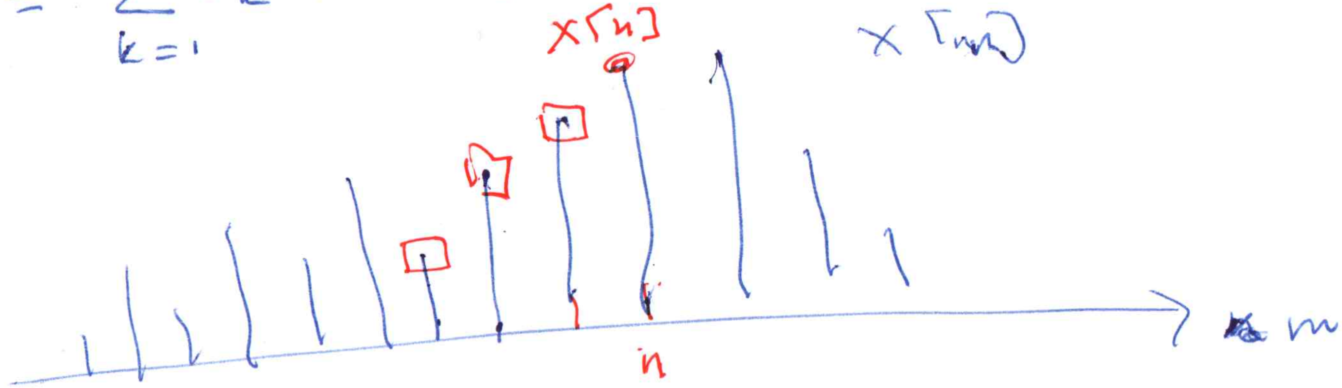
Data $x[n]$ $n = 0, \dots N-1$ $(N = 1000)$

$$x[n] = \sum_{k=1}^{p} a_k \, x[n-k] + e[n] \qquad (1)$$

$p = 3$

Data



Eq(1) $\Rightarrow$ n^{th} sample of $x[n]$ can be written as a linear combination of p past samples.

Define an objective function

$$\mathcal{E}_p = \sum_n e^2[n] \qquad (2)$$

$$\frac{\partial \mathcal{E}}{\partial a_i} = \sum_n 2\, e[n] \frac{\partial e[n]}{\partial a_i} \qquad (3)$$

$$e[n] = x[n] - \sum_{k=1}^{p} a_k\, x[n-k]$$

$$\frac{\partial e[n]}{\partial a_i} = -x[n-i]$$

$$\boxed{\begin{aligned} &\frac{\partial}{\partial a_i}\left(a_k\, x[n-k]\right) \\ &= 0 \qquad k \neq i \\ &= x[n-i] \qquad k = i \end{aligned}}$$

Eq (3) $\quad \dfrac{\partial \mathcal{E}}{\partial a_i} = 0$

$$-2 \sum_n \left(x[n] - \sum_{k=1}^{p} a_k x[n-k] \right) x[n-i] = 0$$

$$\underbrace{\sum_n x[n]\, x[n-i]}_{c[0,i]} - \sum_{k=1}^{p} a_k \underbrace{\sum_n x[n-k]\, x[n-i]}_{c[k,i]} = 0$$

$$c[0,i] = \sum_{k=1}^{p} a_k \; c[k,i] \qquad i = 1, 2, \ldots p$$

$$c[k,i] = \sum_{n} x[n-k] \; x[n-i]$$

Autocovariance -- general term

did not specify limits on n.

- If toy signal $x[n] = \left(\frac{1}{2}\right)^n u[n]$

$$c[k,i] = \sum_{n=-\infty}^{\infty} x[n-k] \; x[n-i]$$

$n - k = m \qquad n = k + m \qquad n - i = k + m - i$

$$c[k,i] = \sum_{m=-\infty}^{\infty} x[m] \; x[k+m-i] \qquad k - i = l$$

$$r[l] = \sum_{m=-\infty}^{\infty} x[m] \; x[m+l] \qquad \text{Autocorrelation}$$

$$= r[-l] \qquad \text{.. even symmetry}$$

Eq. (3)

$$r[+i] = \sum_{k=1}^{p} a_k \, r[k-i] \qquad i = 1, 2, \ldots p$$

$p = 4$

$$\begin{bmatrix} r[1] \\ r[2] \\ r[3] \\ r[4] \end{bmatrix} = \begin{bmatrix} r[0] & r[1] & r[2] & r[3] \\ r[1] & r[0] & r[1] & r[2] \\ r[2] & r[1] & r[0] & r[1] \\ r[3] & r[2] & r[1] & r[0] \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \qquad (4)$$

$$\underline{r} = \underline{R} \; \underline{a}$$

$$\underline{a} = \underline{R}^{-1} \; \underline{r}$$

There is proof that R^{-1} exists.

