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2017

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1. ______________________________________, ______________________ ___.___.20___

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3. ___.___.20___

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Оглавление

. 5

1. 6

2. 9

3. $\mathcal{M}(\epsilon, t)$ 13

3.1. 13

3.2. 19

4. 24

. 28

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, , , (,) (,). , () , , . (-

$\mathbf{r} \otimes t$) . , -, , , , -, , , , . , , .

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: 1; 2 ; 3 , ; 5 ; , .

1.

, , $\Psi(\mathbf{r},t)$:

$$\left[i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\nabla^2-U(r)-V(\mathbf{r},t)\right]\Psi(\mathbf{r},t)=0, \tag{1}$$

$U(r)$ – , $V(\mathbf{r},t)=|e|(\mathbf{F}(t)\cdot\mathbf{r})$ – (e m) , $\mathbf{F}(t)$.[?] - $U(r)$, - $\Psi_0(\mathbf{r},t)$ $E_0=-\hbar^2\kappa^2/(2m)$.

r, $U(r)$ $\Psi(\mathbf{r},t)$, $E=\mathcal{E}(F,\omega)+n\hbar\omega$, $\mathcal{E}(F,\omega)$ – (.. $\mathbf{F}(t)$ Ψ_0), $\hbar\omega$ – , ω.[?] $\Psi(\mathbf{r},t)$ $r\to\infty$ $G(\mathbf{r},t;\mathbf{r}',t')$ $\mathbf{F}(t)$:

$$\Psi(\mathbf{r},t)=-\frac{2\pi\hbar^2}{m\kappa}\int\limits_{-\infty}^{t}e^{-i\epsilon t'/\hbar}G(\mathbf{r},t;0,t')f(t')dt', \tag{2}$$

$f(t)$ – , $\Psi(\mathbf{r},t)$ $r\to 0$, ϵ – $\Psi(\mathbf{r},t)$, E_0 .[?] $G(\mathbf{r},t;\mathbf{r}',t')$

$$\left[i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\nabla^2-V(\mathbf{r},t)\right]G(\mathbf{r},t;\mathbf{r}',t')=\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'), \tag{3}$$

:

$$G(\mathbf{r},t;\mathbf{r}',t')=-\theta(t-t')\frac{i}{\hbar}\left[\frac{m}{2\pi i\hbar(t-t')}\right]^{3/2}e^{iS(\mathbf{r},t;\mathbf{r}',t')/\hbar}, \tag{4}$$

$\theta(x)$ – , S – $\mathbf{F}(t)$:

$$S(\mathbf{r},t;\mathbf{r}',t')=\frac{m}{2(t-t')}\left(\mathbf{r}-\mathbf{r}'-\frac{e}{mc}\int\limits_{t'}^{t}\mathbf{A}(\tau)d\tau\right)^2$$
$$-\frac{e^2}{2mc^2}\int\limits_{t'}^{t}\mathbf{A}^2(\tau)d\tau-\frac{e}{c}[\mathbf{r}\mathbf{A}(t)-\mathbf{r}'\mathbf{A}(t')]. \tag{5}$$

(5) $\mathbf{A}(t)$ – ,

$$\mathbf{F}(t)=-\frac{1}{c}\frac{\partial\mathbf{A}(t)}{\partial t}. \tag{6}$$

$r \to 0$ $\Psi(\mathbf{r}, t)$:

$$\Psi(\mathbf{r}, t) \simeq \left(\frac{1}{r} + B(\epsilon)\right) f(t) e^{-i\epsilon t/\hbar}, \tag{7}$$

$B(\epsilon)$ $U(r)$. , $B = -\kappa = -\sqrt{2m|E_0|}/\hbar = \text{const}.$

(2) (7), $f(t)$,

$$f_n = \frac{1}{\mathcal{T}} \int_0^{\mathcal{T}} f(t) e^{in\omega_\tau t} dt, \quad \omega_\tau = \frac{2\pi}{\mathcal{T}}, \tag{8}$$

$\mathcal{T}$ – ( ).[?] f_n :

$$\sum_{n=-\infty}^{\infty} \mathcal{R}(\epsilon + n\hbar\omega_\tau) f_n e^{-in\omega_\tau t} = \sum_{m=-\infty}^{\infty} e^{-im\omega_\tau t} f_m \mathcal{M}(\epsilon + m\hbar\omega_\tau, t), \tag{9}$$

$$\mathcal{R}(E) = B(E) - i\sqrt{2mE}/\hbar, \tag{10}$$

$$\mathcal{M}(\epsilon, t) = \sqrt{\frac{m}{2\pi i\hbar}} \int_0^\infty \frac{e^{i\epsilon\tau/\hbar}}{\tau^{3/2}} [e^{iS(t,t-\tau)/\hbar} - 1] d\tau. \tag{11}$$

(9) – $\mathcal{M}$. , (11), $\mathcal{M}(\epsilon, t)$. , $|e| = m = \hbar = 1$.

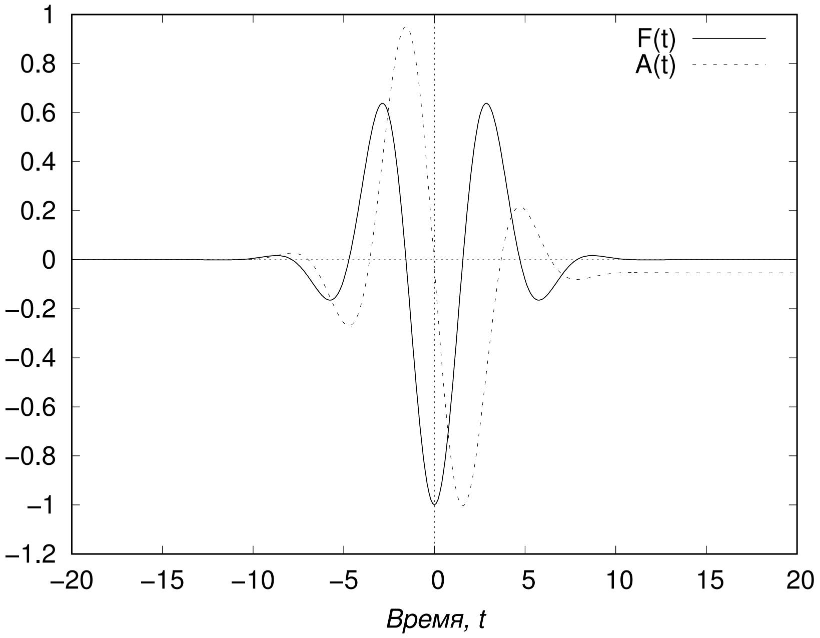
$\mathbf{F}(t) = \mathbf{e}_z F(t)$:

$$S(t, t') = -\frac{1}{2} \int_{t'}^{t} [\alpha(\epsilon, t, t')]^2 d\epsilon, \tag{12}$$

$$\alpha(\epsilon, t, t') = A(\epsilon) - \frac{1}{(t - t')} \int_{t'}^{t} A(\tau) d\tau, \tag{13}$$

$$A(t) = -\int_{-\infty}^{t} F(\tau) d\tau. \tag{14}$$

$F(t)$ $A(t)$ (1)



. 1. $F(t)$ $A(t)$

2.

