(Pages : 4) **K – 3231**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, February 2021

First Degree Programme under CBCSS

Mathematics

Core Course VII

MM 1544 : VECTOR ANALYSIS

(2015 – 2017 Admissions)

Time : 3 Hours

Max. Marks : 80

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Define the directional derivative of f at (x_0, y_0) in the direction of u.

2. Define inverse square field.

3. Along any line segment parallel to the x-axis, $\int_C f(x, y)dy =$ ______.

4. When we say that a vector field is conservative?

5. When we say that a vector is irrotational?

6. State Green's Theorem Area Formula.

7. Negative orientation of the surface $z = g(x, y)$ is ______.

8. Define Divergence of a vector field.

9. Define curl of vector field.

10. Write Laplace equation.

P.T.O.

Answer any **eight** questions from among the questions 11 to 22.

These questions carry **2** marks each.

11. Find the directional derivative of $f(x, y) = xy$ at (1, 2) in the direction of $u = \frac{\sqrt{3}}{2} i + \frac{1}{2} j$.

12. Prove that curl $r = 0$.

13. Integrate $f(x, y, z) = x + \sqrt{y} - z^2$ over the path $C: r(t) = i + j + tk$, $0 \le t \le 1$ from (0, 0, 0) and (1, 1, 1).

14. Is the field $F = yzi + xzj + xyk$ conservative? Justify your answer.

15. Use the Divergence Theorem to find the outward flux of the vector field $F(x, y, z) = x^3 i + y^3 j + z^3 k$ across the surface of the region that is enclosed by the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ and the plane $z = 0$.

16. Evaluate the integral $\int_C xy\,dy - y^2 dx$, where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$.

17. Show that $F = (2x - 3)\hat{i} - z\hat{j} + (\cos z)\hat{k}$ is not conservative.

18. Show that $r^n r$ is solenoidal if $n = -3$.

19. Prove that $\nabla(\nabla \times F) = 0$.

20. If $f = x^3 + y^3 + z^3$, find the directional derivative of f at (1, 2, –1) in the direction of $2i + j + 6k$.

21. Find the work done by the conservative field $F = (yz)i + (xz)j + (xy)k$ along any smooth curve C joining the points (–1, 3, 0) to (1, 6, –4).

22. Evaluate $\int_C 2xy\,dx + (x^2 + y^2)dy$ along the circular arc C given by $x = \cos t$, $y = \sin t$ $(0 \le t \le \pi/2)$.

Answer any **six** questions from the questions 23 to 31.

These questions carry **4** marks each.

23. Show that the divergence of the inverse-square field

$$F(r) = \frac{C}{\|r\|^3} r$$

24. If $r = xi + yj + zk$ and $|r| = r$, then show that $\nabla\left(\frac{1}{r}\right) = \frac{r}{r^3}$.

25. Find the work done in moving a particle once round a circle C in the xy plane : the circle has centre at the origin and radius 3 and the force field is given by

$$F = (2x - y + z)i + (x + y - z^2)j + (3x - 2y + 4z)k.$$

26. Evaluate $\iint x^2 dS$, over the sphere $x^2 + y^2 + z^2 = 1$.

27. Verify Green's theorem for $f = y^2 - 7y$, $g = 2xy + 2x$ and C: the circle $x^2 + y^2 = 1$.

28. Find the mass of the lamina that is the portion of the circular cylinder $x^2 + z^2 = 4$ that lies directly above the rectangle $R = \{(x, y) : 0 \le x \le 1, 0 \le y \le 4\}$ in the xy-plane (Given lamina has constant density δ_0).

29. Find the flux of the vector field $F(x, y, z) = zk$ across the outward-oriented sphere $x^2 + y^2 + z^2 = r^2$.

30. If $F = xyi + xz^3 j + zy^3 k$, using Stokes's theorem, evaluate $\int_C F \cdot T ds$ where C is the circle $x^2 + y^2 = 4$, $z = -3$ oriented counterclockwise as seen by a person standing at the origin and with respect to right handed Cartesian coordinates.

31. Find the outward flux of $F = x^3 i + y^3 j + z^3 k$ across the surface of the region that is enclosed by the circular cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = 2$.

Answer any **two** questions from among the questions 32 to 35.

These questions carry **15** marks each.

32. (a) Prove that

$$div\ grad\ r^n = n(n+1)r^{n-2}.$$

(b) If F, G are differentiable vector functions, then show that

$$\text{curl } (F \times G) = (G \cdot \nabla)F - (F \cdot \nabla)G + F\, div\, G - G\, div\, F$$

33. (a) Using Green's theorem, find the work done by the force field $F = (x^2 + y^2)i - 2xyj$ on a particle that travels once around the rectangle in the xy plane bounded by $x = 0$, $x = a$, $y = 0$, $y = b$.

(b) The temperature of points in space is given by $f(x, y, z) = x^2 + y^2 - z$. A mosquito located at (1, 1, 2) desires to fly in such a direction that it will get cool as soon as possible. In what direction should it move?

34. Verify Stokes' Theorem by evaluating the integral $\int_C F \cdot dr$, where $F(x, y, z) = 3zi + 4xj + 2yk$; C is the boundary of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ with upward orientation.

35. Use the Divergence Theorem to find the flux of **F** across the surface σ with outward orientation : $F(x, y, z) = (x^2 + y)i + z^2 j + (e^y - z)k$; σ is the surface of the rectangular solid bounded by the coordinate planes and the planes $x = 3$, $y = 1$, and $z = 2$.