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TECHNICAL NOTE ON TOTAL AND BOLE VOLUMES, AND ON WOOD DENSITY TO THE CONSEIL SCIENTIFIQUE ET TECHNIQUE DE L'IGN

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Contents

Context

The French National Forest Inventory (NFI) publishes statistics annually, such as basal area or wood resources, on both private and public forest. These statistics are derived from a sampling scheme in which about 6000 new plots are visited each year, along with 6000 plots that were first surveyed five years earlier[1]. The published statistics are calculated using a moving five-year average; for instance, the published results for 2024 are based on campaigns spanning from 2020 to 2024.

[1] These are only the visited plots, more plots are treated by aerial photography

Initially designed to assess forest area and produce estimates of standing timber stock, the French NFI is gradually evolving into more comprehensive tools for forest monitoring, incorporating a variety of indicators. Among this new information, the estimation of forest biomass and carbon is crucial, as it makes it possible to monitor biomass production and its use by different sectors, to estimate the contribution of forests to mitigating the effects of climate change as part of climate commitments monitoring (Citepa), and to develop forest strategies adapted to contemporary environmental and societal challenges (Commission Européenne 2018).

The European Union developed a New EU Forest Strategy for 2030 as part of the plan to adapt to and fight against climate change and make Europe a climate neutral continent by 2050. This strategy relies on improved monitoring of European forests to better understand their condition and respond accordingly. Specifically, it calls for assessing carbon sequestration in forests to evaluate whether or not Europe reached carbon neutrality. One bottleneck is the harmonisation of the forest monitoring methods between European member states, if not within them. The PathFinder project supports member states in implementing a European Forest Monitoring System in order to standardise or harmonise forest data collection and reporting across the EU. This prompted the French NFI to update its methods for assessing forest carbon storage.

Three steps are necessary to estimate above-ground carbon storage from field data (see Fig. 1): (*i*) the total above-ground volume is estimated from diameter at breast height (dbh) and height (Vallet et al. 2006), (*ii*) biomass is then derived by using a coefficient for wood density, and (*iii*) a factor is applied to convert the biomass into carbon content. Below-ground carbon from the roots requires additional coefficients and more research. It is not treated in this document.

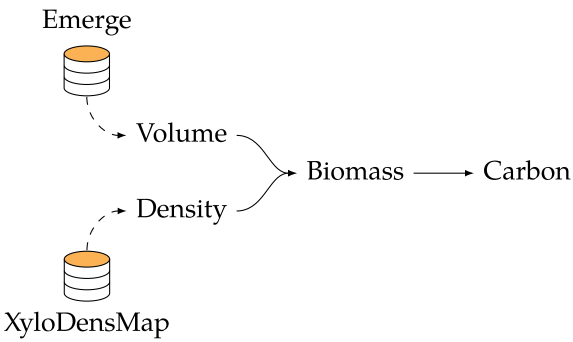


Figure 1: Computation chain for biomass and carbon content.

In this technical note, we explain the whole computation chain we foresee to estimate carbon content. We focus mostly on the base of the chain, *i.e.*, the two volumes computed at the French NFI for each tree (bole and total volumes), and the wood density. In the section Definitions and datasets, we define the different tree components and explain the origins of the data. Then, in the sections Bole volume and Total above-ground volume, we expose the models to compute the individual bole and total volumes, respectively. Lastly, we present the undergoing work on wood density and potential path to convert total above-ground volume into biomass and carbon content in the sections XyloDensMap and Conclusion.

Definitions and datasets

Trees are partitioned into hierarchical elements with definitions that may vary between Forest Inventories and datasets (*e.g.*, Fig. 2).

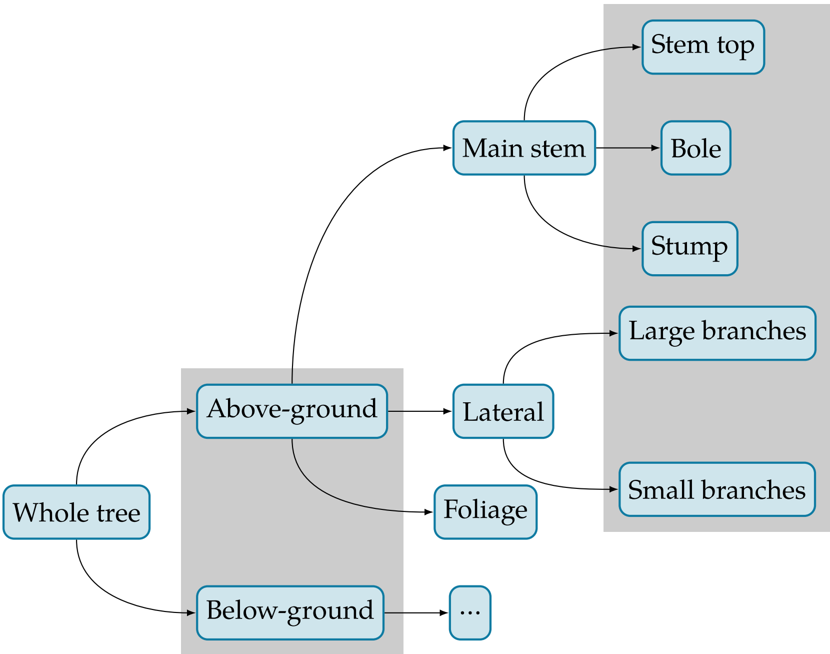


Figure 2: Hierarchical elements of trees, figure inspired by Gschwantner et al. 2009

In our case, the tree data were collected from several institutions, at different periods, and with varying levels of detail regarding recorded tree components:

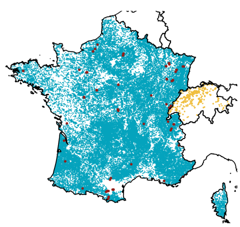


Figure 3: Plot location for French NFI, Emerge, and Swiss dataset.

