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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, July 2024

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1431.1 : MATHEMATICS IV – COMPLEX ANALYSIS, SPECIAL FUNCTIONS AND PROBABILITY THEORY

(2018-2020 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Find the real part of $\frac{1}{z}$.

2. Show that $u(x,y)=e^x \sin y$ is a harmonic function.

3. State whether $f(z)=\overline{Z}$ is analytic or not.

4. Find the residue of the function $e^{1/z}$ at $z=0$.

5. Show that $\int_C \frac{z^3}{z-3} dz=0$ where C is the circle $|z|=2$.

6. State the relation between Beta function and Gamma function.

P.T.O.

7. Define $\Gamma(p)$.

8. If the mean and variance of the Binomial distribution are 5 and 4 respectively. Find the number of trials.

9. A bag contains 3 white and 2 black balls. Find the probability of drawing a white ball.

10. A card is drawn from a well shuffled deck of cards. What is the probability of drawing an ace or a spade?

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each question carries **2** marks.

11. Show that the real and imaginary parts of an analytic function are harmonic.

12. Use Cauchy - Riemann equations to prove that the function $f(z)=\frac{x-iy}{x^2+y^2}$ is analytic.

13. Find the analytic function whose real part is $u=y^3-3x^2y$.

14. Evaluate $\int_0^{1+i}\left(x^2-iy\right)dz$ along the path $y=x$.

15. Verify Cauchy's theorem for the function $f(z)=z^2$ and C is the circle $|z|=1$.

16. Find the residue of $f(z)=\frac{e^z}{(z+1)^3}$ at its poles.

17. Express $\int_0^{\infty} x^{-2/5}e^{-x}\,dx$ as a gamma function.

18. Show that $\int_0^2 \left(8-x^3\right)^{-1/3} dx = (1/3)\,\beta(1/3, 1/3)$.

19. What is the probability that a non-leap year selected at random will contain 53 Sundays?

20. Two students are working independently on the same problem. If the first student has the probability 1/2 of solving it and the second has the probability 3/4 of solving it. Find the probability that at least one of them solves it.

21. Check whether $f(x)=6x(1-x)$ for $0 \le x \le 1$ is a probability density function.

22. A continuous random variable X has a probability density function $f(x)=k\,x(x-2), 0 \le x \le 2$. Find the constant k.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. Each question carries **4** marks.

23. Derive Cauchy- Riemann equations.

24. If a function f is analytic at a point, show that it is continuous there.

25. Find the Taylor series expansion of $\dfrac{1}{(z-1)(z-2)}$ in the region $|z|<1$.

26. Explain the different types of isolated singularities with examples.

27. Evaluate the integral $\int_c \dfrac{5z-2}{z(z-1)}dz$ where C is the circle $|z|=2$, described counter - clockwise.

28. Show that $\beta(p, 1-p) = \dfrac{\pi}{\sin(p\pi)}$.

29. Let X denote the sum of the numbers obtained when two fair dice are thrown. Find the probability density function of X.

30. Find the mean and variance of a random variable X which takes the values 0, 1, 2, 3 with respective probabilities $\frac{5}{12}, \frac{1}{3}, \frac{1}{12}, \frac{1}{6}$.

31. When two dice are thrown, find the probability that the product of numbers on the two dice is 12.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. Each question carries **15** marks.

32. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$.

(b) Using Cauchy's integral formula evaluate the integral $\int_C \frac{e^z}{(2z-1)^2}\,dz$ over the circle $|z|=1$.

33. (a) Represent Beta function in terms of $\sin\theta$ and $\cos\theta$.

(b) Show that $\beta(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$ where $m>0$, $n>0$.

34. (a) Find the mean and variance of the binomial distribution $P(r)=nCrp^r q^{n-r}$.

(b) If the chance that any of the five telephone lines is busy at an instant is 0.001, what is the probability that all lines are busy? What is the probability that not more than 3 lines are busy?

35. (a) Derive the Poisson probability density function $P(x)=\frac{\lambda^x}{x!}e^{-\lambda}$.

(b) A manufacturer knows that the condenser be makes contains an average 1% of defectives. He packs them in boxes of 100. What is the probability that a box picked at random will contain 3 or more faulty condensers?

(2 × 15 = 30 Marks)