



Quantum States: Basic Quantum Mechanics Concepts for Quantum Computing

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Computation is Information Processing

A *model of computation* defines the following:

1. representation of **information**
2. how information is **processed** (valid operations).

	Classical Computing Model	Quantum Computing Model
Unit of Information	Bit	<i>Quantum Bit (Qubit)</i>
Info. Representation	Bit Strings (Binary String)	<i>Quantum State (Multi-Qubits)</i>
Operations	Binary String Operations	Quantum Operations

The focus of this presentation is the introduction of quantum mechanics concepts related to *quantum states*.



Classical Two-State System: Storing One Bit of Information

- A **bit** **b** can either be **0** or **1**.
- Any system that have (at least) two states is needed. It is called **classical two-state system**.
- Let *state* **A** and *state* **B** be the two states of the system.
- Convention: **b = 0** if the system is in state **A**
- Convention: **b = 1** if the system is in state **B**



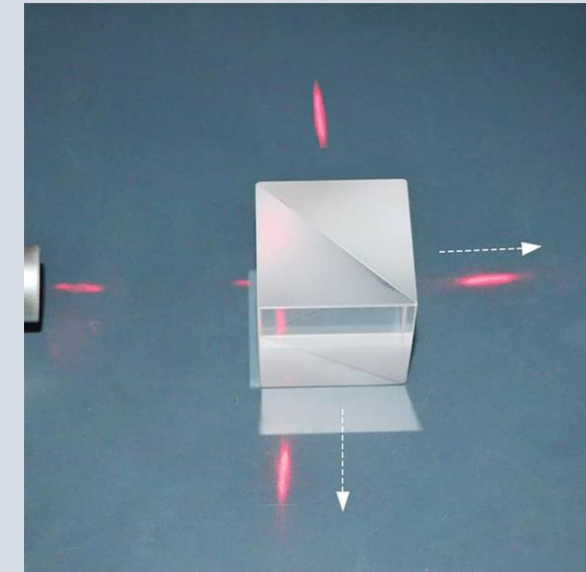
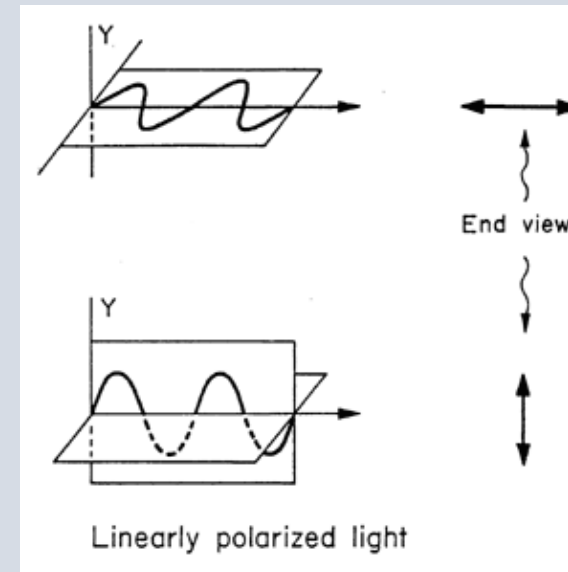
System	State A	State B
Toggle Switch	OFF	ON
Abacus Bead	Bead is down	Bead is up
Punched Tape	No Hole	Has Hole



