(Pages : 4) **L – 3943**

Reg. No. : ....

Name : ....

First Semester B.C.A. Degree Examination, August 2021

Career Related First Degree Programme under CBCSS

Mathematics

Complementary Course I

MM 1131.9 : MATHEMATICS I

(2020 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first questions are compulsory. **Each** question carries **1** mark.

1. Find $\frac{d(\operatorname{sech} x)}{dx}$.

2. Find the derivative of $\sec^{-1} x^2$.

3. Show that $\cosh 2x = 1 + 2(\sinh x)^2$.

4. State Rolle's Theorem.

5. Show that $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}; s > a$.

6. Give an example of second degree first order differential equation.

7. Solve the differential equation $\left(\frac{d^2y}{dx^2} - 1\right)y = 0$.

8. Define Relatively Prime Numbers.

9. State Fermat's Theorem.

10. Express $-1+i\sqrt{3}$ in the form $r(\cos\theta + i\sin\theta)$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions among the questions 11 to 26. They carry **2** marks each.

11. Find $\frac{dy}{dx} = x^y$

12. Show that $\sinh 2x = 2\sinh x \cosh x$

13. Find $\frac{dy}{dx}$ when $x = \cos t$ and $y = \sin t$

14. Verify Rolle's Theorem for the function $f(x) = 16x - x^2$ in then interval [0, 16]

15. Solve the differential equation $\frac{d^2y}{dx^2} + y = 0$

16. Find the integrating factor for the differential equation $\frac{dy}{dx} + y\tan x = \cos^3 x$

17. Find $\mathcal{L}\{te^t\}$

18. Find the sum of divisors of 480

19. Give an example to show that $a^2 \equiv b^2$ (mod n) need not imply that $a \equiv b$ (mod n)

20. Find the remainders when 2^{50} is divided by 7

21. Show that $f(z) = z$ is analytic

22. Separate into real and imaginary parts cosh $(\alpha + i\beta)$

23. Find the derivative of $\cos z + e^z$, where $z = x + iy$

24. Formulate a Linear Programming problem for the following "Maximise the perimeter of a rectangle with sum of length and breadth do not exceed 7 and their difference must be greater than 5"

25. Explain Pure Birth process in Markov Process

26. Write the Fourier Series of the even function $f(x)$ with the period of $2L$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions among the questions 27 to 38. They carry **4** marks each.

27. Find the maxima and minima of $x^2 + xy + y^2 + 3x - 3y + 4$

28. Solve $x\frac{dy}{dx} - y\tan x = \frac{\sin x\cos^2 x}{y^2}$

29. Solve $\frac{d^3y}{dx^3} + y = 0$

30. Find the inverse Laplace Transform of $\frac{3s+1}{(s-1)(s^2+1)}$

31. Find the n the derivative of $e^x \ln x$

32. Show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial z^2} = 0$, for the function $e^{-2y}\cos 2x$

33. Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 10y = e^{2x}$

34. Find the remainder when 2^{1000} is divisible 17

35. If p and q are distinct primes, prove that

$$p^{q-1} + q^{p-1} \equiv 1 \,(mod\ pq)$$

36. Show that 16! + 86 is a multiple of 323

37. Prove that the integer $53^{103} + 103^{53}$ is divisible by 39

38. Expand the function $f(x) = x^2,\ 0 < x < 2\pi$ in a Fourier Series.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions among the questions 39 to 44. They carry **15** marks each.

39. (a) State and prove Leibnitz's Theorem

(b) Show that the maximum value of $\frac{1}{x^x}$ is $e^{1/e}$

40. Find the general solution of the partial differential equation $xz\frac{\partial z}{\partial x} + yz\frac{\partial z}{\partial y} = xy$

41. Show that $a^7 \equiv a \,(\text{mod } 42)$ for any positive integer a

42. Show that 41 divides $2^{20} - 1$

43. (a) Show that $f(x + iy) = x - iy$ is not analytic

(b) Solve $x^5 + 32 = 0$.

44. Maximize $5x-y$ subject to the constraints : $2x + 5y \le 80;\ x + y \le 20,\ x, y \ge 0$.

(2 × 15 = 30 Marks)