

$$\underline{y = x^2}$$

Gradient / First derivative

$$\frac{d(y)}{d(x)} = \frac{d(x^2)}{dx} = 2 \cdot x$$

$$\text{derivative} = 2 \cdot (x)$$

$$\text{derivative} = \frac{\Delta y}{\Delta x}$$

$$\text{when } x=1 \Rightarrow \text{derivative} = 2 \times 1 = 2.$$

We use a learning rate $\eta = 0.1$

Use Gradient \times learning rate to control the weight update

$$\text{derivative} \times \eta = 2 \times (0.1) = 0.2.$$

Gradient Descent learning.

①

$$\text{movement is controlled by: } - \text{derivative} \times \eta = -2 \times (0.1) = -0.2.$$

Current $x = 1$

$$\text{Next} \Rightarrow \textcircled{x=1} - \text{derivative} \times \eta = 1 - 0.2 = 0.8.$$

$$\text{derivative} = 2 \cdot (x) \Rightarrow 2 \cdot (-1) = -2.$$

$x = -1$

②

$$\text{movement is controlled by: } - \text{derivative} \times \eta = -1 \times (-2) \cdot 0.1 = 0.2.$$

Current $x = -1$

$$\text{Next location } \textcircled{x=-1} + \text{derivative} \times \eta = (-1) + 0.2 = -0.8$$

$$\textcircled{1} f(w) = \frac{1}{2}(d-w)^2$$

$$\frac{d(f(w))}{dw} = \frac{1}{2} \times 2(d-w) \cdot \frac{d(d-w)}{dw} = \frac{1}{2} \times 2(d-w) \cdot (-1) = -(d-w)$$

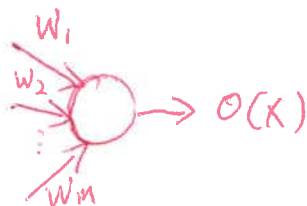
$\textcircled{2}$



$$f(w_1, w_2) = \frac{1}{2}(d - (w_1 \cdot x_1 + w_2 \cdot x_2))^2$$

$$\begin{aligned} \frac{\partial f(w_1, w_2)}{\partial w_1} &= \frac{\partial (d - (w_1 \cdot x_1 + w_2 \cdot x_2))^2}{\partial w_1} = (d - (w_1 \cdot x_1 + w_2 \cdot x_2)) \cdot \frac{\partial (d - (w_1 \cdot x_1 + w_2 \cdot x_2))}{\partial w_1} \\ &= (d - (w_1 \cdot x_1 + w_2 \cdot x_2)) \cdot (-x_1) \\ &= -(d - (w_1 \cdot x_1 + w_2 \cdot x_2)) \cdot x_1 \end{aligned}$$

$\textcircled{3}$



$$f(w_1, w_2, \dots, w_m) = \frac{1}{2}(d - (w_1 \cdot x_1 + w_2 \cdot x_2 + \dots + w_m \cdot x_m))^2$$

$$\frac{\partial f(w_1, w_2, \dots, w_m)}{\partial w_i} = \frac{\partial \left\{ \frac{1}{2} (d - (w_1 \cdot x_1 + w_2 \cdot x_2 + \dots + w_m \cdot x_m))^2 \right\}}{\partial w_i}$$

$$= (d - (w_1 \cdot x_1 + w_2 \cdot x_2 + \dots + w_m \cdot x_m)) \cdot (-x_i)$$

$$= -(d - \sum_{k=1}^m w_k \cdot x_k) \cdot x_i$$

④



$$f(w_1, w_2, \dots, w_m) = \frac{1}{2} \sum_n \left(d(n) - \sum_{i=1}^m w_i \cdot x_i(n) \right)^2$$

$$= \frac{\partial f(w_1, w_2, \dots, w_m)}{\partial w_j} = \frac{\partial \left[\frac{1}{2} \sum_n \left(d(n) - \sum_{i=1}^m w_i \cdot x_i(n) \right)^2 \right]}{\partial w_j}$$

$$= \frac{\partial \left[\frac{1}{2} \left(d(1) - \sum_{i=1}^m w_i \cdot x_i(1) \right)^2 \right]}{\partial w_j} + \frac{\partial \left[\frac{1}{2} \left(d(2) - \sum_{i=1}^m w_i \cdot x_i(2) \right)^2 \right]}{\partial w_j} + \dots$$

$$\Downarrow \quad \dots \quad \frac{\partial \left[\frac{1}{2} \left(d(n) - \sum_{i=1}^m w_i \cdot x_i(n) \right)^2 \right]}{\partial w_j}$$

$$= - \left(d(1) - \sum_{k=1}^m w_k \cdot x_k(1) \right) \cdot x_j(1) - \left(d(2) - \sum_{k=1}^m w_k \cdot x_k(2) \right) \cdot x_j(2) + \dots$$

$$= \left(d(n) - \sum_{k=1}^m w_k \cdot x_k(n) \right) \cdot x_j(n).$$

$$= - \sum_n \left(d(n) - \sum_{k=1}^m w_k \cdot x_k(n) \right) \cdot x_j(n).$$