Quantum Two-State Systems

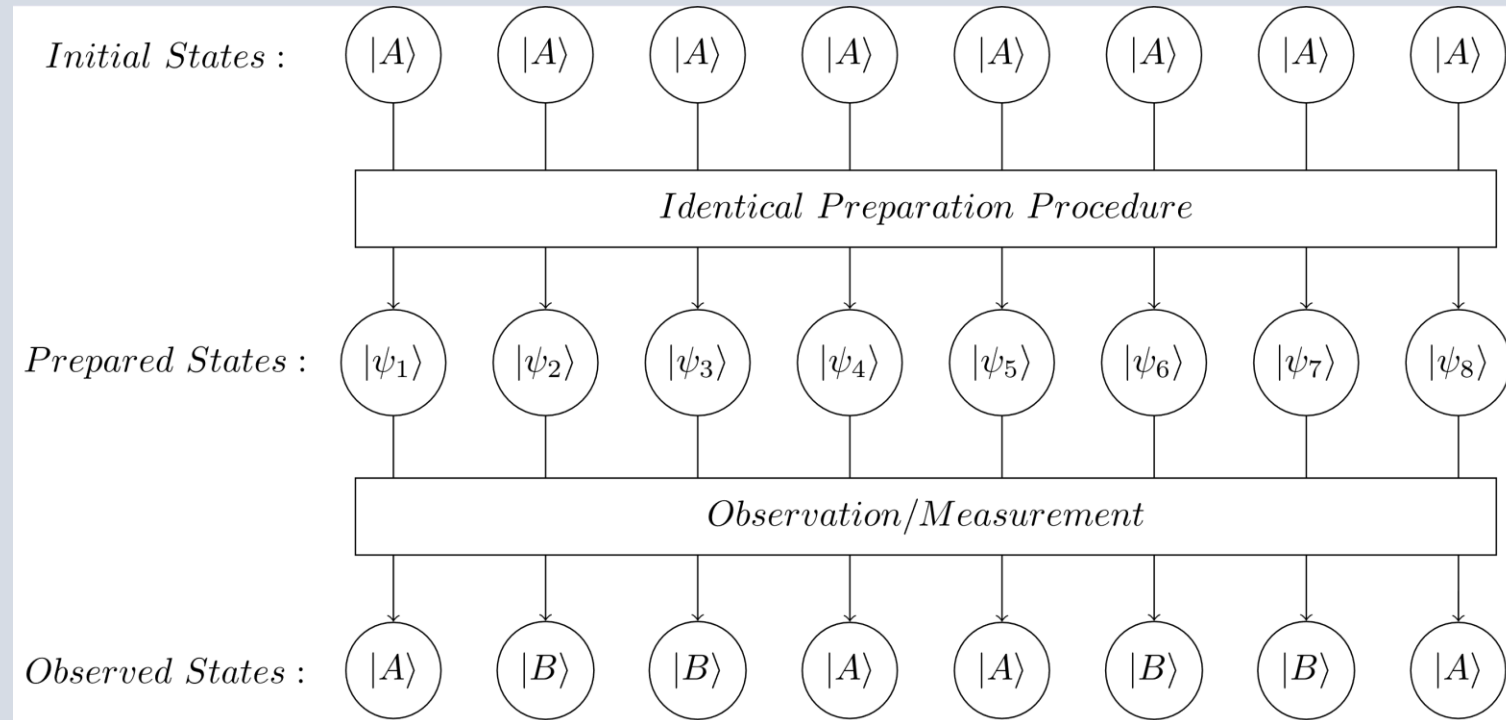
- A qubit q can be $|0\rangle$ or $|1\rangle$.
- $|0\rangle$ - pronounced "ket 0", $|1\rangle$ - pronounced "ket 1"
- In a **quantum two-state system**, there are also two observable states.
- Let the observable states be state $|A\rangle$ and state $|B\rangle$
- state $|A\rangle$ - pronounced "ket A", state $|B\rangle$ - pronounced "ket B"

System	State $ A\rangle$	State $ B\rangle$
Light Polarity	horizontal	vertical
Photon Direction	horizontal	vertical



Quantum Indeterminacy Experiment Concept:

1. Prepare multiple quantum two-state systems. The system should be identical and have the same initial state $|A\rangle$.
2. Apply an identical preparation procedure U to all systems.
3. Observe the state of each system.



Expectation: The observed states should be identical.

Results: For quantum systems and certain procedures, the observed states are not guaranteed to be identical.

The experiment is formulated in a such a way as to eliminate external factors that can affect the state of the quantum system.

Conclusion: After applying the preparation procedure but before observation or measurement, the prepared states are not yet determined (*indeterminate*). Only after observation or measurement will a state be in one of the two definite states.

