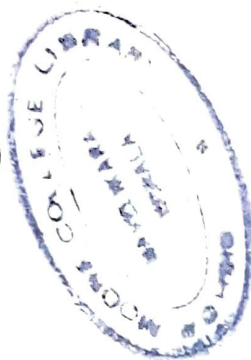


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K – 3229

Reg. No. :

Name :



Fifth Semester B.Sc. Degree Examination, February 2021

First Degree Programme Under CBCSS

Mathematics

Core Course V

MM 1542 : COMPLEX ANALYSIS – I

(2015 – 2017 Admissions)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first ten questions are compulsory. They carry 1 mark each

1. Express $\frac{(3-i)(4+i)}{(2-i)}$ in the form $a+ib$.
2. Find the square roots of $2i$.
3. Show that $\operatorname{Re} z = \frac{z+\bar{z}}{2}$.
4. Represent geometrically $\{z/\operatorname{Re} z > 0\}$.
5. Find $|e^i|$.
6. State triangle inequality.
7. Express $1+i$ in polar form.

P.T.O.

8. Define Cauchy sequence in a complex plane.
9. Write the Cauchy-Riemann equations.
10. Write the power series expansion of e^{2z} .

SECTION – II

Answer any **eight** from among the questions 11 to 22. They carry **2** marks each.

11. Find the sum of the complex numbers $4+i$ and $1-i$ geometrically.
12. Find the fourth roots of unity.
13. Prove that $\{z_n\}$ converges if and only if it is a Cauchy sequence.
14. Use Cauchy-Riemann equations to verify whether $x^2 - y^2 - 2xyi$ is analytic.
15. Does the series $\sum_{k=1}^{\infty} \frac{i^k}{k^2 + i}$ converge or diverge. Justify your answer.
16. Prove that an analytic function with constant modulus is a constant.
17. Evaluate $\int_C x^2 + iy^2 dz$ where $C: z(t) = t + it, 0 \leq t \leq 1$.
18. Evaluate $\int_C \frac{1}{z} dz$ where $C: z(t) = \sin t + i \cos t, 0 \leq t \leq 2\pi$.
19. Find the unique real solution of $x^3 + 6x = 20$ using cubic method.
20. Is the polynomial $x^3 - 3xy^2 - x + i(3x^2y - y^3 - y)$ analytic. Justify your answer.
21. Show that $x^2 + iV(x, y)$ is not analytic for any choice of the real polynomial $V(x, y)$.
22. Define a piecewise differentiable curve.

SECTION – III

Answer any **six** questions from among the questions 23 to 31. They carry 4 marks each.

23. Geometrically represent the following sets.

(a) $\left\{ z : \frac{-\pi}{3} < \arg z < \frac{\pi}{3} \right\}$

(b) $\{ z : |z + 1| < 1 \}$

24. Prove that $|z_1 - z_2| \geq |z_1| - |z_2|$.

25. If $\sum_{n=0}^{\infty} C_n z^n$ is zero at all points of a non-zero sequence $\{z_n\}$ which converges to zero, then prove that the power series is identically zero.

26. Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{z^n}{n^2}$.

27. If C_1 and C_2 are smoothly equivalent then prove that $\int_{C_1} f = \int_{C_2} f$.

28. Prove that $\int_C f(z) dz = F(z(b)) - F(z(a))$ where $f(z) = F'(z)$ and C is a smooth curve.

29. (a) Evaluate $\int_C (z + 2i) dz$ where C is $z(t) = t + it^2$, $-1 \leq t \leq 1$.

(b) Also find the above integral along the straight line from $-1 + i$ to $1 + i$.

30. (a) Show that $f(z) = x^2 + iy^2$ is differentiable at all points on the line $y = x$.

(b) Prove that $e^{z_1 + z_2} = e^{z_1} e^{z_2}$.

31. Show that e^z is an entire function by verifying Cauchy-Riemann equations.

SECTION - IV

Answer any **two** questions from among the questions 32 to 35. They carry 15 marks each.

32. (a) Prove that if $P_y = iP_x$, then the polynomial is analytic.
(b) Hence deduce C.R. equations.
33. (a) Suppose $f(z) = \sum_{n=0}^{\infty} C_n z^n$ converges for $|z| < R$. Then prove that $f'(z)$ exists and equals $\sum_{n=0}^{\infty} nC_n z^{n-1}$ throughout $|z| < R$.
(b) Prove that a power series is infinitely times differentiable in their domain of convergence.
34. (a) Show that the function $f(x, y) = \frac{xy(x+iy)}{x^2+y^2}$, $z \neq 0$ and $f(0) = 0$ satisfies C.R. equations at origin but it is not differentiable at origin.
(b) Suppose $G(t)$ is a continuous complex value function of t . Then, prove that
$$\int_a^b G(t) dt \leq \int_a^b |G(t)| dt.$$
35. State and prove Rectangle Theorem.
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