SEMESTER -6

REAL ANALYSIS

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MATHS CAFE

2025

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For Demo Class

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(Pages : 4) **T – 1605**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2024

First Degree Programme under CBCSS

Mathematics

Core Course IX

MM 1641 : REAL ANALYSIS – II

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. State composition of continuous functions theorem.

2. State extreme value theorem for continuous functions.

3. Define uniform continuity and give an example.

4. State intermediate value theorem.

5. Find the 10^{th} derivative of $f(x) = x^5 + 4x^2 + 1$.

6. Give an example of a monotone function.

7. When we say that a function is Riemann integrable.

8. Give an example of a set of measure 0.

P.T.O.

9. If $\int_a^b f = 10$, then $\int_b^a f = \cdots$.

10. State Lebesgue's Theorem.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. These questions carry **2** marks each.

11. State sequential criterion for functional limits.

12. Evaluate $\lim_{x \to \pi}(x + \sin x)$.

13. Construct two functions f and g, neither of which is continuous at 0 but $f(x) + g(x)$ is continuous at 0.

14. Whether there exists a continuous function defined on a closed interval with range equal to $\{1,2,3\}$.

15. Define Lipschitz Function and give an example of a function which is uniformly continuous but not Lipschitz.

16. Define removable discontinuity with an example.

17. State Darboux's Theorem.

18. State Mean Value Theorem.

19. Find $\lim_{x \to 1}\left(\frac{1-x}{\ln x}\right)$.

20. If P_1 and P_2 are any two partitions of $[a, b]$, then prove that $L(f, P_1) \le U(f, P_2)$.

21. Distinguish between upper integral and lower integral.

22. If $\int_1^4 f = 4$ and $\int_2^4 f = 1$, then find $\int_1^2 f$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

23. Using $\varepsilon - \delta$ definition prove that $\lim_{x \to 2}(3x+4) = 10$.

24. Prove that $f(x) = \sqrt{x}$ is uniformly continuous on $[0, \infty)$.

25. Is the converse of intermediate value theorem true? Justify your claim.

26. Let f be differentiable on an open interval (a, b). If f attains a maximum value at some point $c \in (a,b)$ then prove that $f'(c) = 0$.

27. State and prove Rolle's theorem.

28. If $f : A \to R$ is differentiable at a point $c \in A$, then prove that f is continuous at c. Is the converse true? Justify your answer.

29. If $g : A \to R$ is differentiable on an interval A and satisfies $g'(x) = 0$ for all $x \in A$, then prove that $g(x) = k$ for some constant $k \in R$.

30. Assume that $f_n \to f$ uniformly on $[a, b]$ and that each f_n is integrable. Prove that f is integrable and $\lim_{n \to \infty} \int_a^b f_n = \int_a^b f$.

31. Prove that the Dirichlet's function $g(x) = \begin{cases} 1 \text{ for } x \text{ rational} \\ 0 \text{ for } x \text{ irrational} \end{cases}$ is not integrable.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These questions carry **15** marks each.

32. Let $f : A \to R$ be continuous on A. If $K \subseteq A$ is compact, then prove that $f(K)$ is also compact.

33. State and prove chain rule for derivatives.

34. (a) Prove that a bounded function f is integrable on $[a, b]$ if and only if, for every $\varepsilon > 0$, there exists a partition P_ε of $[a, b]$ such that $U(f, P_\varepsilon) - L(f, P_\varepsilon) < \varepsilon$.

    (b) Prove that if f is continuous on $[a, b]$, then it is integrable.

35. State and prove the fundamental theorem of integral calculus.

(2 × 15 = 30 Marks)

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Sixth Semester B.Sc. Degree Examination, April 2023

First Degree Programme under CBCSS

Mathematics

Core Course IX

MM 1641 : REAL ANALYSIS – II

(2018 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – A

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Evaluate $\lim_{x \to 7/4} \frac{|x-2|}{x-2}$

2. State true or false: Every uniformly continuous function is continuous.

3. Determine the points of discontinuity of the greatest integer function.

4. State the mean value theorem.

5. Define a uniformly continuous function.

6. Define a differentiable function at a point.

7. Give an example of a real valued function which is discontinuous at every point of R.

8. Define upper integral of a function f.

9. When do you say that a bounded real function f is integrable on [a,b]?

10. State true or false: If $|f|$ is integrable on [a,b] then f is also integrable on [a,b].

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each question carries **2** marks.