Question: Why even use quantum two-state systems to store information and do computations if there are situations where the state of the system is indeterminate?

Answer: The indeterminacy of a quantum systems is to **not totally random**. By extending the experiment you extract information about the indeterminate state.



Quantum Indeterminacy & Probabilities

Quantum Indeterminacy Experiment (extended):

Parameter: n – number of identical quantum systems

Parameter: initial state $|\psi\rangle$ (select one: $|\psi\rangle = |A\rangle$ or $|\psi\rangle = |B\rangle$)

Parameter: procedure U

Output: n_A – number of systems observed to be in state $|A\rangle$

Output: n_B – number of systems observed to be in state $|B\rangle$

Compute: $p_A = (n_A/n)$ – portion of the systems observed to be in state $|A\rangle$

Compute: $p_B = (n_B/n)$ – portion of the systems observed to be in state $|B\rangle$

Task: Increase n and observe what values p_A and p_B approach.

Result: For the given parameters, $|\psi\rangle$ and U , p_A and p_B approach some specific values.



Probabilities, Superposition, and the Qubit

The prepared (aka process) quantum state can be **indeterminate** prior to observation. Knowing $|\psi\rangle$ and U , we can still attach the information the following information to the quantum state:

p_A – probability of observing $|A\rangle$

p_B – probability of observing $|B\rangle$

$|\psi'\rangle$ – be prepared/processed state, the state of the system after applying procedure X to initial quantum state $|\psi\rangle$

$|\psi'\rangle = \alpha|A\rangle + \beta|B\rangle$, qubit $|\psi'\rangle$ is a *superposition* of basis states $|A\rangle$ and $|B\rangle$

α – *amplitude* of state $|A\rangle$, $p_A = |\alpha|^2$

β – *amplitude* of state $|B\rangle$, $p_B = |\beta|^2$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, we will now use the logical basis states $|0\rangle$ and $|1\rangle$



Some Examples of Qubits

Qubit	Ampl. of $ 0\rangle$	Ampl. of $ 1\rangle$	Prob. of $ 0\rangle$	Prob. of $ 1\rangle$
$ \psi_1\rangle = 1 0\rangle + 0 1\rangle = 0\rangle$	1	0	$1^2 = 1$	$0^2 = 0$
$ \psi_2\rangle = 0 0\rangle + 1 1\rangle = 1\rangle$	0	1	$0^2 = 0$	$1^2 = 1$
$ \psi_3\rangle = \frac{1}{\sqrt{2}} 0\rangle + \frac{1}{\sqrt{2}} 1\rangle$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$	$\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$
$ \psi_4\rangle = \frac{1}{\sqrt{4}} 0\rangle + \frac{\sqrt{3}}{\sqrt{4}} 1\rangle$	$\frac{1}{\sqrt{4}}$	$\frac{\sqrt{3}}{\sqrt{4}}$	$\left(\frac{1}{\sqrt{4}}\right)^2 = \frac{1}{4}$	$\left(\frac{\sqrt{3}}{\sqrt{4}}\right)^2 = \frac{3}{4}$
$ \psi_5\rangle = \frac{\sqrt{5}}{\sqrt{8}} 0\rangle + \frac{\sqrt{3}}{\sqrt{8}} 1\rangle$	$\frac{\sqrt{5}}{\sqrt{8}}$	$\frac{\sqrt{3}}{\sqrt{8}}$	$\left(\frac{\sqrt{5}}{\sqrt{8}}\right)^2 = \frac{5}{8}$	$\left(\frac{\sqrt{3}}{\sqrt{8}}\right)^2 = \frac{3}{8}$



Rule on Probabilities & the Born Rule

Qubit: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

α , amplitude of state $|0\rangle$

β , amplitude of state $|1\rangle$

$p_0 = |\alpha|^2$, probability of observing state $|0\rangle$

$p_1 = |\beta|^2$, probability of observing state $|1\rangle$

Probability Rule: $p_0 + p_1 = 1$, alternately: $p_0 = (1-p_1)$ or $p_1 = (1-p_0)$

Born Rule: $|\alpha|^2 + |\beta|^2 = 1$, or simply $\alpha^2 + \beta^2 = 1$ if α and β are real numbers.

