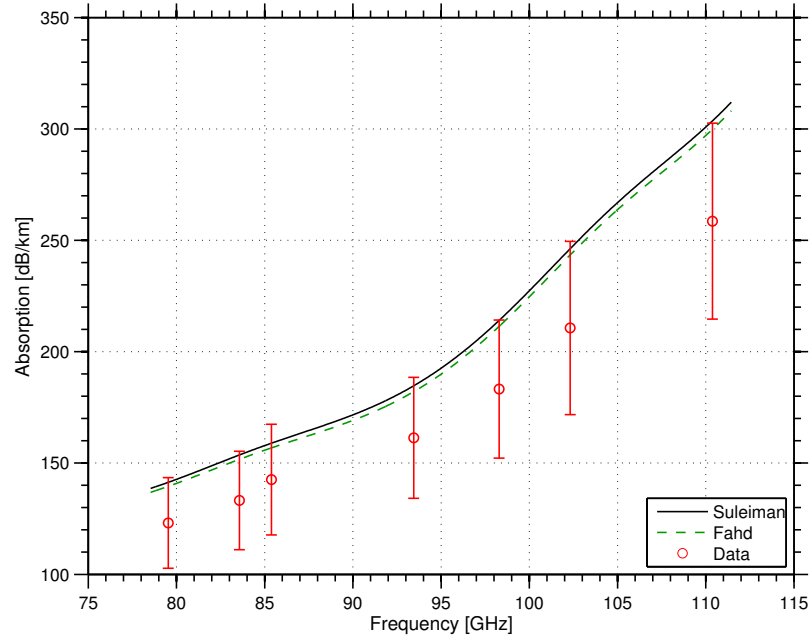
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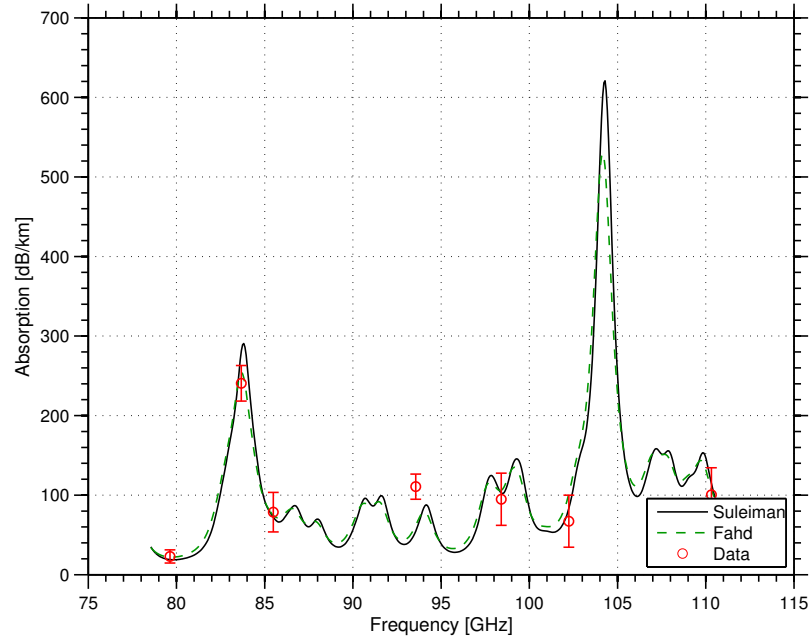
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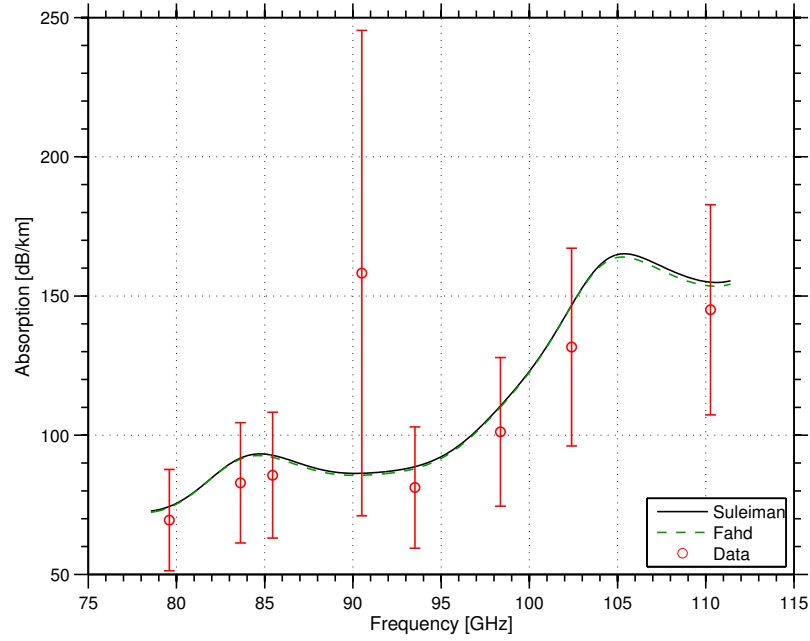
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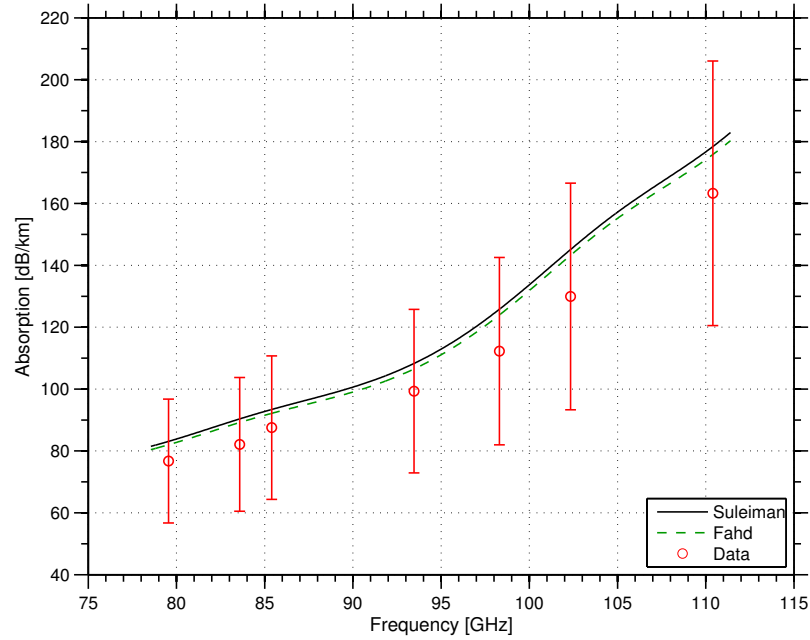
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**Figure 3.18:** Opacity data using the 3-4 mm-wavelength system for a mixture of  $\text{SO}_2 = 84.7\%$  ,  $\text{CO}_2 = 0\%$  at a pressure of 0.060 bar and a temperature of 343.1 K compared to various models



