An Analysis of Fuel Efficiency

Clyde Tressler

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Executive Summary

MotorTrend magazine data from observations of 32 models of cars were analyzed to help understand the characteristics that account for fuel efficiency. Of particular interest is whether there is an effect associated with automatic versus manual transmission. After we account for a strong relationship between vehicle weight and transmission type, we claim there is a statistically significant increase in the mean value of gas mileage attributable to manual transmissions, which we report as **9.23 mpg**, with a **P-value** $\leq$ **0.00378**.

Data Processing and Initial Exploratory Analyses

This report omits some code for brevity. The entire R markdown file can be found here.

Here we show the first several rows of the dataset.

```
##                mpg cyl disp  hp drat    wt  qsec vs am gear carb
## Mazda RX4     21.0   6  160 110 3.90 2.620 16.46  0  1    4    4
## Mazda RX4 Wag 21.0   6  160 110 3.90 2.875 17.02  0  1    4    4
## Datsun 710    22.8   4  108  93 3.85 2.320 18.61  1  1    4    1
```

In the Appendix we present a table explaining the meaning of variable names. Several binary and discrete variables were transformed from numeric vectors to factor variables for this analysis.

Also in the Appendix we present a box and whisker plot of the two transmission types demonstrating that the median mpg of the automatic group lies below that of the manual group.

We report a p-value for a t-test with unequal variances as 0.001374 and therefor reject the null hypothesis at the 95% confidence level, to conclude that the difference between the means of the mileage for the two transmission types, 7.24 mpg, is significant.

This is an important first clue, but, as we will show, it does not present a complete picture.

Developing a Multivariate Linear Model

Next we turn to multivariate regression to develop a model that will help us understand whether confounding variables are contributing **omitted variable bias**. We tabulate the correlation of the numeric variables. Many of these measure closely-related properties, as these values clearly indicate.

Correlation Table of mtcars Data

	mpg	disp	hp	drat	wt	qsec
mpg	1.0000000	-0.8475514	-0.7761684	0.6811719	-0.8676594	0.4186840
disp	-0.8475514	1.0000000	0.7909486	-0.7102139	0.8879799	-0.4336979
hp	-0.7761684	0.7909486	1.0000000	-0.4487591	0.6587479	-0.7082234
drat	0.6811719	-0.7102139	-0.4487591	1.0000000	-0.7124406	0.0912048
wt	-0.8676594	0.8879799	0.6587479	-0.7124406	1.0000000	-0.1747159
qsec	0.4186840	-0.4336979	-0.7082234	0.0912048	-0.1747159	1.0000000

Feature Selection

We first consider the apriori exclusion of certain variables. For example, the quarter second mile time is influenced by driver skill and is heavily biased in favor of manual transmission. This is not meaningful to our goal of quantifying transmission effects under normal driving conditions, so we choose to exclude it.

We also know that 'drat,' the rear axle ratio, is a downstream variable determined by the upstream drive train, including the transmission, so the effect of this variable is captured by other features.

We select weight as our first predictor since it shows the highest correlation with mpg.

When we examine the data sorted by weight, we see that 9 of the top 10 heaviest cars have automatic transmissions, while 9 of lightest 10 have manual. This leads us to consider the addition to our model of an interaction term between weight and transmission type. In the Appendix we show a visualization of this relationship, where the two transmission types exhibit a distinct separation in a plot of weight vs MPG.

