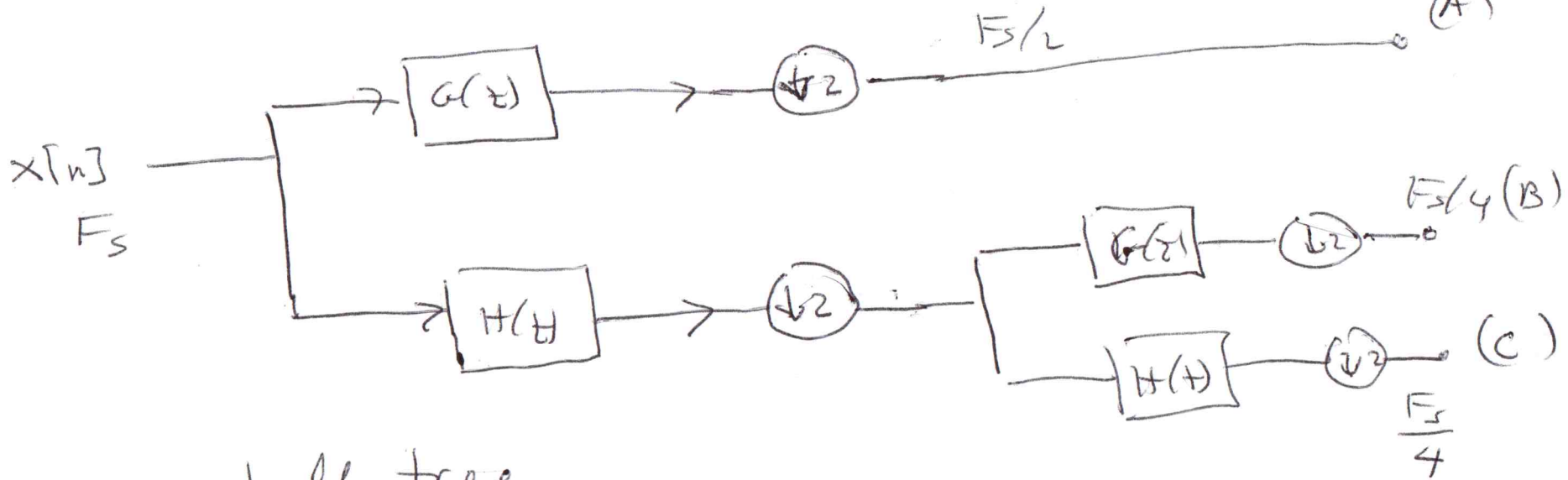


Filter banks

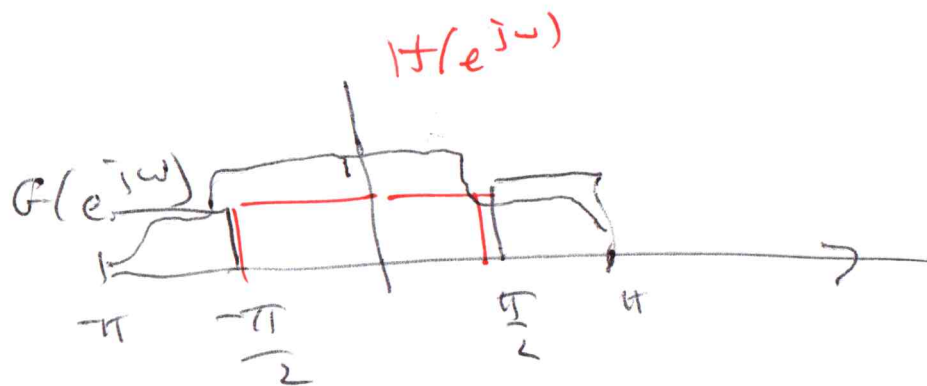
(multi rate signal)

(1)