The *Born Rule* is the probability rule expressed in terms of amplitudes.



The Born Rule & Qubits as Points on a Circle

Qubit: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

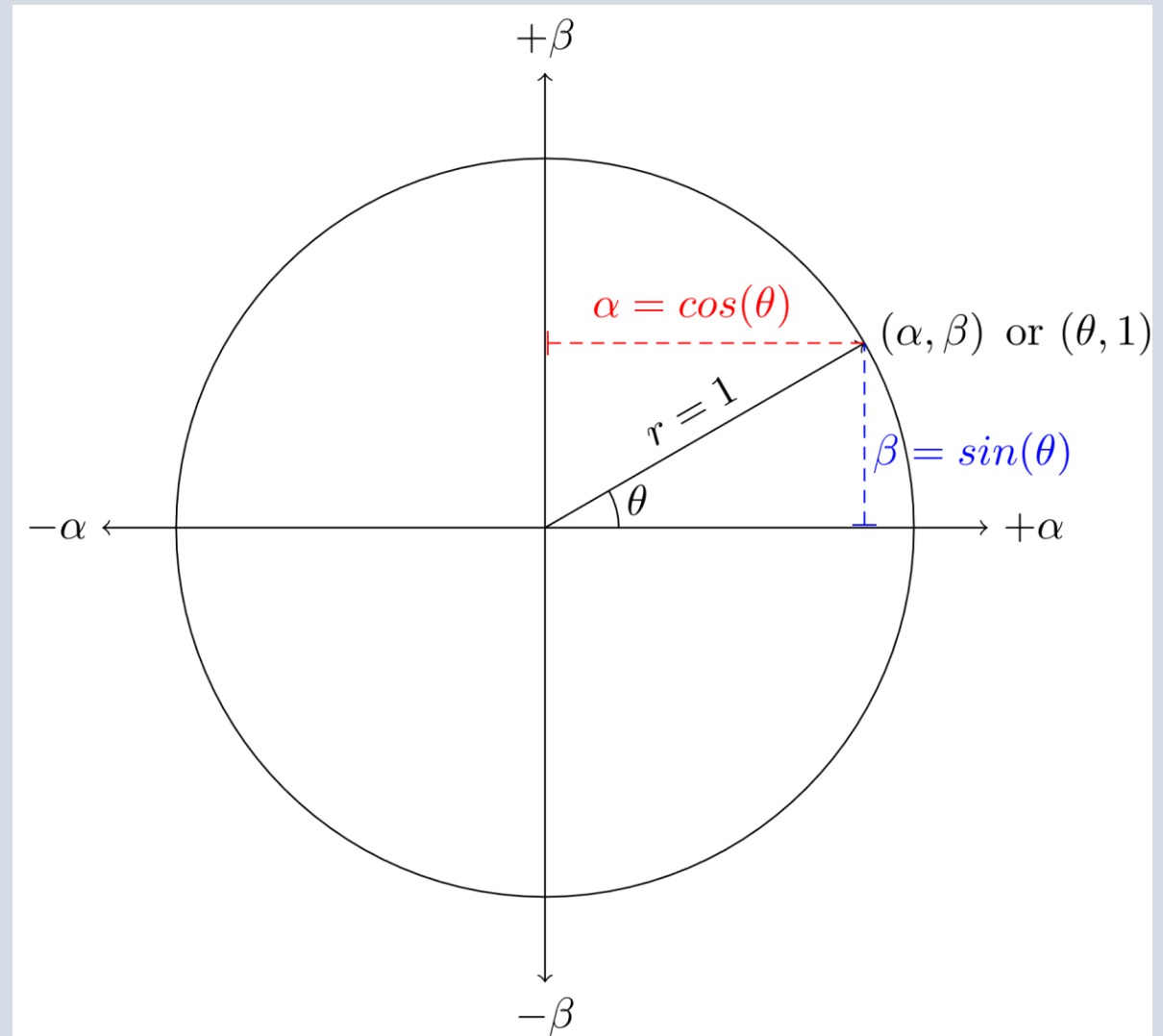
Born Rule: (for real numbers)

