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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, August 2024

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1231.1 : MATHEMATICS II – APPLICATIONS OF CALCULUS AND VECTOR DIFFERENTIATION

(2021 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions. Each carries **1** mark.

1. A function f is increasing on (a, b) if ———— whenever $a < x_1 < x_2 < b$.

2. Define inflection point of a function.

3. State Extreme-Value Theorem.

4. Write the formula for area of the region bounded on the left by $x = v(y)$, on the right by $x = w(y)$, below by $y = c$, and above by $y = d$, where w and v are continuous functions and $w(y) \geq v(y)$ for all y in $|c, d|$.

5. Write an integral expression for the area of the parallelogram bounded by $y = 2x + 8$, $y = 2x - 3$, $x = -1$ and $x = 5$.

6. What is the length of the curve $y = f(x)$ over $[a, b]$ if f is smooth function on $[a, b]$?

P.T.O.

7. State Fubini's Theorem.

8. Write the vector form of the paraboloid $x = u$, $y = v$, $z = 4 - u^2 - v^2$.

9. Write the parametric equations of a line in 3-space that passes through the point (1, 0, 0) and is parallel to the vector (–1, 3, 2).

10. If $r(t) = t^2\hat{i} + e^t\hat{j} + (2\cos\pi t)\hat{k}$. Find $r'(t)$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions from this section. Each carries **2** marks.

11. Find the intervals on which $f(x) = x^3$ is increasing and the intervals on which it is decreasing.

12. Find the inflection points, if any, of $f(x) = x^4$.

13. Find all critical points of $f(x) = x^3 - 3x + 1$.

14. Find the area of the region bounded above by $y = x + 6$, bounded below by $y = x^2$, and bounded on the sides by the lines $x = 0$ and $x = 2$.

15. Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, $x = 1$, $x = 4$, and the x-axis is revolved about the y-axis.

16. Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}$, $y = 2$, and $x = 0$ is revolved about the y-axis.

17. Evaluate $\int_1^3 \int_2^4 (40 - 2xy)\,dy\,dx$.

18. Evaluate double integral $\iint_R y^2 x\,dA$ over the rectangle $R = \{(x, y) : -3 \le x \le 2, 0 \le y \le 1\}$.

19. Find the partial derivatives of the vector-valued function $r = u\hat{i} + v\hat{j} + (4 - u^2 - v^2)\hat{k}$.

20. Evaluate the definite integral $\int_0^2 (2t\hat{i} + 3t^2\hat{j})\,dt$.

21. Let $r(t) = t^2\hat{i} + e^t\hat{j} + (2\cos\pi t)\hat{k}$. Find $\int_0^1 r(t)\,dt$.

22. Find $r(t)$ given that $r'(t) = \langle 3, 2t \rangle$ and $r(1) = \langle 2, 5 \rangle$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. Each carries **4** marks.

23. Find the relative extrema of $f(x) = 3x^5 - 5x^3$.

24. Determine whether the function $f(x) = \dfrac{1}{x^2 - x}$ has any absolute extrema on the interval (0, 1). If so, find them and state where they occur.

25. Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$.

26. Find the volume of the solid generated when the region between the graphs of the equations $f(x) = \dfrac{1}{2} + x^2$ and $g(x) = x$ over the interval [0, 2] is revolved about the *x*-axis.

27. Find the arc length of the curve $y = x^{\frac{3}{2}}$ from (1, 1) to (2, 2, $\sqrt{2}$).

28. Evaluate $\iint_R (2x - y^2)\,dA$ over the triangular region *R* enclosed between the lines $y = -x + 1$, $y = x + 1$, and $y = 3$.

29. Evaluate $\iint_R \sin\theta\,dA$ where *R* is the region in the first quadrant that is outside the circle $r = 2$ and inside the cardioid $r = 2(1 + \cos\theta)$.

30. Find parametric equations of the tangent line to the circular helix $x = \cos t$, $y = \sin t$, $z = t$, where $t = t_0$, and use that result to find parametric equations for the tangent line at the point where $t = \pi$.

31. Find the directional derivative of $f(x, y) = e^{xy}$ at (–2, 0) in the direction of the unit vector that makes an angle of $\dfrac{\pi}{3}$ with the positive *x*-axis.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. Each carries **15** marks.

32. (a) Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), $f(a)=0$ and $f(b)=0$. Then Prove that there is at least one point c in the interval (a, b) such that $f'(c)=0$ **10**

    (b) Find the two x-intercepts of the function $f(x)=x^2-5x+4$ and confirm that $f'(c)=0$ at some point c between those intercepts. **5**

33. (a) Find the area of the region enclosed by $x=y^2$ and $y=x-2$. **8**

    (b) Derive the formula for the volume of a right pyramid whose altitude is h and whose base is a square with sides of length a. **7**

34. (a) Use a double integral to find the volume of the tetrahedron bounded by the coordinate planes and the plane $z=4-4x-2y$. **7**

    (b) The sphere of radius a centered at the origin is expressed in polar coordinates as $r^2+z^2=a$. Use polar double integral to find the volume of the sphere. **8**

35. A heat-seeking particle is located at the point (2, 3) on a flat metal plate whose temperature at a point (x, y) is $T(x, y)=10-8x^2-2y^2$. Find an equation for the trajectory of the particle if it moves continuously in the direction of maximum temperature increase.

(2 × 15 = 30 Marks)