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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, August 2024

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1231.1 : MATHEMATICS II

CALCULUS WITH APPLICATIONS IN PHYSICS — II

(2018-2020 Admission)

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Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions. Each carries **1** mark.

1. Define argument of a complex number.

2. Show that $(z^*)^* = z$.

3. Write $\cosh x$ in terms of exponential function.

4. Define exact differential.

5. Write the limit equation of $\frac{\delta f}{\delta x}$.

6. If f is a function of x and y, and x and y are functions of u. Define df.

P.T.O.

7. Write the jacobian form of $\frac{\delta(x, y)}{\delta(u, v)}$.

8. The infinitesimal change in the position vector v of a particle in an infinitesimal time dt is?

9. Find the gradient of the scalar field $\phi = xy^2z^3$.

10. Find the Laplacian of the scalar field $\phi = xy^2z^3$.

(10 × 1 = 10 Marks)

SECTION – II

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Answer **any eight** questions from this section. Each carries **2** marks.

11. What is the sum of the complex numbers $1+2i$, $3-4i$, $-2+i$.

12. Find modulus of the complex number $z = 2-3i$.

13. Explain multiplication of two complex numbers.

14. Find the complex conjugate of $z = a + 2i + 3ib$.

15. Find $\frac{\delta^2 f}{\delta x^2}$ if $f(x, y) = 2x^3y^2 + y^3$.

16. Find the total differential of the function $f(x, y) = y\exp(x+y)$.

17. Define $\operatorname{sech} x$ in terms of exponential function.

18. Explain Kronecker delta.

19. Express Cartesian coordinates x, y, z in terms of spherical polar coordinates r, θ and ϕ.

20. Show that $\Delta \times (\phi a) = \Delta\phi \times a + \phi \times \Delta a$.

21. Find the divergence of the vector field $a = x^2y^2\hat{i} + y^2z^2\hat{j} + x^2z^2\hat{k}$.

22. Find the curl of the vector field $a = x^2y^2z^2\hat{i} + y^2z^2\hat{j} + x^2z^2\hat{k}$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. Each carries **4** marks.

23. Show that $|z_1 z_2| = |z_1||z_2|$ holds for the product of the functions $z_1 = 3 + 2i$ and $z_2 = -1 - 4i$.

24. Express $\sin 3\theta$ and $\cos 3\theta$ in terms of powers of $\cos\theta$ and $\sin\theta$.

25. Simplify the expression $z = i^{-2i}$.

26. Show that $(y + z)dx + xdy + xdz$ is an exact differential.

27. Find the Taylor expansion, up to quadratic terms in $x - 2$ and $y - 3$, of $f(x, y) = y \exp xy$ about then point $x = 2, y = 3$.

28. Evaluate the double integral $\iint_R x^2 y dx dy$, where R is the triangular area bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$.

29. Find the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 2y$.

30. The position vector of a particle in plane polar coordinates is $r(t) = \rho(t)\hat{e}_p$. Find expressions for the velocity and acceleration of the particle in these coordinates.

31. A curve lying in the xy-plane is given by $y = y(x)$, $z = 0$. Show that the arc length along the curve between $x = a$ and $x = b$ is given by $s = \int_a^b \sqrt{1 + y^2}\, dx$, where $y' = \frac{dy}{dx}$.

(6 × 4 = 24 Marks)

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SECTION – IV

Answer **any two** questions. Each carries **15** marks.

32. Solve the equation $z^6 - z^5 + 4z^4 - 6z^3 + 2z^2 - 8z + 8 = 0$.

33. Find the stationary points of $f(x, y, z) = x^3 + y^3 + z^3$ subject to the following constraints : $g(x, y, z) = x^2 + y^2 + z^2 = 1$ and $h(x, y, z) = x + y + z = 0$.

34. Evaluate the double integral $I = \iint_R (a + \sqrt{x^2 + y^2})\,dx\,dy$, where R is the region bounded by the circle $x^2 + y^2 = a^2$.

35. Find the element of area on the surface of a sphere of radius a, and hence calculate the total surface area of the sphere.

(2 × 15 = 30 Marks)

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