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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, March 2021

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1643 – COMPLEX ANALYSIS – II

(2015 – 2017 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – A

All the first **ten** questions are compulsory. **Each** question carries **1** mark.

1. Find the power series representation of $f(z)=\frac{1}{z}$ in powers $1-z$.

2. Find *the* radius of convergence of the power series $\sum_{n=0}^{\infty}\frac{z^n}{n!}$.

3. What are the isolated singularities of $\frac{1}{\cos\frac{\pi}{2}}$.

4. Define removable singularity.

5. What type of isolated singularity for $f(z)=\sin\left(\frac{1}{z}\right)$ has at z = 0.

6. Find the residue of $f(z)=\frac{1+z}{z-1}$ *at* $z=1$.

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7. Find the pole (poles) of $f(z)=\frac{1}{z(e^z-1)}$ and find their order.

8. Define Cauchy principal value of an improper integral.

9. Find the set of all singular points of the function $f(z) = Re(z)$.

10. Evaluate $\int_C \frac{1}{(z-1)^2} dz$, where C is the circle $|z-1|=1$ taken in counter clockwise direction.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions from this section. **Each** question carries **2** marks.

11. Evaluate $\int_C \frac{z}{(9-z^2)(z+i)} dz$, where C is the circle $|z|=2$ taken in counter clockwise direction.

12. Find a power series representing the function $f(z)=\frac{1}{z^2}$ near $z=3$. Also find the region of Convergence.

13. Evaluate $\int_C \frac{e^z}{z^2+2z+1} dz$, where C is the circle $|z+1|=1$ oriented in counter clockwise direction.

14. If $f(z)$ is an analytic function so that $f(z)=3z+1$ for all z in the unit circle $C:|z|=1$. Show that $|f^5(0)| \leq 480$.

15. Find $\int_C \frac{1}{z^2+5}$, where C is the circle $|z-i|=3$ oriented positively.

16. Define residue of a complex function at an isolated singular point.

17. Find the residues of *cotz* at its poles.

18. State Jordan's lemma.

19. Evaluate $\sum_{n=0}^{n} \binom{n}{k}^2$.

20. State Cauchy's residue theorem.

21. Using Cauchy's residue theorem evaluate $\int_C \frac{e^{-z}}{z^2} dz$, where C is the circle $|z|=3$ oriented positively.

22. Find power series representation of $\cosh z$ in powers of z.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions from this section. **Each** question carries **4** marks.

23. State and prove Cauchy's integral formula.

24. Prove that for any real number a, $\int_0^{\pi} e^{a\cos\theta} \cos(a\sin\theta)\, d\theta = \pi$.

25. Using Cauchy's residue theorem evaluate $\int_C \frac{z^5}{1-z^3} dz$ where C is the circle $|z|=2$ oriented in counter clockwise direction.

26. Prove that an isolated singular point z_0 of a function f is a pole of order m if and only if $f(z)$ can be written in the form $f(z) = \frac{\phi(z)}{(z-z_0)^m}$ where $\phi(z)$ is analytic and nonzero at z_0.

27. Find the residue of $f(z) = \frac{zdz}{(9-z^2)(z+1)}$ at its poles and hence find $\int_C f(z)dz$ where C is the positively oriented circle $|z|=2$.

28. Evaluate the improper integral $\int_0^{\infty} \frac{dx}{(x^2+1)^2}$.

29. Show that $\sum_{n=0}^{\infty} \binom{2n}{n} \frac{1}{5^n} = \sqrt{5}$.

30. Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$.

31. Show that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1} = \frac{-1}{2} + \frac{\pi e^{\pi}}{e^{2\pi}-1}$.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions from this section. **Each** question carries **15** marks.

32. (a) If f is an analytic at a given point, then show that the derivative of all orders of f are analytic there.

(b) Find $g(2)$ if $g(z_0)=\int_C \frac{2s^2-s-2}{s-z_0}ds$, where C is the positively oriented circle $|z|=3$ and $|z_0|\neq 3$.

33. (a) If C is the positively oriented circle $|z|=1$ show that $\int_C z^2 \sin\frac{1}{z}dz=\frac{-\pi i}{3}$.

(b) Prove that, a point z_0 at which the function $f(z)$ is analytic, is a zero of $f(z)$ of order m if and only if $f(z)$ can be written in the form $f(z) = (z - z_0)^m g(z)$ where $g(z)$ is analytic and nonzero at z_0.

(c) If p and q are two complex functions, which are analytic at a point z_0 so that $p(z_0)\neq 0$ and q has a zero of order m at z_0. Show that $\frac{p(z)}{q(z)}$ has a pole of order m at z_0.

34. (a) State and prove Cauchy's residue theorem.

(b) Evaluate $\int_0^\infty \frac{dx}{x^4+1}$.

(c) Evaluate $\int_0^\infty \frac{\cos ax}{(x^2+b^2)^2},(a>b>0)$.

35. (a) Prove that if f is complex function having only a finite number of poles $\{z_k\}$, then $\sum_{n=-\infty, n\neq z_k}^{\infty}(-1)^n f(n)=-\sum_k \text{Res}(f(z)\pi\cot\pi z, z_k)$.

(b) Evaluate $\sum_{n=1}^{\infty}\frac{(-1))^{n+1}}{n^2}$.

(2 × 15 = 30 Marks)