11. Evaluate $\lim_{x \to 0} \frac{x^2}{|x|}$

12. Prove that the Dirichlet's function f defined on R by $f(x) = \begin{cases} 1 \textit{ if } x \textit{ is irrational} \\ -1 \textit{ if } x \textit{ is rational} \end{cases}$

is discontinuous at every point.

13. If f: $A \to$ R and g: $A \to R$ are continuous at a point $c \in A$, show that $f(x)+g(x)$ is also continuous at c.

14. Is the function $f(x) = \frac{1}{x}$ uniformly continuous on (0, 1]? Justify.

15. Prove that $\{f(x_n)\}$ is a Cauchy sequence for every Cauchy sequence $\{x_n\}$ in R where f is a uniformly continuous function.

16. If f is differentiable in (a, b) and $f'(x) \leq 0$ for all $x \in (a,b)$, show that f is monotonically decreasing.

17. Show by an example that a bounded function in [a,b] need not be continuous in [a,b].

18. If f: $A \to R$ is differentiable at a point $c \in A$, then f is continuous at c as well.

19. Find the value of δ for the function $f(x) = x^2 + 4x + 3$ to be uniformly continuous in the interval [-1,1], given $\varepsilon = \frac{1}{10}$.

20. Check whether the following function is integrable over [0,1]: $f(x) = 1$ if $x \in [0,1]$ and x is rational and $f(x) = 0$ if $x \in [0,1]$ and x is irrational.

21. Show that $\int_{\underline{a}}^{b} f\,dx \geq \int_{a}^{\overline{b}} f\,dx$.

22. Show that if f and g are bounded and integrable on [a,b], such that $f \geq g$, then $\int_a^b f\,dx \geq \int_a^b g\,dx$.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each question carries **4** marks.

23. Test the continuity of the function at x = 0 $f(x) = \begin{cases} x \sin \frac{1}{x}, for\ x \neq 0 \\ 0, for\ x = 0 \end{cases}$

24. Explain Lipschitz functions with the geometrical interpretation.

25. Show that a uniformly continuous function preserves Cauchy sequences.

26. Suppose f is a real differentiable function on [a,b] and suppose $f'(a) < \lambda < f'(b)$. Prove that there is a point $x \in (a,b)$ such that $f'(x) = \lambda$.

27. State and prove chain rule of differentiation.

28. State and prove Darboux's theorem.

29. Prove that, if f is monotonic in [a,b], it is integrable in [a,b].

30. If f and g are integrable in [a,b] then show that fg is also integrable in [a,b].

31. Show that the Dirichlet's function $g(x) = \begin{cases} 1 \text{ for } x \text{ rational} \\ 0 \text{ for } x \text{ irrational} \end{cases}$ is not integrable.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. Each question carries **15** marks.

32. Let f: $A \to R$ be continuous on A. If $K \subseteq A$ is compact, then prove that f (K) is compact as well.

33. State and prove Intermediate value theorem. Is the converse true? Justify your answer.

34. Prove that a bounded function f is integrable on [a,b] if and only if for every $\varepsilon > 0$ there exists a partition P such that U (P,f) – L (P, f) < ε.

35. If f is bounded and integrable on [a,b] and k is a number such that $|f(x)| \le k$ for all $x \in [a,b]$. Prove that $\left|\int_a^b f\,dx\right| \le k(b-a)$.

N – 1285

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme Under CBCSS

Mathematics

Core Course IX

MM 1641 – REAL ANALYSIS – II

(2018 & 2019 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – A

All the first ten questions are compulsory. They carry 1 mark each.

1. Evaluate $\lim_{x \to \infty} \left(1 + \frac{1}{n}\right)^n$.

2. State true or false: A function that is continuous on a compact set K is uniformly continuous on K.

3. Define a bounded function.

4. Determine the points of discontinuity of the Dirichlet's function.

5. State Rolle's theorem.

6. State intermediate value property.

7. When do you say that a function is differentiable on an interval?

8. State true or false: If f is differentiable in $[a,b]$ and $f'(x)=0$ for all $x \in (a,b)$, then f is continuous.

9. Define lower integral of a function f.

10. Compute $\int_0^3 [x]\, dx$, where $[x]$ denotes the greatest integer function.

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. Show that the limit $\lim_{x \to 2} \frac{|x-2|}{x-2}$ doesn't exist.