λ

$$I(\lambda) = \int\limits_C \phi(t) e^{\lambda f(z)} dz, \tag{15}$$

$f(z)$ $\phi(z)$, . (15) , , , . .[?]

, -

$$I(\lambda) = \int\limits_a^b \phi(t) e^{\lambda f(t)} dt. \tag{16}$$

.

, $f(t)$ (a, b) . λ, , , λ .

:

:

$$I(\lambda) = \int\limits_0^a \phi(t) e^{-\lambda t^\alpha} dt \qquad (0 < a \leq \infty, \alpha > 0),$$

$\phi(t)$ $|t| < 2h$

$$\phi(t) = t^\beta (c_0 + c_1 t + \cdots + c_n t^n + \ldots), \ \beta > -1,$$

$\int_0^a |\phi(t)| e^{-\lambda_0 t^\alpha} dt \leq M$ λ_0.

$$I(\lambda) \sim \sum_{n=0}^{\infty} \frac{c_n}{\alpha} \Gamma\left(\frac{\beta + n + 1}{\alpha}\right) \lambda^{-\frac{\beta+n+1}{\alpha}}, \tag{17}$$

Γ - - .

(16).

1. (16) $\lambda = \lambda_0$, . .

$$\int\limits_a^b |\phi(t)| e^{\lambda_0 f(t)} dt \leq M,$$

$f(t)$ t_0 $(,b)$, $|t-t_0|<\delta$ $f(t)$

$$f(t)=f(t_0)+a_2(t-t_0)^2+\dots+a_n(t-t_0)^n+\dots \quad (a_2<0),$$

$h>0$, $f(t_0)f(t)>h$. $t=\psi(\tau)$ $\tau=0$ $f(t_0)f(t)=\tau^2$,

$$\phi[\psi(\tau)]\psi'(\tau)=\sum_{n=0}^{\infty}c_n\tau^n. \tag{18}$$

(16)

$$I(\lambda)=\int_a^b\phi(t)e^{\lambda f(t)}dt\sim e^{\lambda}f(t_0)\sqrt{\frac{\pi}{\lambda}}\sum_{n=0}^{\infty}\frac{c_2n}{\lambda^n}\frac{(2n)!}{4^n n!}.$$

, $f(t)$ (a,b).

2. (16) $\lambda=\lambda_0$ (1) $f(t)$ $t=a$, $(f'(a)\neq 0)$, $h>0$, $f(a)-f(t)>h$ a. $t=\psi(t)$ $\tau=0$ $f(a)-f(t)=\tau$, (18).

$$I(\lambda)=\int_a^b\phi(t)e^{\lambda f(t)}dt\sim\frac{e^{f(a)}}{\lambda}\sum_{n=0}^{\infty}\frac{n!c_n}{\lambda^n}. \tag{19}$$

, λ

$$I(\lambda)=\int_C\phi(t)e^{\lambda f(z)}dz$$

C, $|e^{\lambda f(z)}|=e^{\lambda \mathrm{Re} f(z)}$, . . $\mathrm{Re}f(z)$. , $\mathrm{Re}f(z)$, $\widetilde{C}$, , .[?]

, $z=\ +iy$,

$$u=\mathrm{Re}f(z),$$

S $(,y,u)$. , S , , $f'(z)=0$, (, . **??**).

$\widetilde{C}$, $\mathrm{Re}f(z)$, f(z) , , $\mathrm{Im}f(z)=\mathrm{const}$.

, $\widetilde{C}$ z_0, $\mathrm{Re}f(z)$ $\widetilde{C}$. , $f'(z_0)=0$, $\mathrm{Im}f(z)=\mathrm{const}$, $\mathrm{Re}f(z)$, .

, z_0, $\mathrm{Re} f(z)$ $u=0$ $\widetilde{C}$ 0, . . $\frac{\partial}{\partial s}\mathrm{Re} f(z) = 0$, $\mathrm{Im} f(z) = \mathrm{const}$ $\widetilde{C}$, $\frac{\partial}{\partial s}\mathrm{Im} f(z) \equiv = 0$,

$$f'(z_0) = \frac{\partial}{\partial s}\mathrm{Re} f(z) + i\frac{\partial}{\partial s}\mathrm{Im} f(z) = 0.$$

(15) $\widetilde{C}$, z_0 $\mathrm{Im} f(z) = \mathrm{const}$ *(. **??**).*

, : $\widetilde{C}$ $\arg e^{f(z)} = \mathrm{Im} f(z) = \mathrm{const}$, (15) , (16). [**?**]

1 2.

, $\widetilde{C}$, z_0, $f'(z_0) = 0$, $f''(z_0) \neq 0$, z_0 $\mathrm{Im} f(z) = \mathrm{const}$, $\widetilde{C}$ $\mathrm{Re} f(z) < \mathrm{Re} f(z_0) - h$ $(h > 0)$. , , (15) λ. , 1. $z = z(t)$ $\widetilde{C}$; ,

$$I(\lambda) = \int\limits_C \phi(z)e^{\lambda f(z)}dz = e^{\lambda i \mathrm{Im} f[z(t)]}\int\limits_a^b \phi[z(t)]e^{\lambda \mathrm{Re} f[z(t)]}z'dt. \tag{20}$$

16 , , [**?**]:

$$I(\lambda) = \int\limits_a^b \phi(t)e^{\lambda f(t)}dt \sim \frac{e^{f(a)}}{\lambda}\sum_{n=0}^{\infty}\frac{n!c_n}{\lambda^n}.$$

. $\phi[z(t)]z' = \widetilde{\phi}(t)$, $\mathrm{Re} f[z(t)] = \widetilde{f}(t)$ (19) :

$$\int\limits_a^b \widetilde{\phi}(t)e^{\lambda\widetilde{f}(t)}dt \sim e^{\lambda\widetilde{f}(t_0)}\sqrt{\frac{\pi}{\lambda}}\widetilde{c}_0, \tag{21}$$

$\widetilde{c}_0$ - $\widetilde{\phi}[\widetilde{\psi}(\tau)]\widetilde{\psi}'(\tau)$.

