(Pages : 4) **S – 3420**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, February 2024

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1331.1 : MATHEMATICS III – CALCULUS AND LINEAR ALGEBRA

(2018–2020 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the questions are compulsory. They carry **1** mark each.

1. Find the gradient of the function $F(x, y, z) = 7x^2 + 9y^2 + 7xyz$.

2. Prove that for any scalar function $F(x,y,z)$ whose second partial derivatives are continuous, prove that $curl(gradF) = 0$.

3. Find the general solution of $\frac{dy}{dx} = (1+x^2)(1+y^2)$.

4. Form the differential equation of the family of circles touching the χ axis at the origin.

5. Find the Wronskian of $\sin x$, $\sin 3x$.

6. Find the general solution of the differential equation $ydx - (x+2y^2)dy = 0$.

P.T.O.

7. Find the rank of $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 8 & 12 \\ 2 & 4 & 6 \end{pmatrix}$.

8. Find the eigen values of $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 9 \end{pmatrix}$.

9. Evaluate the determinant of $\begin{pmatrix} 5 & 6 & 7 \\ -2 & -1 & 0 \\ 1 & 2 & -3 \end{pmatrix}$.

10. Find the average value of $f(x) = x^3$ on $(-1, 2)$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. These question carries **2** marks each.

11. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then prove that $div\,\vec{r} = 3$ and $curl\,\vec{r} = 0$.

12. What is the matrix corresponding to the quadratic form $x^2 + 2y^2 - 2xy + 3xz - 2yz + z^2$?

13. Use double integral to find the volume of the tetrahedron bounded by the coordinate planes $z = 4 - 4x - 2y$.

14. Determine whether $ax^2 + by^2 = 1$ is a solution of $xy\dfrac{d^2y}{dx^2} + x\left(\dfrac{dy}{dx}\right)^2 - y\dfrac{dy}{dx} = 0$.

15. Is $(x^2 - y^2)dx + 2xydy = 0$ is a homogeneous differential equation? Solve this differential equation.

16. Solve $(e^y + 1)\cos x\,dx + e^y \sin x\,dy = 0$.

17. Solve $(y^2 + 1) + \left(x - e^{-\tan^{-1} y}\right)\dfrac{dy}{dx} = 0$.

18. What is the complementary function of $(D^3 + 3D^2 - 4)y = xe^{-2x}$.

19. Are the vectors (3,–2,8), (1,1,0), (1,0,1), (0,1,1) linearly independent.

20. If λ is a nonzero eigenvalue of a nonsingular matrix A, then show that λ^{-1} is an eigen value of A^{-1}.

21. Find the complimentary function of $(D^2 + a^2)y = Cot\, ax$.

22. Represent the following equations in the matrix form

$$y + z - 2w = 0$$
$$2x - 3y - 3z + 6w = 2$$
$$4x + y + z - 2w = 4$$

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

23. Find the fourier series of the function $f(x)$ of period 4 where $f(x) = \begin{cases} 0, & -2 \le x \le 0 \\ 1, & 0 < x < 2 \end{cases}$

24. Find the area enclosed between the parabola $y = \frac{1}{2}x^2$ and the line $y = 2x$.

25. Show that the integral of $\mathcal{F} = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$ is path independent. Also find the integral over C from (0,0,0) to (2,2,2).

26. Show that $F = (e^x \cos y + yz)\hat{i} + (xz - e^x \sin y)\hat{j} + (xy + z)\hat{k}$ is conservative and find a potential function for it.

27. Evaluate $\iint_R \frac{\sin x}{x}\, d\mathcal{A}$ where R is the triangle in the xy-plane bounded by the x-axis, the line $y = x$ and the line x=1.

28. Integrate $f(x,y,z) = x + \sqrt{y} - z^2$ over the path from (0,0,0) to (1,1,1) given by $C_1 : r(t) = t\hat{i} - t^2\hat{j}, 0 \le t \le 1$.

29. Row reduce the matrix $\begin{pmatrix} 1 & 2 & -3 & -4 \\ 1 & 3 & 1 & -2 \\ 2 & 5 & -2 & -5 \end{pmatrix}$.

30. Solve $(D^2 - 3D + 2)y = \sin 3x$.

31. Define Bernoullis and Legendres equation with examples.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These questions carry **15** marks each.

32. Verify Strokes theorem when $F = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ around S where S is the upper half surface of the unit sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

33. Solve $(2x - 3y + 1)dx + (6y - 4x + 3)dy = 0$.

34. Diagonalisze the matrix $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$.

35. Solve by using the method of variation of parameter $\frac{d^2y}{dx^2} + a^2y = \cos ax$.

(2 × 15 = 30 Marks)
