(Pages : 4) **N – 7754**

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, August 2022.

First Degree Programme under CBCSS

Mathematics

Core Course – III

MM 1441 – ELEMENTARY NUMBER THEORY AND CALCULUS – II

(2019 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

All the first **ten** questions are compulsory. Each carries **1** mark:

1. Represent 'n is divisible by 5' using congruence symbol.
2. Find the digital root of 1976.
3. Write the self-invertible least residues modulo 7.
4. State Wilson's theorem.
5. Evaluate $\int_0^1\int_0^2 (x+3)\,dy\,dx$.
6. Write the vector form of the paraboloid $x=u, y=v, z=4-u^2-v^2$.
7. Find the partial derivatives of the vector valued function $r=u\hat{i}+v\hat{j}+(4-u^2-v^2)\hat{k}$.
8. Define a vector field.

P.T.O.

9. If $F(x,y,z) = f(x,y,z)\hat{i} + g(x,y,z)\hat{j} + h(x,y,z)\hat{k}$. Write divergence of F.

10. Write the line integral of a continuous vector field F along a smooth oriented curve C.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. Each carries **2** marks.

11. Prove that $a \equiv b (\text{mod}\, m)$ if and only if $a = b + km$ for some integer k.

12. Write the congruence classes modulo 5.

13. Find the remainder when 1! + 2! + 3! + ... + 100! is divided by 15.

14. Show that a palindrome with even number of digits is divisible by 11.

15. Determine whether 52 is a solution of the system of linear congruences

$x \equiv 2 (\text{mod}\, 5)$

$x \equiv 3 (\text{mod}\, 7)$

16. Prove that, if a positive integer a is self-invertible modulo p then a $\equiv \pm 1$(mod p).

17. Evaluate $\int_1^3 \int_2^4 (40 - 2xy)\, dy\, dx$.

18. Define a simple polar region.

19. Evaluate the double integral $\iint_R y^2 x\, dA$ over the rectangle $R = \{(x,y) : -3 \le x \le 2, 0 \le y \le 1\}$.

20. Evaluate $\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2}\, dx\, dy$

21. Use a polar double integral to find the area enclosed by the three-petaled rose $r = \sin 3\theta$.

22. Find the curl of the vector field $F(x,y,z) = x^2 y\hat{i} + 2y^3 z\hat{j} + 3z\hat{k}$.

23. Using the parametrization $C : r(t) = t\hat{i} + 2t\hat{j} (0 \le t \le 1)$, evaluate the integral $\int_C (1 + xy^2)\, ds$.

24. Evaluate the line integral of the continuous vector field $F(x,y) = \cos x\hat{i} + \sin x\hat{j}$ along the curve $C : r(t) = -\frac{\pi}{2}\hat{i} + t\hat{j} (-1 \le t \le 2)$

25. Using conservative field test determine whether the vector field $F(x,y) = (y+x)\hat{i} + (y-x)\hat{j}$ is conservative on some open set.

26. Let $F(x,y) = e^y\hat{i} + xe^y\hat{j}$. Verify whether the force field F is conservative on the entire xy-plane.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each carries **4** marks.

27. Find the positive integers n for which $\sum_{k=1}^{n} k!$ is a square.

28. Prove that the digital root of the product of twin primes, other than 3 and 5, is 8.

29. Solve the linear system,

$x \equiv 1 \pmod 3$

$x \equiv 2 \pmod 4$

$x \equiv 3 \pmod 5$

30. Prove that, if p is a prime, then $(p-1)! \equiv -1 \pmod p$.

31. Using double integral find the volume of the solid bounded above by the plane $z = 4 - x - y$ and below by the rectangle $R = [0,1] \times [0,2]$.

32. Evaluate $\iint_R \sin\theta \, dA$ where R is the region in the first quadrant that is outside the circle $r = 2$ and inside the cardioid $r = 2(1+\cos\theta)$.

33. Find the surface area of the portion of the paraboloid $z = x^2 + y^2$ below the plane $z = 1$.

34. Use triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$.

35. Show that the divergence of the inverse-square field $F(x,y,z) = \frac{c}{(x^2+y^2+x^2)^{\frac{3}{2}}}(x\hat{i} + y\hat{j} + z\hat{k})$ is zero.

36. Evaluate $\int_C 2xy\,dx + (x^2+y^2)\,dy$ along the circular arc C given $x = \cos t, y = \sin t \left(0 \le t \le \frac{\pi}{2}\right)$.

37. Use a line integral to find the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

38. Evaluate the surface integral $\iint_\sigma x^2 dS$ over the sphere $x^2+y^2+z^2=1$.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions

39. Prove that the linear system of congruences $x \equiv a_i (\bmod m_i)$ where the moduli are pairwise relatively prime and $1 \le i \le k$, has a unique solution modulo $m_1 m_2, \ldots, m_k$.

40. (a) Let p be a prime and a any integer such that $p \nmid a$. Then prove that $a^{p-1} \equiv 1 (\bmod p)$. 7

(b) Find the primes p for which $\frac{2^{p-1}-1}{p}$ is a square. 8

41. Use cylindrical coordinates to evaluate $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{9-x^2-y^2} x^2 \, dz\, dy\, dx$.

42. Evaluate $\iint_R e^{xy} \, dA$ where R is the region enclosed by the lines $y=\frac{1}{2}x$ and $y=x$ and the hyperbolas $y=\frac{1}{x}$ and $y=\frac{2}{x}$.

43. Evaluate the integral $\oint_C \frac{-y dx + x dy}{x^2+y^2}$ if C is a piece—wise smooth simple closed curve oriented counterclockwise such that,

(a) c does not enclose the origin 5

(b) c encloses the origin 10

44. Let G be simultaneously a simple xy-solid, a simple yz-solid and a simple zx-solid whose surface σ is oriented outward, If $F(x,y,z)=f(x,y,z)\hat{i}+g(x,y,z)\hat{j}+h(x,y,z)\hat{k}$ where f, g, and h have continuous first partial derivatives on some open set containing G, and if $\hat{n}$ is the outward unit normal on σ, then prove that $\iint_\sigma F\cdot\hat{n}\, dS = \iiint_G divF \, dV$.

(2 × 15 = 30 Marks)

____