12. Let f and g be real valued functions then prove that

$$\lim_{x \to c} \{f(x)+g(x)\} = \lim_{x \to c} f(x) + \lim_{x \to c} g(x).$$

13. Show that $|x|$ is continuous everywhere.

14. Let $[x]$ denote the largest integer containing in x and $(x) = x - [x]$ denote the fractional part of x. What discontinuity do the function (x) has?

15. If, f, g be two functions continuous at a point c then the function f-g is also continuous at c.

16. Show that the function $f(x) = x^2$ is uniformly continuous on $[-1, 1]$.

17. Suppose that $\{x_n\}$ is a Cauchy sequence in R. Prove that $f(x_n)$ is a Cauchy sequence where f is a uniformly continuous function.

18. State Squeez theorem.

19. Give an example to show that continuous function need not be differentiable.

20. If f is differentiable in $(a,b)$and $f'(x) \geq 0$ for all $x \in (a,b)$, show that f is monotonically increasing.

21. Suppose f and g are defined on $[a, b]$ and are differential at a point $x \in [a,b]$. Prove that $f+g$ is differentiable.

22. Show that $\underline{\int_a^b} f dx \le \overline{\int_a^b} f dx$.

23. Show that the function $f(x)$ defined on $\mathbf{R}$ by $f(x)=\begin{cases} x \text{ if } x \text{ is irrational} \\ -x \text{ if } x \text{ is rational} \end{cases}$ is continuous only at $x=0$.

24. If P_1 and P_2 are any two partitions of $[a,b]$,then $L(f,P_1) \le U(f,P_2)$.

25. If a function f is integrable on $[a,b]$ and $m \le f(x) \le M$ for $x \in [a,b]$, prove that $m(b-a) \le \int_a^b f \le M(b-a)$.

26. Show that if f and g are bounded and integrable on $[a,b]$ such that $f \le g$ then $\int_a^b f\, dx \le \int_a^b g\, dx$.

SECTION – C

Answer **any six** questions. Each question carries **4** marks.

27. Show that $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$ does not exist.

28. Assume f and g are defined on all of R and that $\lim_{x\to p} f(x)=q$ and $\lim_{x\to q} g(x)=r$. Give an example to show that it may not be true that $\lim_{x\to p} g\,(f(x))=r$.

29. Prove that a function which is continuous on a closed interval is uniformly continuous on that interval.

30. If f is a real differentiable function on $[a,b]$ and suppose $f'(a) < \lambda < f'(b)$ then prove that there is a point $x \in (a,b)$ such that $f'(x)=\lambda$.

31. State and prove extreme value theorem.

32. Prove that if f is differentiable on an open interval in (a, b) and f attains a maximum value at some point c in (a, b), then $f'(c)=0$.

33. Show that $\int_3^4 f dx = \frac{41}{2}$ where $f(x) = 5x + 3$.

34. If $g : A \to R$ is differentiable on an interval A and satisfies $g'(x) = 0$ for all $x \in A$, then prove that $g(x) = k$ for some constant $k \in R$.

35. Prove that a continuous function in a closed interval is integrable in that interval.

36. Prove that if f is monotonic in $[a, b]$ then f is integrable in $[a, b]$.

37. If f is bounded and integrable in $[a, b]$, prove that there exists a number μ lying between a and b such that $\int_a^b f(x)\, dx = \mu(b - a)$.

38. Assume f is integrable function on the interval $[a, b]$, then show that $|f|$ is also integrable and $\left|\int_a^b f\right| \le \int_a^b |f|$.

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

39. State and prove intermediate value theorem. Is the converse true? Justify.

40. Define Lipschitz functions. Show that every Lipschitz function is uniformly continuous. Is the converse statement true? Justify.

41. State and prove chain rule for differentiation.

42. (a) State and prove Mean value theorem.

    (b) Prove that if f is continuous in $[a, b]$, then f is integrable in $[a, b]$.

43. If $f : [a, b] \to R$ is bounded, and f is integrable on $[c, b]$ for all $c \in (a, b)$, then prove that f is integrable on $[a, b]$.

44. State and prove fundamental theorem of calculus.

(Pages : 4)

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, March 2020

First Degree Programme Under CBCSS

Mathematics

Core Course IX

MM 1641 : REAL ANALYSIS – II

(2014 admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **10** questions are compulsory. Each carries 1 mark :

1. Is the function $f(x)=\frac{1}{x}$ continuous on R?

2. Prove that the sum of two real valued continuous functions is continuous.

3. If $f : A \to R$, $A \subseteq R$ is continuous on A, then prove that $|f|$ is continuous on A.

4. Write the sequential criterion for continuity.

5. Is the derivative of the function $f(x)=|x|$, $x \in R$ exists at 0? Justify.

6. Differentiate and simplify $g(x)=\sqrt{5-2x+x^2}$.

7. State Rolle's Theorem.

P.T.O.

