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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1544 : VECTOR ANALYSIS

(2014 – 2017 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

Answer **all** questions from this section. Each question carries 1 mark.

1. Define the divergence of $\overline{F}$.

2. Find the directional derivative of $f(x,y,z) = x^2y - yz^3 + z$ at (1, –2, 0) in the direction of the vector $2\hat{i} + \hat{j} - 2\hat{k}$.

3. Find the gradient of $\overline{F}(x,y,z) = e^z - \ln(x^2 + y^2)$.

4. State Gauss theorem.

5. Define curl of a vector field.

6. Find the divergence of $\overline{F} = 6x^2z\hat{i} + 2x^2y\hat{j} - yz^2\hat{k}$.

7. Show that $\overline{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$ is solenoidal.

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8. A vector that is perpendicular to both the vectors $\vec{A}$ and $\vec{B}$ is ________.

9. Calculate the curl of the vector field $\bar{F}$, $\bar{F}(x,y) = (x^2 - 2y)\hat{i} + (xy - y^2)\hat{j}$.

10. Div $(\bar{F} \times \bar{G}) =$ ________.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** from the following. Each question carries **2** marks.

11. Find the curl $\bar{F}$, $\bar{F} = x^2 z\hat{i} - 2y^3 z^2\hat{j} + xy^2 z\hat{k}$ at (1, 1, 1).

12. Find the directional derivative of $f(x, y, z) = x^3 - xy^2 - z$ at (1, 1, 0) in the direction of $A = 2\hat{i} - 3\hat{j} + 6\hat{k}$.

13. Define inverse square field. State Gauss law of inverse square field.

14. Show that $\bar{F} = (2x - 3)\hat{i} - z\hat{j} + (\cos z)\hat{k}$ is not conservative.

15. Prove that if ϕ is a scalar function $curl\,(grad\ \phi) = 0$.

16. Let $\bar{F}(x,y) = 2x\hat{i} + 3y\hat{j} + 4z\hat{k}$. Find the potential function.

17. Evaluate $\oint_C (x - y)dx + x\,dy$, where C is the unit circle $\bar{r}(t) = \cos t\hat{i} + \sin t\hat{j}$, $0 \leq t \leq 2\pi$.

18. Find the gradient field of $f(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$.

19. Evaluate $\int_C (xy + y + z)ds$ along the curve $\bar{r}(t) = 2t\hat{i} + t\hat{j} + (2 - 2t)\hat{k}$ $0 \leq t \leq 1$.

20. Find the workdone by $\bar{F} = (y - x^2)\hat{i} + (z - y^2)\hat{j} + (x - z^2)\hat{k}$ over the curve $\bar{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$, $0 \leq t \leq 1$, from (0, 0, 0) to (1, 1, 1).

21. Find the circulation of the field $\bar{F} = (x - y)\hat{i} + x\hat{j}$ around the circle $x^2 + y^2 = 1$.

22. Prove that $div\,(curl\,\bar{F}) = 0$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** from the following. Each question carries **4** marks.

23. Find the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

24. A fluids velocity field is $\bar{F}=x\hat{i}+z\hat{j}+y\hat{k}$. Find the flow along the helix $\bar{r}(t)=\cos t\hat{i}+\sin t\hat{j}+t\hat{k}$, $0\leq t\leq\frac{\pi}{2}$.

25. Evaluate $\oint_C x^2y\,dx+(y+xy^2)dy$, where C is the region enclosed by $y=x^2$, $x=y^2$.

26. Find the potential function of $\bar{F}=x^2y\hat{i}+5xy^2\hat{j}$.

27. Evaluate $\iint_S(7x\hat{i}-z\hat{k})\cdot\bar{n}\,d\sigma$ over the sphere $x^2+y^2+z^2=4$ using divergence theorem.

28. Evaluate $\int_C \bar{F}\cdot d\bar{r}$, where $F=(x^2+y^2)\hat{i}-2xy\hat{j}$, C is the rectangle in the XY – plane bounded by $x=0$, $x=a$, $y=0$, $y=a$.

29. Prove that $div\,(f\bar{V})=f\cdot div\,\bar{V}+\bar{V}\cdot grad\,f$, where f is a scalar function.

30. Find the flux of $\bar{F}=xy\hat{i}+yz\hat{j}+xz\hat{k}$ outward through the surface of cube cut from first octant by planes $x=1$, $y=1$, $z=1$.

31. If $\bar{F}=(3x^2+6y)\hat{i}-14yz\hat{j}+20xz^2\hat{k}$. Evaluate $\int_C \bar{F}\cdot d\bar{r}$, where C is the straight line joining (0, 0, 0) and (1, 0, 0).

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** from the following. **Each** question carries **15** marks.

32. (a) State Stoke's theorem.

(b) Verify Stokes theorem for $F = y\hat{i} - x\hat{j}$ for the hemisphere $S : x^2 + y^2 + z^2 = 9$, $z \geq 0$, its bounding circle $C : x^2 + y^2 = 9,\ z = 0$.

33. Evaluate the surface integral $\iint_{\sigma} x^2\, ds$ over the sphere $x^2 + y^2 + z^2 = 1$.

34. Prove that

(a) $div(\phi \overline{F}) = grad\, \phi \overline{F} + \phi\, div\, \overline{F}$

(b) $curl(\phi \overline{F}) = (grad\, \phi) \times \overline{F} + \phi\ curl\, \overline{F}$

35. Verify divergence theorem for the function $\overline{F} = (x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}$, where σ is the spherical surface $x^2 + y^2 + z^2 = 1$.

(2 × 15 = 30 Marks)