1. 'Protocole Oudin' dataset – Preserved by INRAE and covering the period 1930–1980 (shown in red in Fig. 3). This dataset includes measurements of bole volume, large branches, and small branches. Hereafter, it is referred to as 'Emerge',

after the 2008 digitisation project of the same name (Deleuze, Morneau, Longuetaud et al. 2013).

2. Experimental Forest Management Project dataset (EFM) – Described by Didion et al. (2024) and covering the period 1888–1974 (shown in yellow in Fig. 3). It includes measurements of bole volume, large branches, and small branches.

3. French National Forest Inventory (NFI) – Data collected between 1988 and 2007 (shown in blue in Fig. 3). Earlier data (before 1988) recorded diameter instead of circumference and are discarded in this study. This dataset contains measurements of bole volume, following the NFI protocol (less detailed than Emerge and EFM).

4. 'Office National des Forêts (ONF)' – Data collected with protocols from 1972 and from 1983. So far, these data are not used due to the absence of geographic coordinates.

5. Institut Technologique Forêt, Cellulose, Bois-construction, Ameublement (FCBA) – Data available but not yet used for above-ground volume estimation.

6. Institut pour le Développement Forestier (IDF) – The R&D branch of the Centre National de la Propriété Forestière (CNPF). Data not used.

The terminology follows that of Gschwantner et al. 2009 (see Figs. 2 and 4) to ensure consistency in wording across the different datasets:

- Main stem: The stem of a tree is the above-ground part of the main (off) shoot with apical dominance
  - Stem top: topmost part of the stem from an over-bark base-diameter of 7 cm (French NFI) to the stem tip
  - Bole: above-ground part of the stem between stump and the stem top
  - Stump: above-ground base part of the stem which would remain after a tree was cut under normal felling practices
- Lateral parts:
  - Large branches: portion of the above-ground lateral parts with a diameter of more than or equal to 7 cm (French NFI)

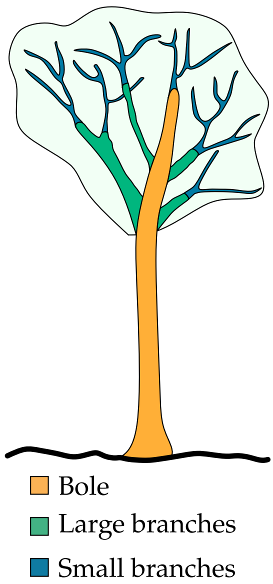


Figure 4: Scheme of tree components.

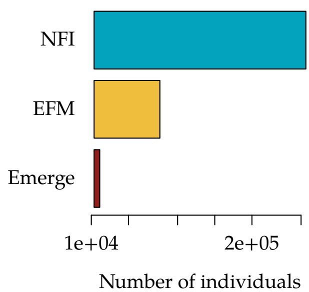


Figure 5: Composition of the used dataset for the bole volume and total above-ground volume.

– Small branches: portion of the above-ground lateral parts with a diameter of less than 7 cm (French NFI)

The combination of the Emerge, EFM, and NFI datasets provides a total of 594 616 individuals, with 98% coming from the NFI (bole volume only), 6% from the EFM (bole, large and small branches), and 2% from Emerge (same components as EFM; see Fig. 5). The challenges in fitting the data are threefold: (*i*) the data exhibit strong heteroskedasticity (see Fig. 6); (*ii*) the datasets are unbalanced; and (*iii*) the trees from the Emerge and EFM datasets do not provide exhaustive forest coverage, implying that total volume estimates for the NFI are often based on extrapolation.

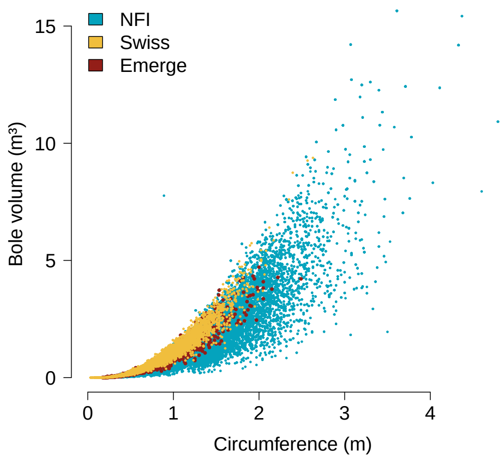


Figure 6: Typical response of bole volume to circumference (here displayed for *Fagus sylvatica*). We can see that both the Emerge and Swiss datasets (Deleuze, Morneau, Longuetaud et al. 2013; Didion et al. 2024) are quite similar and covers elongated trees, while the French NFI covers less regular trees.

