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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, August 2021.

First Degree Programme under CBCSS

Mathematics

Complementary Course I for Chemistry and Polymer Chemistry

MM 1131.2: MATHEMATICS I - CALCULUS WITH APPLICATIONS IN CHEMISTRY I

(2020 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each:

1. Find the first derivative of x^2e^x.

2. Find the 100^{th} derivative of x^8+3x^5+10.

3. State Leibnitz's Theorem.

4. Express $\frac{1}{i}$ in the form $a+bi$.

5. Define modulus of a complex number.

6. Find the complex conjugate of $2+i$.

P.T.O.

7. Two particles have velocities $v_1 = i + 3j + 6k$ and $v_2 = i - 2k$, respectively. Find the velocity u of the second particle relative to the first.

8. Find the unit vector corresponding to the vector $i + 3j + k$.

9. Evaluate $\int \ln x \, dx$.

10. Evaluate $\int x \cos x \, dx$.

SECTION – II

Answer **any eight** questions from among the questions 11 to 26. These questions carry **2** marks each.

11. Find the first derivative of $\frac{x-1}{x-2}$.

12. Define stationary point.

13. If $x = t^2 - 4$ and $y = t^3 - 3t$, find $\frac{dy}{dx}$.

14. Find the derivative of $3^{\sqrt{x}}$ with respect to x.

15. Prove that $\overline{z_1 z_2} = \overline{z_1} . \overline{z_2}$.

16. Find argument of $2 + 2i$.

17. State Demoivre's Theorem.

18. Find the solutions to the equation $z^3 = 1$

19. Find the angle between the vectors $i + j + k$ and $i + 2j + 3k$.

20. If $a = 4i - 8j + 2k$, $b = 6i + 3j + 7k$ and $c = 9i + 6j + 3k$, find $[a, b, c]$.

21. If $v = 3i - 4j$ is a velocity vector, then find the speed.

22. Find the direction of the line of intersection of the two planes $x + 3y - z = 5$ and $2x - 2y + 4z = 3$.

23. Evaluate $\int x \sin 3x \, dx$.

24. Evaluate $\int_0^{\infty} \frac{x}{(x^2 + a^2)^2} dx$.

25. Evaluate $\int_0^2 \frac{1}{(2 - x)^{\frac{1}{4}}} dx$.

26. Find the mean value m of the function $f(x) = 3x^2 - 3$ between the limits $x = 0$ *and* $x = 1$.

SECTION – III

Answer **any six** questions from among the questions 27 to 38. These questions carry **4** marks each.

27. Find $\frac{dy}{dx}$, if $4x^2y^7 - 2x = x^5 + 4y^3$.

28. Verify Mean Value Theorem for differentiation for the function $x^2 + 2x - 1 = 0$ on $[0,1]$.

29. Prove that $Sinh 2x = 2 \sinh x \cosh x$.

30. Find the value of $Ln(-2)$.

31. Find the value of p for which the vectors $3i+2j+9k$ and $i+pj+3k$ are

(a) Perpendicular

(b) Parallel

32. Find the area of the parallelogram with sides $i+2j+3k$ and $3i-2j+k$.

33. Four non-coplanar points A, B, C, D are positioned such that the line AD is perpendicular to BC and BD is perpendicular to AC. Show that CD is perpendicular to AB.

34. Find the minimum distance from the point P with coordinates (1,6,3) to the line $r=(j+2k)+\lambda(i+2j+3k)$

35. The equation *in* polar coordinates of an ellipse with semi-axes a *and* b is $\frac{1}{\rho^2}=\frac{\cos^2\phi}{a^2}+\frac{\sin^2\phi}{b^2}$ Find the area of the ellipse.

36. Find the length of the curve y = In sec x between the points given by x = 0 and $x = \frac{\pi}{3}$.

37. Find the area of surface formed by rotating the curve $y=2\sqrt{x}, 1\le x\le 2$ about the x-axis.

38. Find the volume of a cone enclosed by the surface formed by rotating the curve y = 2x about the x – axis the line between x = 0 *and* $x=h$.

SECTION – IV

Answer **any two** questions from among the questions 39 to 44. These questions carry 15 marks each.

39. (a) Find the magnitude of radius of curvature at a point (x,y) on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

    (b) Find the positions and stationary points of the function $f(x)=2x^3-3x^2-36x+2$.

40. (a) State Rolle's Theorem.

    (b) Which of the following functions satisfies the conditions for Rolle's Theorem on the given interval and which do not.

    (i) $f(x)=x^{\frac{2}{3}};[-1,5]$

    (ii) $g(x)=x^{\frac{3}{4}};\ [0,2]$

    (iii) $h(x)=\sqrt{x(1-x)};[0,1]$

41. (a) Find the value of i^{2i}.

    (b) Express $\tanh^{-1}x$ in logarithmic form.

42. Evaluate the following

    (a) $\int e^{ax}\cos bx\,dx$

    (b) $\frac{d}{dx}(\sinh^{-1}x)$.

43. (a) The vertices of triangle ABC have position vectors a, b *and* c relative to some origin *0*. Find the position vector of the centroid G of the triangle.

(b) Find the shortest distance between the lines $r_1 = (4i - j) + \lambda(i + 2j - 3k)$ and $r_2 = (i - j + 2k) + \mu(2i + 4j - 5k)$.

44. (a) Using integration by parts, find a relationship between I_n and I_{n-1} where $I_n = \int_0^1 (1 - x^3)^n dx$ and n is any positive integer. Hence evaluate $I_2 = \int_0^1 (1 - x^3)^2 dx$.

(b) Show that the value of the integral $I = \int_0^1 \frac{1}{(1 + x^2 + x^3)^{\frac{1}{2}}} dx$ lies between 0.810 and 0.882.
