Mumbai Mills Redevelopment

Theory Notes

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Contents

1 Basic Model

The aim of these notes is develop the building blocks of the full model we use in the paper. We start by outlining a simplified version of Ahlfeldt. al. al. (2016). We begin by setting up the model, outlining and algorithm for simulation, and running an estimation algorithm on the simulated data to check the correct parameters can be recovered

1.1 Model

Workers. There are I discrete locations in the city. There is a continuum of individual workers indexed by ω. Workers choose pairs of locations (i, j) to live and work which have indirect utilility

$$U_{ij}(\omega) = \frac{u_i w_j r_{Ri}^{\beta-1}}{d_{ij}} \epsilon_{ij}(\omega)$$

Here u_i is an amenity (exogenous for now), w_j is a wage, r_{Ri} is the price of residential housing and $d_{ij} = \exp(\kappa t_{ij})$ is a disutility cost of commuting where t_{ij} is the commute time.

Each worker has an idiosyncratic preference for each pair $\epsilon_{ij}(\omega)$ drawn iid from a Frechet distribution with shape θ. This means the number of workers choosing commute ij is

$$L_{ij} = \bar{L}(\gamma/\bar{U})^\theta (u_i w_j r_{Ri}^{\beta-1}/d_{ij})^\theta$$

where $\bar{L}$ is the total population, $\gamma \equiv \Gamma\left(1 - \frac{1}{\theta}\right)$ is a constant that depends on the gamma function, and $\bar{U} = \gamma \left[\sum_{rs}(u_r w_s r_{Rr}^{\beta-1}/d_{rs})^\theta\right]^{1/\theta}$ is expected utility.

The number of residents and workers in each location can then be computed by summing over origin and destination locations:

$$L_{Ri} = \sum_j L_{ij} = \bar{L}(\gamma/\bar{U})^\theta (u_i r_{Ri}^{\beta-1})^\theta \sum_j (w_j/d_{ij})^\theta \tag{1}$$

$$L_{Fj} = \sum_i L_{ij} = \bar{L}(\gamma/\bar{U})^\theta w_j^\theta \sum_i (u_i r_{Ri}^{\beta-1}/d_{ij})^\theta \tag{2}$$

Worker income in location i is given by

$$\bar{w}_i = \sum_j \pi_{j|i} w_j \tag{3}$$

where

$$\pi_{j|i} = \frac{\pi_{ij}}{\pi_i} = \frac{(w_j/d_{ij})^\theta}{\sum_s (w_s/d_{is})^\theta}$$

is the probability of working in j conditional on living in i, computing using the probabilities $\pi_{ij} = L_{ij}/\bar{L}$ and $\pi_i = \sum_j \pi_{ij}$ above.

Cobb-Douglas preferences then imply that total demand for housing is given by

$$r_{Ri}H_{Ri} = (1-\beta)\bar{w}_i L_{Ri} \tag{4}$$

Firms. In each location, a representative firm produces the numeraire good under perfect competition using labor and land. The firm takes the price of labor and land as given and solves

$$\max_{H_{Fj},L_{Fj}} A_j L_{Fj}^{\alpha} H_{Fj}^{1-\alpha} - r_{Fj}H_{Fj} - w_j L_{Fj}.$$

The first order conditions imply

$$L_{Fj} = \left(\frac{\alpha A_j}{w_j}\right)^{\frac{1}{1-\alpha}} H_{Fj} \tag{5}$$

$$H_{Fj} = \left(\frac{(1-\alpha)A_j}{r_{Fj}}\right)^{\frac{1}{\alpha}} L_{Fj} \tag{6}$$

Finally, the price of commercial floorspace adjusts so that firms earn zero profits

$$r_{Fj} = (1-\alpha)A_j^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w_j}\right)^{\frac{\alpha}{1-\alpha}}. \tag{7}$$

