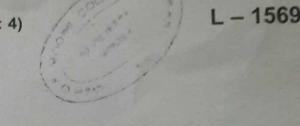
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# Sixth Semester B.Sc. Degree Examination, March 2021 First Degree Programme under CBCSS

# **Mathematics**

MM 1661.1 : Graph Theory (Elective)
(2018 Admission Regular)

Time: 3 Hours

Max. Marks: 80

### SECTION - 1

All the first ten questions are compulsory. They carry 1 mark each.

- Define a simple graph.
- 2. How many edges do K<sub>20</sub> have?
- 3. Define an empty graph.
- 4. Define a connected graph.
- 5. A tree with n vertices has ... edges.
- 6. Define a cut vertex of a graph.
- Define a Hamiltonian graph.
- 8. Is K<sub>4</sub> Eulerian?
- 9. How many regular polyhedra are there?
- 10. State Euler's formula in a connected plane graph.

 $(10 \times 1 = 10 \text{ Marks})$ 

#### SECTION - II

Answer any eight questions from among the questions 11 to 26. These questions carry 2 marks each.

- 11. Define a bipartite graph with example.
- 12. Define a regular graph. Draw a 2-regular graph.
- 13. Let G be a graph in which there is no pair of adjacent edges. What can you say about the degrees of the vertices in G?
- 14. Define complement of a graph. Give an example.
- 15. Define join of two graphs.
- 16. Define adjacency matrix of a graph G. Give an example.
- 17. Draw all non-isomorphic trees with 4 vertices.
- 18. Define vertex connectivity of a graph with example.
- 19. Define a maximal non-Hamiltonian graph. Give example.
- 20. Define an Euler tour. Give an example.
- 21. Define a Jordan curve.
- 22. State Cayley's theorem on spanning trees.
- 23. Define a plane graph. Give an example.
- 24. Define a polyhedral graph.
- 25. State Kuratowski's theorem on planar graphs.
- 26. How can we obtain a subdivision of a graph G.

 $(8 \times 2 = 16 \text{ Marks})$ 

# SECTION - III

Answer any six questions from among the questions 27 to 38. These questions carry 4 marks each.

- 27. Prove that in any graph there is an even number of odd vertices.
- 28. Given two vertices u and v of a graph G. Prove that every u-v walk contains a u-v path.
- 29. Define the following in a connected graph
  - (a) Distance between two vertices
  - (b) Eccentricity of a vertex
  - (c) Radius of a graph
  - (d) Diameter of a graph.
- 30. Let u and v be distinct vertices of a tree T. Prove that there is precisely one path from u to v.
- 31. Prove that an edge e of a graph G is a bridge if and only if e is not part of any cycle in G.
- 32. Let G be a graph with n vertices where  $n \ge 2$ . Prove that G has at least two vertices which are not cut vertices.
- 33. Describe Konigsberg bridge problem.
- 34. Define closure of a graph. Prove that a simple graph G is Hamiltonian if and only if its closure c(G) is Hamiltonian.
- 35. Let G be a graph in which the degree of every vertex is at least two. Then prove that G contains a cycle.
- 36. Prove that the complete graph K<sub>5</sub> is non planar.

- 37. If G is a simple planar graph then prove that G has a vertex v of degree less than 6.
- 38. Let G be a simple graph with at least 11 vertices. Prove that either G or its complement  $\overline{G}$  must be non-planar.

 $(6 \times 4 = 24 \text{ Marks})$ 

# SECTION - IV

Answer any two questions from among the questions 39 to 44. These questions carry 15 marks each.

- 39. Let G be a non-empty graph with at least two vertices. Prove that G is bipartite if and only if it has no odd cycles.
- 40. Define a spanning tree with example. Prove that a graph G is connected if and only if it has a spanning tree.
- 41. Let G be a simple graph with at least three vertices. Then prove that G is 2-connected if and only if for each pair of distinct vertices u and v of G there are two internally disjoint u-v paths in G.
- 42. Prove that a connected graph is Euler if and only if the degree of every vertex is even.
- 43. (a) Describe travelling salesman problem.
  - (b) If G is a simple graph with n vertices where  $n \ge 3$  and the degree  $d(v) \ge n/2$  for every vertex V of G then prove that G is Hamiltonian.
- 44. Let P be a convex polyhedron and G be its corresponding polyhedral graph. For each  $n \ge 3$  let  $v_n$  denote the number of vertices of G of degree n and let  $f_n$  denote the number of faces of G of degree n. Prove that.
  - (a)  $\sum_{n\geq 3} nv_n = \sum_{n\geq 3} nf_n = 2e$ , where e is the number of edges of G.
  - (b) The polyhedron P and so the graph G has at least one face bounded by a cycle of length n = 3, 4 or 5.

 $(2 \times 15 = 30 \text{ Marks})$