SPML Sept 6

Recap – F ~~T~~Transform (FT, DTFT, DFT)

FT $x(t) \leftrightarrow X(\Omega)$

$t \to \Omega$ cont. t, Ω

DTFT $x[n] \leftrightarrow X(e^{j\omega})$

$n \to \omega$ n: disc., ω = cont

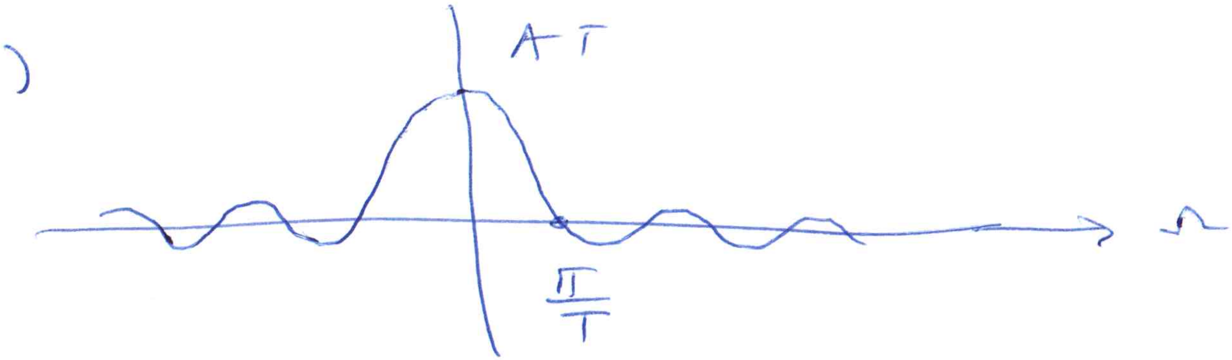
$X(e^{j\omega})$ 2π-periodic in ω

$\omega = \frac{\Omega}{F_s}$ F_s = sampling rate

DFT $x[n] \leftrightarrow X[k]$

$n \to k$ Discrete n, k

$X(\Omega)$



Band width of $x(t)$ or $X(\Omega)$? ∞

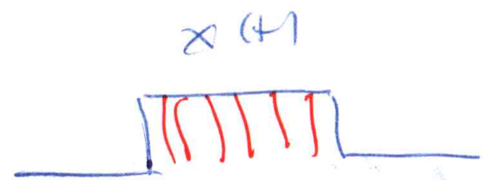
$x(t)$ is <u>not</u> a band-limited signal.

$T_s = \frac{1}{F_s}$ = sampling pd.

DTFT

$x[n] = x(nT_s)$

is an aliased version of $x(t)$.



$$X(e^{j\omega}) = \sum_{n=-N}^{N} 1 \cdot e^{-j\omega n}$$

sin ω

$$\boxed{\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}}$$

$$\sum_{-M}^{N} a^n$$

$$X(e^{j\omega}) = \sum_{n=-N}^{N} e^{-j\omega n}$$

$$= e^{j\omega N} + e^{j\omega(N-1)} + \cdots e^{-j\omega N}$$

$$e^{j\omega} X(e^{j\omega}) = e^{j\omega(N+1)} + e^{j\omega N} + \cdots e^{-j\omega(N-1)}$$

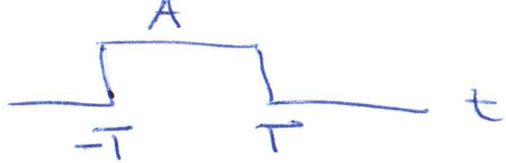
$$(1-e^{j\omega}) X(e^{j\omega}) = e^{-j\omega N} - e^{-j\omega(N+1)}$$

$$X(e^{j\omega}) = \frac{e^{-j\omega N} - e^{+j\omega(N+1)}}{1-e^{j\omega}} \cdot \frac{e^{-j\frac{\omega}{2}}}{e^{-j\frac{\omega}{2}}}$$

$$= \frac{e^{-j\omega\left(\frac{2N+1}{2}\right)} - e^{+j\omega\frac{2N+1}{2}}}{e^{-j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}}$$

$$= \frac{\sin\left(\omega\,\frac{2N+1}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$$