**Figure 3.19:** Opacity data using the 3-4 mm-wavelength system for a mixture of  $\text{SO}_2 = 5.5\%$  ,  $\text{CO}_2 = 93.5\%$  at a pressure of 0.927 bar and a temperature of 343.6 K compared to various models



**Figure 3.20:** Opacity data using the 3-4 mm-wavelength system for a mixture of  $\text{SO}_2 = 2.5\%$  ,  $\text{CO}_2 = 97\%$  at a pressure of 2.004 bar and a temperature of 343.9 K compared to various models

## CHAPTER IV

### MODEL FITTING AND MODIFICATIONS

#### 4.1 *Measurement Uncertainties*

There are five uncertainties for any absorptivity measurements using the millimeter wavelength system: instrumentation errors and electrical noise ( $Err_{inst}$ ), errors in dielectric matching ( $Err_{diel}$ ), errors in transitivity measurement ( $Err_{trans}$ ), errors due to resonance asymmetry ( $Err_{asym}$ ), and errors in measurement conditions ( $Err_{cond}$ ) resulting from uncertainties in temperature, pressure, and mixing ratio. The term  $Err$  is used for representing uncertainties instead of the more frequently used  $\sigma$  to avoid confusion between  $1\sigma$ ,  $2\sigma$ , and  $3\sigma$  uncertainties.

Instrumental errors and electrical noise are caused due to the sensitivity of the electrical devices and their ability to accurately measure bandwidth ( $BW_{measured}$ ) and the center frequency ( $f_o$ ). Electrical noise arises from the frequency references and the noise of the internal electronics. Since electrical noise is uncorrelated, it's best estimate of the uncertainty is the mean of multiple measurements. The variance of the best error estimate is given by the sample variance ( $S_N^2$ ) weighted by the confidence coefficient ( $B$ ) as

$$\sigma_N^2 = B \frac{S_N^2}{N_{samples}} \quad (4.1)$$

where  $N_{samples}$  is the number of independent measurements of the sample. For the millimeter-wavelength system, five sets of independent measurements of each resonance are taken. A confidence coefficient ( $B$ ) of 2.776 is used. This corresponds to the 95% confidence interval. The center frequency standard deviation is very small and its effect on the uncertainty in  $Q$  is negligible. Therefore,  $S_N$  is the sample standard deviation of the bandwidth of the measurements.

The HP 8564E spectrum analyzer is used for measuring the resonances in the millimeter-wavelength system. It's manufacturer specified instrumental uncertainties are the  $3\sigma$  values [?]. The  $3\sigma$  standard deviation for the center frequency and bandwidth are estimated by

$$Err_o \leq \pm(f_o \times f_{ref\ acc} + 0.05 \times SPAN + 0.15 \times RBW + 10)(Hz) \quad (4.2)$$

$$Err_{BW} \leq \pm(BW_{measured} \times f_{ref\ acc} + 4 \times N_H + 2 \times LSD)(Hz) \quad (4.3)$$

where  $f_{ref\ acc}$  is given as

$$f_{ref\ acc} = (aging \times time\ since\ calibration) + inital\ achievable\ accuracy + temperature\ stability \quad (4.4)$$

and  $f_o$ , SPAN, RBW,  $N_H$ , and LSD are the center frequency, frequency span, resolution bandwidth, harmonic number, and least significant digit of the bandwidth measurement, respectively. LSD is calculated as  $LSD = 10^x$  for the smallest positive integer value of  $x$  such that  $SPAN < 10^{x+4}$ . For  $SPAN \leq 2\text{ MHz} \times N_h$ , the SPAN multiplication factor of 0.05 is replaced with 0.01. For the spectrum analyzer used,  $f_{ref\ acc}$  reduces to

$$f_{ref\ acc} = (10^{-7} \times years\ since\ calibrated) + 3.2 \times 10^{-8} \quad (4.5)$$

The worst case scenario is used to transform the uncertainty in center frequency and bandwidth for both loaded and dielectrically matched measurements into an uncertainty in absorptivity as described in DeBoer and Steffes [?].

$$Err_{\Psi}^2 = \langle F_l^2 \rangle + \langle F_m^2 \rangle - \langle F_l F_m \rangle \quad (4.6)$$

where

$$\langle F_i^2 \rangle = \frac{\Upsilon_i^2}{f_{oi}^2} \left[ \frac{Err_o^2}{Q_l^2} + Err_{BW}^2 + Err_{Ni}^2 + \frac{2Err_o Err_{BW}}{Q_i} \right], i = l, m \quad (4.7)$$

$$\langle F_l F_m \rangle = -\frac{\Upsilon_l \Upsilon_m}{f_{ol} f_{om}} \left[ \frac{Err_o^2}{Q_i Q_m} + Err_{BW}^2 + \frac{Err_o Err_{BW}}{Q_l} + \frac{Err_o Err_{BW}}{Q_m} \right] \quad (4.8)$$

$$Q_i = \frac{f_{oi}}{f_{BW_i}}, i = l, m \quad (4.9)$$

$$\Upsilon_i = 1 - \sqrt{t}, i = l, m \quad (4.10)$$

where  $l, m$  denote loaded and dielectrically matched cases, respectively and  $f_{ol,om}$  and  $f_{BWl,BWm}$  represent center frequency and bandwidth of loaded and dielectrically matched cases respectively. The  $2\sigma$  uncertainty of the measured gas absorption due to instrumental errors and electrical noise is given by

$$Err_{inst} = \pm \frac{8.686\pi}{\lambda} Err_{\Psi} (dB/km) \quad (4.11)$$

where  $\lambda$  is the wavelength in km.

Errors in dielectric matching occur when the when the center frequency of the matched measurements are not precisely aligned with the center frequency of the loaded measurement. Since the Q of the resonator can vary slightly this causes an uncertainty in the Q of the matched measurement at the true center frequency of the loaded measurement. The method used to calculate the magnitude of this effect is similar to Devaraj [?]. While this error is the most trivial due to the high precision of the software controlled matching it is important to calculate and account for. The magnitude of this effect is calculated by comparing the Q of the three vacuum measurements to that of the dielectric matched measurements

$$\left( \frac{dQ}{df} \right)_i = \left| \frac{Q_{vac,i} - Q_{matched,i}}{f_{vac,i} - f_{matched,i}} \right| \text{ for } i = 1, 2, 3 \quad (4.12)$$

The maximum of the three values is used to calculate a  $dQ$  value

$$dQ = \left( \frac{dQ}{df} \right)_{max} \times |f_{loaded} - f_{matched}| \quad (4.13)$$

where  $f_{loaded}$  and  $f_{matched}$  are the center frequencies of the resonances under loaded and matched conditions. The error in absorbtivity due to imperfect dielectric matching

is then computed by propagating  $\pm dQ$  through Equation ??.