: $\widetilde{\phi}(t_0) = \phi(z_0)z'(t_0)$, , $f[z(t)] = \mathrm{Re} f[z(t)] + i\mathrm{Im} f[z(t)] = \widetilde{f}(t) + \mathrm{const}$ $\widetilde{C}$,

$$\widetilde{f}''(t_0) = \frac{d^2}{dt^2}f[z(t)]\mid_{t=t_0} = f''(z_0)z'^2(t_0),$$

$f'[z(t)]z''(t) = 0$ $t = t_0$. , $z'(t_0) = ke^{i\theta}$, $\widetilde{f}'' = -|f''(z_0)|k^2$. ,

$$\widetilde{c}_0 = \widetilde{\phi}(t_0)\sqrt{-\frac{2}{\widetilde{f}''(z_0)}} = \phi(z_0)e^{i\theta}\sqrt{\frac{2}{|f''(z_0)|}}.$$

(21), (20),

$$I(\lambda) \sim e^{\lambda f(z_0)} \sqrt{\frac{2\pi}{|f''(z_0)|}} \phi(z_0) e^{i\theta} \frac{1}{\sqrt{\lambda}}. \tag{22}$$

, z_0 - , $\mathrm{Re} f(z)$. - $\widetilde{C}$, $\mathrm{Re} f(z)$, (22) .

, z_0, 2.

, , $\phi(z)$ $f(z)$, , $\lambda \to \infty$

$\mathcal{M}(\epsilon, t)$ (11) λ :

$$\lambda = \frac{1}{2\mathcal{T}} \int\limits_{-\mathcal{T}/2}^{\mathcal{T}/2} A^2(\tau) d\tau. \tag{23}$$

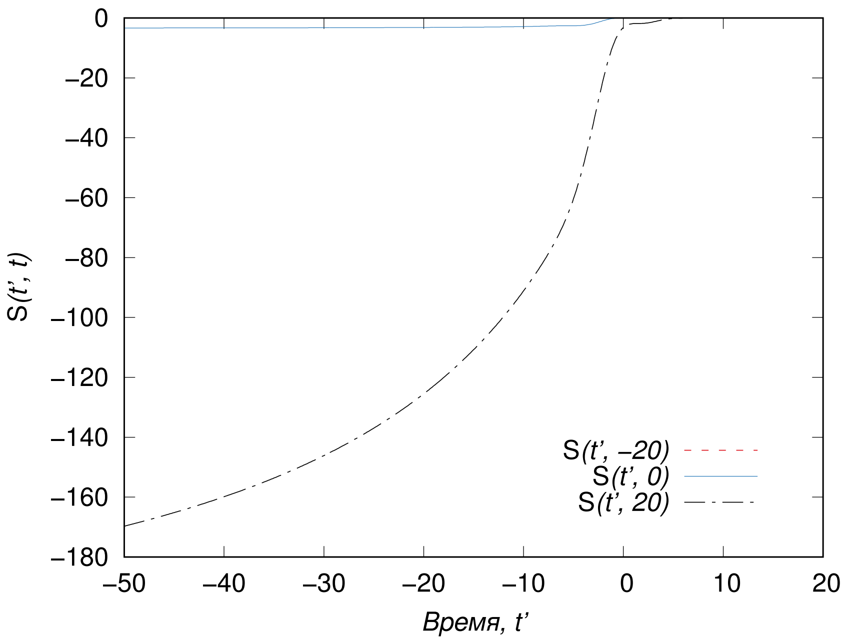
3. $\mathcal{M}(\epsilon, t)$

3.1.

(11) .

$\frac{1}{(t-t')^{3/2}}$, , $t \to t'$, . ., , , , $\lambda \gg 1$, λ (23).

$S(t', t)$ t :



. 2. $S(t', t)$ t=-20, 0 20

(2) , S $|t-t'| \gg 1$. $t = t'$, , . $(t \to t')$ S

$$S(\tau, t) \sim \frac{1}{24}\left(\frac{d}{dt}A(t)\right)^2 (t'-t)^3 + \frac{1}{24}\frac{d}{dt}A(t) \cdot \frac{d^2}{dt^2}A(t)(t'-t)^4 + \ldots$$

$$S(t', t) \sim \frac{1}{24}\left(\dot{A}(t)\right)^2 (t'-t)^3. \tag{24}$$

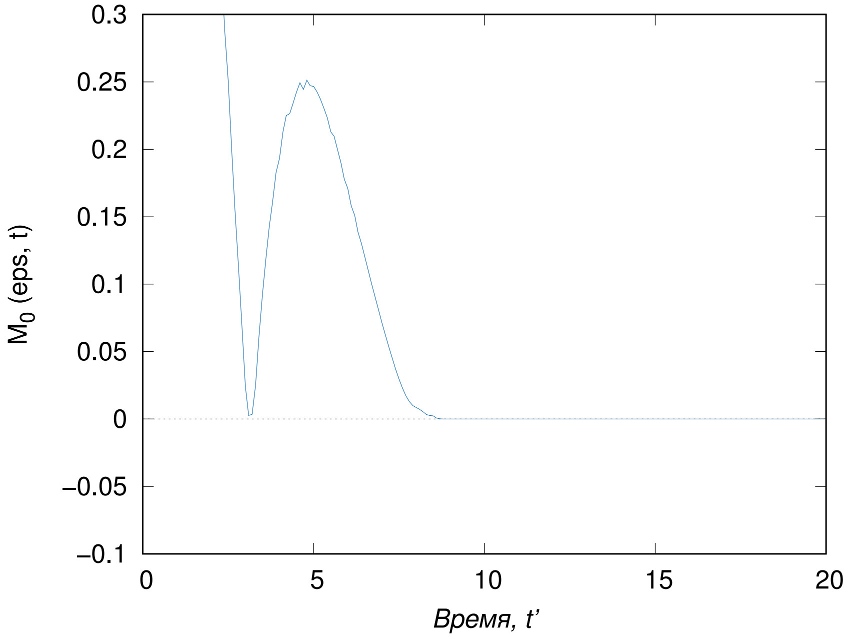
S $\mathcal{M}$ $t - t' = \tau$,

$$\mathcal{M}_{\tau=0} = \frac{1}{\sqrt{2\pi i}} \int\limits_0^\infty \frac{e^{i\epsilon\tau}}{\tau^{3/2}} \left(e^{-i\frac{\dot{A}(t)^2}{24}\tau^3} - 1 \right) d\tau =$$

