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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1541 : REAL ANALYSIS I

(2014-2017 Admissions)

Time : 3 Hours

Max. Marks : 80

SECTION I

All the first **ten** questions are compulsory.

1. State the order properties of real numbers.
2. Give an example of a set that contains its supremum and infimum.
3. Define a sequence of nested intervals.
4. Define E-neighborhood of an element.
5. Define m-tail of a sequence.
6. Give an example of a monotonically increasing bounded sequence.
7. Find $\lim\limits_{n\to\infty}\left(\dfrac{\sqrt{n}-1}{\sqrt{n}+1}\right)$.
8. State the n^{th} term test for series.
9. Give an example of a set in which every point is a cluster point.
10. Give an example of a function that has a right-hand limit but not a left-hand limit at a point.

(10 × 1 = 10 Marks)

P.T.O.

SECTION II

Answer any **eight** questions. **Each** question carries **2** marks.

11. State and prove Bernoulli's inequality.

12. Find all $x \in \mathbb{R}$ that satisfy both $|2x-3|<5$ and $|x+1|>2$ simultaneously.

13. Let $J_n = \left(0, \frac{1}{n}\right)$ for $n \in \mathbb{N}$. Prove that $\bigcap_{n=1}^{\infty} J_n = \phi$.

14. Prove that $\sup(a+S) = a + SupS$.

15. Let (x_n) converge to x. Prove that $(|x_n|)$ converges to $|x|$.

16. Prove that if a sequence converges to a real number L, then any subsequence of this sequence converges to L.

17. Show that if (x_n) is an unbounded sequence, then there exists a properly divergent subsequence.

18. Discuss the convergance of $\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$.

19. Using sequential criteria prove that $\lim_{n \to 0} \operatorname{sgn}(x)$ does not exist, where $\operatorname{sgn}(x)$ is the signum function.

20. Find $\lim_{x \to 0} \frac{\sqrt{1+2x} - \sqrt{1+3x}}{x + 2x^2}$ where $x > 0$.

21. Find $\lim_{x \to 0^+} \frac{(x+2)}{\sqrt{x}}$, $x > 0$.

22. Prove that if $F : A \to \mathbb{R}$ has a limit at $c \in \mathbb{R}$, then F is bounded on some neighborhood of C.

(8 × 2 = 16 Marks)

SECTION III

Answer any **six** questions. **Each** question carries **4** marks.

23. Define least upper bound for a non empty subset of $\mathbb{R}$. State and prove a necessary and sufficient condition for a real number to be the least upper bound of a set.

24. State and prove density theorem.

25. If a set $S \subseteq \mathbb{R}$ contains one of its upper bounds, show that this upper bound is the supremum of S.

26. State and prove squeeze theorem for sequences.

27. Prove that $\lim\left(C^{1/n}\right) = 1$ for $C > 1$.

28. Prove that the alternating harmonic series converges.

29. Prove that every convergent sequence is bounded. Is the converse true. Justify your answer.

30. Show that $\lim\left(\frac{n^2}{n!}\right) = 0$.

31. Prove that $\lim\limits_{x \to 0} \frac{\cos x - 1}{x} = 0$.

(6 × 4 = 24 Marks)

SECTION IV

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) Using the definition of limits prove that $\lim\limits_{x \to c} x^2 = c^2$.

(b) Prove that $\lim\limits_{x \to 0} x \cos\left(\frac{1}{x}\right) = 0$.

33. (a) Let $I_n = [a_n, b_n]$, $n \in \mathbb{N}$, be a nested sequence of closed bounded intervals. PT there exists a number $\xi \in \Re$ such that $\xi \in I_n$ for all $n \in \mathbb{N}$.

(b) Under what conditions will ξ be unique. Prove your argument.

34. (a) State and prove monotone subsequence theorem.

(b) State and prove the limit comparison test for convergence of series.

35. (a) State and prove Cauchy Convergence criterion for sequences.

(b) Let (x_n) be a Cauchy sequence such that x_n is an integer for every $n \in \mathbb{N}$. Show that (x_n) is ultimately a constant.

(2 × 15 = 30 Marks)