(Pages : 4)

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Reg. No. : ....

Name : ....

Third Semester B.Sc Degree Examination, January 2023

First Degree Programme Under CBCSS

Mathematics

Core Course – II

MM 1341 – ALGEBRA AND CALCULUS – I

(2014 – 2017 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

Answer **all** questions. They carry **1** mark each.

1. Find all zero divisors of $Z/6Z$.

2. Find all unit elements of Z.

3. State Abstract Fermat' s Theorem.

4. Find the order of $[2]$ in $Z/7Z$.

5. The plane which consists of all points of the form (x, y, 0) is called ____________

6. Write distance formula in 3-space.

7. By definition, a "cylindrical surface" is a right circular cylinder whose axis is parallel to one of the coordinate axes. True or False?

P.T.O.

8. Describe the parametric curve represented by the equations $X = 1 - t$, $y = 3t$, $z = 2t$.

9. Write the component functions of $r(t) = t\,i + t^2 j + t^3 k$.

10. Express the parametric equations $x = \frac{1}{t}$, $y = \sqrt{t}$, $z = \sin^{-1} t$ as a single vector equation.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. These questions carry **2** marks each.

11. Suppose R is a ring with no zero divisors and S is a subring of R. Show that S has no zero divisors.

12. Prove that a filed has no zero divisors.

13. In Z/mZ, prove that $[a] = 0$, if m divides a.

14. If f is a homomorphism, prove that $f(0) = 0$.

15. Let $f : R \to S$ be a homomorphism where R is a field and $1 \neq 0$ in S. Prove that f is one-to-one.

16. Show that in a commutative ring R, if $a.b = b.a = 1$ and $a.c = 1$, then $b = c$.

17. Find the distance between the points $(1, -2, 0)$ and (4, 0, 5).

18. Find the equation of the sphere having center (–1, 3, 2) and passing through the origin.

19. If $v =< -2, 0, 1 >$ and $w =< 3, 5, -4 >$, then find $v + w$ and $3v$.

20. Sketch the graph and a radius vector of $r(t) = \cos t\,i + \sin t\,j,\ 0 \leq t \leq 2\pi$.

21. Find the domain of $r(t) = \cos t\,i - 3t\,j$.

22. Let $r(t) = t2i + et\,j - (2\cos \pi t)k$. Find $\lim_{t \to 0} r(t)$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. These questions carry **4** marks each.

23. Let R be a commutative ring with at least 5 elements. Prove that the equation $x^2 - rx + z = 0$ has at most two solutions in R for every r, s in R, if and only if R has no zero divisors.

24. Prove that Z/mZ is a field if and only f m is prime.

25. Find the exponent of $G = U_{15}$, the group of units of $Z/15Z$.

26. Find the center and radius of the sphere $x^2 + y^2 + z^2 - 2x - 4y + 8z + 17 = 0$.

27. Find the angle that the vector $v = -\sqrt{3i} + j$ makes with the positive x-axis.

28. Find the angle between the diagonal of a cube and one of its edges.

29. Describe vector form of a line segment.

30. Sketch the graph of $r(t) = \cosh t\ i + \sinh t\ j, 0 \leq t \leq 2\pi$ and show the direction of increasing t.

31. If $r(t)$ is a vector-valued function, then prove that r is differentiable at t if and only if each of its component functions is differentiable at t, in which case the component functions of $r(t)$ are the derivatives of the corresponding component functions of $r(t)$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. These questions carry **15** marks each.

32. (a) If R is a finite commutative ring with identity, and a is any nonzero element of R, then a is either a unit or a zero divisor.

    (b) Define the order of an element. Find the order of all non-zero elements of $Z/7Z$.

33. (a) State and prove Chinese Reminder Theorem.

(b) Solve: $x \equiv 3 \pmod{11}$, $x \equiv 6 \pmod{8}$, $x \equiv -1 \pmod{15}$

34. (a) Show that the direction cosines of a vector satisfy $\cos^2 a + \cos^2 b + \cos^2 c = 1$.

(b) Find the orthogonal projection of $v = i + j + k$ on $b = 2i + 2j$ and find the vector component of v orthogonal to b.

35. (a) Show that the graph of $r = \sin t\, i + 2\cos t\, j + \sqrt{3}\, \sin tk$ is a circle, and find its center and radius.

(b) Show that the $r = t\cos t\, i + t\sin t\, j + tk,\ t \geq 0$, lies on the cone $z = \sqrt{x^2 + y^2}$. Describe the curve.

(2 × 15 = 30 Marks)