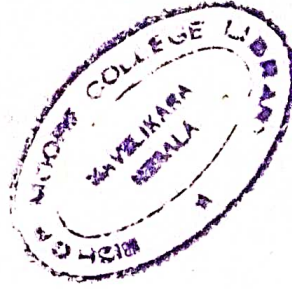


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M – 1455

Reg. No. :

Name :

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Mathematics

Core Course – VIII

MM 1545 : ABSTRACT ALGEBRA – I

(2014 , 2016 & 2017 Admission)

Time : 3 Hours

Max. Marks : 80

PART – A

Answer all the ten are compulsory. They carry 1 mark each.

1. Define Dihedral Group D_n .
2. What is the order of the permutation $(1\ 2\ 4)(3\ 5\ 7)$?
3. Write the following permutations into product of disjoint cycles
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 5 & 2 & 4 & 3 & 1 & 8 & 9 & 7 \end{pmatrix}$$
4. Find the number of elements in the cyclic subgroup of \mathbb{Z}_{30} generated by 25.
5. How many groups are there of order 17 upto isomorphism?
6. True or False "The odd permutations in S_8 form a subgroup of S_8 ". Why?
7. Define the Alternating Group.

P.T.O.

8. What is the order of the cycle (1 4 5 7)?
9. Find the generators of $\mathbb{Z}_4 \times \mathbb{Z}_3$.
10. Define an automorphism.

PART – B

Answer **any eight** questions from this section. Each question carries 2 marks.

11. Show that every group G with identity e and such that $x * x = e$ for all $x \in G$ is abelian.
12. Prove that 'Every cyclic group is abelian'.
13. Find the number of generators of cyclic groups of orders 6, 8, 12 and 60.
14. Find the multiplication table of S_3 .
15. Show that "Every group of prime order is cyclic."
16. Show that the identity element and the inverse element of an element in group G are unique.
17. Show that if $a \in G$, where G is a finite group with identity e , then there exists $n \in \mathbb{Z}^+$ such that $a^n = e$.
18. Show that if a group G with identity e has finite order n , then $a^n = e$ for all $a \in G$.
19. Show that if H is a subgroup of an abelian group G , then every left coset of H is also a right coset of H .
20. Define a transposition. Write the cycle (2,3,5,6,8,9) as the product of transposition.
21. Show that if $n \geq 3$, then the only element σ of S_n , satisfying $\sigma\gamma = \gamma\sigma$ for all $\gamma \in S_n$ is $\sigma = \iota$, the identity permutation.
22. Give an example to which converse of Lagrange's Theorem fails.

PART – C

Answer any six questions from this section. Each question carries 4 marks.

23. Prove that a subset H of a group G is a subgroup of G if and only if
- (a) H is closed under the binary operation of G .
 - (b) the identity e of G is in H
 - (c) for all $a \in H$ it is true that $a^{-1} \in H$.
24. Show that if H and K are subgroups of an abelian group G , then $\{hk \mid h \in H \text{ and } k \in K\}$ is a subgroup of G .
25. Prove that "Every permutation σ of a finite set A is a product of disjoint cycles".
26. Show that if r and s are relatively prime, then G contains a cyclic group of order rs .
27. Prove the theorem "Let A be a nonempty set, and let S_A be the collection of all permutations of A . Then S_A is a group under permutation multiplication".
28. Prove that "Let H and K are subgroups of a group G such that $K \leq H \leq G$, and suppose $(H:K)$ and $(G:H)$ are both finite. Then $(G:K)$ is finite and $(G:K) = (G:H)(H:K)$ ".
29. Show that S_n is non abelian for $n \geq 3$.
30. State and Prove Lagrange's Theorem.
31. Show that every permutation in S_n can be written as a product of at most $n-1$ transpositions.

PART – D

Answer any two questions from this section. Each question carries 15 marks.

32. Prove that "No permutation of a finite set can be expressed both as a product of an even number of transposition and as a product of an odd number of transpositions".
33. Let $\phi: A \rightarrow B$. A map $\phi^{-1}: B \rightarrow A$ is called an inverse of ϕ if $\phi(\phi^{-1}(x)) = x$ for all $x \in B$ and $\phi^{-1}(\phi(y)) = y$ for all $y \in A$.
- (a) Show that ϕ is a bijection if and only if it has an inverse.
- (b) Show that the inverse of a bijection ϕ is unique.
34. Show that, if $n \geq 2$, the collection of all even permutations of $\{1, 2, 3, \dots, n\}$ forms a subgroup of order $\frac{n!}{2}$ of the symmetric group S_n .
35. (a) Let A be an infinite set. Let H be the set of all $\sigma \in S_A$ that move only finite number of elements of A . Show that H is a subgroup of S_A .
- (b) Prove that "Let H be a subgroup of G . The relations
- $$\begin{aligned} &a \sim_L b \text{ if and only if } a^{-1}b \in H \\ \text{and} &a \sim_R b \text{ if and only if } ab^{-1} \in H \end{aligned}$$
- are equivalence relations on G ".
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