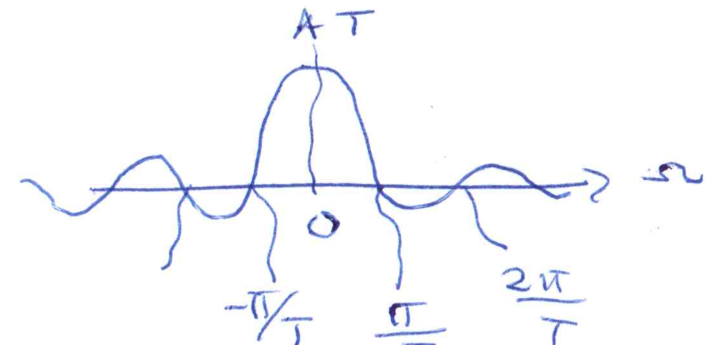
Ex

$x(t) =$ 

$$X(\Omega) = \int_{-T}^{T} A\, e^{-j\Omega t}\, dt$$

$$= \frac{A}{-j\Omega}\left(e^{-j\Omega T} - e^{j\Omega T}\right)$$

$$X(\Omega) = AT\, \frac{\sin(\Omega T)}{\Omega T}$$



- $AT \propto A$
- $X(0) \propto A, T$
- First zero x-ings $\frac{\pi}{T} \to 0 \quad (T \to \infty)$

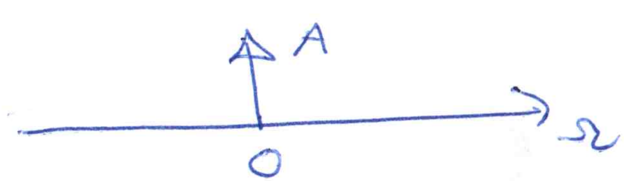
$\lim_{T\to\infty} x(t) = A$ for all T

$$\lim_{T\to\infty} X(\Omega) = A\,\delta(\Omega)$$

$$\delta(\Omega) = \begin{cases} 0 & \Omega \neq 0 \\ \infty & \Omega = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(\Omega)\, dt = 1$$

