EECE 574

ADAPTIVE CONTROL PROJECT

Adaptive Control of Mean Arterial Pressure

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I. Introduction

Drug administration is a critical aspect of patient treatment, which in most cases requires experienced medical personnel, as is the case of post-surgical regulation of the mean arterial pressure (MAP) using vasodilators and infusion of insulin in diabetic patients. Such a task not only necessitates proficient personnel, but continuous patient monitoring over long periods of time. This process is tedious and time consuming, which calls for its automation.

This project consists of developing an adaptive algorithm that attempts to resolve the aforementioned issue. Before starting, the MAP and change in blood pressure equations ought to be mentioned.

The mean arterial pressure is define as:

$$MAP(k) = P_0 - \Delta P(k)$$

where P_0 is the initial blood pressure (also known as background pressure), and $\Delta P(k)$ is the change in pressure, defined below.

$$\Delta P(k) = \frac{q^{-d}(b_1 + b_{m+1}q^{-m})}{1 + a_1 q^{-1}} I(k) + \frac{1}{1 + a_1 q^{-1}} e(k)$$

where $e(k)$ is a zero-mean white noise with variance $\sigma_e^2 = 4mmHg^2$, and $I(k)$ the infusion rate is limited in the range $0 \leq I(k) \leq 250ml/h$.

It is also important to note that model parameters are highly variable, and are summarized in the table below.

Table 1: Model variable parameters

Parameter	Minimum	Maximum	Nominal
b_1	0.053	3.546	0.187
b_{m+1}	0	1.418	0.075
a_1	-0.799	-0.606	-0.741
d	2	5	3
m	2	5	3

II. Design considerations

The adaptive controller has to respect the following specifications:

- For a decrease from 150mmHg to 100mmHg (the normal setpoint), the settling time should be between 5 and 20 minutes for all patients;

- The overshoot should be no more than 10mmHg;

- No steady-state error within a tolerance of ± 5 mmHg;

- Adequate rejection of a +20mmHg step output disturbance once you have reached the 100mmHg setpoint;

- Under no circumstances should the MAP go below 80 mmHg.

This project performs two distinct simulations:

- Simulation for the three patient types each for a period of 3 hours;

- Simulation for a patient who under a course of four hours starts with the maximum parameters (i.e. starts as a sensitive patient) and linearly progresses toward the nominal parameters.

III. Simulation A: Three patients with different but constant parameters

1. Patients' profiles

The patients' profiles are defined as follow:

- **Patient 1:**

This patient is chosen to have the minim parameters' values.

Table 2: Patient 1 profile

b_1	0.053
b_{m+1}	0
a_1	-0.799
d	2
m	2

- **Patient 2:**

This patient is chosen to have the maximum parameters' values.

Table 3: Patient 2 profile

b_1	3.546
b_{m+1}	1.418
a_1	-0.606
d	5
m	5

- **Patient 3:**

This patient is chosen to have the nominal parameters' values.

Table 4: Patient 3 profile

b_1	0.187
b_{m+1}	0.075
a_1	-0.741
d	3
m	3

2. Adaptive algorithm: Model predictive control (MPC)

After testing a set of adaptive algorithms, best results were obtained using model predictive control (MPC). Predictive controllers look beyond the process dead time, which may be useful to regulate mean arterial pressure in patients over an extended time period. The diagram below reminds us of the basic structure of such controllers.

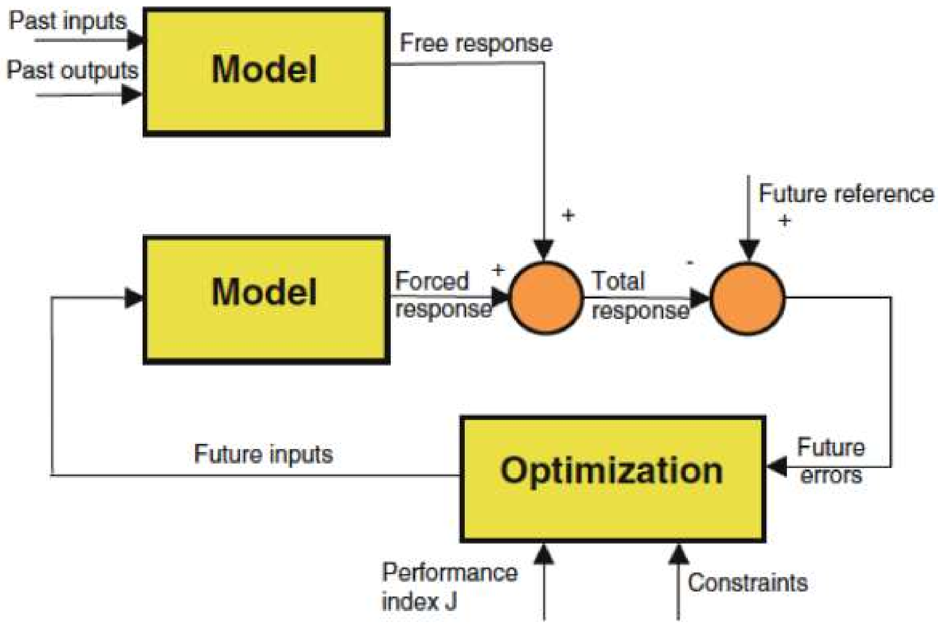

Figure 1: Model predictive control basic structure

MPC consists of iterative finite-horizon optimization as explained below:

a. Update the system parameters:

This step consists of updating the regressor matrix, covariance matrix, gain update, and then the parameters.

b. Compute the controller matrices:

This is done by solving the set of Diophantine equations. The arguments here are the system transfer function polynomials, delay and control horizon.

c. Minimize the quadratic performance index:

The quadric performance index is defined as:

$$J = E\left(\sum_{j=N_1}^{N_2} [y(t+j) - w(t+j)]^2 + \sum_{j=N_1}^{N_U} \rho\,[\Delta u(t+j-1)]^2\right)$$

whose minimization is:

$$\frac{\partial J}{\partial \tilde{u}} = R^T(R\tilde{u} + f - w) + \rho I \tilde{u} = 0$$

which results in the control law below:

$$\tilde{u}(t) = [R^T R + \rho I]^{-1} R^T (w - f)$$

d. Compute the total response:

Using the Diophantine results and optimization block above, free and forced responses are generated and added together to produce the total response.

e. Compute future input:

After subtracting the total response from future reference, future error is generated and fed to the optimizer mentioned above to update the future controller input.

f. Update system output

g. Repeat steps a-f after every sampling instance

After some tweaking, the quadratic performance index was minimized with the following parameters:

- The prediction horizon: $N_p = 20$
- The minimum prediction horizon $N_1 = d$
- The maximum prediction horizon $N_2 = d + N_p$
- The control horizon N_U was set equal to the prediction horizon $N_p = N_U = 20$
- The control weighting factor $\rho = 30$ (shown in the code as lambda)

3. Results

This part depicts the results obtained in terms of parameter estimation, mean arterial pressure, and infusion rates temporal evolution.