$$= \frac{1}{\sqrt{2\pi i}} \int\limits_0^\infty e^{i\epsilon\tau} \tau^{3/2} d\tau \cdot \left(-i \frac{\dot{A}(t)^2}{24} \right)$$

, $\mathcal{M}_{\tau=0}$, (-1), .

3 $\mathcal{M}_{\tau=0}$.



. 3. $\mathcal{M}_{\tau=0}(\epsilon, t)$

, , . $(t \approx 0)$ $S(t', t)$, $\mathcal{M}_{\tau=0}(\epsilon, t)$, (. . $t = (10, \infty)$ $\mathcal{M}_{\tau=0}(\epsilon, t) = 0$).

:

$$\mathcal{M} \sim \int\limits_{-\infty}^{t} \frac{1}{(t-t')^{3/2}} e^{i[\epsilon(t-t') + S(t,t')]} dt'.$$

$t - t' = \tau$,

$$\mathcal{M} \sim \int\limits_0^\infty \frac{1}{\tau^{3/2}} e^{i[\epsilon\tau + S(t, t-\tau)]} d\tau.$$

$f(t,\tau)=\epsilon\tau+S(t,t-\tau)$ $\phi(\tau)=\frac{1}{\tau^{3/2}}$, :

$$M\sim\int\limits_0^\infty\phi(\tau)e^{if(t,\tau)}d\tau.$$

, (15).

:

$$M_0(\epsilon,t)\simeq\sqrt{\frac{1}{2\pi i}}\sum_{t0}e^{f(t,t_0)}\sqrt{-\frac{2}{\frac{\partial f(t,\tau)}{\partial\tau}|_{\tau=t_0}}}\phi(t_0),$$

t_0 - $f'(t)=0$.

$f''(t)$.

(t_0), τ

$$f(t,\tau)=\epsilon\tau+S(t,t-\tau),$$
$$\frac{\partial f(t,\tau)}{\partial\tau}=\epsilon+\frac{\partial S'(t,t-\tau)}{\partial\tau},$$
$$\frac{\partial f(t,\tau)}{\partial\tau}=0\Rightarrow\frac{\partial S'(t,t-\tau)}{\partial\tau}=-\epsilon. \tag{25}$$

S

$$S(t,t-\tau)=-\frac{1}{2m}\int\limits_{t-\tau}^{t}\alpha(\epsilon,t,t-\tau)^2d\epsilon$$

$S(t,\tau)$ τ,

$$\frac{\partial S(t,t-\tau)}{\partial\tau}=-\frac{1}{2m}\alpha(t-\tau,t,t-\tau)^2.$$

$t_0=t-\tau$, $\frac{\partial S(t,t-\tau)}{\partial\tau}=-\epsilon$ (25), :

$$\alpha^2(t_0,t,t_0)=2\epsilon m. \tag{26}$$

$\frac{\partial^2 f(t,\tau)}{\partial\tau^2}$:

$$\frac{\partial^2 f(t,\tau)}{\partial\tau^2} = -\frac{1}{2}\left[\alpha(t-\tau,t,t-\tau)^2\right]' =$$
$$= -\alpha(t_0,t,t_0)\alpha(t_0,t,t_0)'.$$

α:

$$\alpha(t_0,t,t_0) = \frac{|e|}{c}\left[A_\tau(t_0) - \frac{1}{(t-t_0)}\int\limits_{t_0}^{t} A(\tau)d\tau\right].$$

, $f''(t,\tau)$

$$\frac{\partial^2 f(t,\tau)}{\partial\tau^2} = |e|\alpha(t_0,t,t_0)\times$$
$$\times\left[F(t_0) - \tfrac{1}{(t-t_0)^2}\int_{t_0}^{t} A(\tau)d\tau + A(t_0)\tfrac{1}{t-t_0}\right].$$

$D = f''(t,t_0)$

$$\widetilde{S} = \epsilon\tau + S(t,t-\tau) = \epsilon(t-t_0) + S(t,t_0).$$

, :

$$M_0(\epsilon,t) \simeq \sum_{t_0}\frac{e^{i\widetilde{S}(t,t_0)}}{\sqrt{D}(t-t_0)^{3/2}}. \tag{27}$$

, , . t. .

(26). , ϵ, , (9) $\mathcal{M}$, , , . .

$$\mathcal{M}(\epsilon + m\omega_\tau, t).$$

ϵ, (11) $\epsilon + m\omega_\tau$. $\epsilon > 0$.

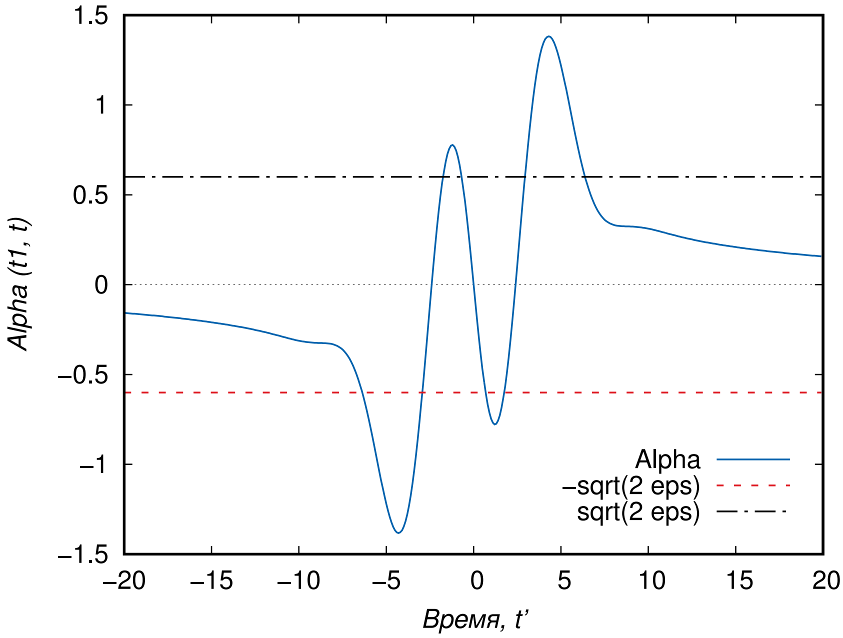
2 :

$$\alpha(t', t, t') - \sqrt{2\epsilon} = 0,$$

$$\alpha(t', t, t') + \sqrt{2\epsilon} = 0.$$

(t') (t_0).

