Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, May 2022

Physics

PH 212 — MATHEMATICAL PHYSICS

(2018 & 2019 Admission)

Time : 3 Hours   Max. Marks : 75

PART – A

Answer any **five** questions. **Each** question carries **3** marks.

I.   (a)   Find   constants   a,   *b,*   and   *c*   such   that   the   vector   field
$\overline{A} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational.

   (b)   Find the principal value of $\log(-3)$.

   (c)   Explain the concept of Fast Fourier transform. Discuss one application.

   (d)   A radioactive material emits on an average 10 particles per minute. Calculate the probability that the material will emit at least 1 particle in a given minute.

   (e)   Explain Hermitian and Unitary matrices. Give an example of each.

   (f)   Write down the Legendre differential equation. Find and classify its singular points.

   (g)   What is meant by contravariant and covariant vectors? Give an example for each.

   (h)   What is a group? Is the set of all integers excluding 0 form a group under ordinary multiplication? Explain.

(5 × 3 = 15 Marks)

PART – B

Answer **all** questions. **Each** question carries **15** marks.

II. (A) (a) Define general orthogonal curvilinear coordinates. **3**

    (b) Derive the expression for Gradient in general orthogonal curvilinear coordinates. **7**

    (c) Obtain the expression for Gradient in cylindrical coordinate system. **5**

OR

(B) (a) Evaluate $\oint \dfrac{\exp(2z)}{(z+1)^4} dz$ where the integral is over a closed circle in the complex plane around the point $z = -1$. **10**

    (b) Derive the Cauchy - Riemann conditions for analyticity of a complex function. **5**

III. (A) Solve the transport equation $\dfrac{\partial u}{\partial t} + 3\dfrac{\partial u}{\partial x} = 0$ with $u(x,0) = f(x)$ where $f(x)$ is an arbitrary function of x.

OR

(B) Solve the first order differential equation $y' + 2xy = 0$ using Frobenius series method.

IV. (A) (a) Define metric tensor and show that it is covariant in both its indices. **5**

    (b) What is meant by raising and lowering of indices of a tensor? **5**

    (c) Obtain metric tensor for cylindrical polar coordinate system. **5**

OR

(B) (a) Discuss the relation between symmetry and groups. **3**

    (b) What is meant by reducible and irreducible representations of a group? **5**

    (c) Discuss $SO(2)$ group. **7**

(3 × 15 = 45 Marks)

PART – C

Answer any **three** questions. **Each** question carries **5** marks.

V.   (a)   A coin has probability of Heads 0.7 and probability of Tails 0.3. If 5 such coins are tossed, calculate the probability of getting at least two Heads.

   (b)   Find all values of $16^{1/4}$.

   (c)   Solve the differential equation $y'' - 5y' + 6y = 0$ with $y(0) = 2$ and $y'(0) = 2$ using Laplace transform.

   (d)   Show that the Legendre function satisfies the relation $P_5'(x) = 9P_4(x) + 5P_2(x) + P_0(x)$.

   (e)   Find the equation of the straight line which fits the data points (1, 1), (2, 3), (3, 5), (4, 5) (6, 6) by the method of least-squares.

   (f)   Starting from the metric tensor components $g_{i,j} = \delta_{i,j}$ for the $2D$ space in the Cartesian system, obtain the components of the metric tensor for the polar coordinate system. (Hint : Write down the polar basis vectors in terms of the Cartesian basis vectors making use of the relations $x = r\cos(\theta)$ and $y = r\sin(\theta)$).

(3 × 5 = 15 Marks)

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