Bole volume (volume bois-fort tige)

The bole volume is the 'reference' volume since the creation of the French NFI (1958). Its definition is largely driven by the requirements of the wood industry, for which the estimation of standing timber volume is an essential tool for resource management and planning.

Notations

We gather in the table 1 the notations and acronyms used in this document.

Symbol	Definition	Unit
c	Circumference at breast height	cm
h	Total tree height	m
h_{dec}	Height to the stem top h or to a sharp narrowing ($\geqslant$ 10% decrease within 1 m)	m
$\mathbb{E}[]$	Expected value operator	diverse
EFM	Experimental Forest Monitoring (Didion et al. 2024)	-
Emerge	Emerge project or dataset (Deleuze, Morneau, Longuetaud et al. 2013)	-
NFI	(French) National Forest Inventory	-
r	Ratio of bole volume over total above-ground volume	-
X, Y	Explanatory and dependent variables, *resp.*	-
V_{bole}	Bole volume	m^3
V_{tot}	Total above-ground volume	m^3
α, β, γ	Parameters	diverse
$\boldsymbol{\theta}$	Vector of parameters	diverse
μ	Mean parameter (often a function of explanatory variables and parameters)	-
ϕ	Precision of the beta distribution	-
ρ	Correlation parameter (between -1 and 1)	-
$\boldsymbol{\Sigma}$	Variance-covariance matrix for the multivariate model	diverse

Table 1: Used notations, sorted by alphabetical order with Latin symbols first.

Bole volume

The bole volume is considered the reference volume of the French NFI and is computed via species-specific allometries. The formulæ changed from time to time, the last one dating from 2016 (Morneau 2016). The allometric equations were fitted on the NFI dataset, which contains the volume of the butt log (from the stump to 2.6 m), the second log (from 2.6 m to h_{dec} — defined later), and upper log sections if any (see Fig. 7). These three volumes relies on the circumferences at breast height, 2.6 m, h_{dec}, d_{dec}, and the median diameter of the second log d_m and are derived using cylinder and Newton-Simpsons formulæ (Gohon 2024).

In what follows, we sum up the equations currently used in the "Two sets of equations" section and compare them with new allometries recently developed (Gohon 2024) in the section New bole models. All the notations and definitions are gathered in Tab. 1.

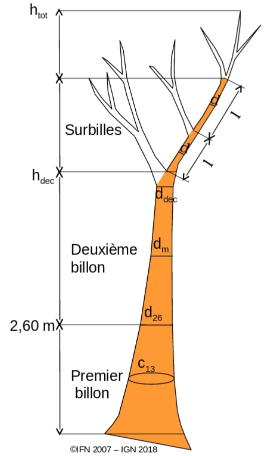


Figure 7

Two sets of equations

The annual living tree sample surveyed by French NFI is divided into two subsets:

Complete trees where girth (at breast height), c, total height, h, and taper height, h_{dec} are measured. These trees are randomly designated, ensuring that there is one individual per girth class, per species, and per plot.

Simplified trees where only girth is measured.

This leads to two sets of equation, one per subset. For the so-called 'complete trees', the bole volume is predicted using an allometric equation with three input variables, as described in Morneau 2016:

$$\begin{cases} \hat{V}_{\text{bole}} = \dfrac{c^2 h}{4\pi\left(1 - \frac{1{,}3}{h}\right)^2} \hat{f}_{\text{new}} \\ \hat{f}_{\text{new}} = \left(\alpha + \beta c + \gamma \dfrac{\sqrt{c}}{h} + f\left(h_{\text{dec}}\right) + \delta \dfrac{1}{c^{\epsilon}}\right)\left[1 - b\left(\dfrac{0{,}07\pi}{c}\right)^3 \left(1 - \dfrac{1{,}3}{h}\right)^3\right], \end{cases} \tag{1}$$

where f is a function of h_{dec} among the following:

$$f\left(h_{\text{dec}}\right) = f_{\max}\left(\frac{h_{\text{dec}}}{h_{\text{dec}} + k}\right)^{1+\rho} \qquad f\left(h_{\text{dec}}\right) = \eta \ln\left(\frac{h_{\text{dec}}}{h}\right)$$

$$f\left(h_{\text{dec}}\right) = \eta \ln\left(h_{\text{dec}}\right) \qquad f\left(h_{\text{dec}}\right) = \eta h_{\text{dec}}$$

$(\alpha, \beta, \gamma, \eta, f_{\max}, k, \rho, \delta, \epsilon, b)$ is a set of coefficients. Those and $f\left(h_{\text{dec}}\right)$ are both depending on the tree species. $(\alpha, \beta, \gamma, \eta, f_{\max}, k, \rho, \delta)$ also depend on the kind of terminal cut of the tree, either the stem top (*regular cut*, *i.e.*, $h_{\text{dec}} = h$) or a sudden decrease in stem diameter (*shape cut*, $h_{\text{dec}} < h$).

For simplified trees, the bole volume is based on the predicted volume of a neighbouring complete tree. A one-variable allometric equation is used to adjust for the girth difference between the reference tree and the simplified tree when assigning bole volume.