3.1. Parameter estimation

- **Patient 1**

While the below plots depict the evolution of the parameter estimation throughout the 3-hour long period, the following table compares the mean of the estimates during 10 minutes before injection the disturbance, and the corresponding mean during the whole period after said injection.

Table 5: Patient 1 parameter estimation

	a_1	b_1	b_{m+1}	Average difference*
Actual	-0.799	0.053	0	
Before disturbance	-0.7467	0.0540	0.0070	0.0201
After disturbance	-0.9978	0.0318	-0.0296	0.0832

** Since b_{m+1} has a reference of zero, it is not possible to compute its percent error as the formula will have a zero in the denominator, and therefore an average percent error is not feasible. Instead. The average difference is used as an appropriate metric to analyze the parameter estimation results.*

The average difference is computed as follows:

$$AD = \frac{\left(d_{a_1} + d_{b_1} + d_{b_{m+1}}\right)}{3}$$

when the individual parameter estimation is computed as:

$$d_{param} = \left|estimated_{param} - actual_{param}\right|$$

The plots of the evolution of Patient 1 estimated parameters are rolled below.

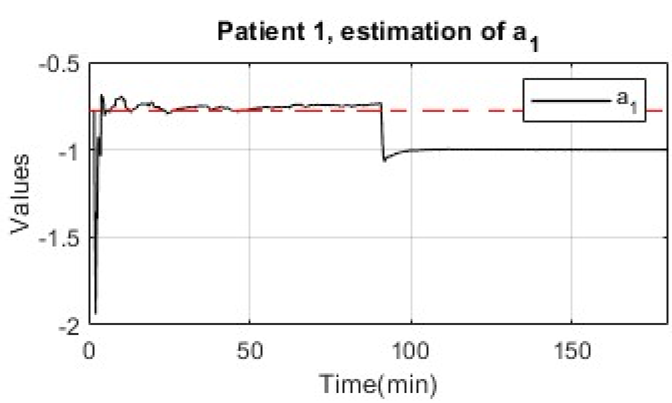

Figure 2: Patient 1, estimation of a_1

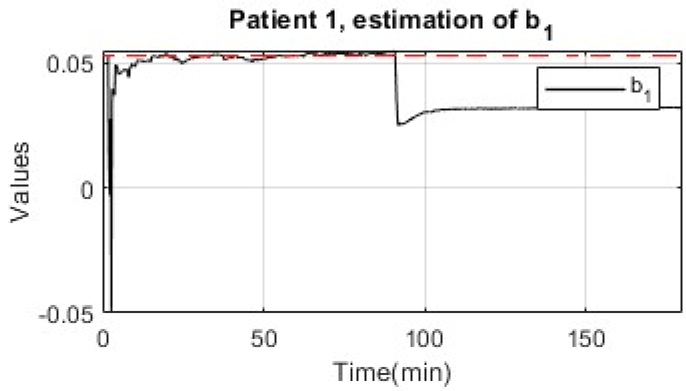

Figure 3: Patient 1, estimation of b_1

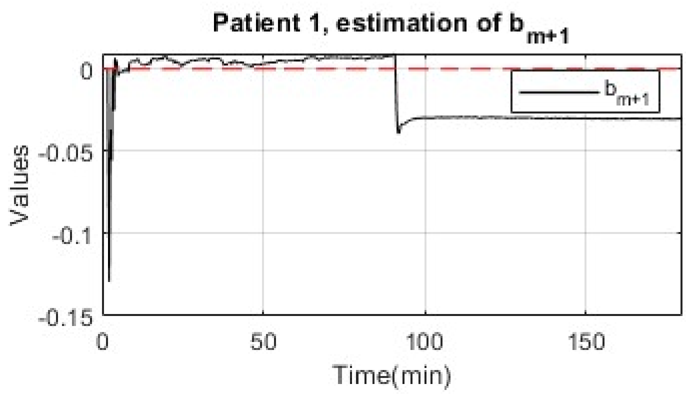

Figure 4: Patient 1, estimation of b_m+1

The graphs clearly details what the table summarizes: The algorithm is performing in a respectable fashion, but the same could not be said post-disturbance injection (an average difference of 0.0832 versus only 0.02101 pre-injection).

- **Patient 2**

Similarly, this patient's results are summarized below.

Table 6: Patient 2 parameter estimation

	a_1	b_1	b_{m+1}	Average percent error*
Actual	-0.6066	3.546	1.418	
Before disturbance	-0.5780	3.8149	1.5123	6.31%
After disturbance	-0.7900	2.6657	0.9188	39.89%

* where average percent error is computed as follows:

$$APE = \frac{\left(e_{a_1} + e_{b_1} + e_{b_{m+1}}\right)}{3}$$

and the individual parameter estimation percent error is computed as:

$$e_{param} = \left|\frac{estimated_{param} - actual_{param}}{actual_{param}}\right| \times 100$$

The plots of the evolution of Patient 2 estimated parameters are rolled below.