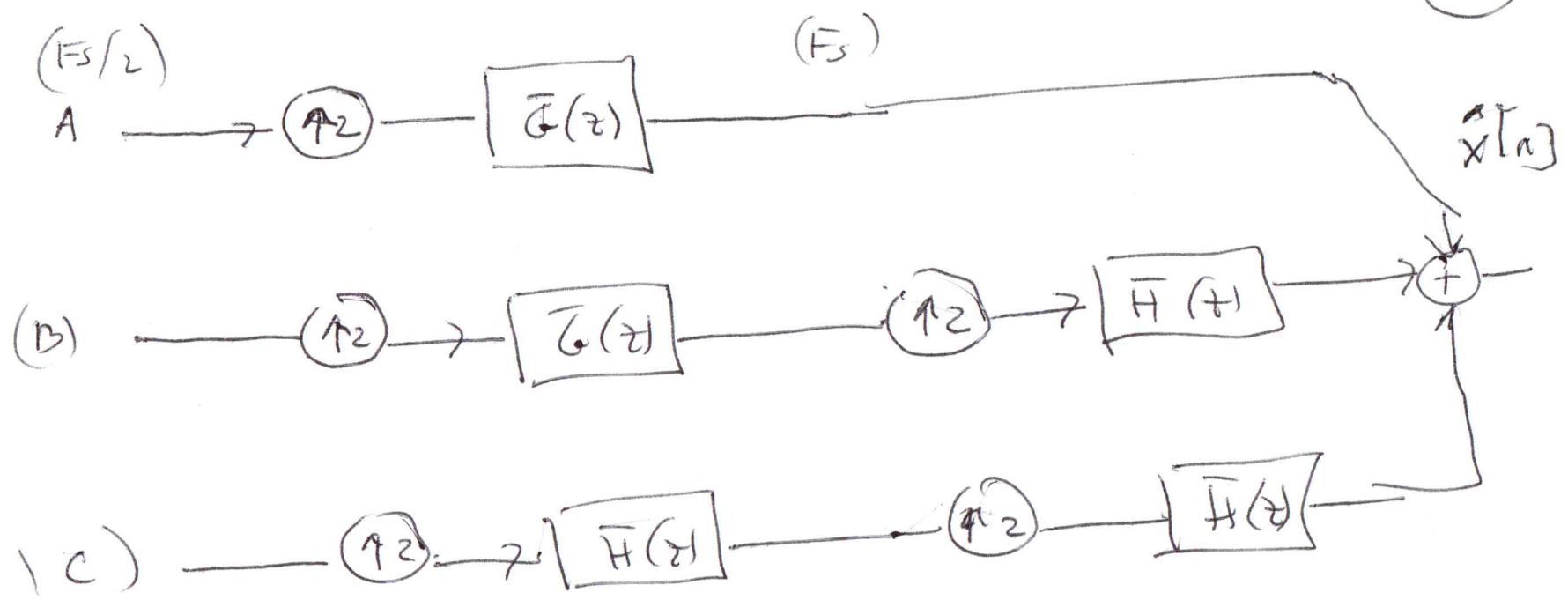
(A)



Binary half tree
(full)