$\alpha(t', t, t')$ $t' = [-20..20]$ $t = 0$ (. 4):



. 4. α $t = 0$

, Oy. , $t' = [-20, t]$. , ϵ . ϵ 0.6.

, , . ($t = const$):

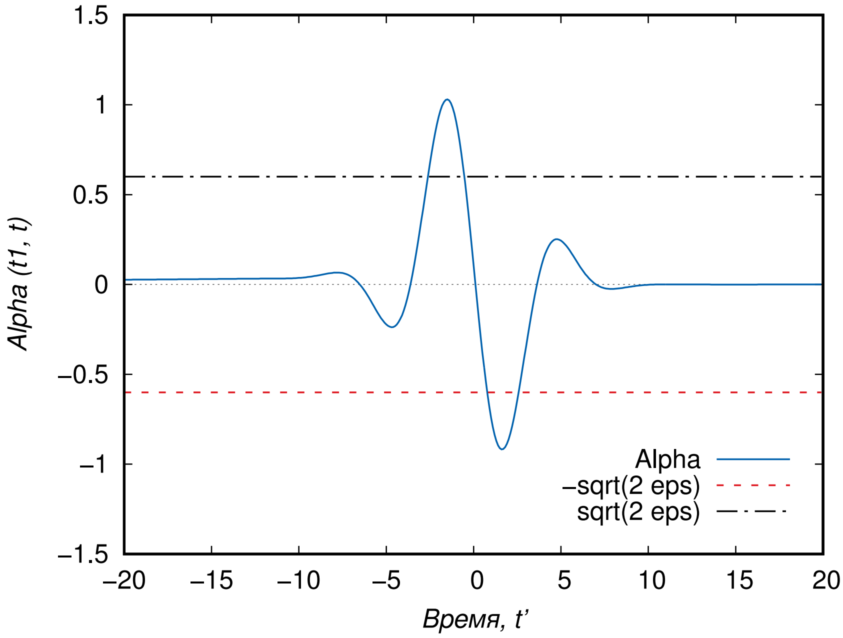
$$\begin{cases} \mathrm{Re}\,\alpha(x+iy, t, x+iy) = 2\epsilon \\ \mathrm{Im}\,\alpha(x+iy, t, x+iy) = 0 \end{cases}$$

ϵ - . , . . , . , .

$\epsilon > 0$. $\alpha(t', t, t')$, , . , , , . , . .[?] , , .

(. 5) $\alpha(t', t, t')$ $t' = [-20..20]$ $t = 20$

, . $\epsilon = 0.6$ (. 4) , $t >> 0$. ϵ , . . [?]



1.5
1
0.5
0
−0.5
−1
−1.5
Alpha (t1, t)
−20
−15
−10
−5
0
5
10
15
20
Время, t'
Alpha
−sqrt(2 eps)
sqrt(2 eps)

. 5. α $t = 20$

3.2.

(11)

, $-\infty$. , , (14), **(??)** . ,

$$|e^{-\frac{t_b^2}{\alpha^2}}| < \epsilon,$$

$[-\infty; 0]$, , t_b, .

(14)

$$A(t) = -c\int\limits_{t_b}^{t} F(\tau)d\tau.$$

. ω_i, .

$$\int\limits_{x_1}^{x_2} f(x)dx = \sum_{j=1}^{N} \omega_j f(x_j).$$

, . , , (14) , , , , , . GNU GSL.

GNU GSL - , C .

GSL: C (C++) , , , C. GSL , . , , , C, , . .

- , / . :

1.

2.

3.

4.

5.

6.

7.

8.

9. (QUADPACK)

10. . .

, $gsl\_integration\_qags$, $gsl_root_fsolver$, , [?]

, , (14) GSL 10^{-9}, .

, α (13), (12), $S(t', t)$:

$$S(t, t') = \frac{1}{2} \int_{t'}^{t} (A(\epsilon) - A_1(t', t))^2 \, d\epsilon, \tag{28}$$

$A_1(t', t) = \frac{1}{t'-t} \int_{t'}^{t} A(\tau) d\tau$. ϵ $A_1(t', t)$, 1 t', t. , M. .

() , .

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi} \, dx$$

, (DFT).

(DTFT), . , DTFT, . DFT - , DTFT DTFT. , DFT . , DTFT ( ). - , DFT DTFT.

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-i2\pi kn/N} =$$

$$= \sum_{n=0}^{N-1} x_n \cdot [\cos(2\pi kn/N) - i \cdot \sin(2\pi kn/N)],$$

DFT (FFT).

(FFT) (DFT) . ( ) . FFT , DFT ( ) . DFT $O(n^2)$, , DFT $O(n \log n)$, n .

, . 1965, 1805. 1994, Gilbert Strang FFT " ".

(11)

$t - t' = \tau,$

$$\mathcal{M}(t,\xi) = \int\limits_0^\infty \frac{e^{i\epsilon\tau}}{\tau^{3/2}}(e^{i[S(t,t-\tau)} - 1)d\tau \tag{29}$$

$$\hat{F}(\xi) = \int\limits_\infty^\infty e^{i\xi\tau}F(\tau)d\tau$$

, , .

FFTW.

FFTW (DFTs).[?]

FFTW (FFT) ( ). , $O(nlogn)$, .

, ( ), . , , ($O(n, \log n)$). , (CooleyTukey FFT), / , Rader Bluestein FFT. - , FFTW , FFTs , (,) .

. , " .

FFTW (MPI). - .