$$\begin{aligned}
Err_{diel} &= \frac{8.686\pi}{\lambda} \\
&\times \left| \left( \frac{1 - \sqrt{t_{loaded}}}{Q_{loaded}^m} - \frac{1 - \sqrt{t_{matched}}}{Q_{matched}^m + dQ} \right) - \left( \frac{1 - \sqrt{t_{loaded}}}{Q_{loaded}^m} - \frac{1 - \sqrt{t_{matched}}}{Q_{matched}^m - dQ} \right) \right| \\
&(dB/km)
\end{aligned} \tag{4.14}$$

Transmissivity errors are due to the uncertainties in the measurement amplitude. This is caused by loss in the millimeter-wavelength instruments (signal generators and spectrum analyzer), cables, adapters, and waveguides used in this system. Measuring this is done taking multiple tests of the system without the FPR and finding the standard deviation ( $S_N$ ) and weighing it by it's confidence coefficient

$$Err_{msl} = \frac{4.303}{\sqrt{3}} S_N \tag{4.15}$$

For the millimeter-wavelength system, the signal level measurements involve sampling the RF power with a WR-10 20 dB directional coupler to feed the harmonic mixer for down-conversion and detection. While this ensures that the input to the harmonic mixer does not exceed its maximum allowed input power of -10 dBm, the WR-10 does not uniformly sample the input signal throughout the entire frequency range. To compensate for this an additional 1.5 dB uncertainty is added to insertion loss error. The signal generator has a temperature stability of 1 dB/10° C, but an internal temperature equilibrium is reached after two hours [?]. Since the measurements units are stored at a constant temperature this uncertainty can be disregarded. The total uncertainty in insertion loss for the millimeter-wavelength system is calculated by

$$Err_{ins loss} = Err_{msl} + 1.5 (dB) \tag{4.16}$$

The error in insertion loss is used to compute the transmissivity error

$$Err_{t,i} = \frac{1}{2} (10^{-S_i - Err_{ins loss}} - 10^{-S_i + Err_{ins loss}}), i = l, m \tag{4.17}$$



where l,m are the loaded and matched cases, respectively, and S is the insertion loss of the resonator. This is used to compute the  $2\sigma$  uncertainties in opacity and is expressed as

$$Err_{trans} = \frac{8.686\pi}{2\lambda} \times \left| \left( \frac{\sqrt{t_l + Err_{t,l}} - \sqrt{t_l - Err_{t,l}}}{Q_{loaded}^m} - \frac{\sqrt{t_m - Err_{t,m}} - \sqrt{t_m + Err_{t,m}}}{Q_{matched}^m} \right) \right| \quad (dB/km). \quad (4.18)$$

Errors from asymmetry are due to the asymmetric nature of the resonances. These are more prominent at low temperatures and short wavelength. Errors due the asymmetry results from the disproportionate asymmetric broadening of the loaded measurements compared to the matched measurements. Equivalent full bandwidths based on assuming symmetry of the high and low sides of the resonances are calculated as

$$BW_{high} = 2 \times (f_{high} - f_{center}) \quad (4.19)$$

$$BW_{low} = 2 \times (f_{center} - f_{low}) \quad (4.20)$$

where  $BW_{high}$ ,  $BW_{low}$ ,  $f_{high}$ ,  $f_{center}$ , and  $f_{low}$  are the high bandwidth, low bandwidth, higher frequency half power point, center frequency, and lower frequency half power point, respectively. The difference between the opacities calculated using  $BW_{high}$  and  $BW_{low}$  is defined as  $Err_{asym}$  and is calculated by

$$Err_{asym} = \frac{8.686\pi}{\lambda} \times \left| \left( \frac{1 - \sqrt{t_{loaded}}}{Q_{loaded,high}^m} - \frac{1 - \sqrt{t_{matched}}}{Q_{matched,high}^m} \right) - \left( \frac{1 - \sqrt{t_{loaded}}}{Q_{loaded,low}^m} - \frac{1 - \sqrt{t_{matched}}}{Q_{matched,low}^m} \right) \right| \quad (dB/km) \quad (4.21)$$

Where  $Q_{matched,high/low}^m$  and  $Q_{loaded,high/low}^m$  are the measured Qs evaluated using the high and low bandwidths for loaded and matched cases.

The measured uncertainties in temperature, pressure, and concentration contribute to the total uncertainties due to the measurement conditions ( $Err_{cond}$ ). While this does not affect the measurements it still needs to be accounted for during the creation of the models for opacity based on experimental data. It is computed by

$$Err_{cond} = \sqrt{Err_{temp}^2 + Err_p^2 + Err_c^2 + Err_{leak}^2} (dB/km) \quad (4.22)$$

with  $Err_{temp}^2$ ,  $Err_p^2$ ,  $Err_c^2$ , and  $Err_{leak}$  representing the  $2\sigma$  uncertainties in the proposed opacity model corresponding to the uncertainties in temperature, pressure, concentration, and leakage.

Measuring temperature was done using a T type thermocoupler along with a Wavetek 23XT voltmeter. The voltmeter has a temperature accuracy of  $\pm(1\% + 2^\circ C)$ . Since the voltmeter has a Cold Compensation circuitry it is unnecessary to modify the temperature read from ambient. Also since a test takes an hour to run the temperature drift is insignificant. The uncertainty in temperature reading is calculated by

$$T = T_{read} \pm (T_{read} \times 1\% + 2) \quad (4.23)$$

Where  $T_{read}$  is the temperature readout from the Voltmeter.

Pressure was measured using an Omega DPG-7000 which has an accuracy of  $\pm 0.05\%FS$ . Since this pressure gauge measures pressure relative to ambient it is necessary to take a measurement before and after each test. The average change in pressure during a test was at most 2 mbar. The way a vacuum was measured was by comparing the Omega DPG-7000 reading to that of an absolute pressure gauge (Druck DPI 104). The Druck has an accuracy of  $\pm 0.05\%FS$  as well but a resolution of  $\pm 1$  mbar. The uncertainty in pressure reading is calculated by

$$P = P_{read} \pm (P_{FS} \times .05\% + 3) \quad (4.24)$$

Where  $P_{FS}$  is the Full Scale pressure of the Omega DPG-7000.

Since  $Err_{cond}$  is dependent on the opacity model, this uncertainty is maintained separately from  $Err_{tot}$ . Thus the total 95% confidence for the measurement uncertainty is expressed in dBkm as per Hanley [?]

$$Err_{tot} = \sqrt{Err_n^2 + Err_{diel}^2 + Err_{trans}^2 + Err_{asym}^2} \text{ (dB/km)}. \quad (4.25)$$

#### 4.1.1 Absorptivity

### 4.2 *Model Fitting Process*

### 4.3 *Suggested Model*

## CHAPTER V

# RADIATIVE TRANSFER MODEL OF THE VENUS ATMOSPHERE

One of the newest aspects of this research has been to model the microwave and millimeter-wave emission spectra from the surface of Venus and its atmosphere. This is accomplished using a radiative transfer equation. The radiative transfer model (RTM) computes the brightness temperature of Venus given a temperature-pressure profile, vertical abundance profiles, and the opacity formalism for the atmospheric constituents. The developed RTM is written in a modular way such that any input can be easily changed without changing other aspects. The RTM has the ability to do pencil-beam, disk average, and antenna patterns. This new model provides a more accurate representation of the microwave and millimeter-wave emission spectrum of Venus.