8. Evaluate $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)$.

9. Define Riemann integrable functions on $[a, b]$.

10. Prove that every constant function on $[a, b]$ is Riemann integrable.

SECTION – II

Answer **any eight** questions from this section. Each question carries **2** marks.

11. Let $f(x)=\begin{cases}1 \text{ if } x \text{ is rational}\\ 0 \text{ if } x \text{ is irrational}\end{cases}$. Show that f is not continuous at any point of R.

12. Let $K>0$ and let $f: R\to R$ satisfy the condition $|f(x)-f(y)|\le K|x-y|$ for all $x, y\in R$. Show that f is continuous at every point $c\in R$.

13. Give an example of a function $f:[0, 1]\to R$ that is discontinuous at every point of $[0, 1]$ but $|f|$ is continuous on $[0, 1]$.

14. Find a closed bounded interval in which the equation $f(x)=xe^x-2=0$ has a root.

15. Give an example to show that the continuous image of an open interval need not be an open interval.

16. Define monotone functions. Give an example of a monotone function which is not continuous.

17. Prove that all differentiable functions are continuous.

18. Let $I=\left(0, \frac{\pi}{2}\right)$. Then find $\lim_{x\to 0+}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$.

19. State Taylor's Theorem. Using Taylor's theorem approximate the function $\sqrt[3]{1+x}$, $x>-1$ with $n=2$.

20. If $I = [0, 4]$, calculate the norm $||\mathcal{P}||$ of the partition $\mathcal{P} = (0, 1, 2, 4)$. Also find the Riemann sum $S(f; \mathcal{P})$ where $f(x) = x^2$ and tags at the right endpoints of the subintervals.

21. Prove that if $f \in \mathcal{R}$ [a, b] then the value of the integral is unique.

22. Prove that if $f, g \in \mathcal{R}$ [a, b] then the sum $f + g \in \mathcal{R}$ [a, b].

SECTION – III

Answer **any six** questions from this section. Each question carries **4** marks.

23. Let $I = [a, b]$ be a closed bounded interval and let $f : I \to R$ be continuous on 1. Then prove that *f* is bounded on *I*.

24. Let I be an interval and let $f : I \to R$ be continuous on I. If $a, b \in I$ and $k \in R$ satisfies $f(a) < k < f(b)$,then prove that there exists a point $c \in I$ between *a* and *b* such that $f(c) = k$.

25. Let *I* be an interval and let $f : I \to R$ be continuous on *I*. Prove that the set $f(I)$ is an interval.

26. Let $f : I \to R$ be increasing on *I*. Suppose that $c \in I$ is not an end point of *I*. Prove that $\lim_{x \to c-} f(x) = \sup\{f(x) : x \in I, x < c\}$.

27. Define derivative of a real valued function *f* at *c*. State and prove product rule of differentiation.

28. Define relative extremum of a real valued function. Prove that if the function $f : I \to R$ has a relative extremum at an interior point *c* of *I* and the derivative $f'(c)$ exists, then $f'(c) = 0$.

29. Prove that if $f \in \mathcal{R}$ [a, b] then *f* is bounded on [a, b].

30. Define a step function. Prove that if $\varphi : [a, b] \to R$ is a step function then $\varphi \in \mathcal{R}$ [a, b].

31. Define antiderivative of a function $f : [a, b] \to R$. State fundamental theorem of Calculus (First Form) and apply the theorem for the function $f(x) = x$ for all $x \in [a, b]$.

SECTION – IV

Answer **any two** questions from this section. Each question carries **15** marks.

32. (a) Define absolute minimum and absolute maximum of a real valued function f. Prove that if f is continuous on a closed bounded interval I, then f has an absolute maximum and an absolute minimum on I.

    (b) Prove that if $f: I \to R$ be continuous on $I = [a, b]$ then $f(I)$ is a closed bounded interval.

33. (a) Define jump of an increasing function $f: I \to R$ at an interior point $c \in I$. Prove that the set of point $D \subseteq I$ at which f is discontinuous is a countable set.