- . , , , , , FFTW , , . , , , .

(29),

$$F(\tau) = \frac{1}{\tau^{3/2}}(e^{i[+S(t,t-\tau)]} - 1),$$

$$\hat{F}(\psi) = \int\limits_{-\infty}^\infty e^{i\phi\psi}F(\phi)d\phi.$$

$\psi = \epsilon,$ $[0, \infty],$ $\hat{F}(0)$

$$\hat{F}(\epsilon) = \mathcal{M}(t,\epsilon).$$

, , $\epsilon,$ ϵ . , $x_1 < \epsilon < x_2,$ $\mathcal{M}(t,\epsilon)$:

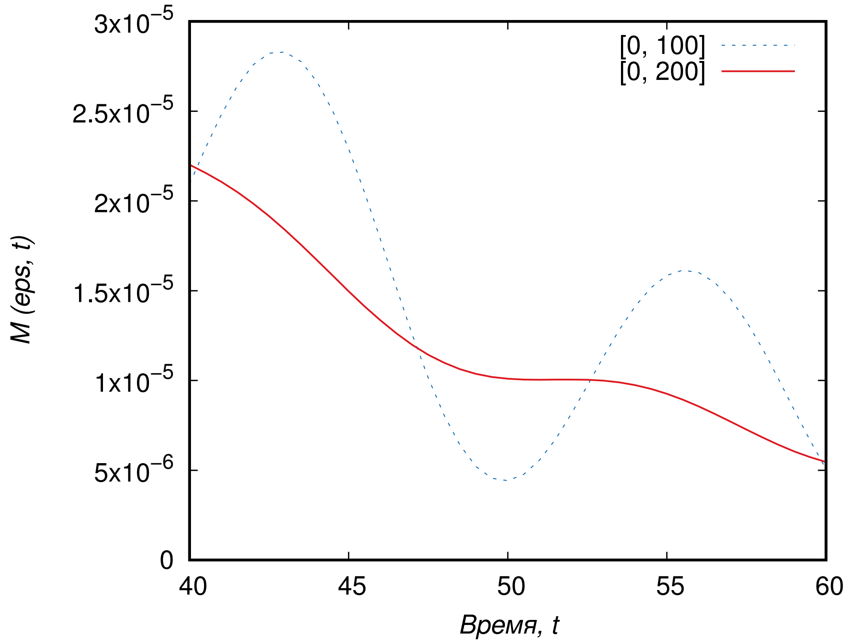
$$\mathcal{M}(t,\epsilon) = \frac{(\epsilon - x_1)\cdot(F(x_2) - F(x_1))}{x_2 - x_1} + F(x_1) \tag{30}$$

, $\mathcal{M}$ ϵ, , ϵ, ( ).

, , .

, , 0 ∞, , .

(6) FFTW (100 200).



. 6. FFTW

, , 100. , , $[0, 350]$ /.

, , . , , , , $(f(x_0) \approx f(x_{end}))$.

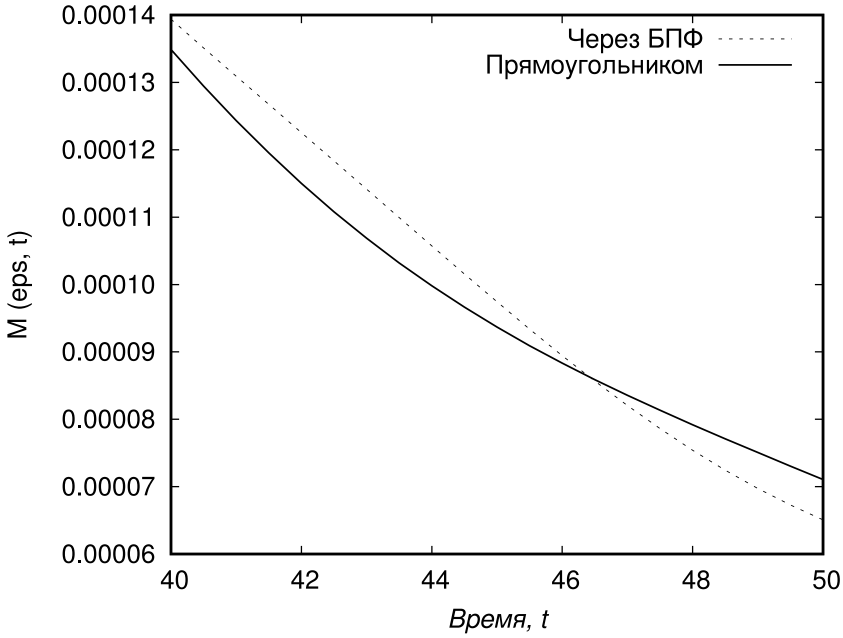
0 , , , .

, , . ϵ, , .[?]

, fftw (. 7)

, , . , , ϵ - . (30), (ϵ). , , , .

, , GSL.



0.00014
0.00013
0.00012
0.00011
0.00010
0.00009
0.00008
0.00007
0.00006
M (eps, t)
40
42
44
46
48
50
Время, t
Через БПФ
Прямоугольником

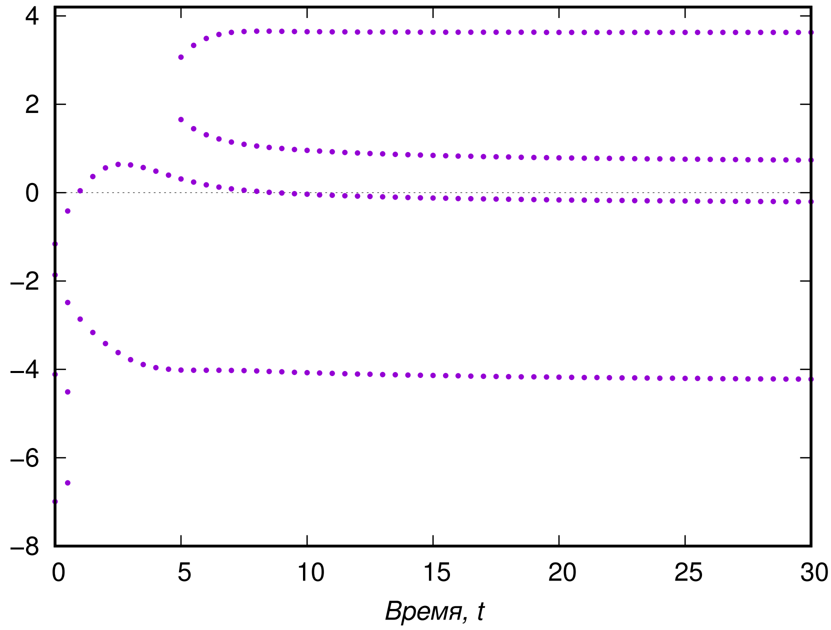
. 7.