### *5.1 Theoretical Background*

The emission from the surface of Venus and its atmosphere can be computed using a Radiative Transfer Model (RTM). Radiative transfer is a method to solve for the distribution of electromagnetic energy in a medium. The two assumptions in RTM is that the solution for intensity (or brightness temperature) is along an infinitely thin beam (pencil beam). The second assumption is that the atmosphere is in local thermodynamic equilibrium (LTE). LTE assumes that for a given moment or snapshot in time the atmosphere is static; that is the model does not consider atmospheric dynamics when solving the radiative transfer equation. The differential form of the

radiative transfer equation is

$$dI_\nu = -\alpha I_\nu ds + \alpha J ds \quad (5.1)$$

where  $dI_\nu$  is the change in intensity at a given frequency  $\nu$ , over a path length  $ds$ ,  $\alpha$  is the absorption coefficient or attenuation over a path length  $ds$ , and  $J$  is the source function[?].

In the microwave and millimeter wave regime, effects from scattering approach the Rayleigh, which can be neglected without introducing significant error. Therefore the source function  $J$  becomes the Plank function.

$$J_\nu = B_\nu(T) = \frac{h\nu^3}{c^2} \frac{1}{\exp(\frac{h\nu}{kT}) - 1} \approx \frac{2kT\nu^2}{c^2} \quad (5.2)$$

where  $T$  is the temperature in Kelvin,  $h$  is Planck's constant,  $k$  is Boltzman's constant, and  $c$  is the speed of light. The approximation for equation ?? is for cases where  $h\nu \ll kT$  and is known as the Rayleigh-Jeans approximation. The brightness temperature of the black body can then be directly related to the radiation intensity

$$T_b(\nu) = \frac{\lambda}{2k} I_\nu \quad (5.3)$$

If equation ?? is integrated over the path  $s$  it becomes

$$I_\nu(s) = I_\nu(s_0)e^{-\tau_\nu(s_0)} + \int_0^{s_0} \alpha_\nu(s)B_\nu(T)e^{-\tau_\nu(s)}ds \quad (5.4)$$

where the first term is the intensity at the boundary of the integration and represents contributions to emissions form sources other thean those over the path of integration, such as background or surface emission, and  $\tau$  is the optical depth defined by

$$\tau_\nu(s) = \int_0^s \alpha_\nu(s')ds' \quad (5.5)$$

For the terrestrial inner planets, the surface term is

$$I_\nu(s_0) = I_{surf} + I_{cmb}(s_0) + I_{down}(s_0) \quad (5.6)$$

where the first term is the surface emission, the second is the cosmic microwave background, and the final term is the downwelling of each atmospheric layer attenuated by the atmospheric absorption encountered on its way up out of the atmosphere.

If we model the atmosphere as having  $N$  layers, we can combine equations ?? and ?? and rewrite them as a function of  $B'_\nu$ , the brightness emanating from the planet at a particular microwave frequency and an impact parameter  $a$  (computed using a Ray Tracing algorithm).

$$\begin{aligned}
B'_\nu(a) = & \epsilon(\theta)B_{surf}e^{-\tau_0 \rightarrow \infty} + [1 - \epsilon(\theta)]B_{cmb}e^{-2\tau_0 \rightarrow \infty} \\
& + \sum_{i=1}^N B_i(1 - e^{-\tau_i})e^{-\tau_{i+1} \rightarrow \infty} \\
& + \sum_{i=1}^N B_i(1 - e^{-\tau_i})[1 - \epsilon(\theta)]e^{-\tau_0 \rightarrow i-1}e^{-\tau_0 \rightarrow \infty}
\end{aligned} \tag{5.7}$$

where  $B_{surf}$  is the surface brightness,  $B_i$  is the brightness of the atmospheric layer  $i$ ,  $B_{cmb}$  is the cosmic microwave background brightness (2.7 K in brightness temperature units),  $\epsilon$  is the surface emissivity at emission angle  $\theta$  (function of  $a$ ). The terms containing  $\tau$  are opacities integrated along different sections of the ray path for impact parameter  $a$ . The brightnesses are related to the physical temperature of the surface and atmospheric layers by a factor of unity (i.e.  $B_{surf} = T_{surf}$ ) due to the Rayleigh-Jeans approximation.

The first term in equation ?? is the brightness at the surface of the planet multiplied by the planet's emissivity then attenuated by its atmosphere. The second term is the cosmic background attenuated by two passes through the atmosphere and a reflection from the surface. The third term represents the upwelling radiation from each layer in the atmosphere attenuated from each layer's atmospheric absorption encountered on its way up from the atmosphere. The final term is the downwelling radiation from each atmospheric layer attenuated by every layer below it as well as the whole atmosphere, finally it is modified by the reflection of the surface of the planet.

Optical depths are integrated over the distance that the ray travels in each layer of the atmosphere. The optical depth of each layer is given by

$$\tau_i = \int_{s(z=z_i)}^{s(z=z_{i+1})} \alpha(s) ds \quad (5.8)$$

where  $z_i$  is the height of the  $i^{th}$  layer. To calculate the optical depth of a length of layers the following formula is used

$$\tau_{j \rightarrow k} = \sum_{i=j}^k \tau_i \quad (5.9)$$

where  $j$  is the first layer and  $k$  is the final layer of interest.

To compute the surface emissivity  $\epsilon$  the following formula can be used

$$\epsilon(\theta) = 1 - R_{surf}(\theta) \quad (5.10)$$

where

$$\begin{aligned} R_{surf}(\theta) = & \frac{1}{2} \left[ \frac{\cos \theta - \sqrt{\epsilon_d/\eta_1^2 - \sin^2 \theta}}{\sin \theta + \sqrt{\epsilon_d/\eta_1^2 - \sin^2 \theta}} \right]^2 \\ & + \frac{1}{2} \left[ \frac{\epsilon_d/\eta_1^2 \cos \theta - \sqrt{\epsilon_d/\eta_1^2 - \sin^2 \theta}}{\epsilon_d/\eta_1^2 \cos \theta + \sqrt{\epsilon_d/\eta_1^2 - \sin^2 \theta}} \right]^2 \end{aligned} \quad (5.11)$$

where  $\theta$  is the transmission angle through the first atmospheric layer and  $\eta_1$  is the index of refraction for the first atmospheric layer. The dielectric constant  $\epsilon_d$  is assumed to have a uniform value of 4.0.