Market Clearing. We assume fixed supplies of commercial and residential housing. Market clearing for commercial housing is simply the zero profit equation (7) above. Market clearing for residential housing can be taken from the cobb-douglas demand for housing (4) above. Finally, market clearing for wages implies that supply has to equal demand, which is determined by adding (5) to the system of equations along with the supply curve (2)

Equilibrium. Given parameters $\alpha, \beta, \kappa, \theta$ and location characteristics $A_i, u_i, H_{Ri}, H_{Fi}, t_{ij}$, equilibrium is a vector of endogenous variables $L_{Ri}, L_{Fi}, w_i, r_{Ri}, r_{Fi}$ that satisfy the following system of equations

$$L_{Ri} = \bar{L}(\gamma/\bar{U})^{\theta}(u_i r_{Ri}^{\beta-1})^{\theta} \sum_j (w_j/d_{ij})^{\theta} \tag{L_{Ri}}$$

$$L_{Fj} = \bar{L}(\gamma/\bar{U})^{\theta} w_j^{\theta} \sum_i (u_i r_{Ri}^{\beta-1}/d_{ij})^{\theta} \tag{L_{Fj}}$$

$$L_{Fj} = \left(\frac{\alpha A_j}{w_j}\right)^{\frac{1}{1-\alpha}} H_{Fj} \tag{w_j}$$

$$r_{Ri}H_{Ri} = (1-\beta)\bar{w}_i L_{Ri} \tag{r_{Ri}}$$

$$r_{Fj} = (1-\alpha)A_j^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w_j}\right)^{\frac{\alpha}{1-\alpha}} \tag{r_{Fj}}$$

where the label indicates which endogenous variable each equation pins down condition on the others. Auxiliary variables, such as $\bar{w}_i$, are defined as functions of the endogenous variables and their form is given above.

Introducing Agglomeration Forces[1]**.** We allow final goods productivity to depend on production fundamentals ($\bar{A}_j$) and production externalities (Υ_j). Specifically we use the standard approch in urban economics of modeling these externatilies as depending on the travel-time weighted sum of workplace employment density in surrounding blocks:

$$A_j = \bar{A}_j \Upsilon_i^{\mu_A} \quad \Upsilon_j = \sum_s e^{(-\delta_A t_{js})} (\frac{L_{Fs}}{K_s}) \tag{8}$$

where $(\frac{L_{Fs}}{K_s})$ is workplace employment density per unit of land area; production externalities decline with travel time (t_{js}) through the iceberg factor $e^{-\delta_A t_{js}} \in (0,1]$; δ_A determines their rate of spatial decay; and μ_A controls their relative importance in determining overall productivity. We model the externalities in workers' residential to depend on residential fundamentals $\bar{u_i}$. We adopt a similar specification as for production externalities, and we model residential externalities as depending on the travel-time weighted sum of residential fundamentals in surrounding blocks:

$$u_i = (\sum_r e^{(-\delta_u t_{ir})} * \bar{u_r})^{\mu_u} \tag{9}$$

where residential externalities decline with travel time(t_{ir}) through the iceberg factor $e^{-\delta_u t_{ir}} \in (0,1]$; δ_u determines their rate of spatial decay; and μ_u controls their relative importance in overall residential amenities. We define the residential amenities differently from Ahlfeldt et al., 2015. They adopt externalities in workers' residential choices analogously to the externalities in firms' production choices. For our parameters, the main change is in the residential fundamentals. Changes in residence employment density are small relative to changes in $\bar{u}_i$, which makes it hard to get amenities changing close by under their model.

1.2 Simulation

We begin by simulating a hypothetical city to build code that simulates the model in matlab. Choose parameters $\alpha = \beta = 0.7, \theta = 3, \kappa = 0.01$; For μ_u, we choose 3 different values centered in $\mu_u = 0.3$ and three value for δ_u centered in $\delta_u = 0.01$. Transport times are measured in minutes for a hypothetical city with 400 locations. For now, we will define symetric location areas and characteristics, and we set productivity spillover to zero ($\mu_A = 0$).