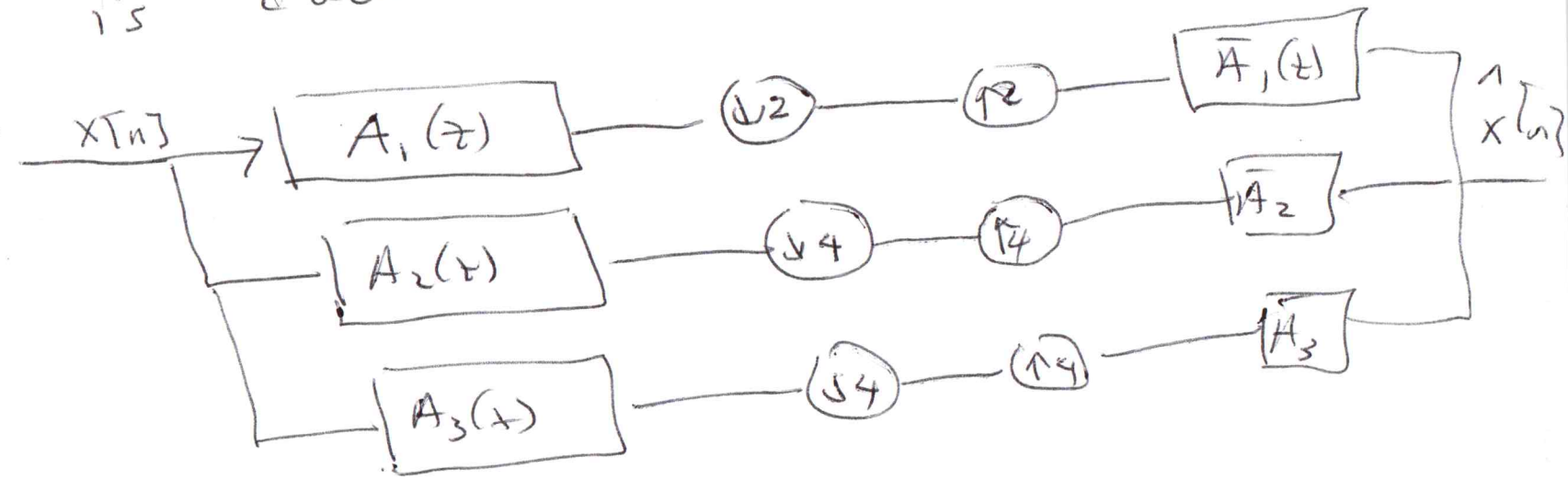


2



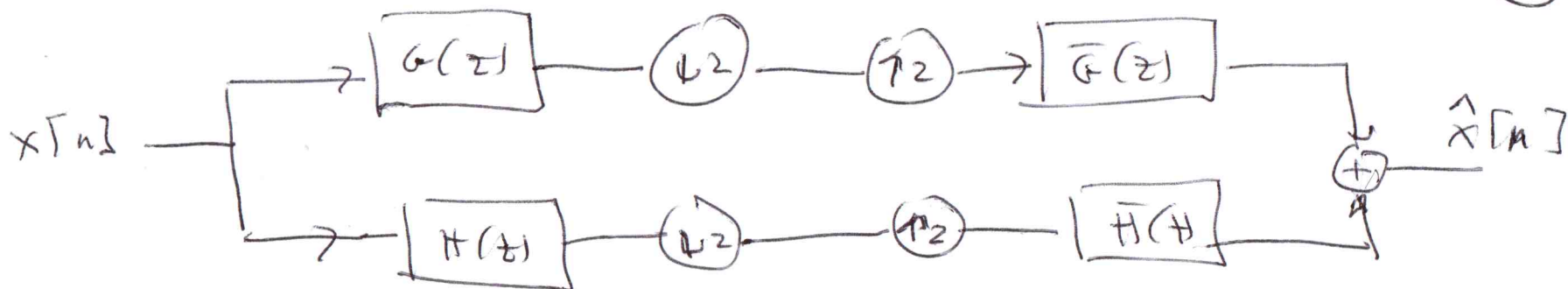
If $\hat{x}[n] = x[n]$, then the FB is called a perfect reconstruction FB.

Alternate

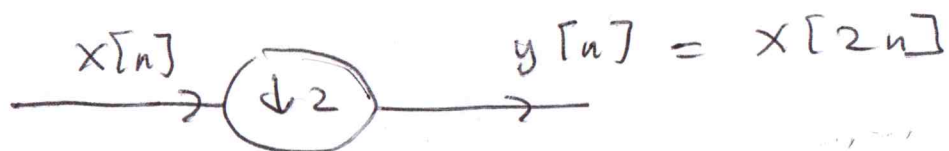


21 even F B

(3)



general



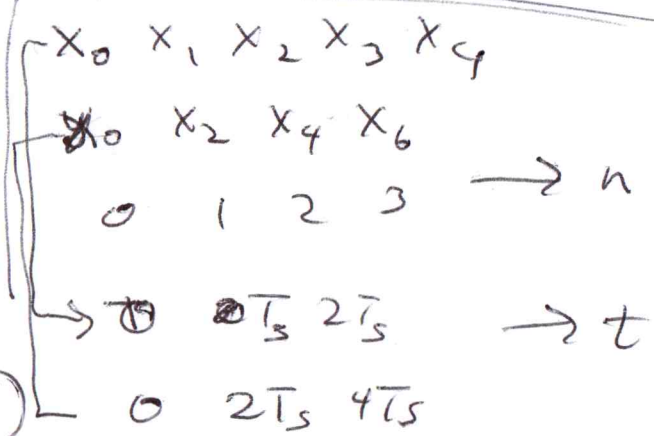
$$Y(e^{j\omega}) = \sum_n y[n] e^{-j\omega n}$$

$$= \sum_n x[2n] e^{-j\omega n}$$

$$= \sum_{\text{even } m} x[m] e^{-j\frac{\omega}{2}m}$$

$$= \sum_{\substack{m \\ \text{all}}} x[m] \left(\frac{1 + (-1)^m}{2} \right) e^{-j\frac{\omega}{2}m}$$

$m = 2n$
even



$$a_k = \frac{1 + (-1)^k}{2}$$

$$= \begin{cases} 1 & k \text{ even} \\ 0 & k \text{ odd} \end{cases}$$

(4)

$$Y(e^{j\omega}) = \sum_m x[m] \frac{1 + (-1)^m}{2} e^{-j\frac{\omega}{2}m}$$

$$= \frac{1}{2} \sum_m x[m] e^{-j\frac{\omega}{2}m} + \frac{1}{2} \sum_m x[m] (-1)^m e^{-j\frac{\omega}{2}m}$$

$$\boxed{X(e^{j\omega}) = \sum_m x[m] e^{-j\omega m}} + \frac{1}{2} \sum_m x[m] \left(-e^{-j\frac{\omega}{2}} \right)^m$$

$$Y(e^{j\omega}) = \frac{1}{2} X(e^{j\frac{\omega}{2}}) + \frac{1}{2} \sum_m x[m] e^{-j\frac{\omega - 2\pi}{2}m}$$