Sorting the Data by Vehicle Weight

rownames	wt	am	hp	mpg
Lincoln Continental	5.424	0	215	10.4
Chrysler Imperial	5.345	0	230	14.7
Cadillac Fleetwood	5.250	0	205	10.4
Merc 450SE	4.070	0	180	16.4
Pontiac Firebird	3.845	0	175	19.2
Camaro Z28	3.840	0	245	13.3
Merc 450SLC	3.780	0	180	15.2
Merc 450SL	3.730	0	180	17.3
Duster 360	3.570	0	245	14.3
Maserati Bora	3.570	1	335	15.0

rownames	wt	am	hp	mpg
Ferrari Dino	2.770	1	175	19.7
Mazda RX4	2.620	1	110	21.0
Toyota Corona	2.465	0	97	21.5
Datsun 710	2.320	1	93	22.8
Fiat 128	2.200	1	66	32.4
Porsche 914-2	2.140	1	91	26.0
Fiat X1-9	1.935	1	66	27.3
Toyota Corolla	1.835	1	65	33.9
Honda Civic	1.615	1	52	30.4
Lotus Europa	1.513	1	113	30.4

We systematically examine the inclusion of other variables and choose to include horsepower, which will therefor serve as a proxy for the remaining collinear engine performance variables. The reader is referred to the full R markdown file for the code demonstrating this search procedure.

Diagnostics ANOVA testing of nested models is employed to choose which terms are significant in the presence of others.

```
## Analysis of Variance Table
##
## Model 1: mpg ~ wt
```

```
## Model 2: mpg ~ wt + hp
## Model 3: mpg ~ wt + hp + am
## Model 4: mpg ~ wt + hp + am + wt:am
## Model 5: mpg ~ wt + hp + am + wt:am + hp:am
##   Res.Df    RSS Df Sum of Sq       F   Pr(>F)
## 1     30 278.32
## 2     29 195.05  1    83.274 15.9318 0.000478 ***
## 3     28 180.29  1    14.757  2.8232 0.104888
## 4     27 146.84  1    33.446  6.3988 0.017820 *
## 5     26 135.90  1    10.945  2.0940 0.159833
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Once we satisfy the required inclusion of transmission type, we see that the addition of the interaction term (wt*am) is also needed.

High Influence Points

In the Appendix we plot residuals vs hat-values for the model **mpg~wt+hp+am+wt*am** and identify the Maserati Bora as a high influence point. This is a high-performance vehicle and is the only vehicle in the data set with 8 carburetors. Furthermore, the dfbeta of this point for the coefficient of transmission type, -2.2929, lends motivation to our choice to remove it.

We note that the response for mpg vs horsepower (see Appendix) appears less quadratic after the removal of the Maserati data point.

The residuals vs Fitted and Q-Q plots shown in the Appendix indicate the distribution of the residuals is close to normal.

```
##
## Call:
## lm(formula = mpg ~ wt + hp + am + wt * am, data = mtcars2)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -3.015 -1.452 -0.393  1.341  4.897
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 30.79263    2.57244  11.970 4.43e-12 ***
## wt          -2.09632    0.82265  -2.548  0.01708 *
## hp          -0.03584    0.01020  -3.515  0.00163 **
## am1         13.84775    3.95675   3.500  0.00170 **
## wt:am1      -4.61832    1.45195  -3.181  0.00378 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.202 on 26 degrees of freedom
## Multiple R-squared:  0.8853, Adjusted R-squared:  0.8677
## F-statistic: 50.18 on 4 and 26 DF,  p-value: 7.415e-12
```

```
##                  2.5 %      97.5 %
## (Intercept) 25.5049130 36.08035486
## wt          -3.7873067 -0.40533831
```