The simulation algorithm is as follows:

1. Guess an initial vector $w_i^0, r_{Ri}^0, r_{Fi}^0, u_i^0$

2. Given $w_i^t, r_{Ri}^t, r_{Fi}^t, u_i^t$:

[1] I copied this section from the Berlin Wall paper and just changed the notation

(a) Compute

$$L_{Ri}^t = \bar{L}\sum_j \frac{(u_i^t w_j^t (r_{Ri}^t)^{\beta-1}/d_{ij})^\theta}{\sum_{rs}(u_r^t w_s^t (r_{Rr}^t)^{\beta-1}/d_{rs})^\theta}$$
$$L_{Fj}^t = \bar{L}\sum_i \frac{(u_i^t w_j^t (r_{Ri}^t)^{\beta-1}/d_{ij})^\theta}{\sum_{rs}(u_r^t w_s^t (r_{Rr}^t)^{\beta-1}/d_{rs})^\theta}$$
$$\bar{w}_i^t = \sum_j \frac{(w_j^t/d_{ij})^\theta}{\sum_s (w_s^t/d_{is})^\theta} w_j^t$$

(b) Update variables

$$\tilde{w}_j^t = \alpha A_j \left(H_{Fj}/L_{Fj}^t\right)^{1-\alpha}$$
$$\tilde{r}_{Ri}^t = (1-\beta)\bar{w}_i^t L_{Ri}^t/H_{Ri}$$
$$\tilde{r}_{Fj}^t = (1-\alpha)A_j^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w_j^t}\right)^{\frac{\alpha}{1-\alpha}}$$
$$\tilde{u}_i = \bar{u}_i\left[\sum_r e^{(-\delta_u t_{ir})}(\frac{L_{Rr}^t}{K_r})\right]^{\mu_u}$$

(c) If $||(\tilde{w}_i^t, \tilde{r}_{Ri}^t, \tilde{r}_{Fi}^t, \tilde{u}_i) - (w_i^t, r_{Ri}^t, r_{Fi}^t, u_i^t)||_\infty < \epsilon_{tol}$ then stop, otherwise update $x_i^{t+1} = \zeta\tilde{x}_i^t + (1-\zeta)x_i^t$ and continue. [2]

The welfare is $\bar{U} = \gamma\left[\sum_{rs}(u_r w_s r_{Rr}^{\beta-1}/d_{rs})^\theta\right]^{1/\theta}$.

Amenity shock. We simulate a shock to the residential amenities. We increase the value of location 1[3] by 10%, 20% and 50% and simulate the model for all combinations of μ and δ, and compare the results with the initial equilibrium.

Partial equilibrium. First, we simulate the model in partial equilibrium holding wages and productivities fixed. We use the initial equilibrium values as starting values for the counterfactuals. Figures 1, 2, and 3 show the results for different shocks and parameters. All graphs plot the log change in population and amenities in the y-axis and the distance to location 1 in th x-axis. Since wages are constant and the log change in house prices is the same as the log change in population, we only present the results for L_r. The counterfactual analysis in Partial Equilibrium shows that an amenity shock attracts more residents to move into that location, increasing house prices, and due to spillovers, amenities also grow in places close by. Then, people are also attracted to these neighborhoods, and as a result, amenities increase more in nearby locations compare to other areas of the city. Figure 1 shows that larger shocks to location 1 increase (i) the number of residents

[2]Note from Nick: A larger updating weight ζ will allow for faster convergence, but also tends to lead to divergence for large values. I would start with a small one aorund 0.1, and then increase once you have things working to understand where the optimal lies. I've found around 0.3 to be best from vague recollection.

[3]Location 1 is the location in the center f the city.