$$= \frac{1}{2} \underbrace{X(e^{j\frac{\omega}{2}})} + \frac{1}{2} X(e^{j\frac{\omega - 2\pi}{2}})$$

$$X(e^{j\frac{\omega}{2}}) = B(\omega) \quad \left| \quad X(e^{j\frac{\omega - 2\pi}{2}}) = X(e^{j(\frac{\omega}{2} - \pi)})$$

$$X(e^{j\frac{\omega - 2\pi}{2}}) = B(\omega - 2\pi) \quad \left| \quad \begin{array}{l} \textcircled{1} X(e^{j(\omega - \pi)}) = A(\omega) \\ \textcircled{2} X(e^{j(\frac{\omega}{2} - \pi)}) = A(\frac{\omega}{2}) \end{array} \right.$$

$$\boxed{e^{j2\pi} e^{-j\frac{\omega}{2}} = e^{j(\frac{\omega - 2\pi}{2})}}$$

PLTL 0 + 7242 = XLSN



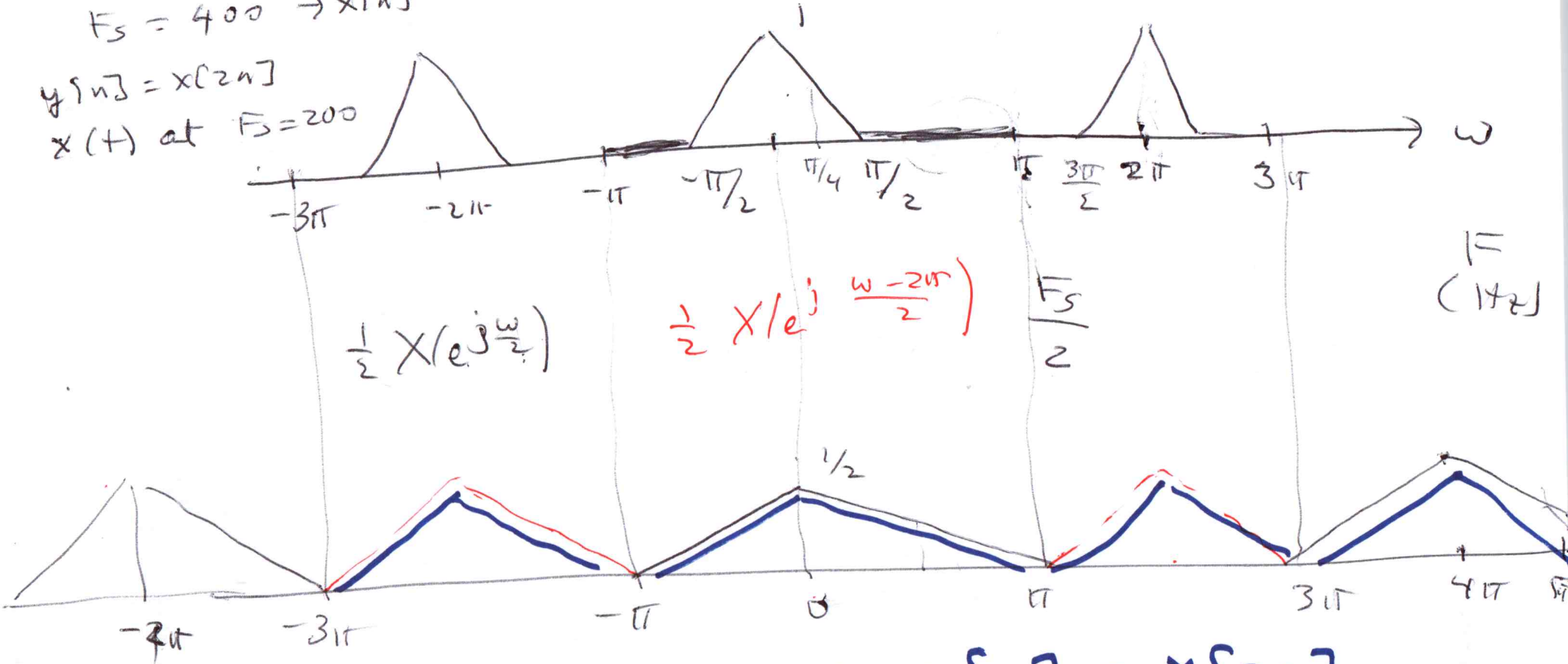
$X(t)$ BW 100 Hz
 $F_s = 400 \rightarrow X[n]$

