(Pages : 4)

K – 3229

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, February 2021

First Degree Programme Under CBCSS

Mathematics

Core Course V

MM 1542 : COMPLEX ANALYSIS – I

(2015 – 2017 Admissions)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each

1. Express $\dfrac{(3-i)(4+i)}{(2-i)}$ in the form $a+ib$.

2. Find the square roots of $2i$.

3. Show that $\operatorname{Re} z = \dfrac{z+\overline{z}}{2}$.

4. Represent geometrically $\{z/\operatorname{Re} z > 0\}$.

5. Find $|e^{i}|$.

6. State triangle inequality.

7. Express $1+i$ in polar form.

P.T.O.

8. Define Cauchy sequence in a complex plane.

9. Write the Cauchy-Riemann equations.

10. Write the power series expansion of e^{2z}.

SECTION – II

Answer any **eight** from among the questions 11 to 22. They carry **2** marks each.

11. Find the sum of the complex numbers $4+i$ and $1-i$ geometrically.

12. Find the fourth roots of unity.

13. Prove that $\{z_n\}$ converges if and only if it is a Cauchy sequence.

14. Use Cauchy-Riemann equations to verify whether $x^2 - y^2 - 2xyi$ is analytic.

15. Does the series $\sum_{k=1}^{\infty} \frac{i^k}{k^2+i}$ converge or diverge. Justify your answer.

16. Prove that an analytic function with constant modulus is a constant.

17. Evaluate $\int_C x^2 + iy^2 \, dz$ where $C : z(t) = t + it$, $0 \le t \le 1$.

18. Evaluate $\int_C \frac{1}{z} \, dz$ where $C : z(t) = \sin t + i \cos t$, $0 \le t \le 2\pi$.

19. Find the unique real solution of $x^3 + 6x = 20$ using cubic method.

20. Is the polynomial $x^3 - 3xy^2 - x + i(3x^2y - y^3 - y)$ analytic. Justify your answer.

21. Show that $x^2 + iV(x, y)$ is not analytic for any choice of the real polynomial $V(x, y)$.

22. Define a piecewise differentiable curve.

SECTION – III

Answer any **six** questions from among the questions 23 to 31. They carry 4 marks each.

23. Geometrically represent the following sets.

    (a) $\left\{z : \frac{-\pi}{3} < \arg z < \frac{\pi}{3}\right\}$

    (b) $\{z : |z+1| < 1\}$

24. Prove that $|z_1 - z_2| \geq |z_1| - |z_2|$.

25. If $\sum_{n=0}^{\infty} C_n z^n$ is zero at all points of a non-zero sequence $\{z_n\}$ which converges to zero, then prove that the power series is identically zero.

26. Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{z^n}{n^2}$.

27. If C_1 and C_2 are smoothly equivalent then prove that $\int_{C_1} f = \int_{C_2} f$.

28. Prove that $\int_C f(z)dz = F(z(b)) - F(z(a))$ where $f(z) = F'(z)$ and C is a smooth curve.

29. (a) Evaluate $\int_{\underset{\sim}{C}} (z+2i)dz$ where C is $z(t) = t + it^2$, $-1 \leq t \leq 1$.

    (b) Also find the above integral along the straight line from $-1+i$ to $1+i$.

30. (a) Show that $f(z) = x^2 + iy^2$ is differentiable at all points on the line $y = x$.

    (b) Prove that $e^{z_1+z_2} = e^{z_1}e^{z_2}$.

31. Show that e^z is an entire function by verifying Cauchy-Riemann equations.

SECTION – IV

Answer any **two** questions from among the questions 32 to 35. They carry 15 marks each.

32. (a) Prove that if $P_y = iP_x$, then the polynomial is analytic.

    (b) Hence deduce C.R. equations.

33. (a) Suppose $f(z) = \sum_{n=0}^{\infty} C_n z^n$ converges for $|z| < R$. Then prove that $f'(z)$ exists and equals $\sum_{n=0}^{\infty} nC_n z^{n-1}$ throughout $|z| < R$.

    (b) Prove that a power series is infinitely times differentiable in their domain of convergence.

34. (a) Show that the function $f(x, y) = \dfrac{xy(x+iy)}{x^2+y^2}$, $z \neq 0$ and $f(0) = 0$ satisfies C.R. equations at origin but it is not differentiable at origin.

    (b) Suppose $G(t)$ is a continuous complex value function of t. Then, prove that $\int_a^b G(t)\, dt \leq \int_a^b |G(t)|\, dt$.

35. State and prove Rectangle Theorem.