It is also useful to know how each portion of the atmosphere affects the brightness temperature, this can be found through calculation of the weighting function,

$$W_i = (1 - e^{\tau, i}) e^{-\tau_{i+1, N}} \quad (5.12)$$

## 5.2 *Parameters of the Radiative Transfer Model*

The parameters of the radiative transfer model (RTM) are the temperature-pressure profiles, the opacity formalisms for the various atmospheric constituents, index of refraction for each atmospheric layer, and the vertical abundance profiles for the absorbing constituents. Together these make up the Thermo-Chemical Model of the atmosphere.

### 5.2.1 Temperature-Pressure Profiles

The temperature-pressure profiles for the atmosphere of Venus have been obtained from the data collected using the Pioneer-Venus sounder and north probes [?]. Figure ?? shows the temperature as a function of altitude in the Venus atmosphere as reported by the Pioneer-Venus sounder and north probes. Figure ?? shows the pressure as a function of altitude in the Venus atmosphere as reported by the Pioneer-Venus sounder and north probes.

The sounder probe temperature represents the temperature-pressure profile in the equatorial regions of the Venus atmosphere. It is used for latitudes between  $-45^\circ$  and  $+45^\circ$ . The north probe is representative of the polar regions of Venus and it used between  $\pm 45^\circ$  and  $\pm 90^\circ$ . A physical surface temperature of 730 K is used in this RTM.

### 5.2.2 Opacity Formalisms

There are several major absorbing constituents at the microwave and millimeter-wave frequencies in the Venus atmosphere. The major constituents are gaseous  $\text{CO}_2\text{-N}_2$  mixture, gaseous  $\text{SO}_2$ , gaseous  $\text{H}_2\text{SO}_4$ , and liquid  $\text{H}_2\text{SO}_4$  in the form of clouds. The formalisms used in this RTM are described below.

#### *Gaseous $\text{CO}_2\text{-N}_2$*

Although  $\text{CO}_2$  is a non-polar molecule, collision induced absorption by gaseous  $\text{CO}_2$  [?] is the dominate source of centimeter- and millimeter-wavelength absorption at low altitudes of the Venus atmosphere. The opacity from gaseous  $\text{CO}_2$  was derived based on laboratory measurements of gaseous  $\text{CO}_2\text{-N}_2$  from Ho et al. 1996 [?]. The  $\text{CO}_2\text{-N}_2$  opacity formalism used in this RTM is

$$\alpha_{\text{CO}_2} = 1.15 \times 10^8 (q_{\text{CO}_2}^2 + 0.25 q_{\text{CO}_2} q_{\text{N}_2} + 0.0054 q_{\text{N}_2}^2) f^2 p^2 T^{-5} \quad (5.13)$$





**Figure 5.1:** Temperature as a function of altitude in the Venus atmosphere obtained using the Pioneer-Venus sounder and north probes



**Figure 5.2:** Pressure as a function of altitude in the Venus atmosphere obtained using the Pioneer-Venus sounder and north probes

where  $f$  is the frequency in GHz,  $p$  is the pressure in atm,  $T$  is the temperature in kelvins,  $q$  is the number mixing ratio, and  $\alpha$  is the absorption in units of dB/km.

#### *Gaseous SO<sub>2</sub>*

The second major opacity contribution comes from gaseous SO<sub>2</sub>. In the developed RTM the opacity formalism developed by Fahd et al 1992 [?] is described below. The formalism developed by Fahd was chosen over the Ben Reuven formalism by Sulieman due to it's better performance when compared to laboratory measurements as shown previously in this work.

This formalism is based on the Van Vleck-Weisskopf formalism where the absorption at a particular frequency can be expressed for each rotational resonant line.

$$\alpha = \alpha_{max} \left( \frac{f}{f_0} \right)^2 \gamma [((f_0 - f)^2 + \gamma^2)^{-1} + ((f_0 + f)^2 + \gamma^2)^{-1}] \quad (5.14)$$

where  $\alpha_{max}$  is the absorption at the line centers,  $f$  is the frequency of interest,  $f_0$  is the resonant line frequency, and  $\gamma$  is the line width. In this calculation a line width of  $\gamma_{SO_2/CO_2} = 7MHz/Torr$  is used for the CO<sub>2</sub> broadening of SO<sub>2</sub> and a line width of  $\gamma_{SO_2/SO_2} = 16MHz/Torr$  is used for the self broadening of SO<sub>2</sub>. Thus the formalism includes the effects of bot CO<sub>2</sub> and SO<sub>2</sub> self-broadening so

$$\gamma = \gamma_{SO_2/CO_2} P_{CO_2} + \gamma_{SO_2/SO_2} P_{SO_2} \quad (5.15)$$

where  $P_{CO_2}$  and  $P_{SO_2}$  are the partial pressures of gaseous CO<sub>2</sub> and SO<sub>2</sub> respectively.

#### *Gaseous H<sub>2</sub>SO<sub>4</sub>*

The next opacity contribution comes from gaseous H<sub>2</sub>SO<sub>4</sub>. The formalism for the opacity of H<sub>2</sub>SO<sub>4</sub> is based on a multiplicative expression from laboratory measurements done by Kolodner et al. 1997 [?]. There are six best fit lines based on the

frequency of the observation. The formalism is listed below

$$\alpha_{H_2SO_4}(f = 2.26) = 106.575 \times q_{H_2SO_4} P^{1.333} \left( \frac{553}{T} \right)^{3.2} \quad (5.16)$$

$$\alpha_{H_2SO_4}(f = 8.4) = 451.760 \times q_{H_2SO_4} P^{1.283} \left( \frac{553}{T} \right)^{3.0} \quad (5.17)$$

$$\alpha_{H_2SO_4}(f = 11.9) = 744.170 \times q_{H_2SO_4} P^{1.309} \left( \frac{553}{T} \right)^{2.9} \quad (5.18)$$

$$\alpha_{H_2SO_4}(f = 21.6) = 1972.783 \times q_{H_2SO_4} P^{1.08} \left( \frac{553}{T} \right)^{3.0} \quad (5.19)$$

$$\alpha_{H_2SO_4}(f < 12) = 33.837 \times q_{H_2SO_4} P^{1.333} f^{1.27} \left( \frac{553}{T} \right)^{3.0} \quad (5.20)$$

$$\alpha_{H_2SO_4}(f) = 55.874 \times q_{H_2SO_4} P^{1.333} f^{1.15} \left( \frac{553}{T} \right)^{3.0} \quad (5.21)$$

where  $f$  is the frequency,  $q_{H_2SO_4}$  is the mixing ratio of gaseous  $H_2SO_4$ ,  $P$  is the pressure in atm, and  $T$  is the temperature in Kelvin. This RTM implements all of the previously listed formalism based on the appropriate frequency.