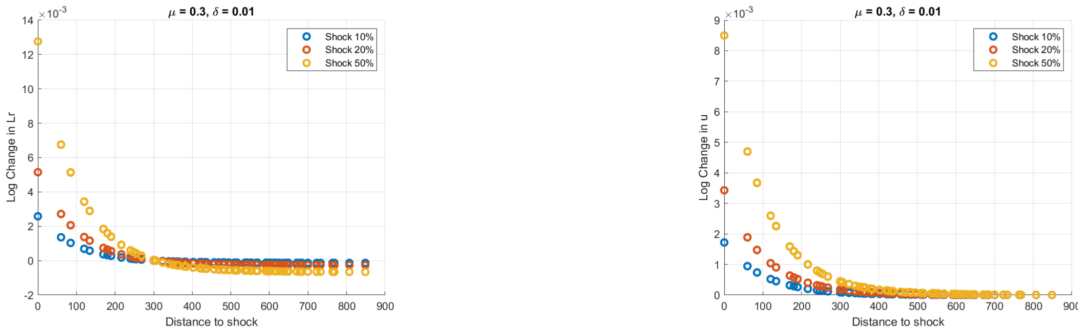

Figure 1: Partial equilibrium - Change in amenity shocks

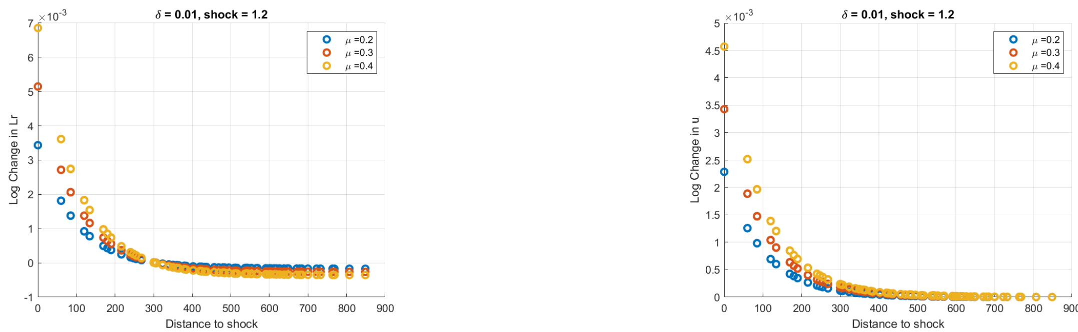

Figure 2: Partial equilibrium - Change in spillover parameter

and (ii) the amenities at a greater extent in nearby places. The results for Figure 2 are very similar to the effects of Figure 1, since μ measures the relative importance of the externality in overall residential amenities. Higher μ amplifies the effect of the shock, which increases the number of residents and amenities in location 1 and nearby neighborhoods. Figure 3 shows the results for changes in the rate of spatial decay. Larger δ have a lower impact on other locations, so only the closest locations have a high increase in population and amenities.