```
## hp          -0.0568056 -0.01488208
## am1          5.7145332 21.98095771
## wt:am1      -7.6028370 -1.63380182
```

Conclusion

We have shown that a linear model using weight, horsepower, transmission type, and an interaction term between weight and transmission fits observations in the data set well, with an R-squared value of 0.868, indicating 86.8% of the total variance in mpg has been explained. The 95% confidence intervals for the coefficients show no instances where the intervals cross zero and the residuals plots show no discernible patterns.

Clearly, fuel efficiency is adversely impacted by vehicle weight, as shown by the negative slope in the linear model. Likewise, increased horsepower, which we also take to be a proxy for other engine size variables, such as displacement and numbers of cylinders and carburetors, also reduces fuel efficiency.

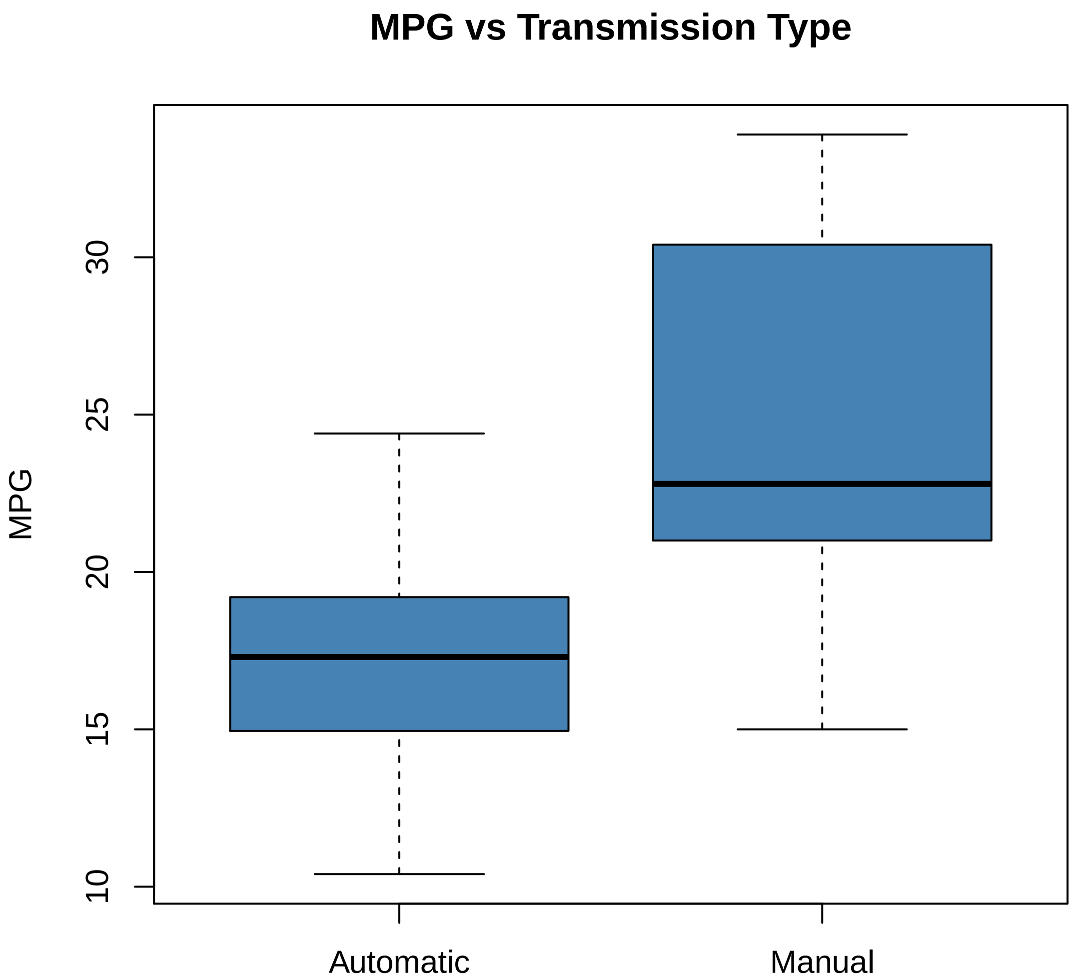
We can report that the increase in mean miles per gallon associated with a manual transmission, as calculated from the values of the transmission type slope coefficient and that of its interaction term with weight, is shown to be **9.23 mpg**, with a maximum **P-value of 0.00378** as given by the interaction term.