$$\alpha^2 + \beta^2 = 1$$

- circle on the $\alpha\beta$ -plane
- center on $(0,0)$
- radius = 1

$$x^2 + y^2 = 1$$

- circle on the xy -plane
- center on $(0,0)$
- radius = 1



Qubits as Points on a Circle

Qubit: $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$

(α, β)

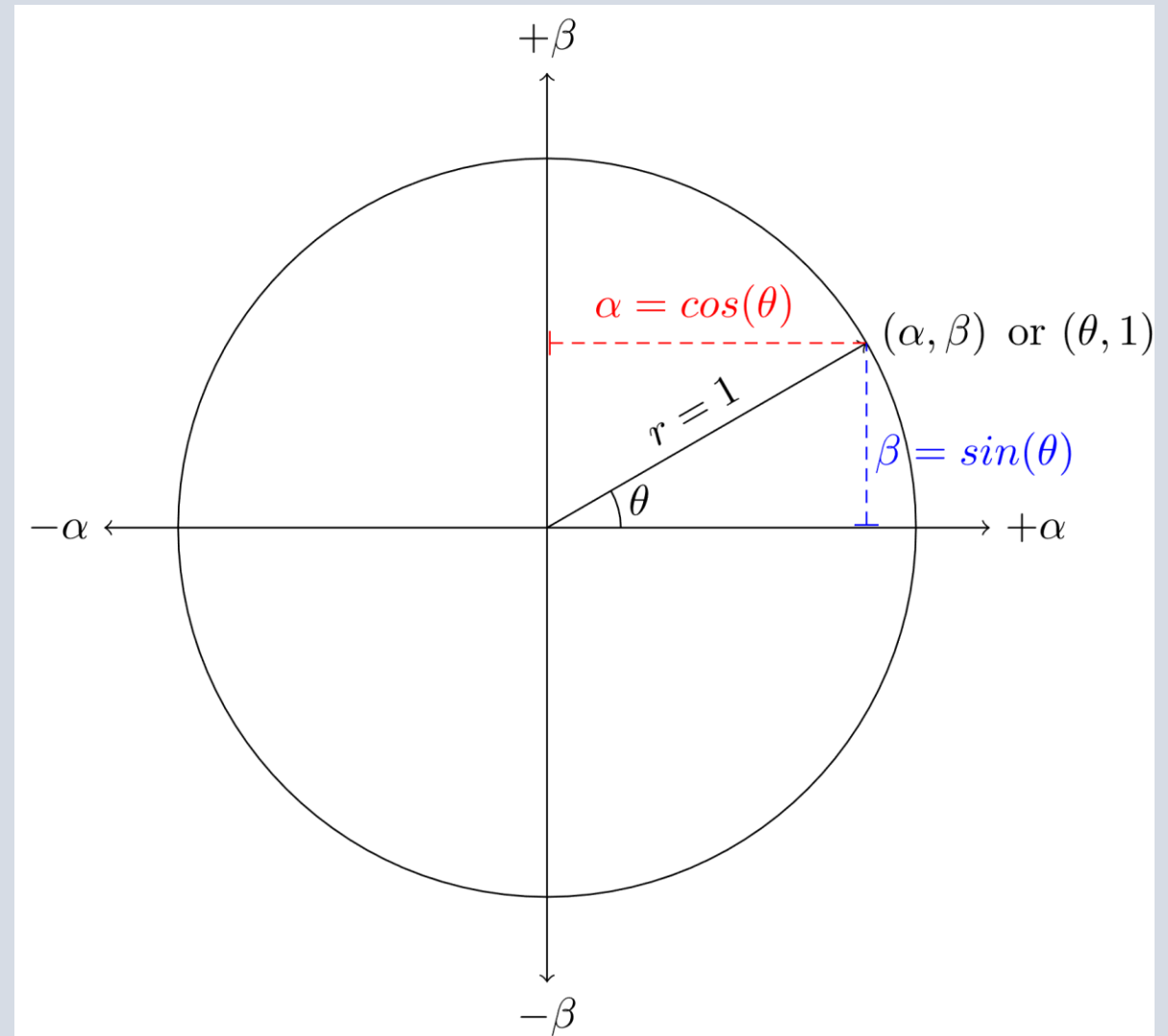
- point on the circle
- represents $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$(\theta, 1)$