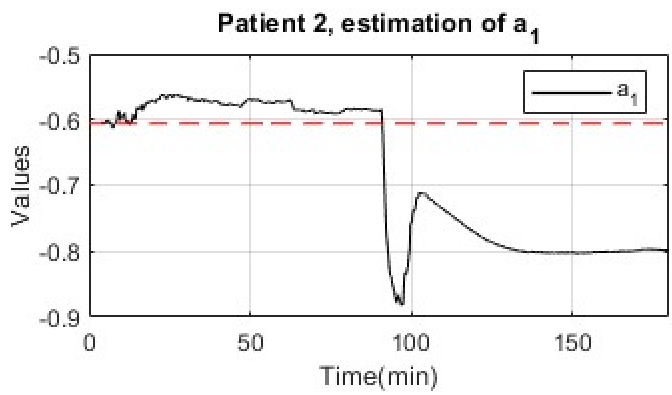

Figure 5: Patient 2, estimation of a_1

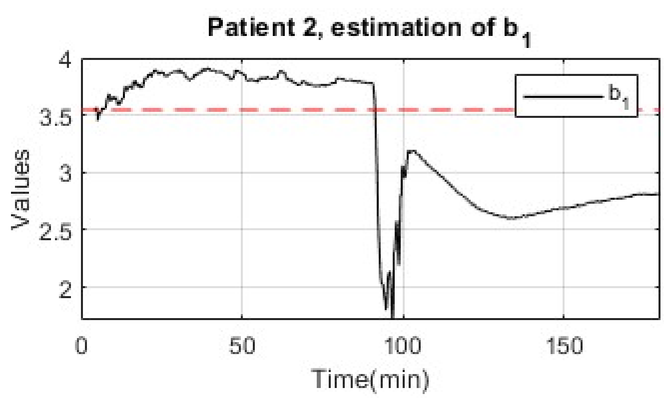

Figure 6: Patient 2, estimation of b_1

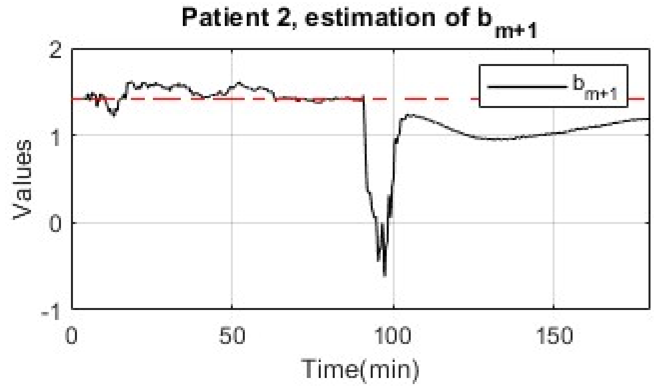

Figure 7: Patient 2, estimation of b_m+1

The graphs clearly details what the table summarizes: The algorithm is performing in a respectable fashion, but the same could not be said post-disturbance injection (an average percent error of 39.89% versus only 6.31% pre-injection).

- **Patient 3**

Similarly, this patient's results are summarized below.

Table 7: Patient 3 parameter estimation

	a_1	b_1	b_{m+1}	Average percent error
Actual	-0.741	0.187	0.075	
Before disturbance	-0.7067	0.2245	0.0730	9.12% (due to high e_{b_1})
After disturbance	-0.9959	2.6657	-0.1557	554% (due to extremely high e_{b_1} and $e_{b_{m+1}}$)

The plots of the evolution of Patient 3 estimated parameters are rolled below.

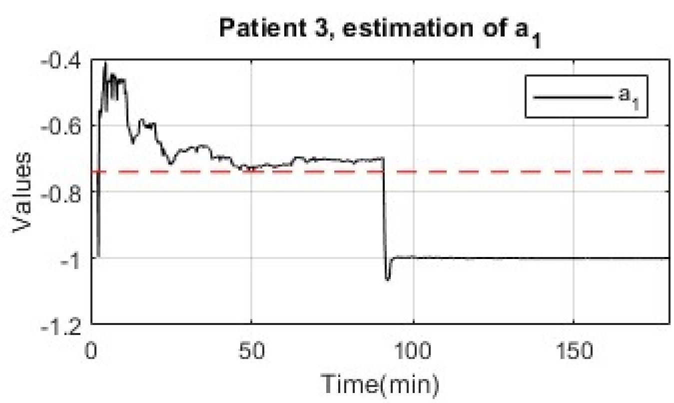

Figure 8: Patient 3, estimation of a_1

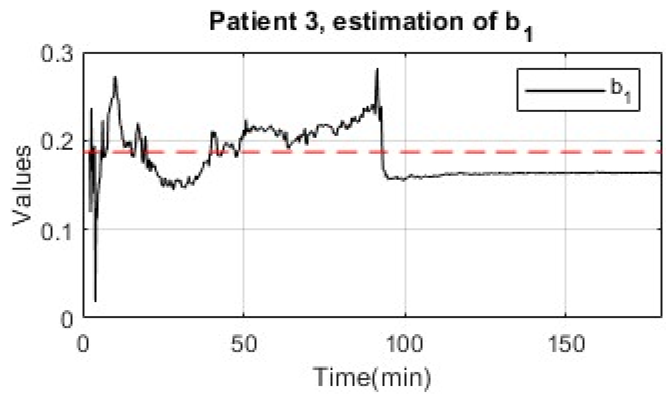

Figure 9: Patient 3, estimation of b_1

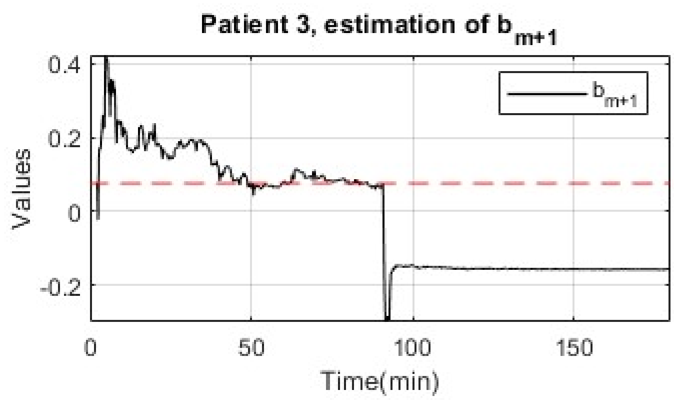

Figure 10: Patient 3, estimation of b_m+1

3.2. Mean arterial pressure output

Important note

Although much thought and effort have been devoted towards the successful completion of this project, due to time constraints, satisfactory results –especially after disturbance injection- were not provided with +20mmHg disturbance. However, promising results were obtained when injecting a +10mmHg or +5mmHg step disturbance.

In the effort to support the above statement, results from +20mmHg, +10mmHg, and +5mmHg step disturbances will be displayed in this section.

The following nomenclature contains the metrics that will summarize the algorithm performance relative to MAP output. This is crucial to verify whether the design specifications were met or not, and to which extent.