Appendix

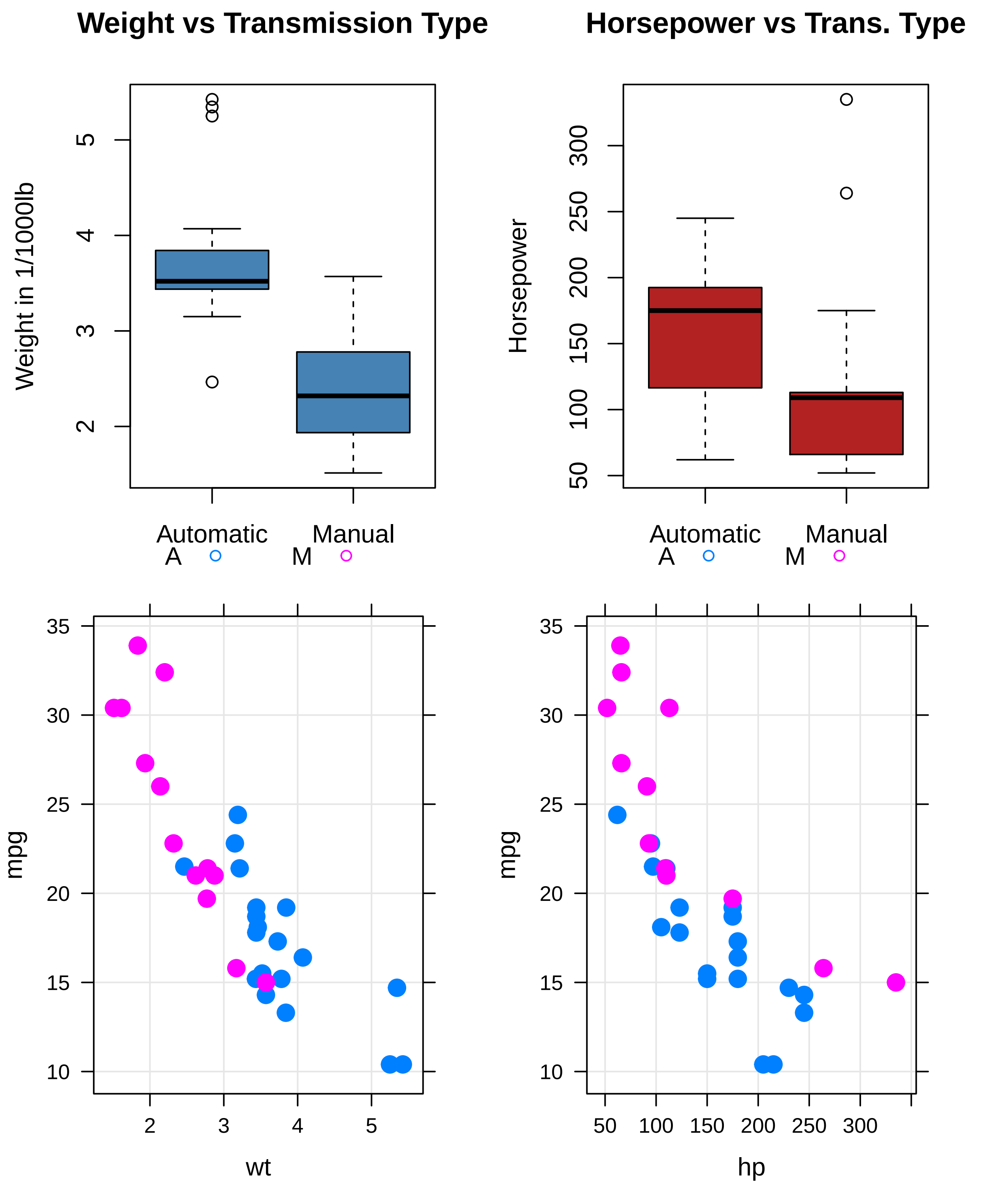
Table 1: Variables

Variable Name	Description
mpg	Miles/(US) gallon
cyl	Number of cylinders
disp	Displacement (cu.in.)
hp	Gross horsepower
drat	Rear axle ratio
wt	Weight(lb/1000)
qsec	1/4 mile time
vs	Cylinder alignment, V or Straight
am	Transmission (0 = automatic, 1 = manual)
gear	Number of forward gears
carb	Number of carburetors

