EEL 6935/ COT 6930 Signal Processing for Machine Learning

Project 2

Due October 3, 2022

1. Consider a 2-tap adaptive filter with coefficients $\mathbf{W} = [w_0 w_1]$ in the interference canceling configuration shown in the figure below. The input signal to the filter (Reference Signal) is $x[n] = sin(2\pi\frac{n}{30})$ and the desired or primary signal is $d[n] = cos(2\pi\frac{n}{30})$. Clearly the two signals have the same frequency but are out of phase with each other. The purpose of the adaptive filter is for the filter output $y[n]$ to match the primary signal $d[n]$ in amplitude and phase so that the error is small. Update coefficients $\mathbf{w}$ using the LMS update equation given by

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu\mathbf{x}(n)e(n).$$

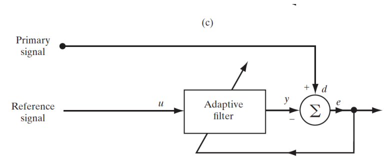

FIGURE 11 Four basic classes of adaptive filtering applications: (a) class I: identificatio (b) class II: inverse modeling; (c) class III: prediction; (d) class IV: interference cancellati

   (a) Implement the adaptive filter using a adaptation step size of $\mu = 0.4$.
   (b) Plot $x[n], d[n], y[n], e[n]$ and $\mathbf{w[n]} = [w_0[n]w_1[n]]$.
   (c) Analytical work:
      i. Write the error signal as $e[n] = d[n] - y[n]$.
      ii. Formulate the expected value of the squared error, i.e.

$$\mathbf{J} = \mathbf{E}[e^2[n]].$$

      iii. Evaluate the derivative of $\mathbf{J}$ with respect each one of the weights w_0 and w_1. Set the two equations equal to zero.
      iv. Solve for the weights. These are your optimal weights.
      v. Use the optimal weights in the mse function $\mathbf{J}$. This is the minimum error.

(d) Using a 3-D plot, plot the error surface $\mathbf{J}$ as a function of the weights w_0 and w_1.

(e) Mark the the value of $\mathbf{J}$ as weights are updated.

2. A discrete time signal consists of a sinusoid plus additive white Gaussian noise and is given by

$$d[n] = s[n] + w[n]$$

where $s[n] = sin(2 * pi * \frac{n}{15} - \frac{\pi}{3})$ for $0 \leq n < 15000$. Let $x[n] = sin(2 * pi * \frac{n}{15})$.Use the interference canceling configuration of the adaptive filter above to separate the signals.

(a) Let SNR= 10 dB, N=2 and $\mu = 0.1$. Plot output $y[n]$, error e[n], filter weights $\mathbf{w[n]}$ and the mean-square-error $J[n]$ versus n.

(b) Vary the SNR choosing snr= 5, 0, -5, -10. Repeat outputs specified in 2.a.

(c) Let SNR=5 dB. Vary $N = 4, 6, 8, 12$. Repeat outputs specified in 2.a.

(d) Let SNR=5 dB. Vary $\mu = 0.2, 0.3, 0.4, 0.60.8$. Repeat outputs specified in 2.a.

(e) For N=2 and SNR=5dB, find the optimal filter coefficients as you did in Problem 1.Compare your results above to the theoretical ones.

(f) Write an analysis of your work for this problem. Comment on the choice of the parameters N, μ and SNR.

3. Use what you have learned in the above exercises to remove the dominant interfering tone from the audio file TONE.wav used in Project 1. I recommend you start with the configuration of Problem 2. You may attempt any creative improvement on this configuration to make your results better. Oh, don't forget to analyze the problems and the successes.