- Maximum overshooting:

$$over_{max}$$

- Mean steady-state error before disturbance injection:

This is the average of all steady-state error instances before disturbance injection. This is used to show that on average, the tolerance range is respected, and that only outputs at few time instances fail to do so.

$$e_{ss\_bef,mean}$$

- Maximum steady-state error before disturbance injection:

$$e_{ss\_bef,max}$$

- Mean steady-state error after disturbance injection:

This is the average of all steady-state error instances after disturbance injection. This is used to show that on average, the tolerance range is respected, and that only outputs at few time instances fail to do so.

$$e_{ss\_aft,mean}$$

- Maximum Steady-state error after disturbance injection:

$$e_{ss\_aft,max}$$

- Minimum mean arterial pressure output:

$$MAP_{min}$$

- **Patient 1**

The table measurements are all in mmHg.

Table 8: Patient 1 MAP results versus disturbance magnitudes

	+20mmHg	**+10mmHg**	**+5mmHg**
$over_{max}$	30.30	30.30	30.30
$e_{ss\_bef,mean}$	6.96	6.96	6.96
$e_{ss\_bef,max}$	14.35	14.35	14.35
$e_{ss\_aft,mean}$	46.24	8.23	3.79
$e_{ss\_aft,max}$	86.41	33.98	16.51
MAP_{min}	13.59	66.0199	83.49

Notice how the MAP does not go under the 80mmHg mark in the +5mmHg disturbance scenario, while it fails in the first two. Unfortunately, overshooting and maximum steady-state errors exceed their tolerated threshold, 10mmHg and +/-5mmHf respectively.

Notice how the average steady-state error after disturbance injection improves as disturbance step magnitude decreases until it actually becomes within tolerable bounds of 5%.

As expected and shown in the three graphs below, post-disturbance steady state error is considerably lowered whith lower disturbance values.

Another important observation to underline is that this specific patient (Patient 1 with minimum parameter values has faces extreme difficulty to settle, and showcases extreme overshooting).

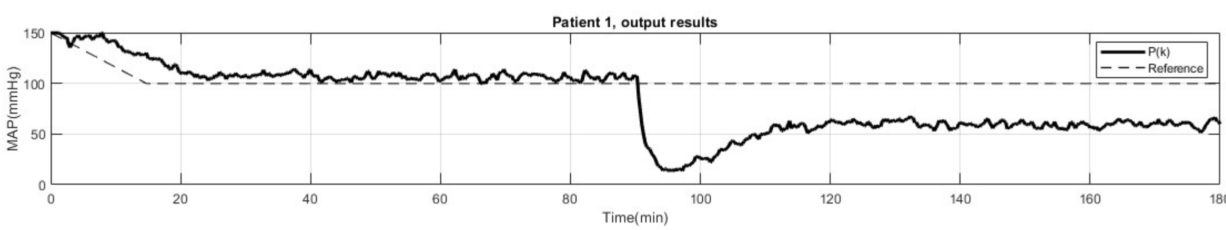

Figure 11: Patient 1, MAP with +20mmHg disturbance

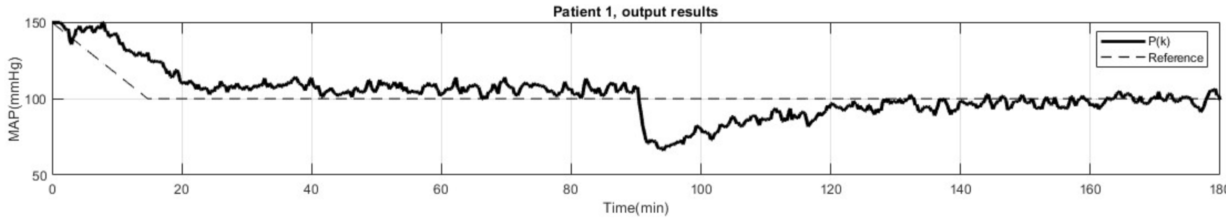

Figure 12: Patient 1, MAP with 10mmHg disturbance

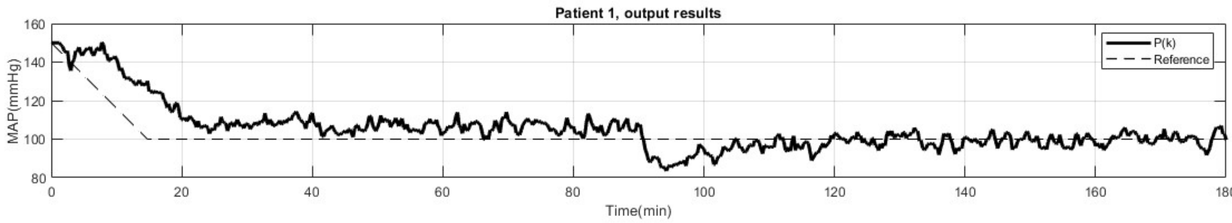

Figure 13: Patient 1, MAP with 5mmHg disturbance

- **Patient 2**

Table 9: Patient 2 MAP results versus disturbance magnitudes

	+20mmHg	**+10mmHg**	**+5mmHg**
$over_{max}$	13.73	13.73	13.73
$e_{ss\_bef,mean}$	2.94	2.94	2.94
$e_{ss\_bef,max}$	12.07	12.07	12.07
$e_{ss\_aft,mean}$	51.83	11.35	4.39
$e_{ss\_aft,max}$	296.8 ((due to disturbance ONLY)	84.26	16.34
MAP_{min}	-196.81 (due to disturbance ONLY)	15.74	83.66

Although pre-disturbance maximum steady-state error falls out of the tolerated range, its mean counterpart fails within.

Notice how mean steady-state error after disturbance injection improves as disturbance step magnitude decreases until it actually becomes within tolerable bounds of 5%.

Below are the detailed output graphs.

