UNC CHARLOTTE
College of Computing and Informatics

PROJECT DOCUMENTATION REPORT
ON

SOLVING Map Coloring Problem using CSP

in the context of USA and Australia maps

PROGRAMMING PROJECT 3

ITCS 6150 - Intelligent Systems

SUBMITTED BY:

Alekhya Majeti (801135655)

Preetham Gowda (801154091)

Sai Teja Sarangu (801100597)

Introduction to CSP

Constraint satisfaction is the art of finding values for variables in a way that satisfies various constraints. Many everyday problems can be usefully couched in these terms.

Constraint satisfaction problems (**CSPs**) are mathematical questions defined as a set of objects whose state must satisfy a number of constraints or limitations. CSPs represent the entities in a problem as a homogeneous collection of finite constraints over variables, which is solved by constraint satisfaction methods. Constraint satisfaction problems on finite domains are typically solved using a form of search. The most used techniques are variants of backtracking, constraint propagation, and local search.

In graph theory, **graph coloring** is a special case of graph labeling; it is an assignment of labels traditionally called "colors" to elements of a graph subject to certain constraints. In its simplest form, it is a way of coloring the edge coloring assigns a color to each edge so that no two adjacent edges are of the same color.

A graph G is called k-colorable if there exists a graph coloring on G with k colors. If a graph is k-colorable, then it is n-colorable for any n>k. A graph has a chromatic number that is at least as large as the chromatic number of any of its subgraphs. A graph has a chromatic number that is at most one larger than the chromatic number of a subgraph containing only one less edge.

Map coloring approaches:

There are three approaches to solve this problem.

- BFS
- BFS + Forward Checking
- BFS + Forward Checking + Propagation through singleton domains

BFS:

The idea is to assign colors one by one to different vertices, starting from the vertex 0. Before assigning a color, we check for safety by considering already assigned colors to the adjacent vertices. If we find a color assignment which is safe, we mark the color assignment as part of solution. If we do not find a color due to clashes then we backtrack and return false.

BFS + Forward Checking:

Forward checking is the easiest way to prevent future conflicts. Instead of performing arc consistency to the instantiated variables, it performs restricted form of arc consistency to the not yet instantiated variables. We speak about restricted arc consistency because forward checking checks only the constraints between the current variable and the future variables. When a value is assigned to the current variable, any value in the domain of a "future" variable which conflicts with this assignment is (temporarily) removed from the domain. The advantage of this is that if the domain of a future variable becomes empty, it is known immediately that the current partial solution is inconsistent. Forward checking therefore allows branches of the search tree that will lead to failure to be pruned earlier than with simple backtracking. Note that whenever a new variable is considered, all its remaining values are guaranteed to be consistent with the past variables, so the checking an assignment against the past assignments is no longer necessary.

DFS WITH FORWARD CHECKING AND PROPAGATION THROUGH SINGLETON DOMAINS

Heuristics used :

The following are the heuristics used while solving the map coloring problem.

Variable and value ordering:

- **Degree heuristic**: assign a value to the variable that is involved in the largest number of constraints on other unassigned variables.
- **Minimum remaining values (MRV)**: choose the variable with the fewest possible values.
- **Least-constraining value heuristic**: choose a value that rules out the smallest number of values in variables connected to the current variable by constraints.

Results:

SIMULATION RESULTS FOR USA:-
FOR DFS

Number of run	Number of backtrack	Time taken
1	1012451	16.342
2	1012451	19.455
3	1012451	19.564
4	1012451	19.356

RESULT SAMPLE:-
VERIFIED ANSWER
(1, {'Maine': 'red', 'Minnesota': 'red', 'South Dakota': 'blue',
'Illinois': 'red', 'Utah': 'red', 'Wyoming': 'green', 'Texas': 'blue',
'Idaho': 'blue', 'Wisconsin': 'blue', 'Connecticut': 'red',
'Pennsylvania': 'red', 'Kansas': 'green', 'West Virginia': 'blue', 'North
Carolina': 'green', 'Colorado': 'blue', 'California': 'red', 'Florida':
'red', 'Vermont': 'red', 'Virginia': 'red', 'North Dakota': 'green',
'Michigan': 'green', 'New Jersey': 'blue', 'Nevada': 'green', 'Arkansas':
'green', 'Mississippi': 'red', 'Iowa': 'green', 'Kentucky': 'green',
'Maryland': 'green', 'Louisiana': 'black', 'Alabama': 'green', 'Oklahoma':
'red', 'New Mexico': 'green', 'Rhode Island': 'blue', 'Massachusetts':
'green', 'South Carolina': 'red', 'Indiana': 'blue', 'Delaware': 'black',
'Tennessee': 'blue', 'Georgia': 'black', 'Arizona': 'black', 'Nebraska':
'red', 'Missouri': 'black', 'New Hampshire': 'blue', 'Ohio': 'black',
'Oregon': 'black', 'Washington': 'red', 'Montana': 'red', 'New York':
'black'})
NUMBER OF BACKTRACKS: 1012451
TIME OF EXECUTION: 19.74671196937561seconds

DFS WITH FORWARD CHECK

Number of run	Number of backtrack	Time taken
1	10238	1.245
2	10238	0.535
3	10238	0.232
4	10238	0.243