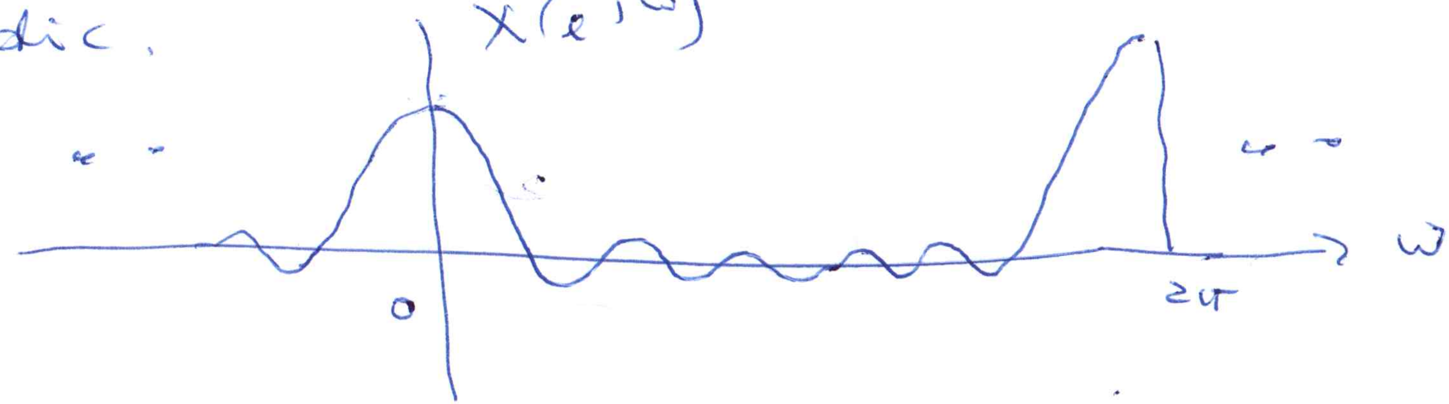
$A\,\delta(\Omega)$



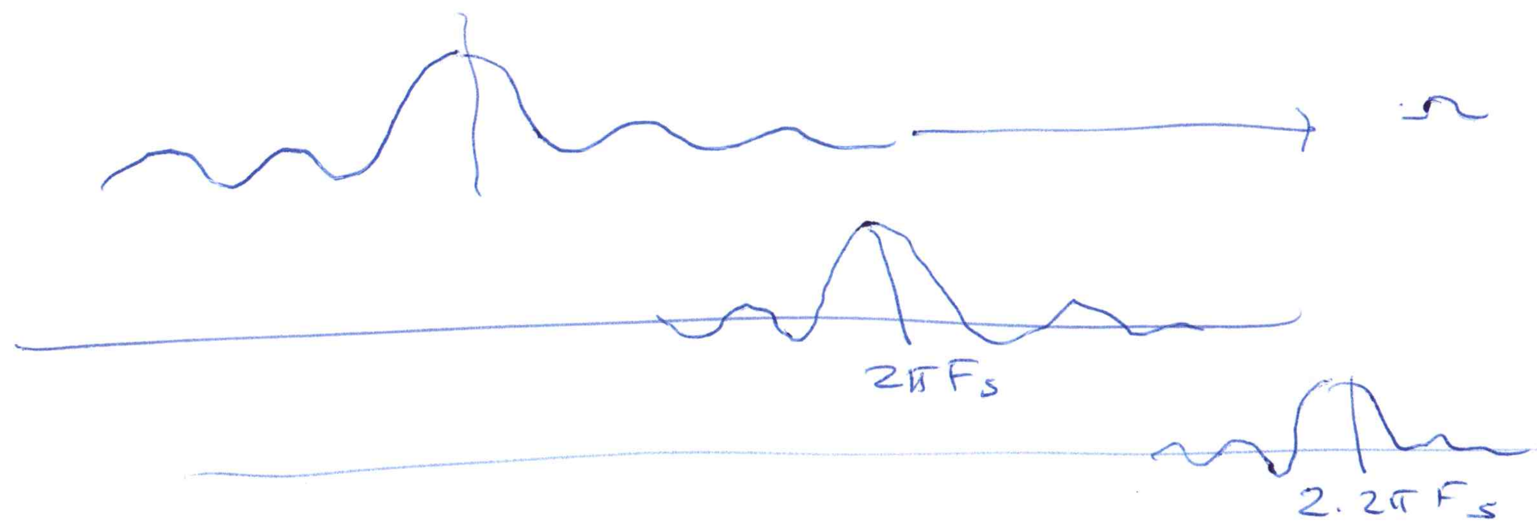
A: weight

$$X(e^{j\omega}) = \frac{\sin\left(\omega \, \frac{2N+1}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$$

- 2π-pdic.



$$X e^{j\frac{\Omega}{T_s}}) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(\Omega - 2\pi F_s n)$$



$x[n] = \ldots \underset{-N}{|}\ | \ | \ \ldots \underset{N}{|} \ldots$

DFT generic DFT

$$Y[k] = \sum_{n=0}^{N-1} y[n]\, e^{-j\frac{2\pi}{N}nk} \qquad k = 0, \ldots N-1 \qquad (1)$$

DFSeries

$$\tilde{Y}[k] = \sum_{n=0}^{N-1} \tilde{y}[n]\, e^{-j\frac{2\pi}{N}nk} \qquad (2)$$

DFT : Compute for $k = 0, \ldots N-1$

You can compute $Y[k]$ for other k

DFS : Allow the RHS of (2) to be computed for $-\infty < k < \infty$

$\tilde{Y}[k]$ is periodic in k w/ pd N.

$$\tilde{Y}[k] = \sum_{n=n_0}^{n_0+N-1} \tilde{y}[n]\, e^{-j\frac{2\pi}{N}nk}$$

.. over any pd.