4.

, , . $t = [0, 70]$. . $\mathcal{M}(\epsilon, t)$ 0 , 10^{-7} 1024 10^{-9} .

α, **(??)** $\sqrt{20}$, , , ϵ, F_0 ω .

, $\epsilon = 0.4$, $F_0 = 2$, $\omega = 0.8$. (. 8):



. 8. $\epsilon = 0.4, \omega = 0.8$

, $(t \gg 0)$, .

ϵ (. 9):

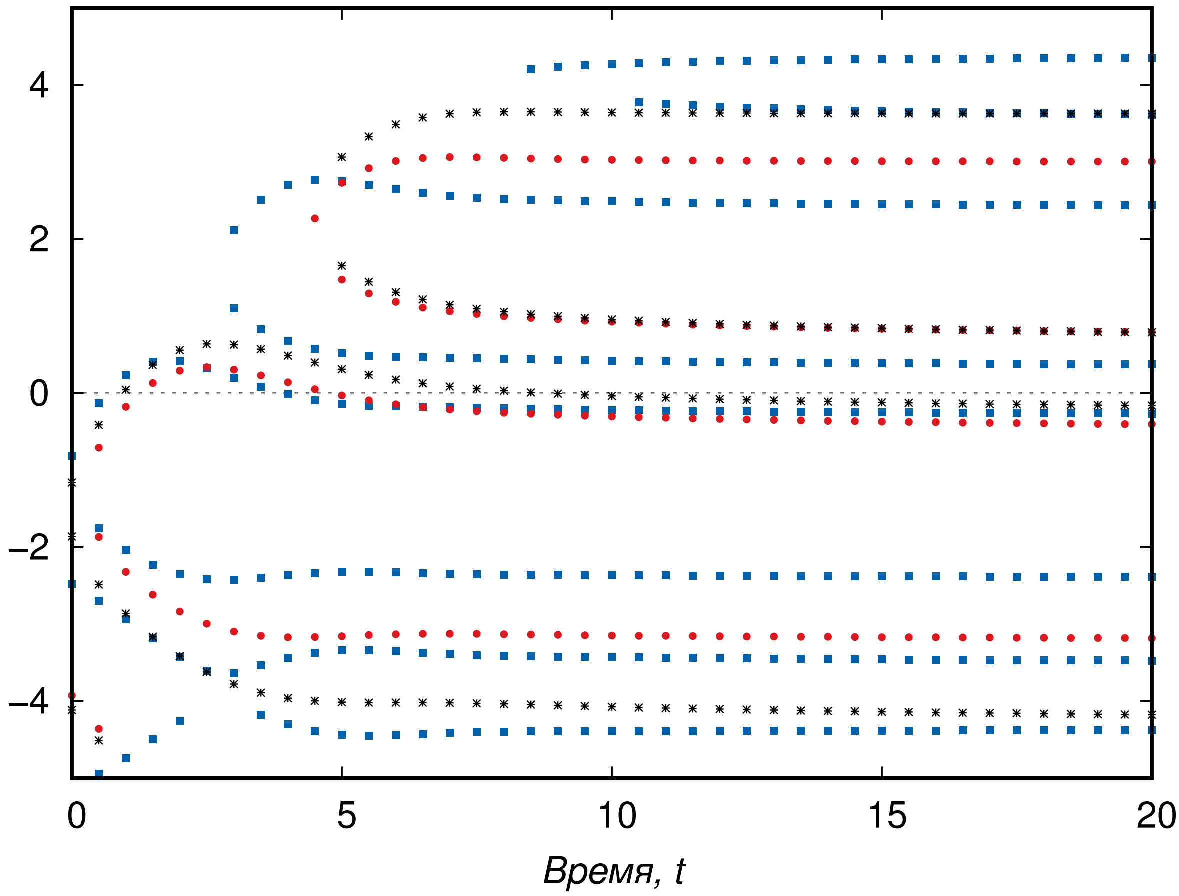
, $(\epsilon = 0.3)$, , 6, $t \approx 11$. (. 5), ϵ .

(10, 11 12) , $\epsilon = 0.4, F_0 = 2, \omega = 0.8$.

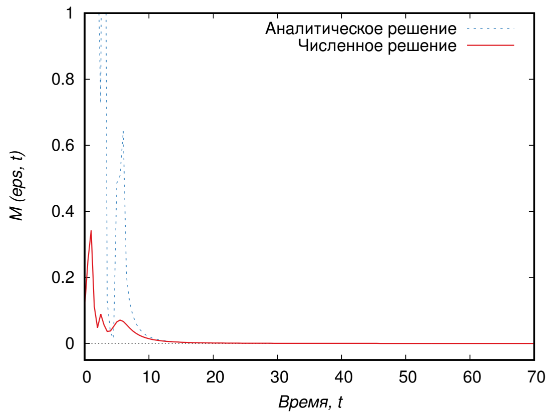
(11) , (. 10) , $t \approx t_0$ (t_0 -), , $\frac{1}{(t-t')^{3/2}}$. , $|t - t_0| \gg 1$, , (. 12).

(13, 14 15) , $\epsilon = 0.4, F_0 = 2, \omega = 0.8$.

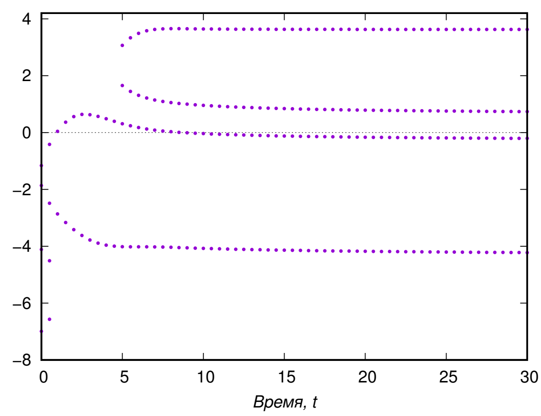
$\mathcal{M}$, , . , $t \approx 0$, $t \approx t_0$. . , 10 . , , .