RESULT SAMPLE:-
VERIFIED ANSWER
(1, {'New Hampshire': 'red', 'Oklahoma': 'red', 'Tennessee': 'red',
'Illinois': 'red', 'New Mexico': 'blue', 'Kentucky': 'blue', 'West
Virginia': 'green', 'Maryland': 'red', 'Maine': 'blue', 'Wisconsin':
'blue', 'Missouri': 'green', 'Minnesota': 'red', 'Montana': 'red',
'Massachusetts': 'blue', 'South Carolina': 'red', 'North Dakota': 'blue',
'Pennsylvania': 'blue', 'Arizona': 'green', 'South Dakota': 'green',
'Ohio': 'red', 'Oregon': 'red', 'Alabama': 'blue', 'Indiana': 'green',
'Rhode Island': 'red', 'Virginia': 'black', 'Idaho': 'green', 'Nevada':
'blue', 'Nebraska': 'red', 'New York': 'green', 'Utah': 'red', 'Michigan':
'black', 'Kansas': 'blue', 'Florida': 'red', 'Connecticut': 'black',
'Iowa': 'black', 'Wyoming': 'blue', 'Louisiana': 'red', 'California':
'black', 'Vermont': 'black', 'Texas': 'green', 'Georgia': 'green', 'New
Jersey': 'red', 'North Carolina': 'blue', 'Washington': 'blue',
'Delaware': 'green', 'Colorado': 'black', 'Mississippi': 'green',
'Arkansas': 'blue'})
NUMBER OF BACKTRACKS: 10238
TIME OF EXECUTION: 0.15990805625915527seconds
10286

DFS WITH SINGLETON AND FORWARD CHECK:-

Number of run	Number of backtrack	Time taken
1	10238	1.422
2	10238	0.187
3	10238	0.162
4	10238	0.138

EXECUTION RESULTS WITH HEURISTICS FORWARD CHECK WITH DFS:-

Number of run	Number of backtrack	Time taken
1	1754	0.0742
2	1754	0.179
3	1754	0.145
4	1754	0.162

RESULT SAMPLE:-
(1, {'Illinois': 'red', 'Oklahoma': 'red', 'California': 'red', 'Utah': 'red', 'Wyoming': 'blue', 'Missouri': 'blue', 'Michigan': 'green', 'Texas': 'blue', 'Iowa': 'green', 'Delaware': 'red', 'Tennessee': 'red', 'Maryland': 'blue', 'Kentucky': 'green', 'Montana': 'red', 'Minnesota': 'red', 'Connecticut': 'red', 'Louisiana': 'red', 'West Virginia': 'red', 'Pennsylvania': 'green', 'Nebraska': 'red', 'Kansas': 'green', 'Indiana': 'blue', 'Rhode Island': 'blue', 'Arizona': 'blue', 'Florida': 'red', 'Massachusetts': 'green', 'South Dakota': 'black', 'Nevada': 'green', 'South Carolina': 'red', 'Ohio': 'black', 'New Hampshire': 'red', 'Idaho': 'black', 'Washington': 'red', 'Colorado': 'black', 'Oregon': 'blue', 'New Jersey': 'blue', 'Mississippi': 'blue', 'Arkansas': 'green', 'Vermont': 'blue', 'Wisconsin': 'blue', 'Alabama': 'green', 'Georgia': 'blue', 'Maine': 'blue', 'New Mexico': 'green', 'North Carolina': 'green', 'New York': 'black', 'Virginia': 'black', 'North Dakota': 'blue'})
NUMBER OF BACKTRACKS: 1754
TIME OF EXECUTION: 0.0969seconds

HEURISTICS FOR DFS:-

Number of run	Number of backtrack	Time taken
1	701287	48.345
2	701287	48.355
3	701287	48.621
4	701287	48.922

Singleton With Heuristic:

Number of run	Number of backtrack	Time taken
1	9691	0.322
2	9691	0.452
3	9691	0.454
4	9691	0.468

TABULATION FOR AUSTRALIA:-

DFS:-

Number of run	Number of backtrack	Time taken
1	0	4.566
2	0	3.553
3	0	2.953e-05
4	0	2.392

WITH HEURISTIC:-

Number of run	Number of backtrack	Time taken
1	0	4.643e-05
2	0	1.976e-05
3	0	2.973e-05
4	0	2.751e-05

DFS WITH FORWARD CHECK:-

Number of run	Number of backtrack	Time taken
1	0	2.543e-05
2	0	3.953e-05
3	0	2.873e-05
4	0	3.245e-05

WITH HEURISTIC:-

Number of run	Number of backtrack	Time taken
1	0	4.087e-05
2	0	1.986e-05
3	0	2.945e-05
4	0	2.474e-05

DFS WITH SINGLETON AND FORWARD CHECK:-

Number of run	Number of backtrack	Time taken
1	0	2.987e-05
2	0	3.954e-05
3	0	2.765e-05
4	0	2.583e-05

WITH HEURISTIC:-

Number of run	Number of backtrack	Time taken
1	0	4.976e-05
2	0	1.943e-05
3	0	2.982e-05
4	0	2.897e-05