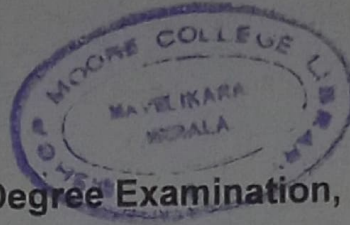


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N – 1290

Reg. No. : .....

Name : .....



Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme under CBCSS

Mathematics

Core Course XI

MM 1643 COMPLEX ANALYSIS II

(2014 & 2017 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

Answer **all** the **ten** questions are compulsory. Each question carries **1** mark.

1. Find the singular points of the function  $\frac{z+1}{z^2(z-i)}$ .
2. Find the power series expansion of  $\frac{1}{z-4}$  in a disk of radius 1 centred at  $z = 5$ .
3. Find  $\int_{|z|=2} \frac{3}{z-3} dz$ .
4. Find the residue at  $z = 0$  of the function  $\frac{1}{z+z^2}$ .
5. Find the essential singularity of  $e^{1/z}$ .
6. Find the order of zero of  $f(z) = z^3 - 8$ .

P.T.O.

7. If  $z_0$  is a pole of a function  $f$  then what is the value of  $\lim_{z \rightarrow z_0} f(z)$  ?
8. Determine the type of singularity of  $f(z) = \sin z / z$ .
9. Define Isolated singular points of a complex function with an example.
10. Determine  $\int_{|z| < 2} \frac{ze^z}{(z^2 + 9)^5} dz$ .

(10 × 1 = 10 Marks)

## SECTION – II

Answer **any eight** questions from this section. Each question carries **2** marks.

11. Find the power series expansion of  $\sin(1/z)$  around  $z = 1$ .
12. Determine the nature of all singularities of  $f(z) = \cos[1/z]$ .
13. Find the residue of the function  $f(z) = \tanh z / z^2$ .
14. Evaluate the integral  $\int_{|z|=2} \tan z \, dz$ .
15. Show that  $\text{Res}_{z=\pi i} \frac{z - \sinh z}{z^2 \sinh z} = \frac{i}{\pi}$ .
16. State Jordan's lemma.
17. Show that  $\int_{|z|=1} \exp\left(\frac{1}{z^2}\right) dz = 0$ .
18. Describe any two different types of singular points with example.
19. Show that 2 is a simple pole of  $f(z) = \frac{z^2 - 2z + 3}{z - 2}$ .
20. Determine the order  $m$  of each pole, the corresponding residue  $B$  for  $f(z) = \left(\frac{z}{2z+1}\right)^{x^3}$ .



21. Find the Cauchy principal value of the integral  $\int_{-\infty}^{\infty} \frac{x \sin x \, dx}{x^2 + 2x + 2}$ .
22. Define residue of a function  $f(z)$  at infinity.

(8 × 2 = 16 Marks)

### SECTION – III

Answer **any six** questions from this section. Each question carries **4** marks.

23. Show that  $z = \pi i / 2$  is a simple pole of  $f(z) = \tan z / z^2$ .

24. Find

(a)  $\int_{|z|=1} \sec z \, dz$

(b)  $\int_{|z|=1} \frac{dz}{z^2 + 4}$

25. Evaluate

(a)  $\oint_{|z|=3} \frac{e^z}{z-2} \, dz$

(b)  $\oint_{|z|=3} \frac{dz}{z-3i}$

26. Using Cauchy Integral formula, evaluate  $\int_{|z|=1} \frac{e^z \cos z \, dz}{\left(z - \frac{\pi}{4}\right)^3}$

27. Let two functions  $p$  and  $q$  be analytic at a point  $z_0$ . If  $p(z_0) \neq 0$ ,  $q(z_0) = 0$ , and  $q'(z_0) \neq 0$ , then show that  $z_0$  is a simple pole of the quotient  $p(z)/q(z)$  and

$$\operatorname{Res}_{z=z_0} \frac{p(z)}{q(z)} = \frac{p(z_0)}{q'(z_0)}.$$

28. Show that  $1+i$  is an isolated singularity of  $\frac{z}{z^4 + 4}$ .

29. Find the poles and residues of  $f(z) = \frac{e^z}{z^2 + \pi^2}$ .

30. Evaluate  $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$ .

31. Using Cauchy Residue theorem, evaluate  $\int_{|z|=2} \frac{\sin z}{z^6} dz$ .

(6 × 4 = 24 Marks)

#### SECTION – IV

Answer **any two** questions. Each question carries **15** marks.

32. (a) State and prove Cauchy Residue theorem.

(b) Using this, evaluate  $\int_{|z|=2} \frac{5z-2}{z(z-1)} dz$ .

33. Evaluate  $\int_0^{\infty} \frac{x^2}{x^6 + 1} dx$ .

34. (a) Find  $\int_0^{2\pi} \frac{d\theta}{5 + 4 \sin \theta}$

(b) Find  $\int_0^{2\pi} \frac{d\theta}{1 + a \cos \theta}$

35. (a) State and prove Cauchy Integral formula.

(b) Evaluate  $\int_C \frac{3z^3 + 2}{(z-1)(z^2 + 9)} dz$  taken counter clockwise around the circle  $|z-2|=2$  and  $|z|=4$ .

(2 × 15 = 30 Marks)