Volume imputation

We associate each simplified tree i with a complete tree j that belongs to the same species, sample plot, and same girth class and adjust the bole volume of i using a ratio relative to its reference tree:

$$\hat{V}^{\text{imp}}_{\text{bole}}(i) = \hat{V}_{\text{bole}}(j)\frac{V_I(i)}{V_I(j)}, \tag{2}$$

where $\hat{V}_{\text{bole}}$ is the three variable equation (1), and V_I is a species-specific allometric equation that takes only c as an input variable (see equation (**??**)).

Current one-entry allometric equations are built on log transformation:

$$V_I = \mathrm{e}^{\alpha + \beta \ln(c) + \gamma \ln(c)^2 + \delta \ln(c)^3 + \zeta \ln(c)^4 + \eta f(g) + \frac{\sigma^2}{2}}, \tag{3}$$

where $f(g)$ is a function of local basal area. The coefficients $\alpha, \cdots, \sigma$ depend on the tree species and contextual parameters (ecological area, altitude zone, property regime).

Motivations for changes

Regarding three-entries allometric equations, one aim at improving model parsimony. Future species models all share the same formula. Also, fitting rely no more on the kind of terminal cut. In relation to it, a decision is to get rid of h_{dec} entry for trees with regular cut: since 2020, h_{dec} is no longer measured on that kind of trees and now it has to be assessed by a third-party model. One achieve it by setting down h'_{dec} as:

$$h'_{\text{dec}} = \begin{cases} h_{\text{dec}} & \text{in case of a shape cut} \\ h & \text{otherwise} \end{cases}$$

For monitoring stock variations over time, the new coming inventory method will assess tree sample data twice. Tree samples are surveyed a second time, but only tree girth is measured again. So, one-entry allometric equations shall be used to assess tree new volume at second survey given old and new girth, the same way than simplified trees (see 2). For that purpose, one-entry volume equations have to be an increasing function of girth. In particular, an explanatory variable $f(g)$ involving local basal area, conveying stand competition, has to be questioned.

The approach of new allometric equations development is detailed in Gohon 2024.

New bole models

Like current equations, future three-entries models are fitted using French NFI data (see 2 3.). It models a shape coefficient f_{new} (see 1) an analogous way than Morneau 2016.

$$\hat{f}_{\text{new}} = \alpha + \beta c + \gamma \frac{\sqrt{c}}{h'_{\text{dec}}} + \delta \frac{\sqrt{h'_{\text{dec}}}}{c^2 h} + \eta \left(1 - \frac{h'_{\text{dec}}}{h}\right)$$

where coefficients $(\alpha, \beta, \gamma, \delta, \eta)$ only depend on the tree species.

New one-entry models are fitted to volume estimates of trees with height measures from recent inventory campaigns.

$$V_I = \mathrm{e}^{\alpha + \beta \ln(c) + \gamma \ln(c)^2 + \frac{\sigma^2}{2}}$$

where $(\alpha, \beta, \gamma, \sigma)$ are still depending on the tree species and same contextual parameters (ecological area, altitude zone, property regime).

New and current models perform similarly. Survey estimates (see for instance 8 and 9 from Gohon 2024) are a go-to output to foresee the forthcoming effect of new allometric equations on NFI results. It shows lower volume estimates for small trees and slightly higher volume estimates for bigger trees, and consequently higher volume growth estimates. Differences never exceed confidence intervals.

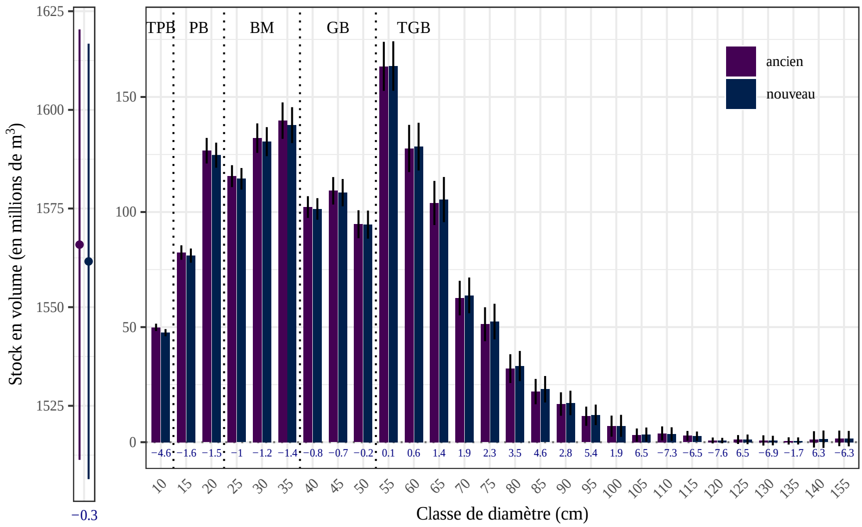


Figure 8: National bole volume estimates for trees with height measures, by rounded girth.