- point on the circle
- polar coordinate of (α, β)

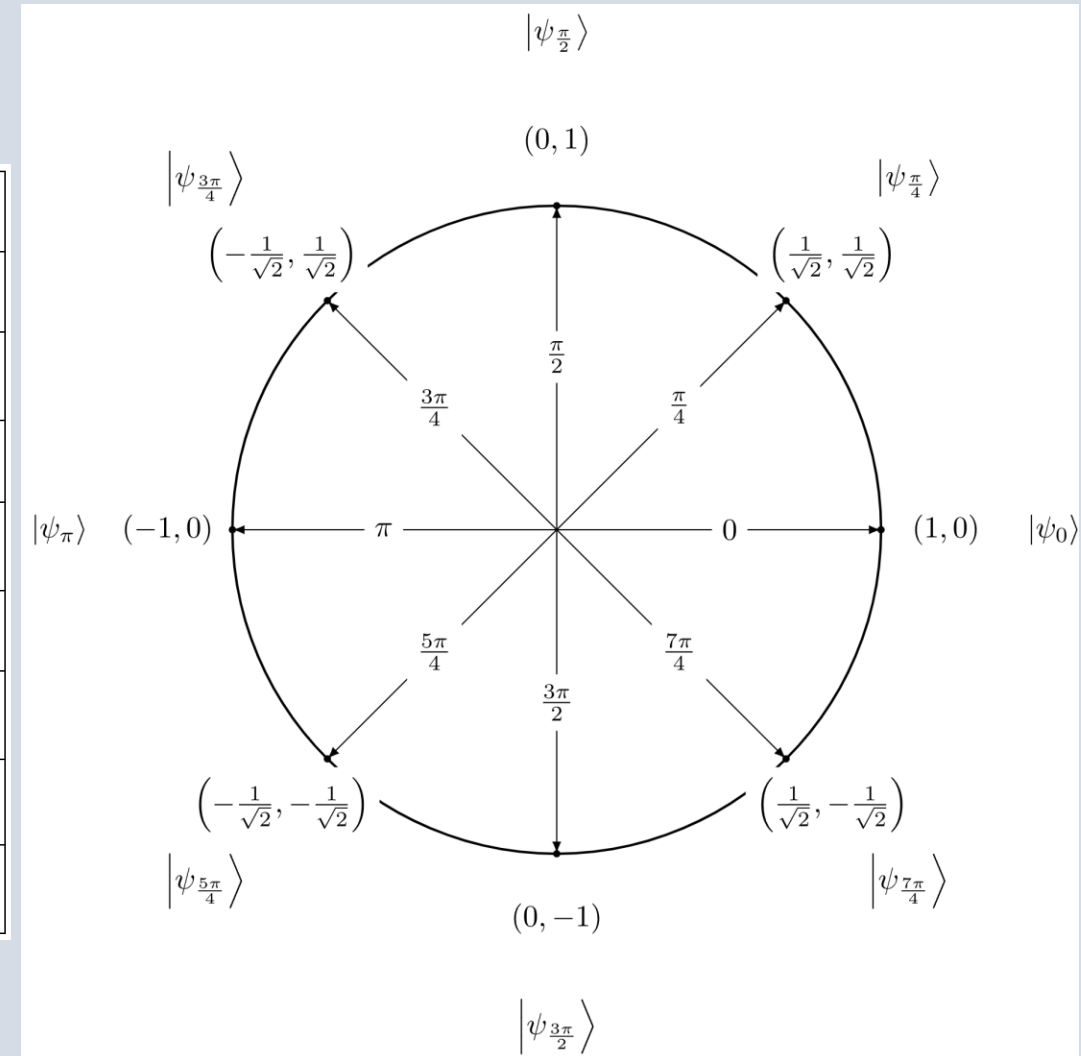
Qubit: $|\Psi_\theta\rangle = \cos(\theta)|0\rangle + \sin(\theta)|1\rangle$