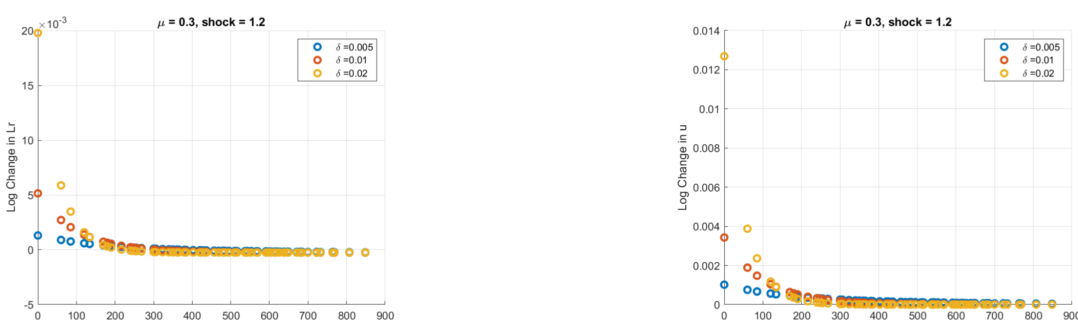

Figure 3: Partial equilibrium - Change in the decay parameter

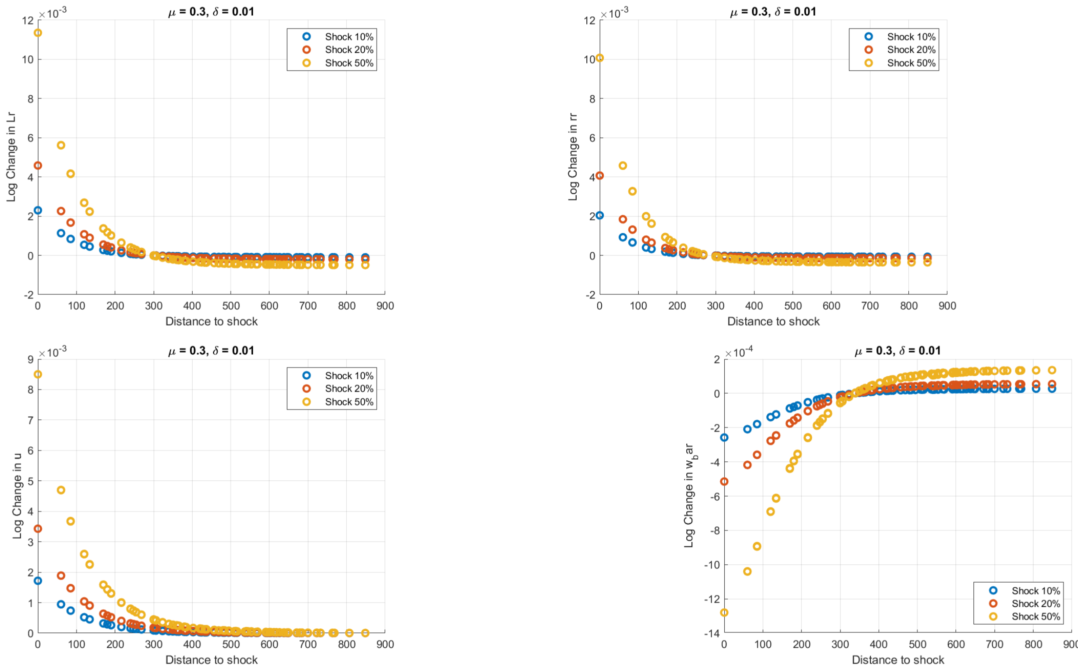

Figure 4: General equilibrium - Change in amenity shocks

General equilibrium. We simulate the model in general equilibrium but holding productivities fixed. Again, we use the initial equilibrium values as starting values for the counterfactuals. The number of residents, the house prices, and amenities respond in the same way that in the partial equilibrium. Then, when more people move into location 1, it increases the labor supply to proximate locations, decreasing wages there. Figures 4 and 5 show that a larger shock or a higher μ increases the labor supply in a higher magnitude, which makes wages to decrease more relative to scenarios with smaller shocks or lower values of μ. Figure 6 shows the spatial spillover; higher decay paraments reduce the spillover on nearby locations, so changes in wages are smaller in distant locations.

