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M – 2364

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021.

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry and Polymer Chemistry

MM 1231.2 MATHEMATICS II – CALCULUS WITH APPLICATIONS IN CHEMISTRY – II

(2018 – 2019 Admission)

Time : 3 Hours | Max. Marks : 80

SECTION – I

All **ten** questions are compulsory, each caries **1** mark:

1. Find f_{yx} of the function $f(x,y) = xe^y + ye^x$.

2. If $w = f(x)$ be differentiable functions of x and $x = \varphi(t)$ be differentiable function of t then write the chain rule formula to find $\frac{dw}{dt}$.

3. Find the sum of even number between 1 and 101.

4. Find the formula to calculate S_N for a Arithmetico-geometric series.

5. Show that $\sum_{n=1}^{\infty}(n^2 + 2)$ is not convergent.

6. If $\vec{r}(t) = 5t^3\hat{i} + (3t+2)\hat{j} + 3t^2\hat{k}$ then $\frac{d\vec{r}}{dt} =$

7. If $F(x,y,z) = (x^2 + y)\hat{i} + (y^2 + z)\hat{j} + (x^2 + z)\hat{k}$. Find $DivF$ at (1,1,2)

P.T.O.

8. If $\varphi(x,y,z)=xyz$, find $grad\varphi$ at $(1,1,1)$.

9. Find $\int_0^a\int_0^b\int_0^c 8xyz\,dx\,dy\,dz$.

10. Suppose $f(x,y)=x+y$ defined over R in xy- plane given by $0\le x,y\le 1$, then find average of f over the region R.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions, each carries **2** marks.

11. If $f(x,y)=x^2+y^2+2xy$ then find $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}$.

12. If $f(x,y)=x^2y^2$ the find the total derivative df.

13. Given that $x(u)=1+au$ and $y(u)=bu^3$, where a and b are constant. Find rate of change of $f(x,y)=xe^{-y}$ with respect u.

14. Show that $\sum_{n=1}^{\infty}\frac{1}{n!}$ converges.

15. Determine the convergence of $\sum_{n=1}^{\infty}\frac{1}{(n!)^2}$.

16. Explain Cauchy's root test for the convergence of a series.

17. If $\varphi(x,y,z)=x^2+y^2+z^2$ then find $div(grad\varphi)$ at $(1,1,-1)$.

18. Find Laplacian of $\varphi(x,y,z)=z+e^x\cos y$.

19. Show that $F(x,y,z)=(e^z+y)\hat{i}+(e^x-y)\hat{j}+(e^y+z)\hat{k}$ is solenoidal.

20. Evaluate $\int_0^1\int_0^2 xy(x-y)dx\,dy$.

21. Evaluate $\int_0^1\int_0^x\int_0^y xyz\,dx\,dy\,dz$.

22. Find the area enclosed between $x=5$, $x=10$ $y=x$ and $y=5+x$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions, each carries **4** marks

23. Find Taylor's theorem to find a quadratic approximation of $f(x,y)=xe^{y}$ about the origin.

24. The temperature of a point (x,y) on a unit circle is given by $T(x,y)=1+xy$ find the temperature of the two hottest point on the circle.

25. Sum the series $s(\theta)=1+\cos\theta+\frac{\cos 2\theta}{2!}+\frac{\cos 3\theta}{3!}+....$

26. Given that the series $\sum_{n=1}^{\infty}\frac{1}{n}$ diverge, determine whether the series converges: $\sum_{n=1}^{\infty}\frac{4n^2-n-3}{n^3+2n}$.

27. Show that $\nabla.(\nabla\varphi)=\nabla^2\varphi$ for any scalar field φ and verify for $\varphi(x,y,z)=e^{xyz}$.

28. If $F(x,y,z)=xyz\hat{i}+(2x+z)\hat{j}+(x+2y)\hat{k}$, then find $\nabla.(\nabla\times F)$.

29. Find unit tangent vector $\hat{t}$ and acceleration $\vec{a}$ of a particle moving along the trajectory $\vec{r}(t)=5\cos t\hat{i}+5\sin t\hat{j}+3t\hat{k}$.

30. Find the centre of mass of the solid hemisphere bounded by the surfaces $x^2+y^2+z^2=a^2$ and *xy plane* assuming that it has a uniform density ρ.

31. Find the moment of inertia of a uniform rectangular lamina of mass M with sides a *and b* about of the side of length *b*.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions, each carries **15** mark each

32. Find the stationary points of $f(x,y,z)=x^3+y^3+z^3$ subject to the following conditions

    (a) $g(x,y,z)=x^2+y^2+z^2=1$

    (b) $g(x,y,z)=x^2+y^2+z^2=1$ and $h(x,y,z)=x+y+z=0$.

33. Determine the convergence of the following series

(a) $\sum_{n=1}^{\infty} \frac{2}{n^2}$

(b) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

(c) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$.

34. (a) Find the volume of tetrahedron bounded by the coordinate surfaces $x = 0, y = 0, z = 0$ and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

(b) Find $\int_0^4 \int_{x=\frac{y}{2}}^{\frac{y}{2}+1} \frac{2x-y}{2} dx\, dy$ by applying the transformation $u = \frac{2x-y}{2}$ and $v = \frac{y}{2}$.

35. (a) Find the length of one turn of a helix $\vec{r}(t) = 5\cos t\hat{i} + 5\sin t\hat{j} + 3t\hat{k}$; $0 \le t \le 2\pi$.

(b) Show that the acceleration of a particle travelling along a trajectory $\vec{r}(t)$ is given by $\vec{a}(t) = \frac{dv}{dt}\hat{t} + \frac{v^2}{\rho}\hat{n}$, where $\hat{t}$ is the unit tangent vector, v is the speed $\hat{n}$ is the principal normal vector and ρ is the radius of convergence.

(2 × 15 = 30 Marks)