- polar coordinate form
- $\alpha = \cos(\theta)$
- $\beta = \sin(\theta)$



Qubits as Points on a Circle: Examples

θ	$\alpha = \cos(\theta)$	$\beta = \sin(\theta)$	$ \psi_\theta\rangle = \alpha 0\rangle + \beta 1\rangle$
0	1	0	$ \psi_0\rangle = 1 0\rangle + 0 1\rangle = 0\rangle$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$ \psi_{\frac{\pi}{4}}\rangle = \frac{1}{\sqrt{2}} 0\rangle + \frac{1}{\sqrt{2}} 1\rangle$
$\frac{\pi}{2}$	0	1	$ \psi_{\frac{\pi}{2}}\rangle = 0 0\rangle + 1 1\rangle = 1\rangle$
$\frac{3\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$ \psi_{\frac{3\pi}{4}}\rangle = -\frac{1}{\sqrt{2}} 0\rangle + \frac{1}{\sqrt{2}} 1\rangle$
π	-1	0	$ \psi_\pi\rangle = -1 0\rangle + 0 1\rangle = - 0\rangle$
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$ \psi_{\frac{5\pi}{4}}\rangle = -\frac{1}{\sqrt{2}} 0\rangle - \frac{1}{\sqrt{2}} 1\rangle$
$\frac{3\pi}{2}$	0	-1	$ \psi_{\frac{3\pi}{2}}\rangle = 0 0\rangle - 1 1\rangle = - 1\rangle$
$\frac{7\pi}{4}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$ \psi_{\frac{7\pi}{4}}\rangle = \frac{1}{\sqrt{2}} 0\rangle - \frac{1}{\sqrt{2}} 1\rangle$



Qubits as Points on a Circle

Due to the born rule and our initial **restriction** that the amplitudes are real numbers (not yet complex numbers), we are able to define **any unique qubit** using a single parameter which is the angle θ .

Qubit	$ \psi\rangle = \alpha 0\rangle + \beta 1\rangle$
Point (Rectangular Coordinate)	(α, β)
Point (Polar Coordinate)	$(\theta, 1)$
Qubit	$ \psi_\theta\rangle = \cos(\theta) 0\rangle + \sin(\theta) 1\rangle$

If you look at the probability rule $p_0 + p_1 = 1$, it can help you see that you only need 1 parameter to define a qubit with real amplitudes. If you specify the qubit's p_0 , due to the probability rule, you can derive $p_1 = (1 - p_0)$ from p_0 . They are not independent values. The same goes for the amplitudes. *i.e.* $\beta = \sqrt{1 - \alpha^2}$



Operations on Single Qubits

- An **operation U** transforms a qubit by changing the qubit's amplitudes.
- The **operation U** should follow the born rule.

$$\begin{aligned} |\psi\rangle = \alpha|0\rangle + \beta|1\rangle &\longrightarrow \boxed{U} \longrightarrow |\psi'\rangle = \alpha'|0\rangle + \beta'|1\rangle \\ (\alpha, \beta) &\longrightarrow \boxed{U} \longrightarrow (\alpha', \beta') \\ (\theta, 1) &\longrightarrow \boxed{U} \longrightarrow (\theta', 1) \end{aligned}$$



Operations on Single Qubits

- Qubit **operations** can be viewed as rotations or reflections.