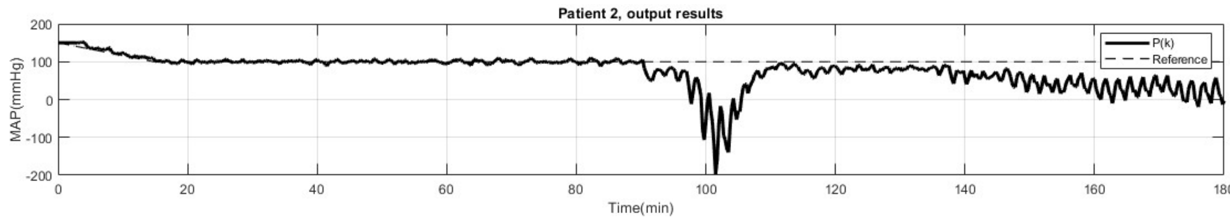

Figure 14: Patient 2, MAP with +20mmHg disturbance

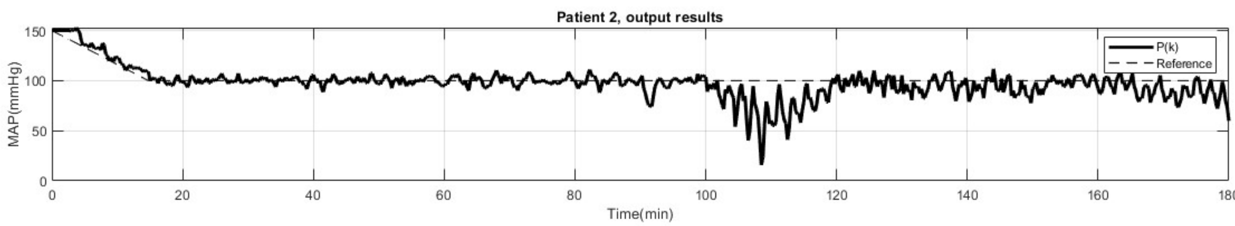

Figure 15: Patient 2, MAP with +10mmHg disturbance

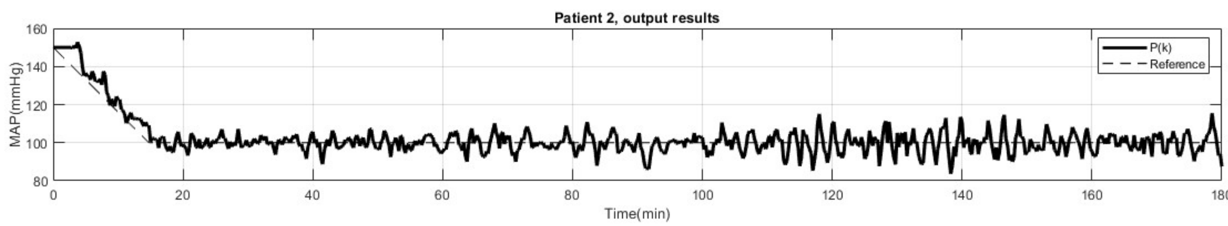

Figure 16: Patient 2, MAP with +5mmHg disturbance

- **Patient 3**

Table 10: Patient 3 MAP results versus disturbance magnitudes

	+20mmHg	**+10mmHg**	**+5mmHg**
$over_{max}$	9.87	9.87	9.87
$e_{ss\_bef,mean}$	2.45	2.45	2.45
$e_{ss\_bef,max}$	8.71	8.71	8.71
$e_{ss\_aft,mean}$	27.62	4.41	3.75
$e_{ss\_aft,max}$	59.81	30.83	16.11
MAP_{min}	40.19	69.17	84.90

This patients shows the most satisfactory results. The overshooting in all cases is below the 10mmHg ceiling; the pre-disturbance mean steady-state error falls within tolerated bounds.

Notice how mean steady-state error after disturbance injection improves as disturbance step magnitude decreases until it actually becomes within tolerable bounds of 5%.

Below are the detailed output graphs.

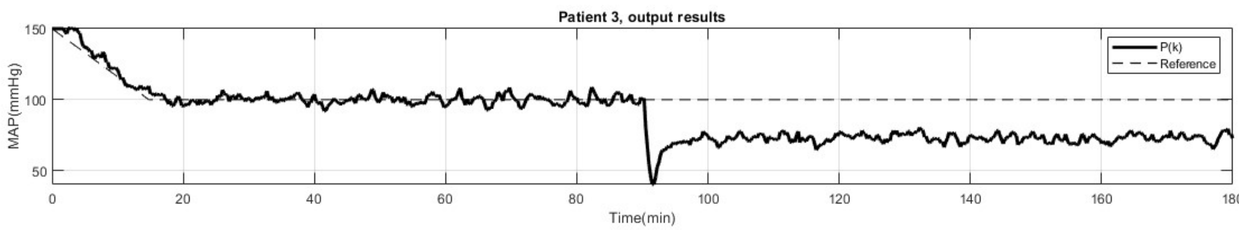

Figure 17: Patient 3, MAP with +20mmHg disturbance

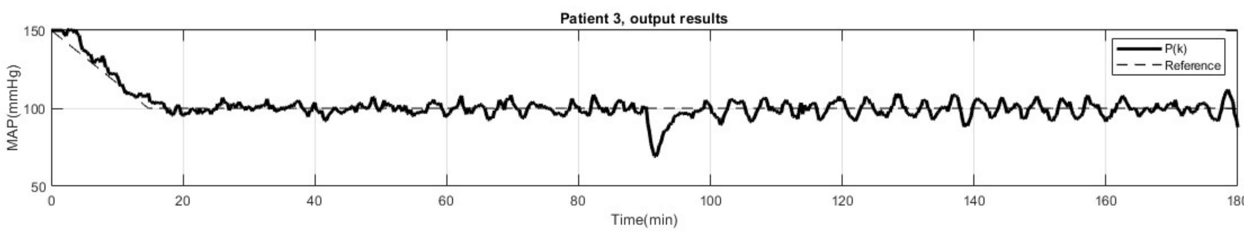

Figure 18: Patient 3, MAP with +10mmHg disturbance

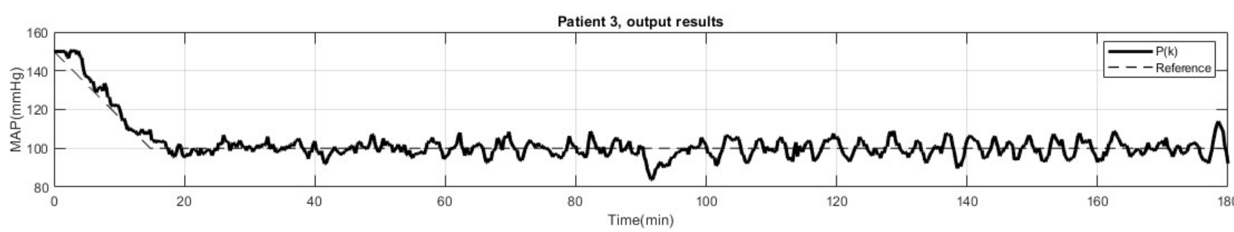

Figure 19: Patient 3, MAP with +5mmHg disturbance

3.3 Infusion rate control effort

To avoid a cumbersome report, only results for +20mmHg case are shown.