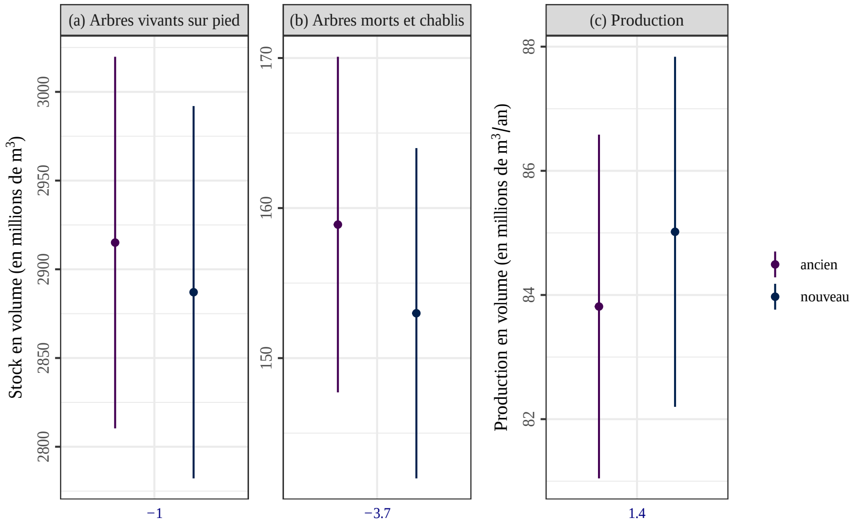


Figure 9: National bole volume and growth estimates for all living trees and dead trees.

Total above-ground volume

In this section, I explore two possible paths to estimate the total above-ground volume, V_{tot}. The first option, based on Longuetaud et al. 2013, uses the ratio:

$$r = \frac{V_{\text{bole}}}{V_{\text{tot}}} \tag{4}$$

and is comprised between 0 and 1 by construction. The second option is to fit a multivariate normal distribution, MVN, to the *transformed* ratio, $\text{logit}(r)$, jointly with the log-transformed total volume, $\log(V_{\text{tot}})$:

$$\begin{pmatrix} \text{logit}(r) \\ \log(V_{\text{tot}}) \end{pmatrix} \sim \text{MVN}\left\{ \begin{pmatrix} f(c, h; \boldsymbol{\theta}) \\ g(c, h; \boldsymbol{\theta}) \end{pmatrix}, \boldsymbol{\Sigma} \right\}, \tag{5}$$

where f and g are functions tailored to our specific needs, c is circumference at breast height, h, is tree height, and θ is a vector of parameters to estimate.

Ratio approach

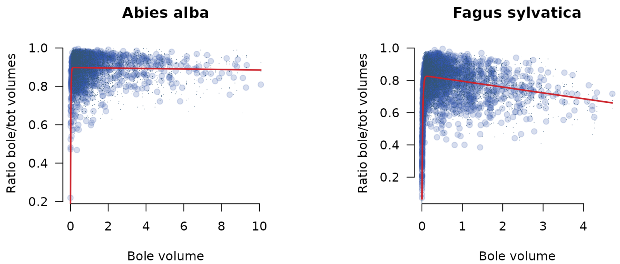
I modelled the ratio r (see equation (3)), which quantifies the percentage of volume represented by the bole, by a Beta-distribution. This distribution respects the bounds and accounts for heteroskedasticity, but its parameters shape_1 and shape_2 are not intuitive. Hence, I reparametrised the Beta-distribution in function of its mean, μ and precision, ϕ:

$$\begin{aligned} \text{shape}_1 &= \phi\mu \\ \text{shape}_2 &= \phi(1-\mu) \end{aligned}$$

Employing the mean and variance is not advisable, since it introduces the interacting constraint $\sigma^2 < \mu(1-\mu)$, while the precision lies between 0 and $+\infty$. The ratio approach has been tested on the 17th most common species in the Emerge dataset. The mean μ is a function of three parameters to estimate (see equation (5)) and V_{bole}, the *predictive variable*:

$$\mu(V_{\text{bole}}; \alpha, \beta, \gamma) = \exp[-\beta V_{\text{bole}}](V_{\text{bole}} - \alpha) + \gamma V_{\text{bole}} + \alpha, \quad (6)$$

where α, β and γ are parameters to estimate. The results are displayed for *Abies alba* and *Fagus sylvatica*, to represent both conifers and broadleaves (see Fig. 10).



(a) *Abies alba* (b) *Fagus sylvatica*

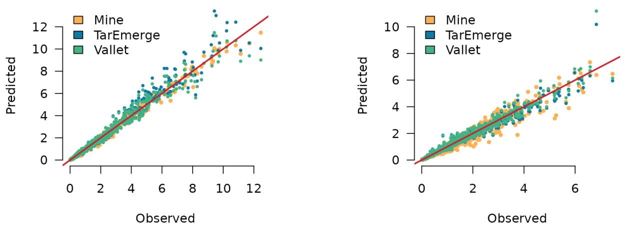
Figure 10: Fit of the ratio data from Emerge only (circles). The dots are posterior predictions, which are in the range of the observations, although residuals show under-dispersion for small values of V_{bole} (see appendix 3). The red curve is the fitted equation (5)

The equation (5) gives similar predictions (see 11) to Vallet et al. 2006 (currently in use at the NFI) or Emerge[2] (Deleuze, Morneau, Renaud et al. 2014). Responses for the non-displayed species are similar to *Abies alba* and *Fagus sylvatica* results.