$$x[n] = \left(\tfrac{1}{2}\right)^n u[n]$$

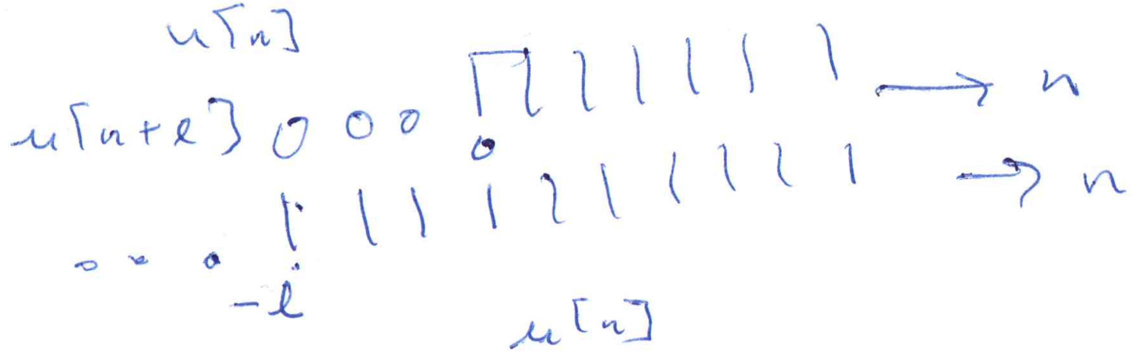
$$r[l] = \sum_{n=0}^{\infty} x[n]\, x[n+l]$$

$$0 \quad 0 \quad 0 \quad 1 \quad \tfrac{1}{2} \quad \tfrac{1}{4} \quad \tfrac{1}{8} \quad \tfrac{1}{16}$$

$l=1 \quad 0 \quad 0 \quad 1 \quad \tfrac{1}{2} \quad \tfrac{1}{4} \quad \tfrac{1}{8} \quad \tfrac{1}{16}$ $\Big\rangle \times \quad + \quad r[1]$

$$r[l] = \sum_{n=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^n u[n] \left(\tfrac{1}{2}\right)^{n+l} u[n+l]$$

$$= \left(\tfrac{1}{2}\right)^l \sum_{n=-\infty}^{\infty} \left(\tfrac{1}{4}\right)^n \quad \underbrace{u[n] \quad u[n+l]}_{u[n]} \quad l > 0$$



$$r[\ell] = \left(\frac{1}{2}\right)^{\ell} \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^{n} = \left(\frac{1}{2}\right)^{\ell} \frac{1}{1-\frac{1}{4}}$$

$$= \left(\frac{4}{3}\right)\left(\frac{1}{2}\right)^{\ell} \qquad \ell \geq 0$$

$$r[-\ell] = r[\ell]$$

$p = 2$

$$\begin{bmatrix} r[1] \\ r[2] \end{bmatrix} = \begin{bmatrix} r[0] & r[1] \\ r[1] & r[0] \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\underline{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$$

$$x[n] = a_1 x[n-1] + a_2 x[n-2] + e[n]$$

$$x[n] = \frac{1}{2} x[n-1] + e[n]$$

$$\left(\frac{1}{2}\right)^{n} u[n] = \frac{1}{2} \cdot \left(\frac{1}{2}\right)^{n-1} u[n] + e[n]$$

$$\left(\frac{1}{2}\right)^{n} u[n] = \left(\frac{1}{2}\right)^{n} u[n] \qquad n \neq 0 \Longrightarrow e[n] = 0$$

$$e[0] = 1$$

$$x[n] = \frac{1}{2} x[n-1] + e[n]$$

$$e[n] = \delta[n]$$

$$\cancel{x[n]\left(1-\frac{1}{2}\right.}$$

$$x[n] - \frac{1}{2} x[n-1] = \delta[n]$$

$$X(z)\left[1-\frac{1}{2}z^{-1}\right] = 1$$

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \longleftrightarrow \left(\frac{1}{2}\right)^n u[n]$$

Ex

Consider $x_o[n] = \cos(w_o n)$

Data we have is ~~$x[n] = \cos(w_o n)\ w[n]$~~

$$x[n] = \begin{cases} \cos(w_o n) & n = 0, \dots \cancel{N-1}\ 19 \\ ? & \text{other } n \end{cases}$$

(- No assumption

- 0 assumed

$N = 20$

$\cancel{r[l] = \sum_{n=?}^{?} x[n]\, x[n+l]}$ $\quad c[k,i] = \sum_n x[n-k]\, x[n-i]$

AC 1) $x[n] = 0 \quad n \notin [0, 19] \Rightarrow r[l]$

A COV. 2) Use only what you have

we have $x[n] \quad n = 0, \dots 19$

$$c[l] = \sum_{n=0}^{19-l} x[n]\, x[n+l] \qquad l \geq 0$$

$$= \sum_{n=-l}^{19} x[n]\, x[n+l] \qquad l < 0$$

$c[-1] \neq c[1]$