$y[n] = x[2n]$

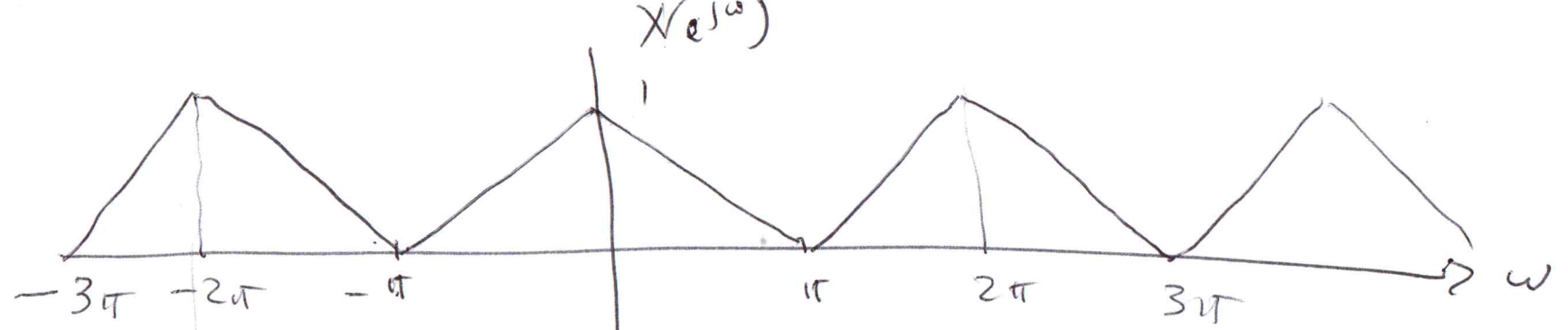
$x(t)$ at $F_s = 200$

$X(e^{j\omega})$

(5)



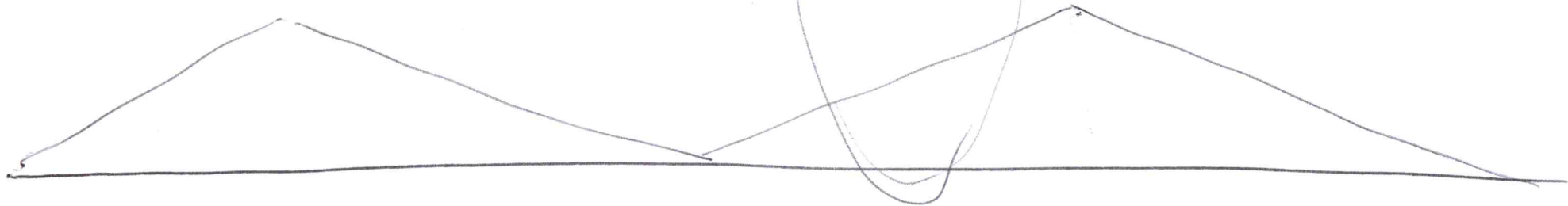
DTFT of $y[n] = x[2n]$



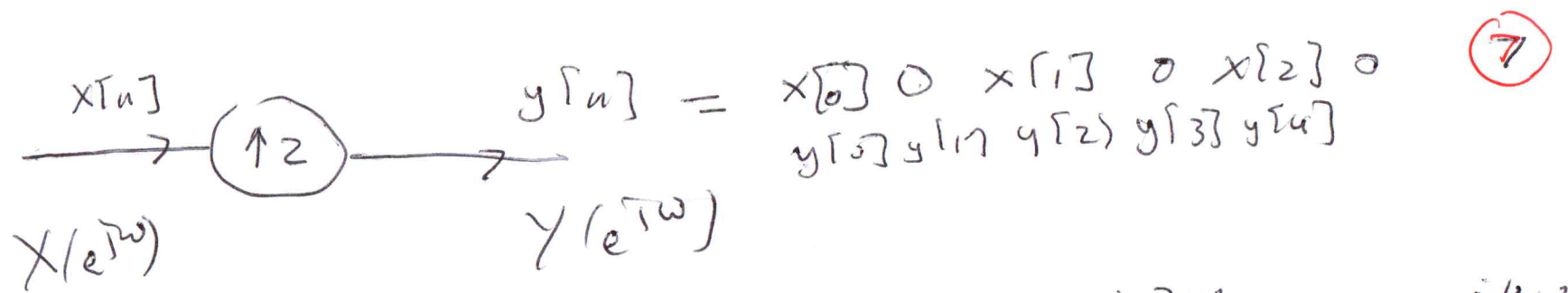
$$\frac{1}{2} X(e^{j\omega/2})$$



$$\frac{1}{2} X(e^{j\frac{\omega-2\pi}{2}})$$



Circle \Rightarrow Aliases.