IDFT

$$y[n] = \frac{1}{N}\sum_{k=0}^{N-1} Y[k]\, e^{j\frac{2\pi}{N}kn}$$

$n = 0, 1, \ldots N-1$

<u>IDFS</u>

$$\tilde{y}[n] = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{Y}[k]\, e^{j\frac{2\pi}{N}nk}$$

$-\infty < n < \infty$

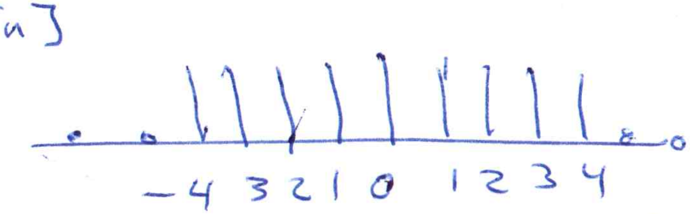
$\tilde{y}[n]$ is pdic in n w/ pd N.

→ DFT is 1 pd of the DFS confined to $k = 0 \ldots N-1$

→ IDFT is 1 pd of the IDFS " to $n = 0, \ldots, N-1$

$$\tilde{y}[n] = \sum_{k=k_0}^{k_0+N-1} \tilde{Y}[k]\, e^{j\frac{2\pi}{N}nk}$$

$x[n]$ w/ $N = 4$



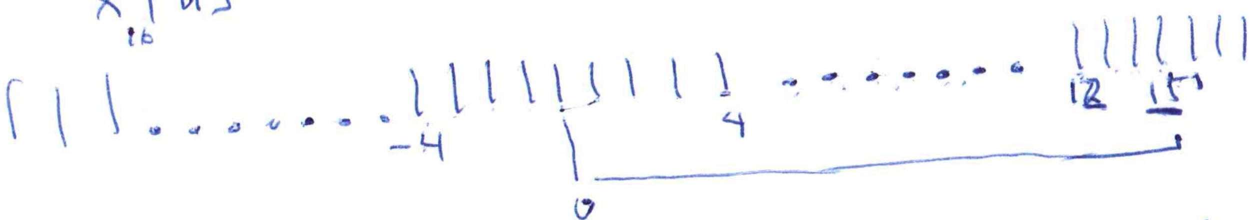
~~x[n]~~ DFT ~~DFT~~ length of $x[n]$ is 9.

N = no. of DFT pd.

Let $M = 16 \geq 9$

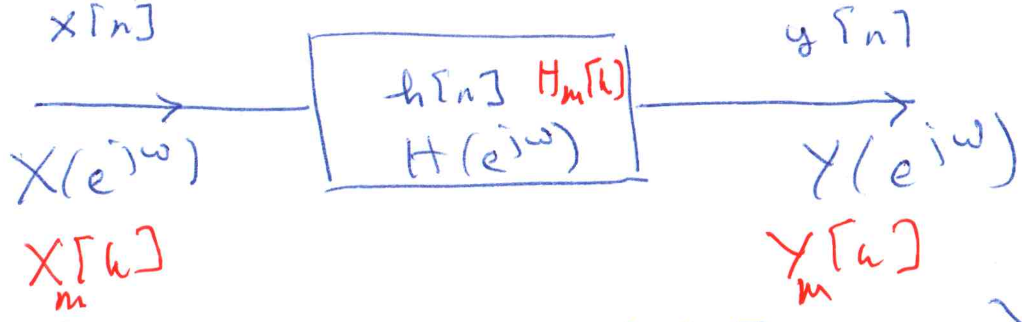
(1) generate $\tilde{x}[n]$, pdic extension of $x[n]$ w/ pd. 16

$\tilde{x}_{16}[n]$



(2) $x_a[n]$ augmented

Filtering.