#### *Liquid $H_2SO_4$*

The formalism for the opacity of clouds is taken from Fahd et al. 1992 [?] and is

$$\alpha_{cloud} = \frac{246M\epsilon_r''}{\rho\lambda[(\epsilon_r' + 2)^2 + (\epsilon_r'')^2]} \quad (5.22)$$

where  $\rho$  is the density of the liquid sulfuric acid (1.84E9 mg/m<sup>3</sup>),  $M$  is the bulk density of the cloud (50 mg/m<sup>3</sup>),  $\lambda$  is the wavelength in km and  $\epsilon_r'$  and  $\epsilon_r''$  are the real and imaginary parts of the complex dielectric constant of the liquid which is found using

$$\epsilon_r = 3.3 + \frac{87.5 - 3.3}{(1 + (2\pi f(1.7 \times 10^{-11}))^{1-0.09})} \quad (5.23)$$

with  $f$  being the frequency in Hz. Since clouds are only formed between 48-50km the absorption is only appropriate for the pressures and temperatures associated with the range of altitudes.

### 5.2.3 Index of Refraction

### 5.2.4 Abundance Profiles

The last parameter of the TCM is the vertical abundance distribution of the absorbing constituents in the atmosphere of Venus. The principal constituents of the Venus atmosphere is gaseous CO<sub>2</sub> which comprises 96.5% of the atmosphere. Gaseous N<sub>2</sub> constitutes about 3.5%. In this RTM these mixing ratios are used for all altitudes of the Venus atmosphere.

For gaseous H<sub>2</sub>SO<sub>4</sub> the developed RTM implements a saturation vapor pressure model as done in Kolodner et al. 1997 [?]. For altitudes less than 48 km it is assumed that the H<sub>2</sub>SO<sub>4</sub> mixing ratio is zero. For altitudes above 48km the mixing ratio is

$$P_{H_2SO_4} = \exp \left( 10156 \left[ -\frac{1}{T} + \frac{0.38}{T_c - T_o} \left( 1 + \ln \frac{T_o}{T} - \frac{T_o}{T} \right) \right] - \frac{\Delta F}{RT} + 16.259 \right) \quad (5.24)$$

where  $P_{H_2SO_4}$  is the partial pressure of H<sub>2</sub>SO<sub>4</sub>,  $T$  is the temperature in Kelvin,  $T_c$  is the critical temperature of 910.5 K,  $T_o$  is the reference temperature of 375 K,  $\Delta F$  is change in chemical potentials (477.60 J/mole) [?], and  $R$  is the gas constant (8.3143 J/mole-K). Different abundance profiles can be used in place of this simple one. See the Appendix for a list of all H<sub>2</sub>SO<sub>4</sub> abundance profiles implemented in the RTM.

Finally a variable abundance profile for gaseous SO<sub>2</sub> is implemented in the developed RTM. A uniform mixing ratio of any value is adopted for altitudes below the main cloud layer (i.e.  $\leq 48$  km). Above the cloud layer the SO<sub>2</sub> abundance profile is assumed to decay exponentially with a scale height of 3.3 km [?]. This is calculated by the following

$$SO_2mr(z) = \begin{cases} SO_2base & : z < 48 \\ SO_2base \times \exp(-(z - 48)/3.3); & : z \geq 48 \end{cases} \quad (5.25)$$

where  $SO_2base$  is the variable mixing ratio of SO<sub>2</sub> at the base of the atmosphere and  $z$  is the location of the current altitude layer.

### **5.3 *Ray-tracing***

While the radiative transfer equation can be used to solve for brightness temperatures from an orbiting spacecraft, the formalism only holds true for an infinitely narrow beamwidth. The formalism also neglects the effects of refraction between atmospheric layers. Here we present the ray tracing approach used in the developed RTM [?]

#### **5.3.1 Ray-tracing Described**

Most radio observations of planets are done by measuring emitted rays originating deep in the atmosphere. However for modeling purposes it is easier to model ray-paths originating from the spacecraft and entering in the planet's atmosphere. These are equivalent by reciprocity.

The initial origin of the ray is in the location of the radiometer (either on the spacecraft or on earth) in a cartesian space with the origin defined as the center of the planet. The initial ray direction is set as the pointing direction of the antenna. First the boresight ray-path is calculated. If this does not intersect the outer most atmosphere of the planet the antenna is not pointed toward the planet. Once the ray intersects the first layer, the vector location of this intersection is recorded. From this, the local normal (ray pointing from the origin to the location of intersection) and the zenith angle, can be calculated. The incidence angle is found and Snell's law is applied to find the vector direction of the transmitted ray. Once the vector direction is determined the vector origin of the ray-segment is set as the initial intersection. A new sphere is defined by the next layer and the ray-sphere intersection algorithm is applied with the new inputs. The algorithm calculated the distance and this is recorded. Using the distance we calculate the new intersection which can be either at the next deeper layer or the previous layer. The latter occurs only when observing from the limb of the planet. This continues until the ray hits the planet, exits from the back of the planet, or becomes so opaque that no significant transmission occurs.

When the ray hits the planet the incidence angle is recorded and is used to find the emissivity of the planet (Equation ??, ??). If the ray does not hit the surface of the planet the incidence angle is not recorded and no surface temperature is calculated. The ray has a possibility of orbiting the planet, this occurs if the next layer causes critical refraction. When this occurs the layers pathlength gets set to infinity which sets the brightness temperature of the layer to the thermal temperature. The emission from this layer is then attenuated by the layers above.

### 5.3.2 Ray-tracing Algorithm Mathematics

The mathematical foundation for the ray-tracing component of the RTM is developed in this section. The ray-sphere intersection algorithm begins with definition of the parametric equation for a ray. A ray is defined as,

$$\begin{aligned} R_{origin} = R_o &= \begin{bmatrix} X_o & Y_o & Z_o \end{bmatrix} \\ R_{direction} = R_d &= \begin{bmatrix} X_d & Y_d & Z_d \end{bmatrix} \end{aligned} \quad (5.26)$$

where

$$\|R_d\|_2^2 = 1 \quad (5.27)$$

which defines a ray as a set of points described by the equation for a line

$$R = R_o + R_d \times t \quad (5.28)$$

where  $T > 0$ . The sphere is defined by,

$$\begin{aligned} S_{center} = S_c &= \begin{bmatrix} X_c & Y_c & Z_c \end{bmatrix} \\ S_{radius} &= S_r \\ S_{surface} = S_s &= \begin{bmatrix} X_s & Y_s & Z_s \end{bmatrix} \end{aligned} \quad (5.29)$$

where

$$\|S_s - S_c\|_2^2 = S_r^2 \quad (5.30)$$

Using equation ?? as the intersection equation for the ray we can substitute that into equation ??, resulting in,

$$\|(R_o + R_d \times t) - S_c\|_2^2 = S_r^2 \quad (5.31)$$

which can be expanded to

$$(X_o + X_d t - X_c)^2 + (Y_o + Y_d t - Y_c)^2 + (Z_o + Z_d t - Z_c)^2 = S_r^2 \quad (5.32)$$