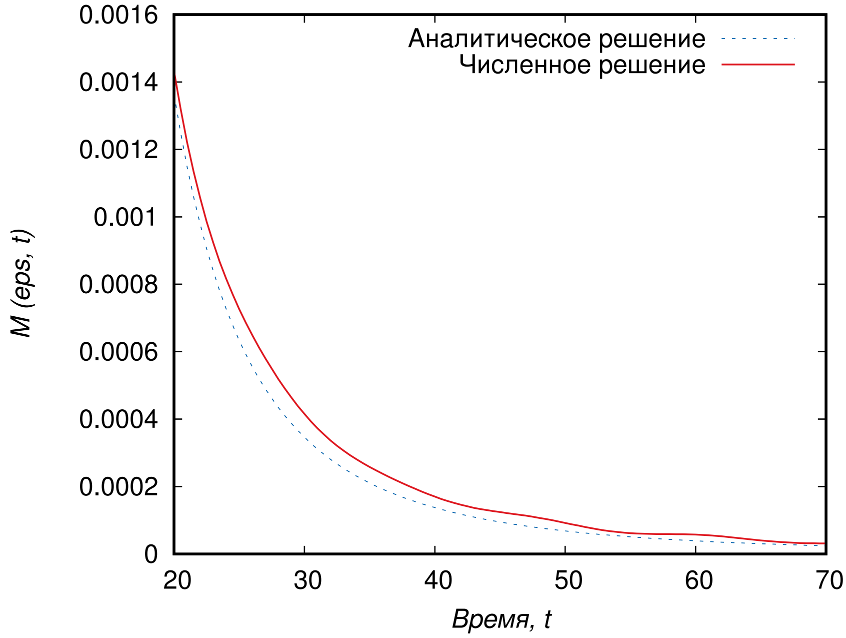
. 9. $\epsilon = 0.3$ (), 0.35 () 0.4 ()



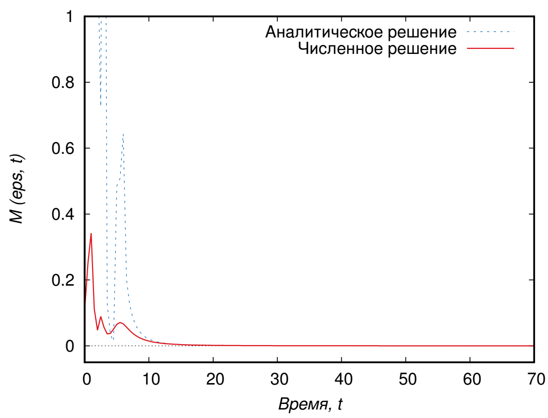
. 10. *( )* *()* . [0, 70]



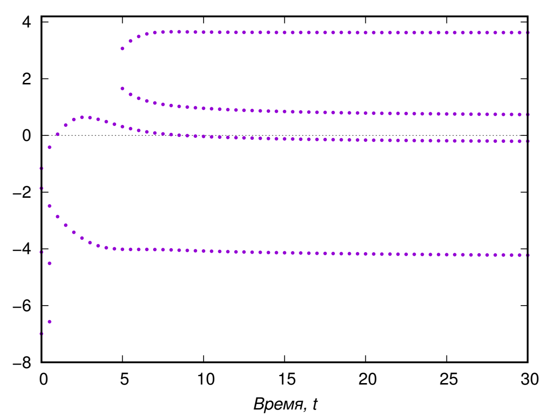
. 11.


Аналитическое решение
Численное решение
M (eps, t)
Время, t
0.0016
0.0014
0.0012
0.001
0.0008
0.0006
0.0004
0.0002
0
20
30
40
50
60
70

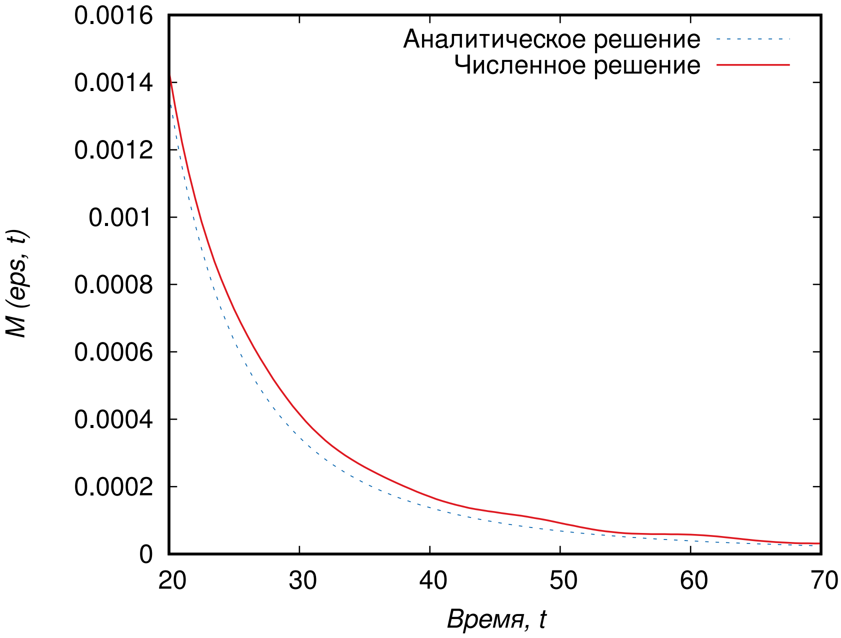
. 12. *( ) ()* . [20, 70]


Аналитическое решение
Численное решение
M (eps, t)
Время, t

. 13. *( ) ()*


Время, t

. 14.


0.0016
0.0014
0.0012
0.001
0.0008
0.0006
0.0004
0.0002
0
20
30
40
50
60
70
M (eps, t)
Время, t
Аналитическое решение
Численное решение

. 15. ( ) ()

$\mathcal{M}$, - , .

, . , $t = 0$. C++, FFTW GNU GSL . , .

, , $t \gg 0$, $t \gg t_0$.

. , $t \approx 0$, $t \approx t_0$, t_0 - $\alpha^2(t', t, t') = 2\epsilon$. , , , .

, , , , $((t - t_0)^{-3/2})$,

, 10 . , , .