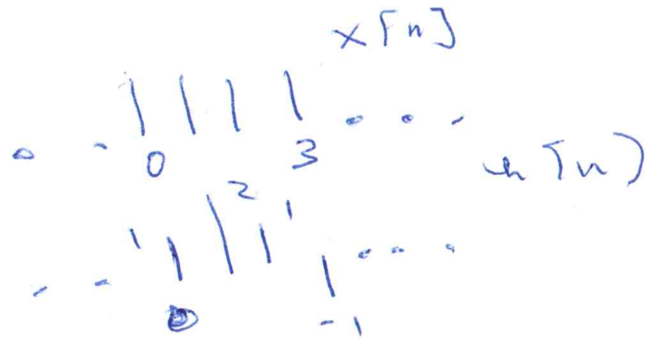


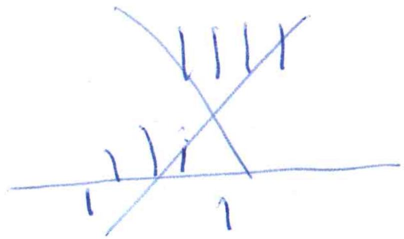
DTFT

$$y[n] = x[n] * h[n]$$

$$Y(e^{j\omega}) = X(e^{j\omega})\,H(e^{j\omega})$$

$$= \sum_{\ell} x[\ell]\,h[n-\ell]$$





1

Length of $y[n] = l_x + l_y - 1$

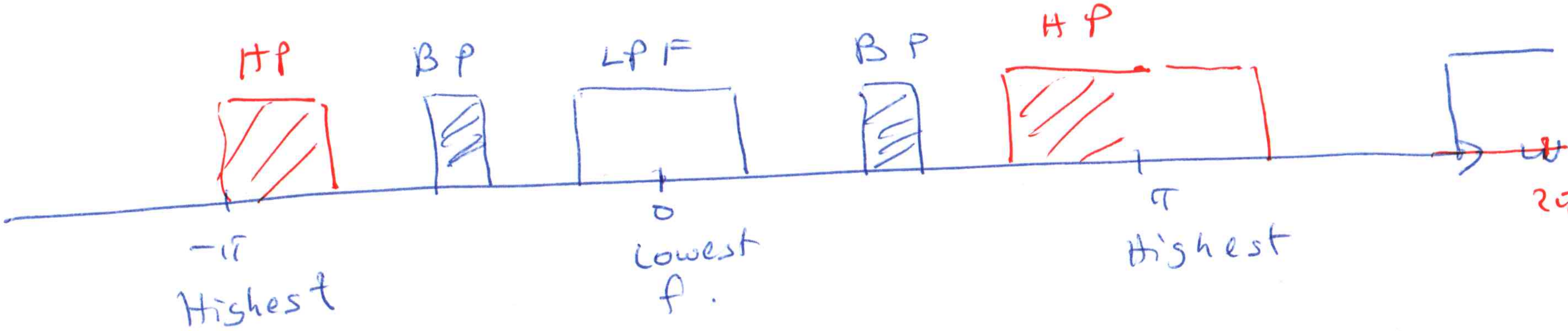
$= 7$

$$Y_m[k] = X_m[k] \cdot H_m[k]$$

$m \geq 7$

0 0 0 1 1 1 1
-1 1 2 1 0 0 0
-1 1 2 1
-1 1 2 1
-1 1 2 1
-1 1 2 1
-1 1 2 1

$y[0] = 1$
$y[1] = 3$
$\vdots$
$y[7] \neq 0$
$y[8] = 0$



Symmetry $\left(\begin{array}{ll} \text{even} & \text{magnitude} \\ \text{odd} & \text{phase} \end{array}\right) \Rightarrow$ signals are real seqs. of n (time)

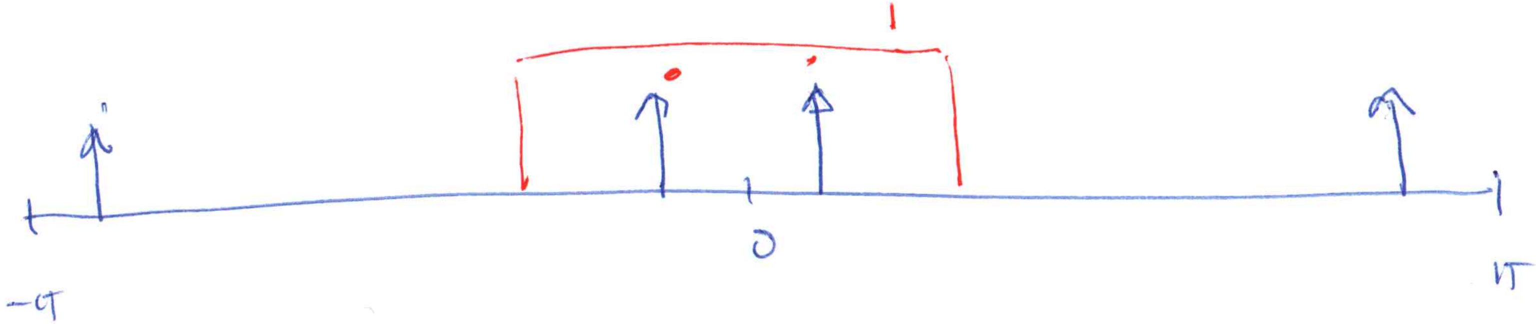
LPF $\quad H_L(e^{j\omega}) \Leftrightarrow h[n]$