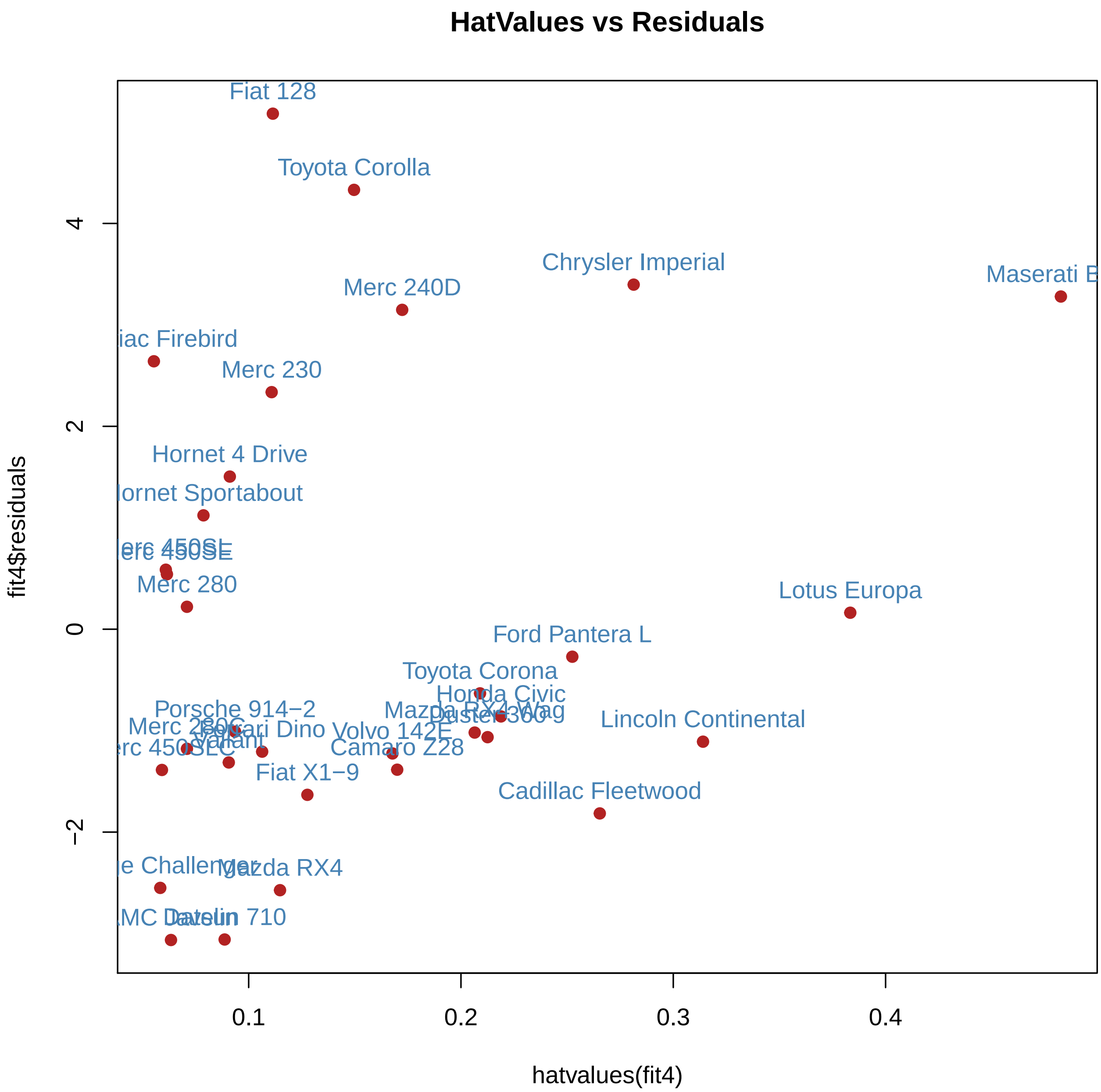
Box and Whisker plot of MPG vs Transmission Type



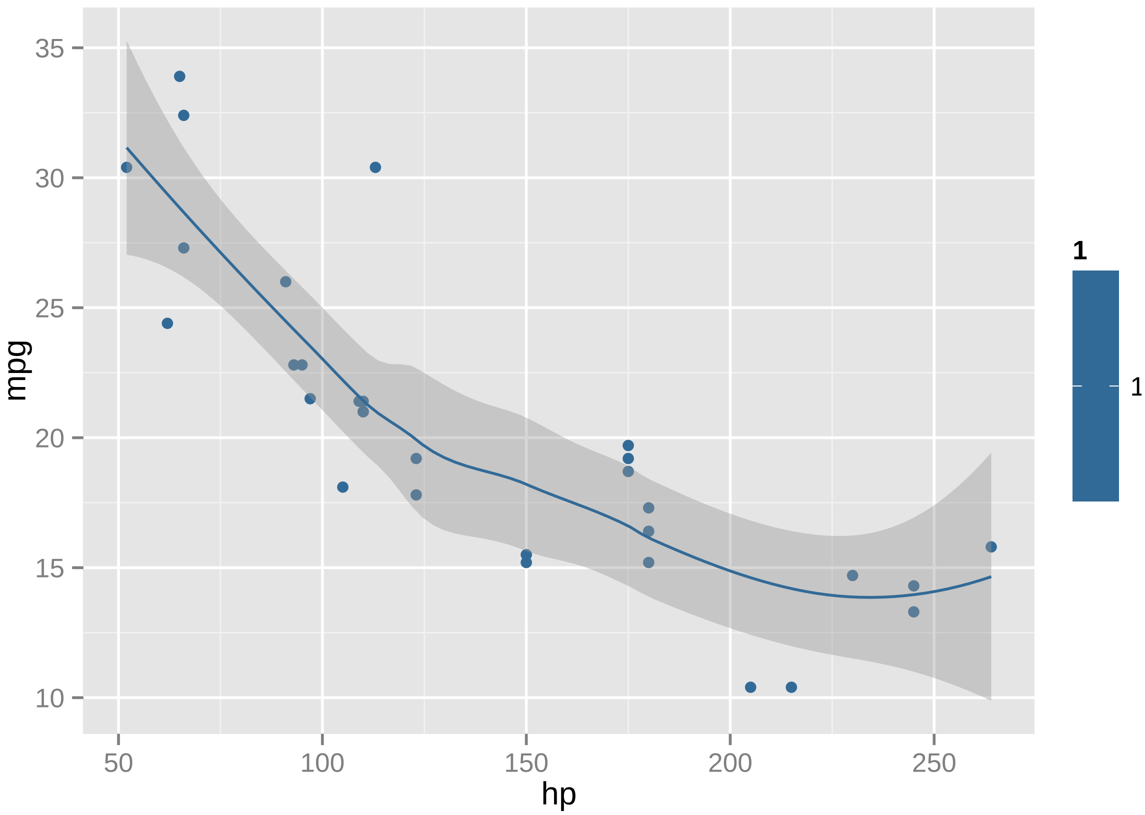
Interactions for Weight and Transmission and for Horsepower and Transmission



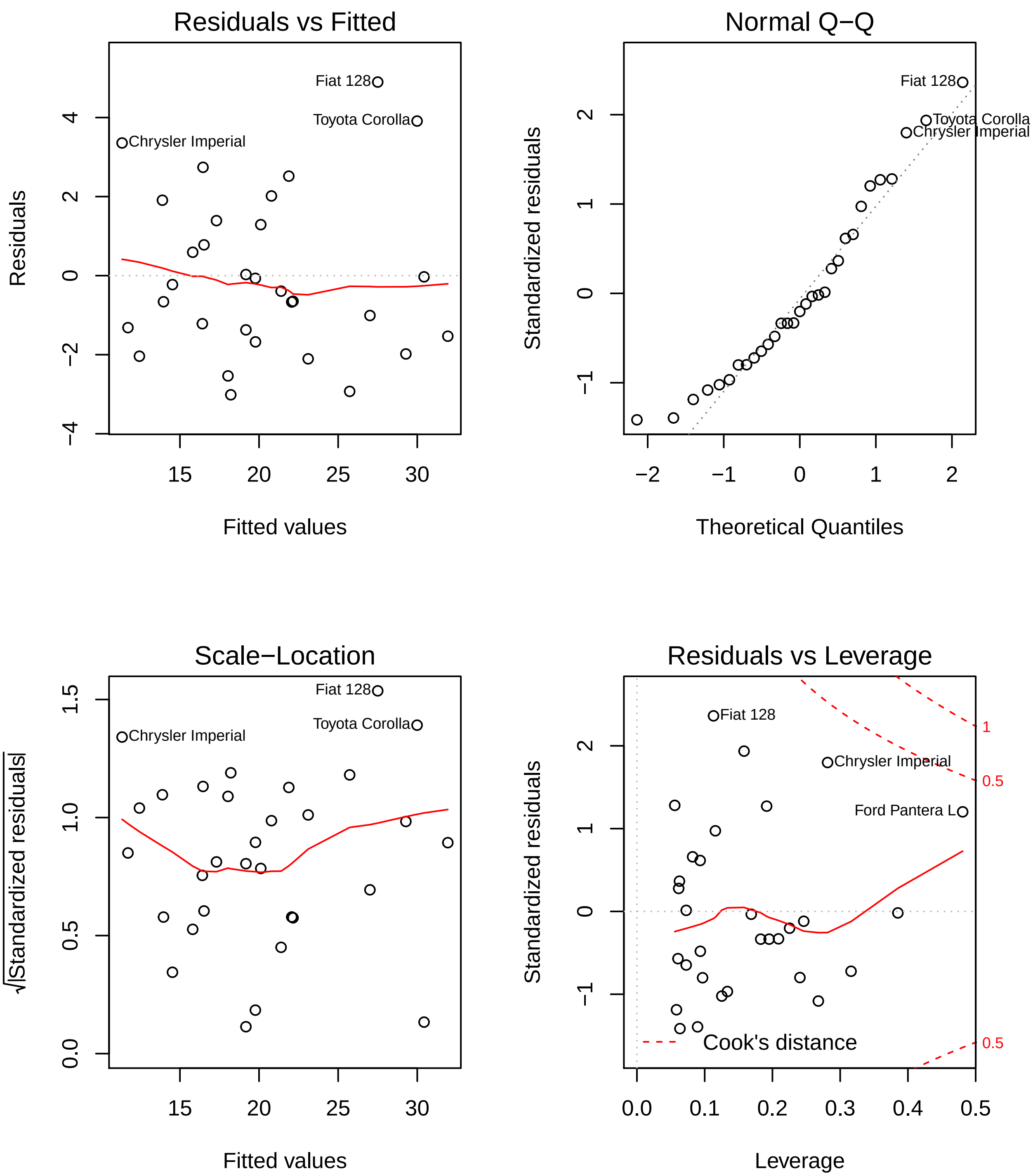
Checking for High Influence Points

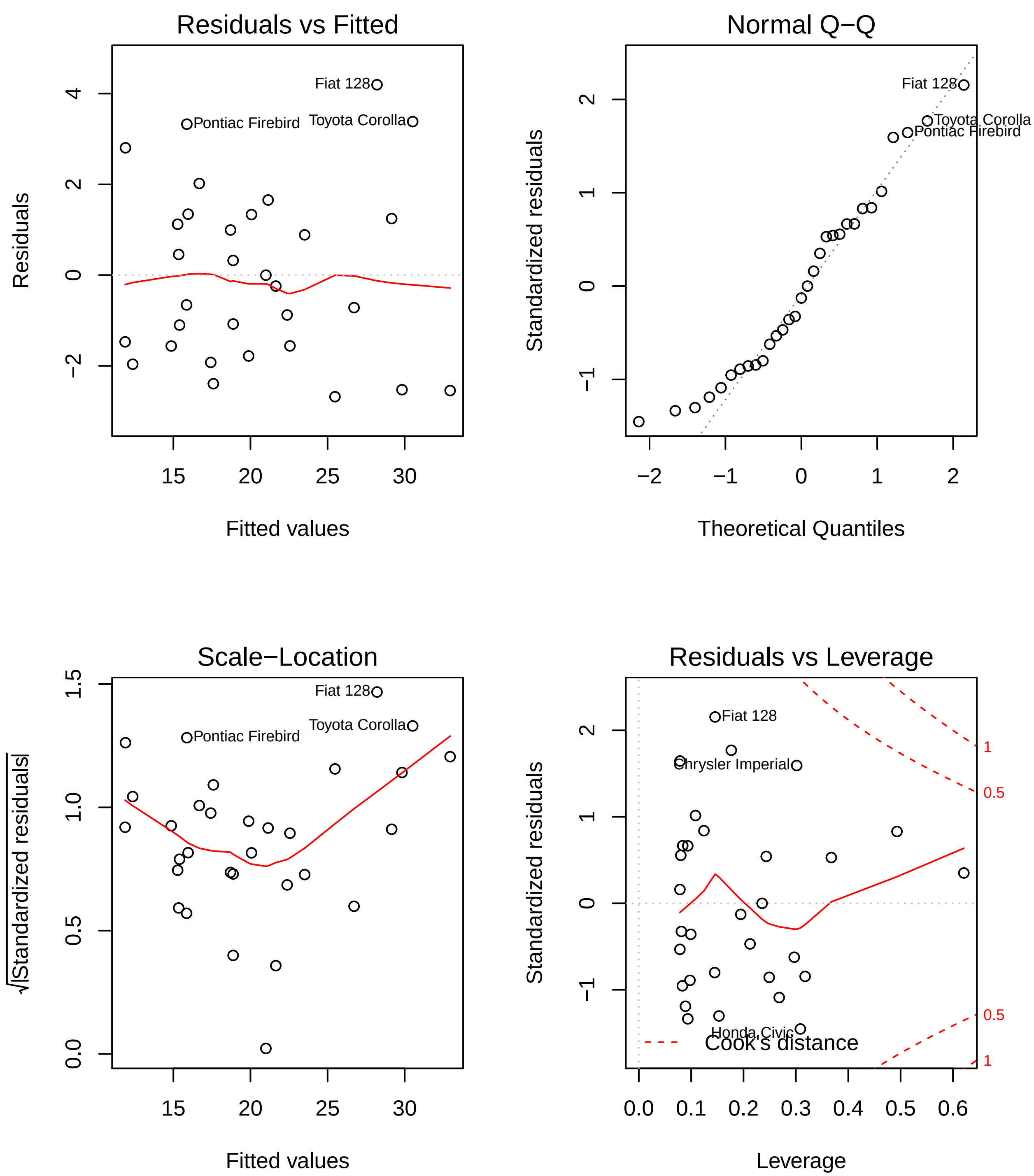


Quadratic Response of MPG with Horsepower



Comparing models with and without Horsepwer Squared Term




Residuals vs Fitted
Residuals
Fitted values
Fiat 128
Pontiac Firebird
Toyota Corolla
Normal Q−Q
Standardized residuals
Theoretical Quantiles
Fiat 128
Toyota Corolla
Pontiac Firebird
Scale−Location
√|Standardized residuals|
Fitted values
Fiat 128
Pontiac Firebird
Toyota Corolla
Residuals vs Leverage
Standardized residuals
Leverage
Fiat 128
Chrysler Imperial
Honda Civic
Cook's distance
1
0.5
0.5
1