This can be simplified into a quadratic equation

$$At^2 + Bt + C = 0 \quad (5.33)$$

where,

$$A = \|R_d\|_2^2 = 1 \quad (5.34)$$

$$B = 2((R_o - S_c) \bullet R_d) \quad (5.35)$$

$$C = \|R_o - S_c\|_2^2 - S_r^2 \quad (5.36)$$

The solutions to this equation are the standard quadratic solutions

$$t_{0,1} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (5.37)$$

where the  $t$ 's (solutions) are the distance to the intersection point from the ray origin. If the discriminant of these equations is negative the ray misses the sphere. For the purpose of the RTM these are the cases where the ray misses the planet or it exits out of the planet's atmosphere. The smallest positive  $t$  value is the correct solution. Once the  $t$  is found the vector location of the intersection is

$$r_{int} = r_i = \begin{bmatrix} x_i & y_i & z_i \end{bmatrix} = \begin{bmatrix} X_o + X_d t & Y_o + Y_d t & Z_o + Z_d t \end{bmatrix} \quad (5.38)$$

and the unit vector normal at the surface is then

$$r_{normal} = r_n = \frac{(r_i - S_c)}{S_r} = \begin{bmatrix} \frac{(x_i - X_c)}{S_r} & \frac{(y_i - Y_c)}{S_r} & \frac{(z_i - Z_c)}{S_r} \end{bmatrix} \quad (5.39)$$



In terms of the RTM, the solution to the quadratic equation ( $t$ ) is the distance the ray travels through a given layer. The origin of the transmitted ray is set at the intersection location  $r_{int}$  and the direction of the transmitted ray is calculated from the intersection  $r_{int}$  and the surface normal  $r_{normal}$ , using Snell's law.

The vector form of Snell's law needs two vector, the incident ray vector ( $\mathbf{I}$ ) and the local surface normal ( $\mathbf{N}$ ). The incident angle is calculated using

$$\cos(\theta_1) = -\mathbf{I} \bullet \mathbf{N} \quad (5.40)$$

From Snell's law, the relative index of refraction ( $\eta$ ) is,

$$\eta = \frac{\sin(\theta_2)}{\sin(\theta_1)} = \frac{\eta_1}{\eta_2} \quad (5.41)$$

The angle of the transmitted ray ( $\theta_2$ ) can be computed from known quantities,

$$\cos(\theta_2) = \sqrt{(1 - \sin^2(\theta_2))} = \sqrt{(1 - \eta^2 \sin^2(\theta_1))} = \sqrt{(1 - \eta^2(1 - \cos^2(\theta_1)))} \quad (5.42)$$

The vector direction of the transmitted ray is computed as,

$$\mathbf{T} = \eta \mathbf{I} + (\eta \cos(\theta_1) - \cos(\theta_2)) \mathbf{N} \quad (5.43)$$

the values of  $\mathbf{I}$  and  $\mathbf{N}$  are the vectors  $R_d$  and  $r_n$  respectively. The output of this formula ( $\mathbf{T}$ ) is the new value for  $R_d$ . Using this algorithm and techniques described in the previous sections we can trace a path through each layer of the atmosphere.

## 5.4 *Vector Radiative Transfer*

A typical method of RTM is to iterate through each layer and calculate everything at each iteration. While going about RTM this way is easier to understand it is extremely inefficient. The following section describes a more efficient way of doing RTM.

#### 5.4.1 Thermo-Chemical Model

The first step is to put together the TCM for the Venus atmosphere. The TCM is dependent on the altitude vector  $\mathbf{a}$  whose size is  $N \times 1$  where  $N$  is the amount of layers in the altitude and the vector of all interested frequencies  $\mathbf{F}$  which is  $1 \times M$  with  $M$  being the number of interested frequencies.  $\mathbf{a}$  is defined as

$$a_i = iz_{step} \quad (5.44)$$

where  $a_i$  is the  $i^{th}$  element in the vector and  $z_{step}$  is the distance between each atmospheric layer.

The TCM for Venus also needs a Latitude of observation. This is due to the different Temperature-Pressure profiles of the planet. Using the appropriate Temperature-Pressure profile, the  $\mathbf{a}$  vector, it is possible to do a one dimension interpolation of the T-P profiles of the interested atmospheric layers. The temperature and pressure vectors are  $\mathbf{T}$  and  $\mathbf{P}$  respectively. Using the  $\mathbf{T}$  and  $\mathbf{P}$  it will be possible to create a vector for all constituents mixing ratio  $\mathbf{Q}_c$ , with  $c$  being the constituent of interest. The refractive index vector  $\mathbf{N}$  can be calculated using the same methods as  $\mathbf{Q}_{con}$ . The vectors  $\mathbf{T}$ ,  $\mathbf{P}$ ,  $\mathbf{Q}_c$ , and  $\mathbf{N}$  are of size  $N \times 1$

Finally the absorption matrix  $\mathbf{A}$  needs to be calculated. This is done by

$$A_{i,f} = \sum_{constituents} \alpha_{i,c}(\mathbf{F}(f)) \quad (5.45)$$

where  $A_i$  is the  $i^{th}$  element in the vector and  $\alpha_{i,c}(f)$  is the absorption of the constituent  $c$  at the  $i^{th}$  layer in the atmosphere with a frequency of  $\mathbf{F}(f)$ .  $\mathbf{A}$  is of size  $N \times M$ .

#### 5.4.2 Ray-Tracing

In this method Ray-tracing is still done iteratively but in this case we start with a distance vector  $\mathbf{d}$  of size  $N \times 1$  such that

$$\mathbf{d} = \vec{0}$$

and for every  $t$  (found using the Ray-Tracing algorithm) calculated in the Ray-tracing algorithm the vector  $\mathbf{d}$  is updated using

$$\mathbf{d}_i = \mathbf{d}_i + t \quad (5.46)$$

This keeps track of the total distance spent in each layer.

### 5.4.3 Radiative Transfer Model

To preform the RTM a few variables must be defined. The first is our  $\vec{\tau}$  matrix which is defined as

$$\vec{\tau}_{i,j} = \alpha_{i,j} \mathbf{d} \quad (5.47)$$

where  $\alpha$  is the opacity at layer  $i$  at frequency  $j$ , and  $\mathbf{d}$  is the distance the ray travels through layer  $i$

Using the  $\vec{\tau}$  matrix it is possible to calculate the weighting matrix for the upwelling and downwelling of the atmosphere,  $\mathbf{W}_{up}$  and  $\mathbf{W}_{down}$  respectively, using the following

$$\mathbf{W}_{up_{i,j}} = (1 - e^{-\tau_{i,j}}) e^{(-\sum_{l=i+1}^N \tau_{l,j})} \quad (5.48)$$

$$\mathbf{W}_{down_{i,j}} = (1 - e^{-\tau_{i,j}}) e^{(-\sum_{l=1}^{i-1} \tau_{l,j})} e^{(-\sum_{l=1}^N \tau_{l,j})} (1 - \epsilon(\theta)) \quad (5.49)$$

where  $i$  is again each layer of the atmosphere,  $j$  is each frequency of interest and  $\epsilon(\theta)$  is the surface emissivity.  $\mathbf{W}_{up}$  calculates the attenuation of the current layer and every layer above it.  $\mathbf{W}_{down}$  calculates the attenuation from the current layer towards the surface and back through the entire atmosphere.