$$y[n] = x[0] \cdot 0 \cdot x[1] \cdot 0 \cdot x[2] \cdot 0 \cdot y[0] \cdot y[1] \cdot y[2] \cdot y[3] \cdot y[4]$$

$$Y(e^{j\omega}) = \dots x[0] + x[1]e^{-j2\omega} + x[n]e^{-j4\omega} + 0 \cdot e^{-j3\omega} + 0 \cdot e^{-j\omega} +$$

$$Y(e^{j\omega}) = \sum_n x[n] e^{-j2n\omega}$$

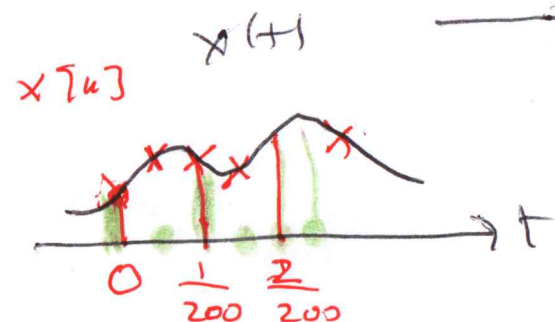
$$= X(e^{j2\omega})$$

8

$x(t)$ BW 100 Hz

$x[n]$ $F_s = 200$

$p[n]$ $F_s = 400$

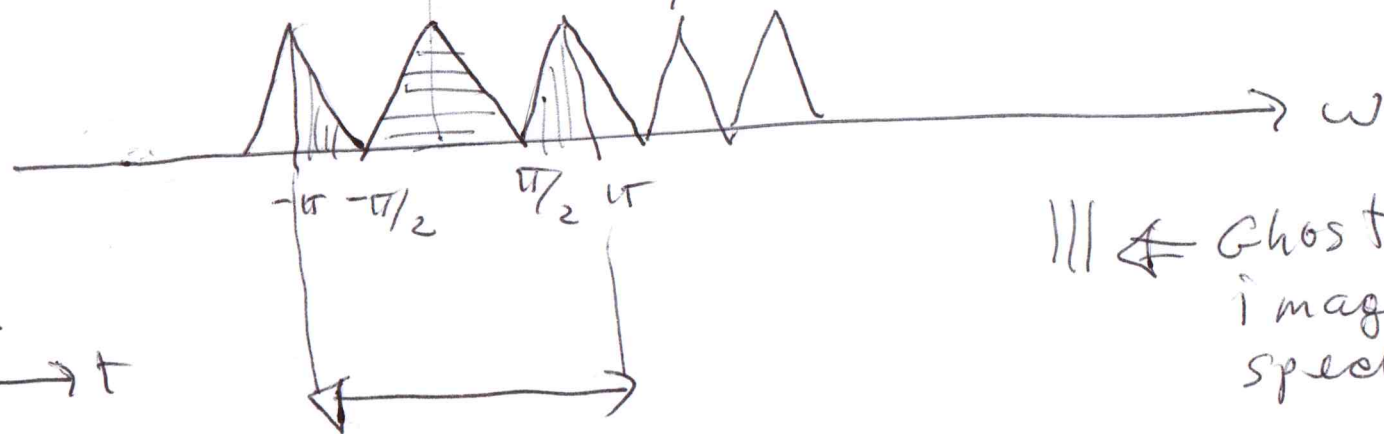


$x; p[n]$

$y[n] = x \uparrow 2$

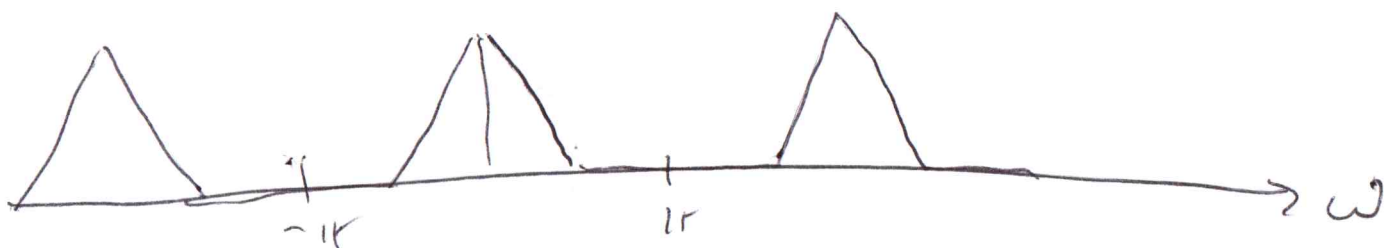


$$X(e^{j2\omega}) = Y(e^{j\omega})$$

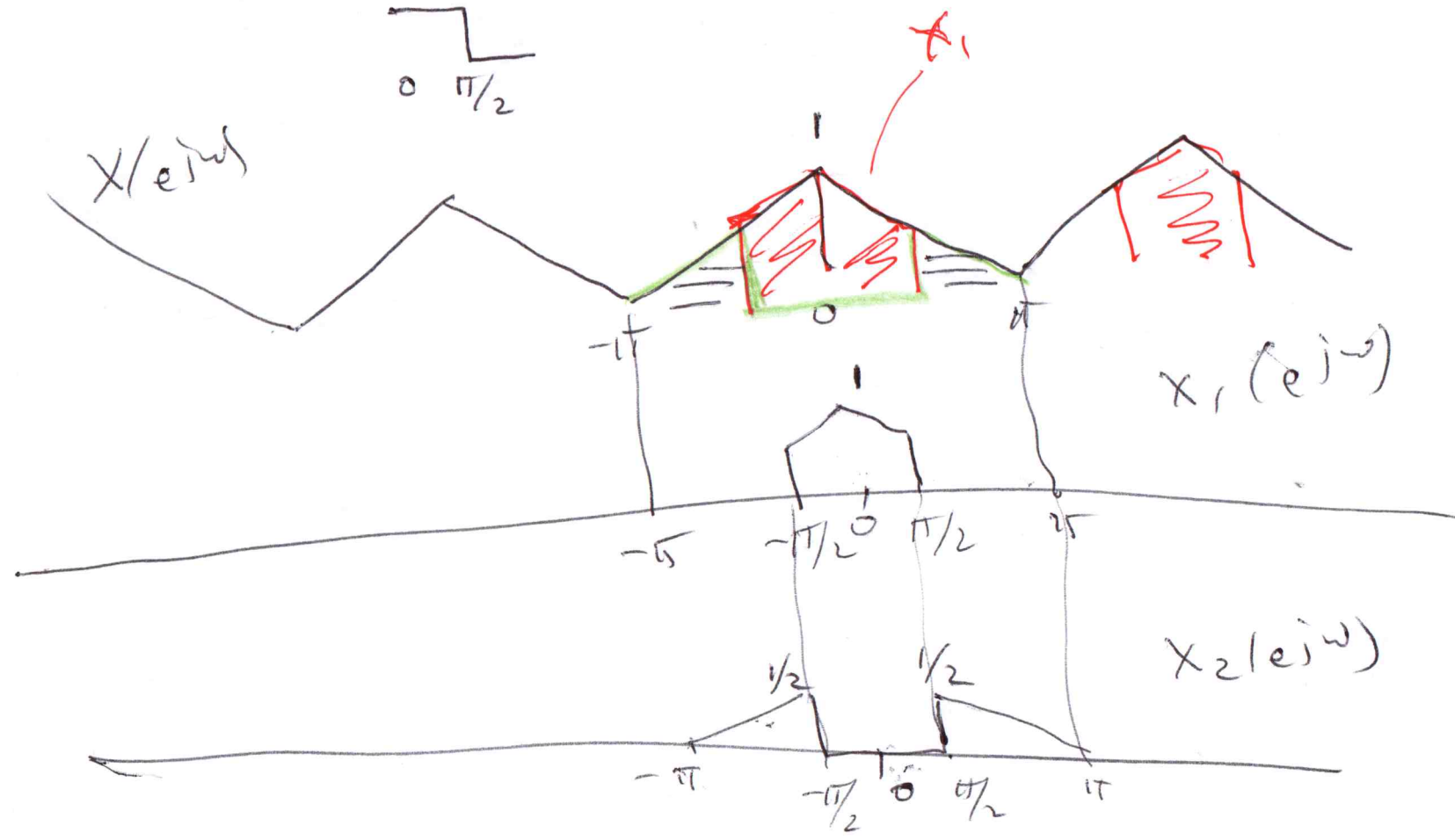
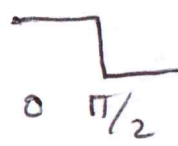
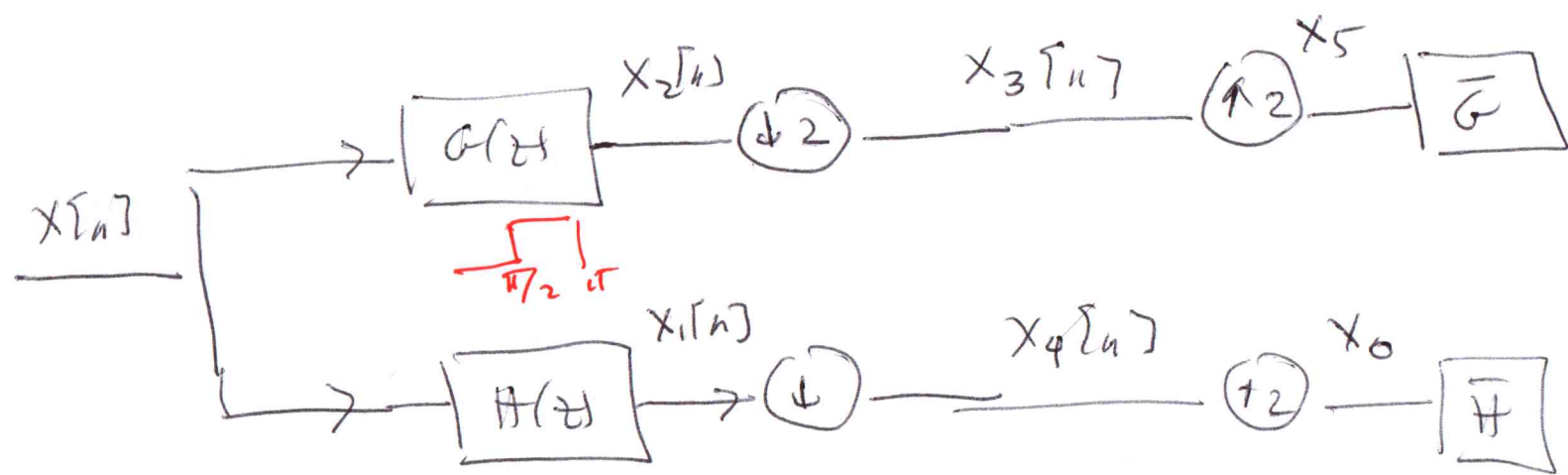