1.3 Estimation

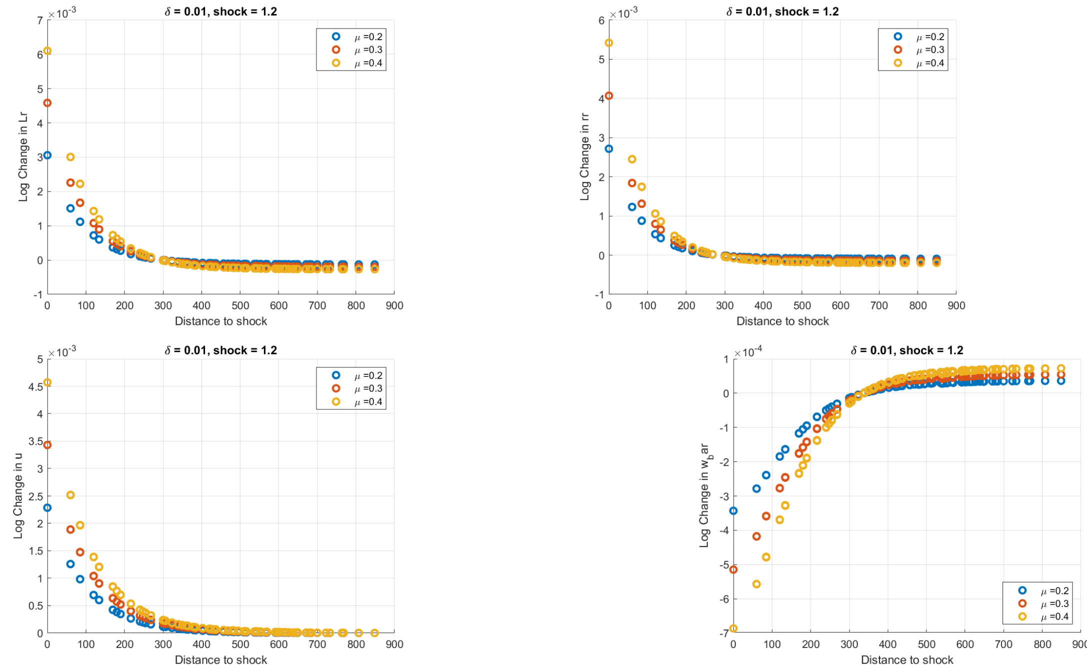

Figure 5: General equilibrium - Change in spillover parameter

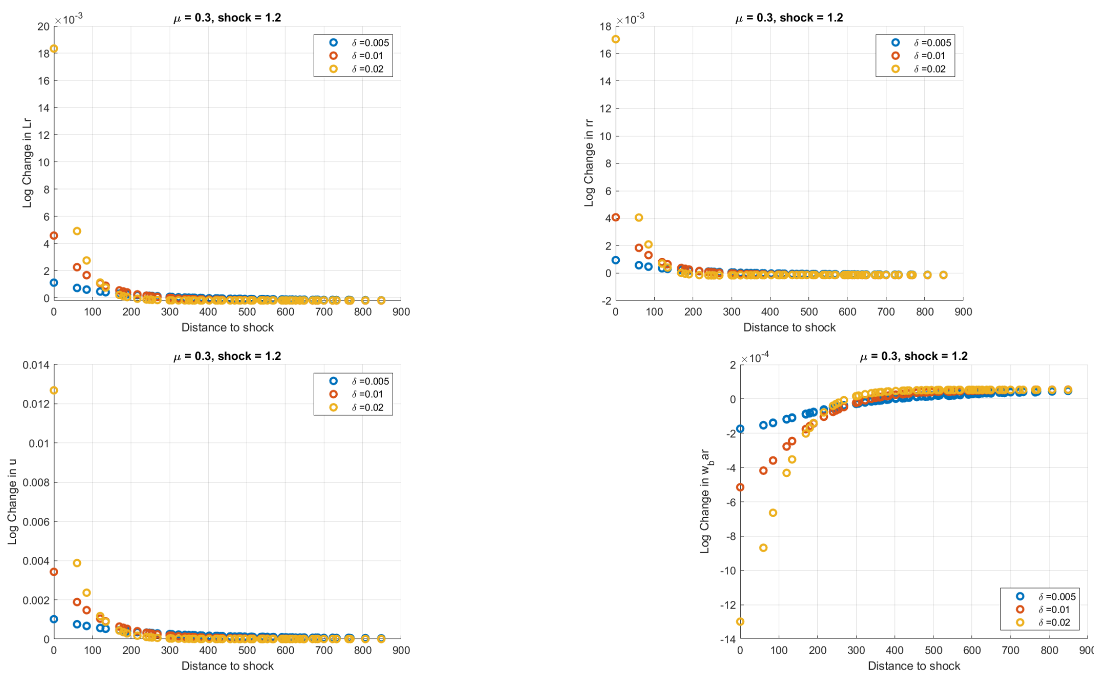

Figure 6: General equilibrium - Change in the decay parameter