[2] In addition to digitising the dataset related to the 'Protocole Oudin', the Emerge project aimed to develop a unified allometry for all tree species in French forests. They arrived at the allometry

$$V_{\text{tot}} = \frac{c^2 h}{4\pi(1 - {}^{1.3}/_{h})}\left(\alpha + \beta\frac{\sqrt{c}}{h} + \gamma\frac{h}{c}\right).$$

This allometry is not used officially by the French NFI



(a) *Abies alba* (b) *Fagus sylvatica*

Figure 11: Predictions of total aerial volume in function of bole volume (the explanatory variable) with my model (equation (5)), the current model used at the French NFI(Vallet et al. 2006), and the allometry developed during the project Emerge.

Multivariate approach

Multivariate models allow pulling information across components, *i.e.*, they describe the variation within and covariation among tree components. I chose to model jointly r and V_{tot} but the alternative modelling bole and crown could also be considered.

> **Intractability**
>
> Note that whatever I try to model, there will always be intractability somewhere. If I model both components – trunk and crown – with a multivariate lognormal, the distribution of the total volume becomes intractable (sum of two lognormals has no closed form distribution). Alternatively, if I model the total volume and the ratio r with transformations, then V_{bole} has no closed-form expression. Indeed, to know the product $V_{\text{bole}} = rV_{\text{tot}}$ requires to know the joined distribution of r and V_{tot}.

Simulated data

In order to demonstrate the relevance of multivariate methods, I simulate two correlated random variables and then recover the generating parameters. Let X be an explanatory variable, and $\mathbf{Y} = (Y_1, Y_2)$ be the dependant variable, such that:

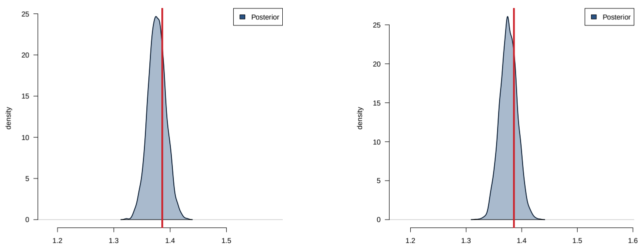
$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \sim \text{MVN}\left\{ \begin{pmatrix} \alpha_1 + \beta_1 X \\ \alpha_2 + \beta_2 X \end{pmatrix}, \mathbf{\Sigma} \right\},$$

where α and β are regression parameters to estimate, and $\mathbf{\Sigma}$ is the variance-covariance matrix defined by:

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}.$$

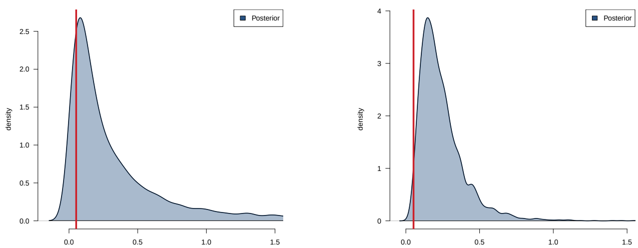
I am able to recover the parameters α, β and the variance-covariance matrix $\mathbf{\Sigma}$ with both a multivariate normal or two separated linear regression as shown in Fig 12. However, when multiplying both variables Y_1 and Y_2, as we would do to compute the bole volume from the total volume and the ratio r, we see that the two separated linear models give a biased result with a smaller credibility interval (see Figs. 13 – 15). This is because the real expected value for the product is $\mathbb{E}[Y_1 Y_2] = \mathbb{E}[Y_1]\,\mathbb{E}[Y_2] + \text{Cov}[Y_1, Y_2]$ in the correlated case, while for the uncorrelated case it assumes $\mathbb{E}[Y_1 Y_2] = \mathbb{E}[Y_1]\,\mathbb{E}[Y_2]$ (since the covariance is 0 in this case). The R code is available on Github.

In this simulated case, the principal drawback of employing a *classical* rather than a *multivariate* model is that it produces narrower credibility intervals and apparently consistent parameter estimates, yet leads to a biased output product $Y_1 Y_2$.



(a) Multivariate model (b) Uncorrelated model

Figure 12: Posterior of the intercept parameter α, with the real value displayed in red. As can be seen, both model can recover the intercept (and same for the other parameters)

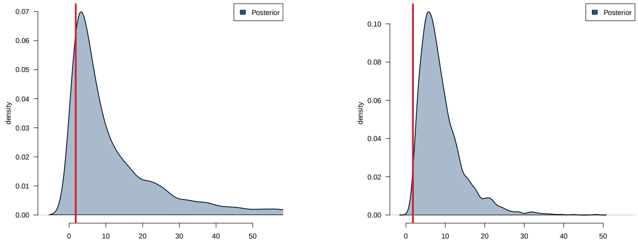


(a) Multivariate model (b) Uncorrelated model

Figure 13: Posterior predictions of the product $Y_1 Y_2$ for a value sitting around the 15th percentile

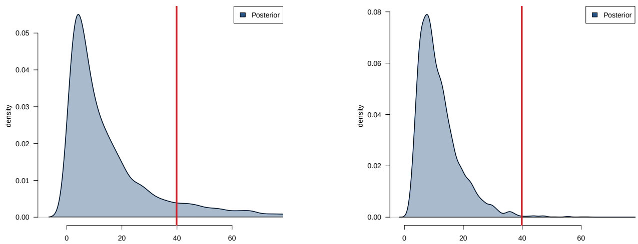
Real case study

I tested a simple multivariate model on $\text{logit}(r)$ and $\log(V_{\text{tot}})$ with only two intercepts and $\mathbf{\Sigma}$ to estimate. I found a positive correlation of 0.28 $(0.24 - 0.31)$ for *Fagus sylvatica* between the two, suggesting that larger total volumes are predominantly concentrated in the bole.