||| \nLeftarrow Ghost image or spectrum

$P(e^{j\omega})$

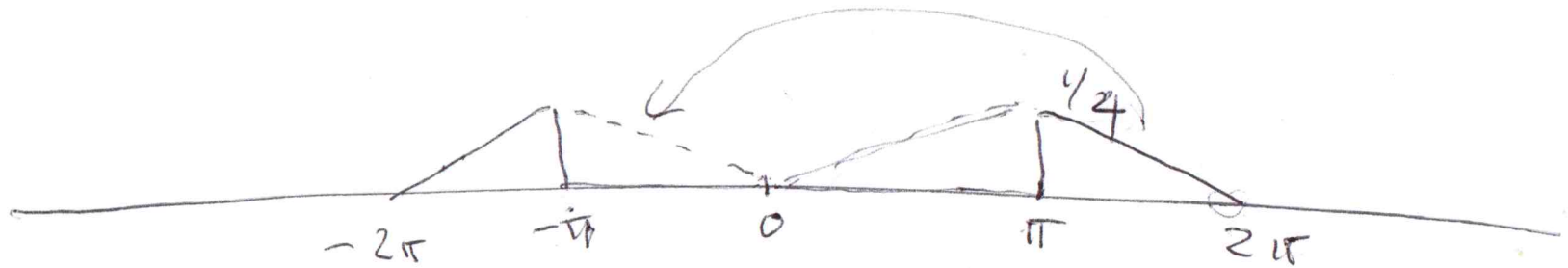
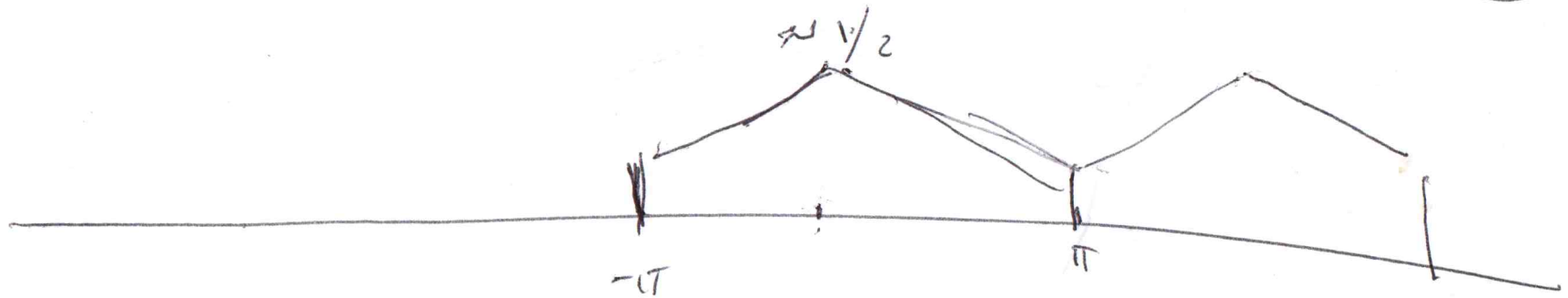


9



$$X_4(e^{j\omega})$$

(10)



$$X_5(e^{j\omega}) = X_4(e^{j2\omega})$$