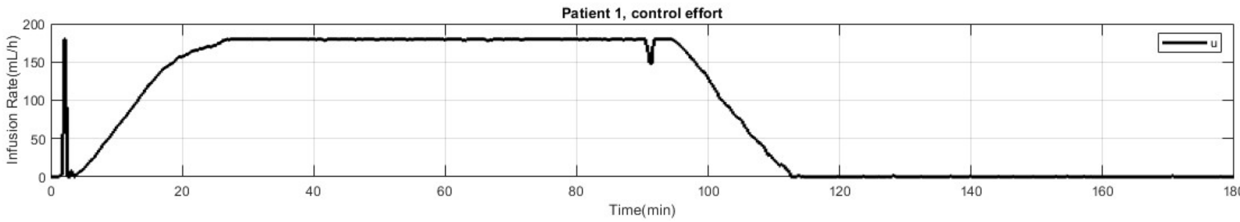

Figure 20: Patient 1, infusion rate control effort

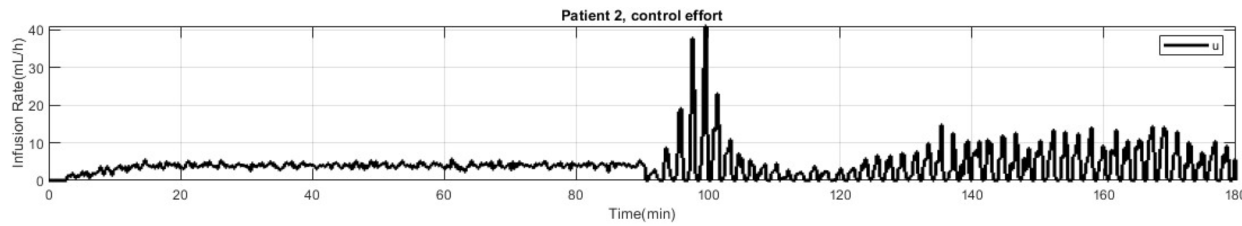

Figure 21: Patient 2, infusion rate control effort

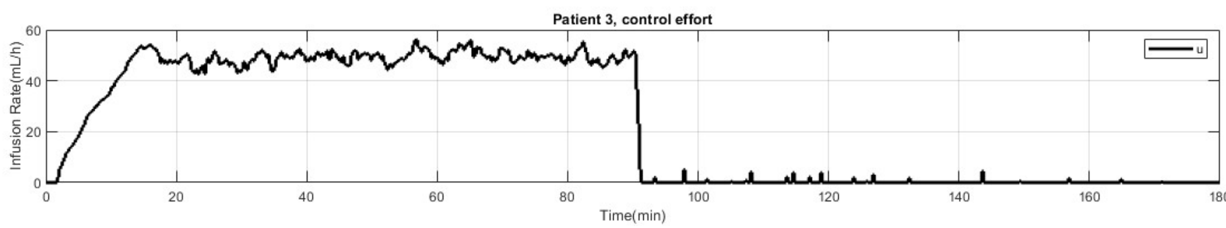

Figure 22: Patient 3, infusion rate control effect

IV. Simulation B: One patient with linearly-varying parameters

1. Patient's profile

This part consists of simulating a patient who under a course of four hours starts with the maximum parameters (i.e. starts as a sensitive patient) and linearly progresses toward the nominal parameters.

Since the same adaptive algorithm is used in this simulation, there is no need to reintroduce it here, and therefore the results shall be displayed directly.

2. Results

Important note

As stated in the previous simulation: although much thought and effort have been devoted towards the successful completion of this project, due to time constraints, satisfactory results – especially after disturbance injection- were not provided with +20mmHg disturbance. However, promising results were obtained when injecting a +10mmHg or +5mmHg step disturbance.

In the effort to support the above statement, results from +20mmHg, +10mmHg, and +5mmHg step disturbances will be displayed in this section.

2.1. Parameter estimation

As shown below, parameters b_1 and b_{m+1} are more successfully followed by the estimator than is the case ofa_1. Also, as expected, estimation is more correct as step disturbance decreases in magnitude.