These weighting vectors along with the temperature vector,  $\mathbf{T}$ , can give us the expected temperature brightness through

$$\begin{aligned} \mathbf{Tb}_j = & T_{surf} \cdot \epsilon(\theta) \cdot e^{(-\sum_{l=1}^N \tau_{l,j})} + T_{cmb} \cdot (1 - \epsilon(\theta)) \cdot e^{(-2\sum_{l=1}^N \tau_{l,j})} \\ & + \sum_{i=1}^N \mathbf{T}_i \cdot \mathbf{W}_{up_{i,j}} + \sum_{i=1}^N \mathbf{T}_i \cdot \mathbf{W}_{down_{i,j}} \end{aligned} \quad (5.50)$$

where the first term is the temperature at the surface multiplied by the emissivity and attenuated by the atmosphere. The second term is the cosmic microwave background

(2.7K) multiplied by the reflectivity of the planet then attenuated by the atmosphere twice (down and back up). The third term is the upwelling of the atmosphere which is the temperature at each level multiplied by the upwelling weighting matrix described previously. The final term is the downwelling of the atmosphere which again is the temperature at each level multiplied by the downwelling weighting matrix defined previously.

### 5.5 *Beam Forming*

Since the ray-tracing algorithm assumes a pencil beam (or ray) it is necessary to form spatial samples of the main beam in order to estimate properly emergent flux of the atmosphere incident on the antenna. This is accomplished by generating a set of vectors that each describe a ray that is offset from the direction of the boresight ray. The boresight may take on any direction. Since the developed RTM is used for earth based observations the problem of mapping an antenna to the planet gets simplified quite a bit. The parameters of this beam forming algorithm are  $R_{proj}$ ,  $BWHM$ ,  $N_c$ , and  $n_0$ .  $R_{proj}$  is the projected radius of the antenna beam pattern onto a planar projection of Venus (in km). This can be thought of as a pixel resolution (1 pixel = 200x200 km). The second parameter is the  $3dB$  beamwidth of the antenna's main beam.  $N_c$  is the number of concentric rings while  $n_0$  is the number of samples in the initial ring. Once the free samples are chosen the number of beamsamples in each ring may be found by

$$N(k) = N(1) \times (2k - 1) \quad (5.51)$$

where  $N$  is the number of samples and  $k$  is the integer multiple of the ring spacing in terms of radius. For example if a ring spacing of  $1/3$  of the half-power beamwidth is chosen, then there will be three concentric rings sampling the beam ( $N_c = 3$ ). Thus if the first ring is sampled at  $90^\circ$ , there will be four beamsamples in the first ring ( $360^\circ/n_0 = 90^\circ$  for  $n_0 = 4$ ).  $\Delta\phi$  is defined as the current spacing between each

beamsample in the current ring and can be found by

$$\Delta\phi(k) = \frac{BWHM}{k}. \quad (5.52)$$

Using  $\Delta\phi$  allows for us to calculate the weight of each beamsample using

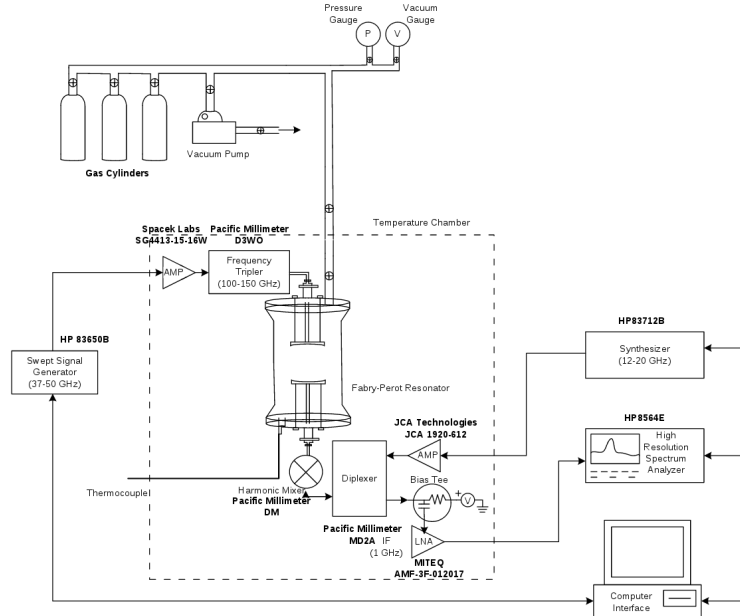
$$beamweight(\Delta\phi) = e^{\left(-2.76 \times \left(\frac{\Delta\phi}{BWHM}\right)^2\right)} \quad (5.53)$$

Combining equation ?? and ?? it is possible to remove the need for  $BWHM$ .

$$beamweight(k) = e^{\left(-2.76 \times \left(\frac{1}{k}\right)^2\right)} \quad (5.54)$$

The spatial resolution of the beamsampling may be increased and is limited by only the memory of the computer and the patience of the user.

## 5.6 Radiative Transfer Results



**Figure 5.3:** Block diagram of the F band measurement system. Solid lines represent the electrical connections and the arrows show the direction of the signal propagation. Valves controlling the flow of gasses are shown by small crossed circles.

## CHAPTER VI

### SUMMARY AND CONCLUSIONS

#### *6.1 Significant Results*

#### *6.2 Application to Venus Observations*

#### *6.3 Suggestions for Future Work*

Venus RTM

## INDEX

## VITA

Perry H. Disdainful was born in an insignificant town whose only claim to fame is that it produced such a fine specimen of a researcher.



# THE MILLIMETER-WAVELENGTH SULFUR DIOXIDE ABSORPTION SPECTRA MEASURED UNDER SIMULATED VENUS CONDITIONS

Amadeo Bellotti

?? Pages

Directed by Professor Paul G. Steffes

The objective of the proposed research is to develop a mathematical model that accurately estimates the opacity of sulfur dioxide in a carbon dioxide atmosphere under conditions characteristic of the Venus troposphere based on extensive laboratory measurements. High precision measurements of the millimeter-wavelength properties of sulfur dioxide are being conducted under multiple pressure and temperatures. These measurements are being conducted in both W-band and F-band (2-3 and 3-4 millimeter-wavelengths). The results of this research will significantly improve the understanding of the millimeter-wavelength emission spectrum of Venus and possibly determine the source of variations in the Venus millimeter-wavelength emissions.