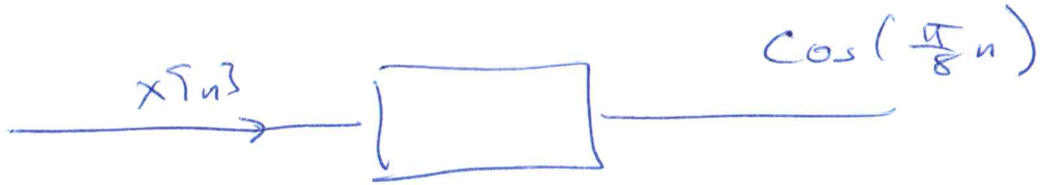
Transfer fn $\quad$ Impulse

$H_L(e^{j\omega}) \Leftrightarrow h[n]$ real

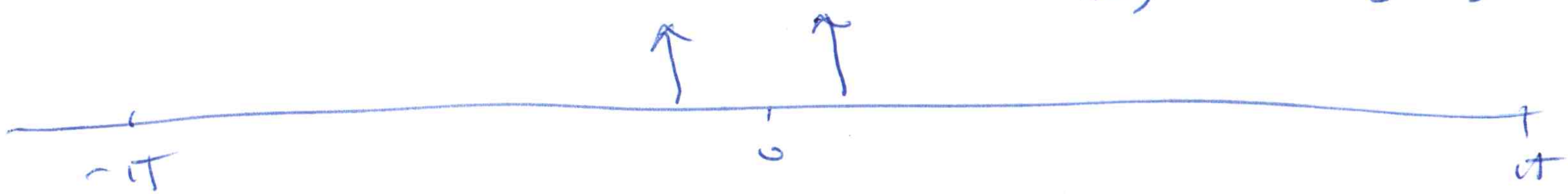
Conjugate symmetries

$$x[n] = \cos\left(\frac{\pi}{8} n\right) + \cos\left(\frac{9}{10}\pi\right)$$





$\rightarrow \cos\left(\frac{\pi}{8} n\right)$



MATLAB

Signal $\quad x = [\ \dots\ ]$

<u>Filter</u> $\quad$ - z - transform.

$$h[n] \overset{z}{\longleftrightarrow} H(z) = \sum_{n=\infty}^{\infty} h[n] z^{-n}$$

$$\overset{DTFT}{\longleftrightarrow} H(e^{j\omega}) = \sum h(n) e^{-j\omega n}$$

$$= H(z)\big|_{z=e^{j\omega}}$$

<u>Ex</u> $\quad h[n] =$

$$H(z) = \underbrace{1 + 1\cdot z^{-1} + 1\cdot z^{-2}}_{\text{used gen.}} = \frac{z^2+z+1}{z^2}$$

All zero filter

FIR

Poles at 0 or ∞ only

$h = [1\ 1\ 1] = b \qquad a = 1$

~~y = filter (x, h)~~ b, x

y = filter (b, a, x)

$$H(z) = \frac{1}{1 - \frac{1}{2} z^{-1}} \Leftrightarrow h[n] = \left(\frac{1}{2}\right)^n u[n]$$

All-pole filter

$$H(z) = \frac{1 \cdot z}{z - \frac{1}{2}}$$

Zeroes at $z = 0, \infty$ only

Poles $z = 1/2$

IIR
$h[n] = 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ ∞ ly long.
Poles other than at 0 or ∞

Num coefs. → $b = 1$

Den → $a = [1 \quad -\frac{1}{2}]$

$y = \text{filter}(\cancel{x, b, a})$
(b, a, x)

Characterize your filter

mag vs f

phas vs f

freqz (b, a, N) ← no of pts "fft"