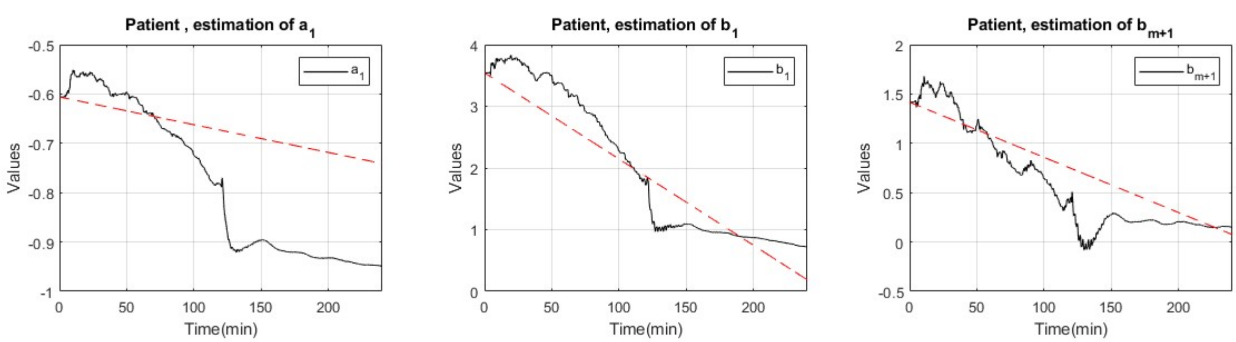

Figure 23: Patient4, parameter estimation with +20mmHg step disturbance

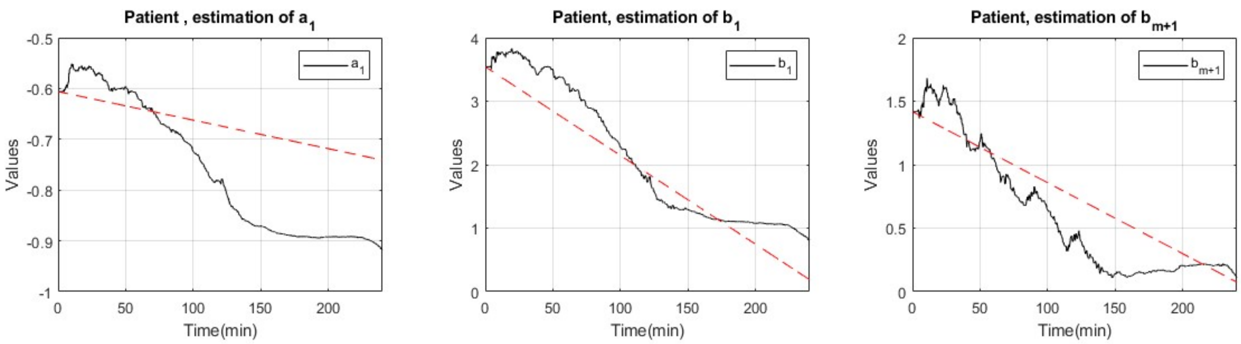

Figure 24: Patient4, parameter estimation with +10mmHg step disturbance

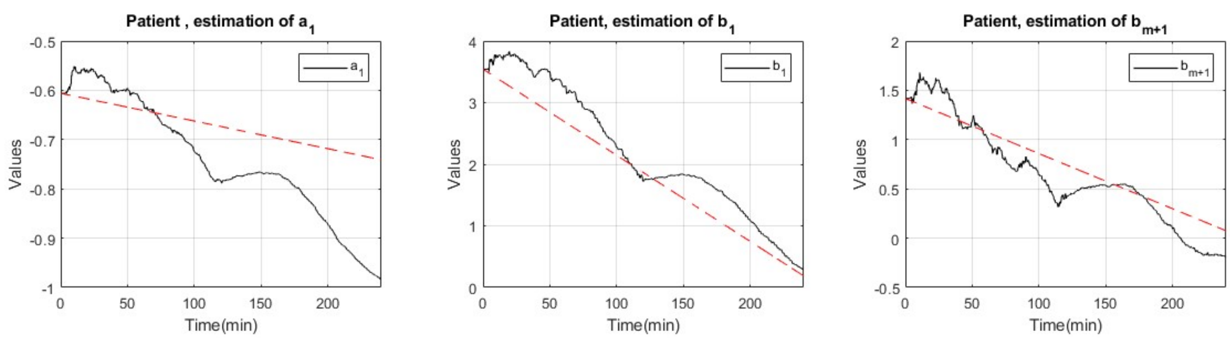

Figure 25: Patient4, parameter estimation with +5mmHg step disturbance

2.2. Mean arterial pressure output

The results of this section are summarized in the table and detailed in the plots below.

Table 11: Patient 4 MAP results versus disturbance magnitudes

	+20mmHg	**+10mmHg**	**+5mmHg**
$over_{max}$	13.87	13.87	13.87
$e_{ss\_bef,mean}$	2.89	2.89	2.89
$e_{ss\_bef,max}$	10.76	10.76	10.76
$e_{ss\_aft,mean}$	30.42	3.86	3.14
$e_{ss\_aft,max}$	82.41	30.84	16.94
MAP_{min}	17.56	69.17	83.06

Although pre-disturbance maximum steady-state error falls out of the tolerated range, its mean counterpart fails within.

Notice how mean steady-state error after disturbance injection improves as disturbance step magnitude decreases, and actually becomes within tolerable bounds of 5%.

Below are the detailed output graphs.

