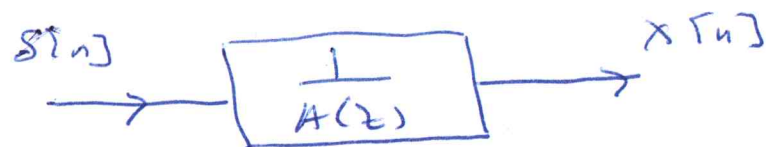


Consider $A(z) = a_0 + a_1 z^{-1} + \dots + a_p z^{-p}$



Find $\{a_k\}_{k=1, \dots, p}$ so that ~~satisfy~~ the filter output is the data that I have.

The filter ΔE is

$$a_0 x[n] + a_1 x[n-1] + \dots + a_p x[n-p] = s[n] \quad \text{①}$$

Let $x[n] = 2^{-n} u[n] = \{ \dots, 0, 0, 0, \underset{\uparrow n=0}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots \}$

and $p = 3, a_0 = 1$

Write eqn. ① for any set of n .

I'll choose $n = 0, 2, 3$.

This exercise is only useful to show the problems with the formulation so don't worry.

$$n=1 \quad x[1] + a_1 x[0] + a_2 x[-1] + a_3 x[-2] = 0$$

$$n=2 \quad x[2] + a_1 x[1] + a_2 x[0] + a_3 x[-1] = 0$$

$$n=3 \quad x[3] + a_1 x[2] + a_2 x[1] + a_3 x[0] = 0$$

I have 3 unknowns (a_1, a_2, a_3) and 3 eqs. Can I solve?

Re write the above in matrix form.

$$\underbrace{\begin{bmatrix} x[1] \\ x[2] \\ x[3] \end{bmatrix}}_{\underline{x}} = - \underbrace{\begin{bmatrix} x[0] & x[-1] & x[-2] \\ x[1] & x[0] & x[-1] \\ x[2] & x[1] & x[0] \end{bmatrix}}_{\underline{X}} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}}_{\underline{a}}$$

or

$$\underline{x} = - \underline{X} \cdot \underline{a}$$

$$\underline{a} = - \underline{X}^{-1} \cdot \underline{x} \quad (?)$$

The proposed solu

$$\underline{a} = - \underline{X}^{-1} \cdot \underline{x} \quad (2)$$

requires that \underline{X} have an inverse.
Let's assume it does & we find \underline{a} .

What about $n = 4, 5, 6$?

$$\begin{bmatrix} x[4] \\ x[5] \\ x[6] \end{bmatrix} = \begin{bmatrix} x[3] & x[2] & x[1] \\ x[4] & x[3] & x[2] \\ x[5] & x[4] & x[1] \end{bmatrix} \underline{a} \quad (3)$$

or $n = 15, 35, 82$? any n ?
with the \underline{a} ~~found~~^{given} by (2) satisfy (3) ?

You can check these with the "toy" signal

$x[n] = \left(\frac{1}{2}\right)^n u[n]$. To find if a matrix
has an inverse, check its rank

If $\text{rank}(\underline{X}) < 3$, then the answer
is no.

Also use other data, e.g. TONE.wav.

pg. ④

Take any 3 values of n , form \underline{X} & \bar{X} .

- Check if X^{-1} exists

- If it does, for one set of n , does your
Soln. satisfy eq (1) for other n ?

The answers will be no most of the time.
That's why we need to relax some requirements.

Rewrite ① replacing $\delta[n]$ by $e[n]$.

$$\sum_{k=0}^P a_k x[n-k] = e[n]$$

and proceed in the same way as above.

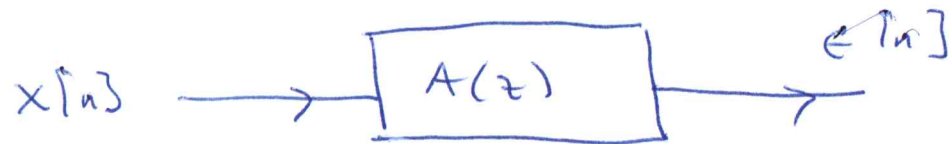
What is $e[n]$?

What we have is a case of more eqs. than unknowns, as well as X^{-1} not guaranteed, Pg. 5

with $a_0 = 1$

$$x[n] + a_1 x[n-1] + \dots + a_p x[n-1] = e[n] \quad (4)$$

can be represented by



so we want $e[n]$ to somehow represent $\delta[n]$ = 1 $n=0$ & 0 for other n .

MSE to the rescue.

$$\text{let } \mathcal{E} = \sum_n e^2[n]$$

Choose a_k s.t. \mathcal{E} is minimized,

$$\mathcal{E} = \sum_n \left(\sum_{k=0}^p a_k x[n-k] \right)^2$$

ps. (8)

$$\frac{\partial \mathcal{E}}{\partial a_i} = 0 \quad i = 1, 2, \dots, p$$

$$\begin{aligned} \frac{\partial \mathcal{E}}{\partial a_i} &= 2 \sum_n \sum_k a_k x[n-k] \cdot x[n-i] \\ &= 2 \sum_{k=0}^p a_k \underbrace{\sum_n x[n-k] x[n-i]}_{c_{xx}[k, i]} \end{aligned}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}_{(1+p) \times 1} = \begin{bmatrix} \cancel{c_{xx}[0]} & \cancel{c_{xx}[1]} & \dots \\ \cancel{c_{xx}[1]} & \dots & \dots \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} a_0 = 1 \\ a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix}_{(1+p) \times 1}$$

$\underline{C}_{(p+1) \times (p+1)}$

(5)

$$\underline{C} = \begin{bmatrix} c_{xx}[0,1] & c_{xx}[1,1] & c_{xx}[2,1] & \dots & c_{xx}[p,1] \\ c_{xx}[0,2] & c_{xx}[1,2] & \dots & c_{xx}[p,2] \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ c_{xx}[0,p] & c_{xx}[1,p] & \dots & \dots & c_{xx}[p,p] \end{bmatrix}$$

\underline{C} is called the auto correlation or auto covariance matrix.

• Explore its properties

• Since $a_0 = 1$, rearranging Eq. (5)

$$\begin{bmatrix} c_{xx}[0,1] \\ c_{xx}[0,2] \\ \vdots \\ c_{xx}[0,p] \end{bmatrix}_{p \times 1}$$

$$= \underline{C}_{p \times p}$$

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix}_{p \times 1}$$

What is \underline{C} ?