(a) Multivariate model (b) Uncorrelated model

Figure 14: Posterior predictions of the product $Y_1 Y_2$ for a value sitting around the 50th percentile (*i.e.*, median)



(a) Multivariate model (b) Uncorrelated model

Figure 15: Posterior predictions of the product $Y_1 Y_2$ for a value sitting around the 85th percentile

XyloDensMap

In France, national estimates of forest biomass and carbon stocks have so far relied on wood density values derived from an unpublished dataset, compiled from a literature review conducted by Jean-Luc Dupouey (INRAE) as part of the CARBOFOR project (Loustau 2004). Some of these values originate from a 170-year-old source (Mathieu 1855), updated to provide a single average value per species, covering around 50 species. However, these estimates are based on small and unbalanced samples—typically fewer than 10 mature trees per species and do not reflect the diversity of conditions found in French forests, particularly in terms of species, tree size, and growth conditions.

To address the lack of precise and comprehensive data on wood density for forest species in mainland France, the XyloDensMap project was launched in 2015 through a collaboration between INRAE and IGN. The resulting open dataset, XyloDensMap, includes individual wood density measurements from 110 763 wood cores taken at breast height across metropolitan France. These data were obtained by combining the spatially systematic sampling design of the French National Forest Inventory (NFI) with a high-throughput method for measuring wood density using X-ray computed tomography (Freyburger et al. 2009; Jacquin et al. 2019). Due to the systematic nature of the IFN's annual sampling framework (Bontemps and Bouriaud 2024; Bouriaud et al. 2023), the XyloDensMap dataset is representative of French forests, particularly in terms of species diversity and tree size.

All details regarding the sampling design, sample processing, and wood density measurement in XyloDensMap are provided in the corresponding data paper (Cuny, Leban et al. 2025). A second publication on wood density modeling in France using the XyloDensMap dataset is currently under review at Biogeosciences, with the article already available as a preprint (Cuny, Bontemps et al. 2025). This work compares the application of different methods (average species coefficients, more detailed linear models incorporating tree size and climatic conditions, random forest) to simulate wood density in IFN data, and discusses the influence of the chosen method on estimating total aboveground biomass stocks at multiple scales in France. The publication thus already includes simulations of wood density across the entire NFI framework (see Fig. 16)—a necessary step for calculating biomass and carbon stocks and fluxes at different scales in France—and may serve as a basis for discussion to define the method to be used for simulating wood density in the NFI dataset.

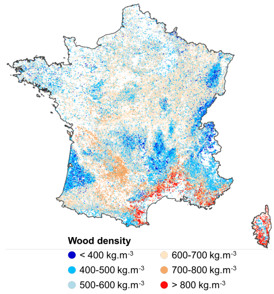


Figure 16: Map of community-wide mean wood density in mainland France on each forest plot inventoried by the French NFI between2005 and 2022.

Conclusion

Above-ground volumes (bole and total)

We presented two alternative approaches for estimating tree volumes. The first approach involves estimating V_{bole} and V_{tot} separately, using a ratio for the total above-ground volume to ensure consistency between both volumes. Indeed, the main so far has been that the formula currently used by the French NFI to predict V_{bole} (Morneau 2016) sometimes yields values greater than the total volume predicted by Vallet et al. 2006. Using the first approach has the advantage of retaining the existing NFI bole volume while providing a coherent total volume such that $V_{\text{tot}} > V_{\text{bole}}$.

The second approach has the advantage of jointly fitting both volumes, which accounts for correlations. It also ensures consistent volumes by construction, but is harder to parametrise and implies a loss of analytical tractability due to the product of two (correlated) random variables. Refitting the bole volume might also imply a break in the time series generated by the French NFI since 1958. We might have to keep both the old and new allometry then, for compatibility reasons with external studies.

A third option, not yet explored, would be to use *copulas* (Nelsen 2006). Copulas also belong to the family of multivariate models, and could be described as functions that join or couple multivariate distribution functions to their one-dimensional marginal distribution functions. In other words, it would allow us to model r directly with a Beta-distribution and V_{tot} with, for example, a Gamma-distribution rather than relying on a multivariate normal model in which all components of the random vector must follow a normal distribution. However, copulas are more complicated and would require a dedicated review of two-dimensional copulas (Nelsen (2006) provides a well written introduction covering all aspects relevant to our application).