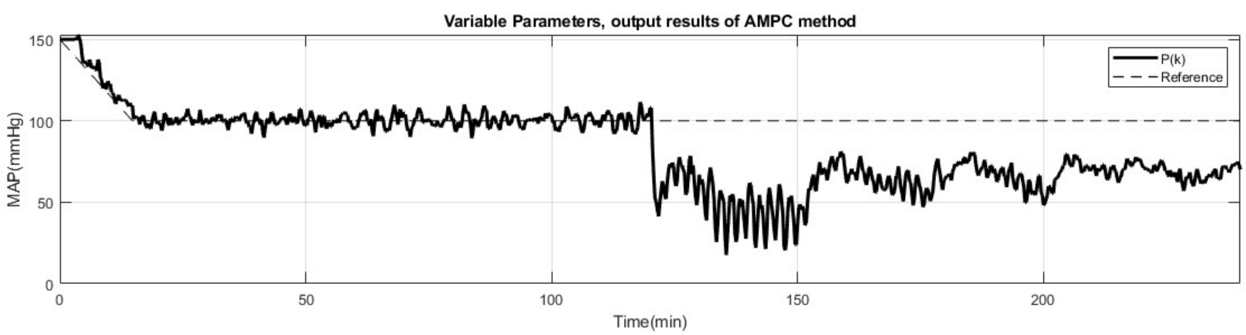

Figure 26: Patient 4, MAP with +20mmHg disturbance

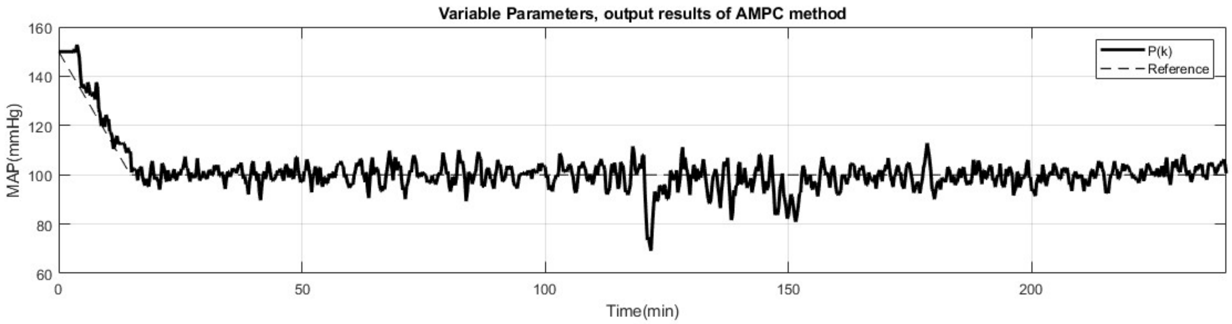

Figure 27: Patient 4, MAP with +10mmHg disturbance

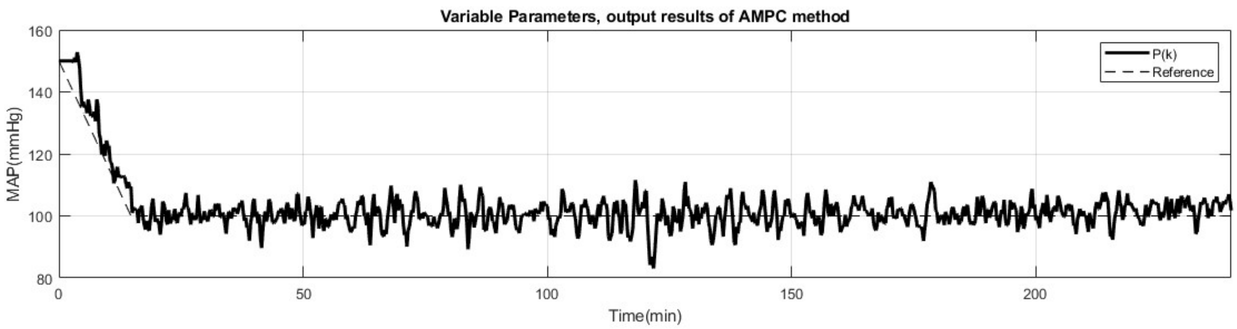

Figure 28: Patient 4, MAP with +5mmHg disturbance

2.3. Infusion rate control effort

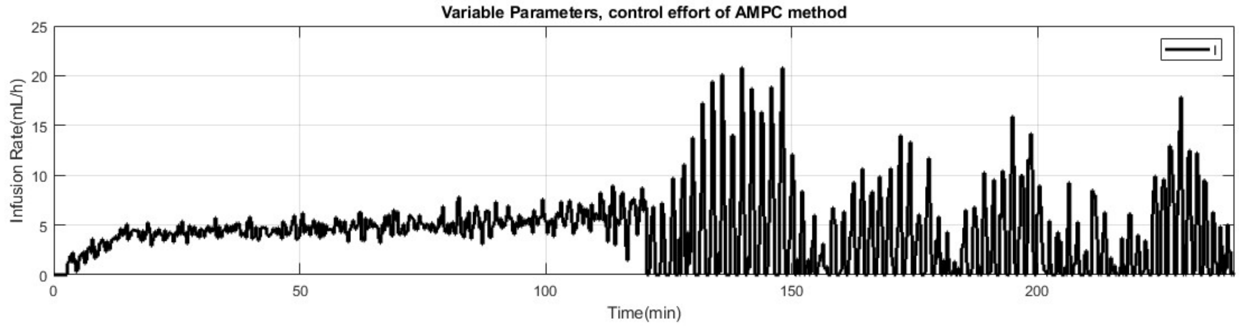

Figure 29: Patient 4, infusion rate control effort with +20mmHg step disturbance

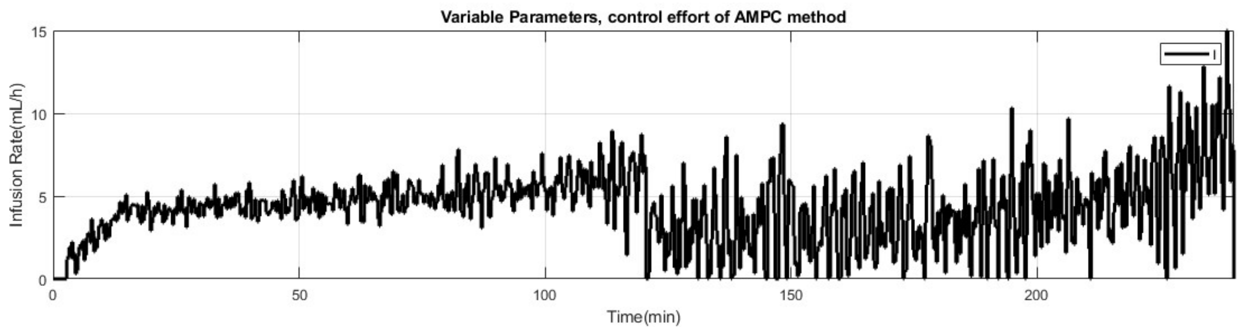

Figure 30: Patient 4, infusion rate control effort with +10mmHg step disturbance

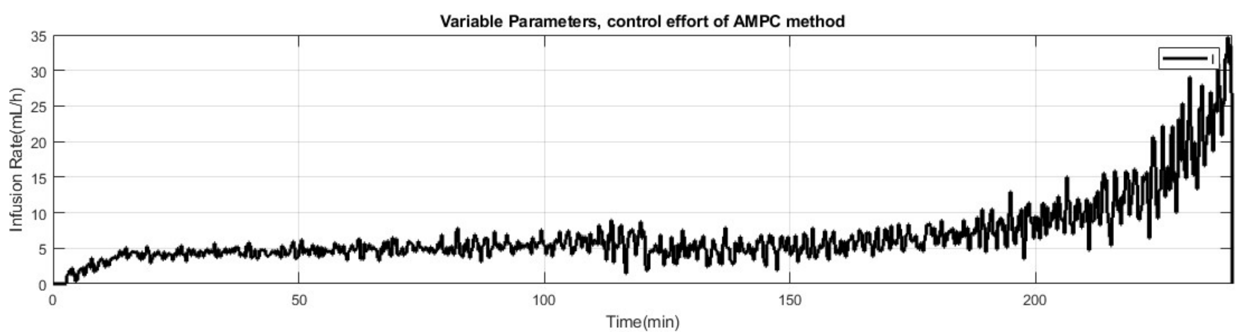

Figure 31: Patient 4, infusion rate control effort with +5mmHg step disturbance

V. Conclusion

Our unsatisfactory results may stem from the following:

- Algorithm not robust enough to adapt to this project constrains and thresholds;
- Noise variance too high;
- Step disturbance magnitude too high (better results were obtained with lower values, and illustrated in the Results section of this report);