The Perturbative Resolvent Method

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The point of this post is for me to understand the paper *The Perturbative Resolvent Method: spectral densities of random matrix ensembles via perturbation theory* by Cui, Rocks and Mehta 2020. The arXiv link for the paper can be found here.

In this paper the authors devise a way to calculate the spectral density of random matrix ensembles that builds off of past approaches using the cavity method to devise a self consistent equation for the resolvent, as discussed in notes here. However one of the differences between this method and previous approaches is that here the method is applicable for matrices with complex eigenvalues, where as the standard resolvent method fails for such matrices.

To recap this discussion of the importance of the resolvent, in general if we can calculate

$$G(z) = \frac{1}{N} \left\langle \mathrm{Tr}\, \frac{1}{zI - A} \right\rangle$$

then the spectral density of this ensemble is given by

$$\rho(x) = \lim_{\epsilon \to 0^+} \frac{1}{\pi} \mathrm{Im}\, G(x - i\epsilon).$$

We can also expand $G(z)$ into a power series in $\mathrm{Tr}\, A$.