Root volume and wood carbon content

While actions regarding aboveground volumes (stem wood and total above-ground biomass) and wood density are already underway, aspects related to root volume and carbon content in wood remain less explored at this stage. Unlike the evaluation of or wood density, the IGN does not currently have datasets that allow for the analysis of root volumes or wood carbon content. For these aspects, a different approach will be implemented, involving a literature review to assess the current state of knowledge.

The objective will be to determine whether the current assumptions used in the IGN method—namely, a root expansion coefficient by botanical class (1.28 for broadleaves and 1.30 for conifers) and a single carbon content coefficient (0.475)—are still relevant in light of current knowledge, or whether they could be improved to better reflect natural variability. The literature review will be complemented by discussions with expert researchers in these fields, such as those at the INRAE center in Champenoux.

An internship was already conducted during the summer of 2025 on this topic. It helped identify a number of bibliographic references for each aspect. Regarding wood carbon content, many articles have recently been published, and a global database was even compiled through a literature review (Doraisami et al. 2022). This database includes carbon content values for several species found in French forests. As for root volume, an interview was conducted with Frédéric Danjon, a root specialist at INRAE, which provided access to numerous relevant references on root volume. A synthesis of the various identified references for each aspect now needs to be carried out.

Should the assumptions regarding root volume and carbon content be revised, an analysis and documentation of the impact of these revisions on carbon estimates will be undertaken via a sensitivity analysis (Iooss and Saltelli 2017).

Residuals

Model on ratio

Residuals for the model (5).

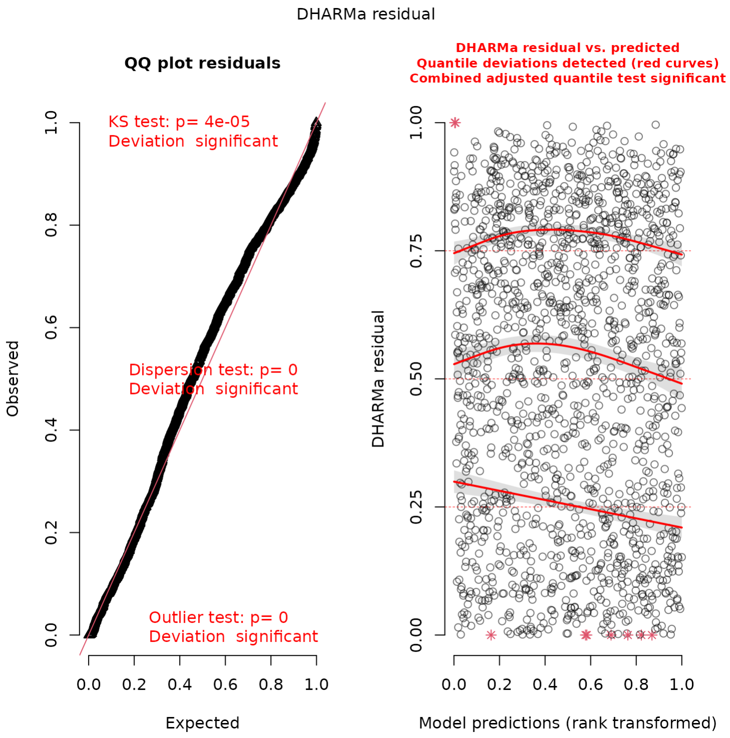


Figure 17: Residuals for *Abies alba*

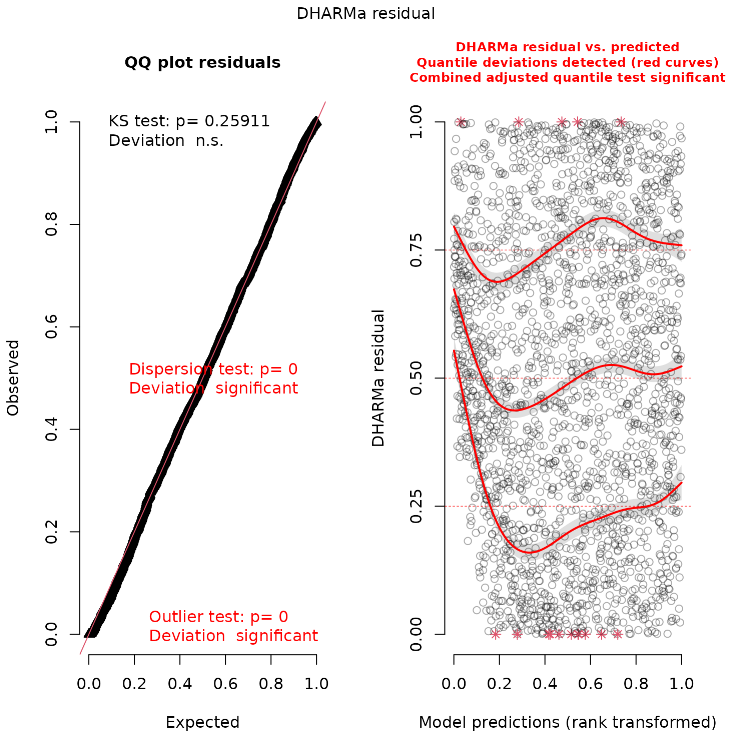


Figure 18: Residuals for *Fagus sylvatica*

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