

Reg. No. :

Name :

First Semester B.Sc. Degree Examination, January 2024

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1131.1 : Mathematics I — CALCULUS AND SEQUENCE AND SERIES

(2021 Admission onwards)

Time : 3 Hours

Max. Marks : 80

I. All the first **ten** questions are compulsory. They carry **1** mark each.

1. Find $\lim_{x \rightarrow 2} (x^2 - x + 1)$.

2. Find x such that $\ln(x+1) = \sqrt{2}$.

3. State the quotient rule of differentiation.

4. Evaluate $\int \cos^2 x \, dx$.

5. Find $f_x(1, 3)$ for the function $f(x, y) = 2x^3y^2 + 2y + 4x$.

6. Compute the differential of function $z = xy^2$.

7. State the extreme value theorem for the function of two variables.

8. Find the limit of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$

9. Show that $\lim_{n \rightarrow +\infty} \sqrt[n]{n} = 1$.

10. Does the series $\sum_{k=1}^{\infty} \frac{1}{k}$ converge or diverge?

(10 × 1 = 10 Marks)

II. Answer any **eight** questions. These questions carry **2** marks each.

11. Show that $|x|$ is continuous everywhere.

12. Find $\lim_{x \rightarrow -\infty} \frac{4x^2 - x}{2x^3 - 5}$.

13. Find the equation of the tangent line to $y = x^2$ at $x = 2$.

14. Find $\int_1^e \frac{1}{x} dx$.

15. Find $\frac{d}{dx} \left[\int_1^x t^3 dt \right]$.

16. Find the area of the region under the curve $y = 3 \sin x$ over the interval $\left[0, \frac{2\pi}{3}\right]$.

17. Find the slope of the surface $z = f(x, y) = x^2y + 5y^3$ in the x -direction at the point $(1, -2)$.

18. Let $f(x, y) = y^2 e^x + y$. Find f_{xyy} .

19. Let $(x, y) = x^2 e^y$. Find the maximum value of a directional derivative at $(-2, 0)$.

20. Find the limit of the sequence $1, -\frac{1}{2}, \frac{1}{2^2}, -\frac{1}{2^3}, \dots, (-1)^n \frac{1}{2^n}, \dots$
21. Find the rational number represented by the repeating decimal 0.784784784..
22. Use the root test to determine whether the series converge or diverge
- $$\sum_{k=2}^{\infty} \left(\frac{4k-5}{2k+1} \right)^k.$$

(8 × 2 = 16 Marks)

III. Answer any six questions. These questions carry 4 marks each.

23. Find $\lim_{x \rightarrow +\infty} (\sqrt{x^6 + 5x^3 - x^3})$.
24. Find $\frac{d}{dx} [\sin \sqrt{1 + \cos x}]$.
25. Evaluate $\int \frac{e^x}{\sqrt{1 - e^x}} dx$
26. Evaluate $\int_0^1 \tan^{-1} x dx$.
27. Find the second-order partial derivatives of $f(x, y) = x^2 y^3 + x^4 y$.
28. Prove that $r(x, y) = x^2 + y^2$ is differentiable at (0, 0).
29. Given that $z = e^{xy}$, $x = 2u + v$, $y = \frac{u}{v}$, find $\frac{dz}{du}$ and $\frac{\partial z}{\partial v}$ using the chain rule.
30. Test for convergence of series $\sum_{k=1}^{\infty} \frac{3k^3 - 2k^2 + 4}{k^7 - k^3 + 2}$.
31. Find the n^{th} Maclaurin polynomials for $\cos x$.

(6 × 4 = 24 Marks)

IV. Answer any **two** questions. These questions carry **15** marks each.

32. Let $f(x) = \begin{cases} \frac{1}{x+2}, & x < -2 \\ x^2 - 5, & -2 < x \leq 3 \\ \sqrt{x+13}, & x > 3 \end{cases}$

Find

(a) $\lim_{x \rightarrow -2} f(x)$

(b) $\lim_{x \rightarrow 0} f(x)$

(c) $\lim_{x \rightarrow 3} f(x)$.

33. (a) Use implicit differentiation to find $\frac{dy}{dx}$ for the Folium of Descartes $x^3 + y^3 = 3xy$.

(b) Find an equation for the tangent line to the Folium of Descartes at the point $\left(\frac{3}{2}, \frac{3}{2}\right)$.

(c) At what point in the first quadrant is the tangent line to the Folium of Descartes horizontal?

34. Locate all relative extrema and saddle points of $f(x, y) = 4xy - x^4 - y^4$.

35. Find the interval of convergence and radius of convergence of the following power series.

(a) $\sum_{k=0}^{\infty} x^k$

(b) $\sum_{k=0}^{\infty} k! x^k$

(c) $\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{3^k (k+1)}$

(2 × 15 = 30 Marks)