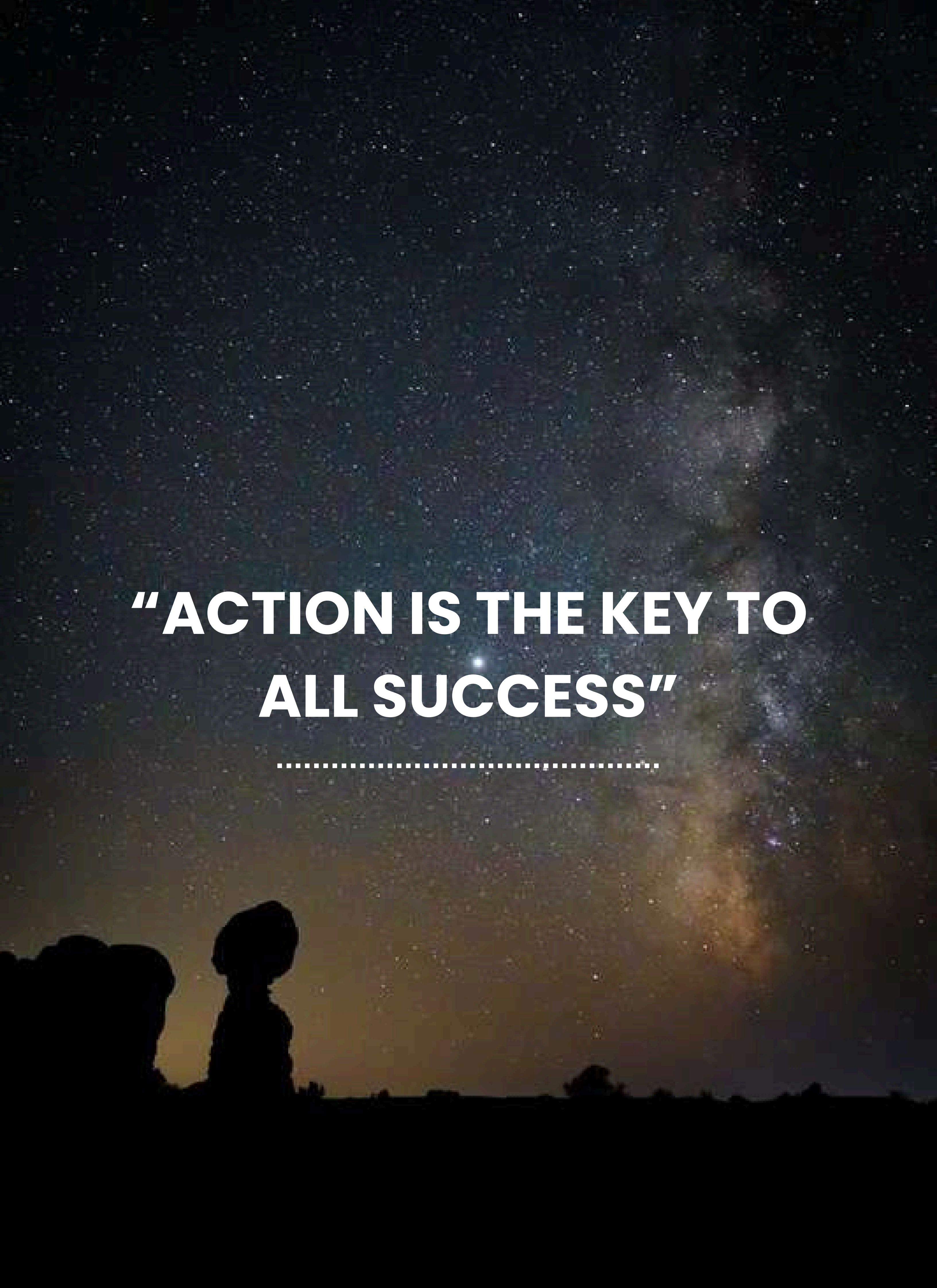
    (b) State and prove Chain Rule of differentiation.

34. (a) State and prove mean value theorem.

    (b) State and prove Cauchy Criterion for Riemann integrability.

35. (a) Prove that if $f: [a, b] \to R$ is continuous on $[a, b]$, then $f \in \mathcal{R}$ [a, b].

    (b) Prove that if $f: [a, b] \to R$ is monotone on $[a, b]$, then $f \in \mathcal{R}$ [a, b].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ALL SUCCESS"