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M – 1455

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Mathematics

Core Course – VIII

MM 1545 : ABSTRACT ALGEBRA – I

(2014 , 2016 & 2017 Admission)

Time : 3 Hours

Max. Marks : 80

PART – A

Answer all the **ten** are compulsory. They carry **1** mark each.

1. Define Dihedral Group D_n.

2. What is the order of the permutation (1 2 4) (3 5 7)?

3. Write the following permutations into product of disjoint cycles $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 5 & 2 & 4 & 3 & 1 & 8 & 9 & 7 \end{pmatrix}$.

4. Find the number of elements in the cyclic subgroup of $\mathbb{Z}_{30}$ generated by 25.

5. How many groups are there of order 17 upto isomorphism?

6. True or False "The odd permutations in S_8 from a subgroup of S_8". Why?

7. Define the Alternating Group.

P.T.O.

8. What is the order of the cycle $(1 \quad 4 \quad 5 \quad 7)$?

9. Find the generators of $\mathbb{Z}_4 \times \mathbb{Z}_3$.

10. Define an automorphism.

PART – B

Answer **any eight** questions from this section. Each question caries 2 marks.

11. Show that every group G with identity e and such that $x * x = e$ for all $x \in G$ is abelian.

12. Prove that 'Every cyclic group is abelian".

13. Find the number of generators of cyclic groups of orders 6, 8, 12 and 60.

14. Find the multiplication table of S_3.

15. Show that "Every group of prime order is cyclic."

16. Show that the identity element and the inverse element of an element in group G are unique.

17. Show that if $a \in G$, where G is a finite group with identity e, then there exists $n \in \mathbb{Z}^+$ such that $a^n = e$.

18. Show that if a group G with identity e has finite order n, then $a^n = e$ for all $a \in G$.

19. Show that if H is a subgroup of an abelian group G, then every left coset of H is also a right coset of H.

20. Define a transposition. White the cycle (2,3,5,6,8,9) as the product of transposition.

21. Show that if $n \geq 3$, then the only element σ of S_n, satisfying $\sigma\gamma = \gamma\sigma$ for all $\gamma \in S_n$ is $\sigma = \iota$, the identity permutation.

22. Give an example to which converse of Lagranges Theorem fails.

PART – C

Answer **any six** questions from this section. Each question caries **4** marks.

23. Prove that a subset H of a group G is a subgroup of G if and only if

    (a) H is closed under the binary operation of G.

    (b) the identity e of G is in H

    (c) for all $a \in H$ it is true that $a^{-1} \in H$.

24. Show that if H and K are subgroups of an abelian group G, then $\{hk \mid h \in H$ and $k \in K\}$ is a subgroup of G.

25. Prove that "Every permutation σ of a finite set A is a product of disjoint cycles".

26. Show that if r and s are relatively prime, then G contains a cyclic group of order rs.

27. Prove the theorem "Let A be a nonempty set, and let S_A be the collection of all permutations of A. Then S_A is a group under permutation multiplication".

28. Prove that "Let H and K are subgroups of a group G such that $K \leq H \leq G$, and suppose $(H : K)$ and $(G : H)$ are both finite. Then $(G : K)$ is finite and $(G : K) = (G : H)(H : K)$".

29. Show that S_n is non abelian for $n \geq 3$.

30. State and Prove Lagrange's Theorem.

31. Show that every permutation in S_n can be written as a product of at most $n-1$ transpositions.

PART – D

Answer **any two** questions from this section. Each question caries **15** marks.

32. Prove that "No permutation of a finite set can be expressed both as a product of an even number of transposition and as a product of an odd number of transpositions".

33. Let $\phi : A \mapsto B$. A map $\phi^{-1} : B \mapsto A$ is called an inverse of ϕ if $\phi(\phi^{-1}(x)) = x$ for all $x \in B$ and $\phi^{-1}(\phi(y)) = y$ for all $y \in A$.

    (a) Show that ϕ is a bijection if and only if it has an inverse.

    (b) Show that the inverse of a bijection ϕ is unique.

34. Show that, if $n \geq 2$, the collection of all even permutations of {1, 2, 3, ... ,n} forms a subgroup of order $\frac{n!}{2}$ of the symmetric group S_n.

35. (a) Let A be an infinite set. Let H be the set of all $\sigma \in S_A$ that move only finite number of elements of A. Show that H is a subgroup of S_A.

    (b) Prove that "Let H be a subgroup of G. The relations

    $$a \sim_L b \quad \textit{if and only if} \quad a^{-1}b \in H$$

    and

    $$a \sim_R b \quad \textit{if and only if} \quad ab^{-1} \in H$$

    are equivalence relations on G".