

Reg. No. :

Name :

Fourth Semester B.Sc. Degree Examination, July 2024

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

**MM 1431.1 : MATHEMATICS IV- FOURIER SERIES, COMPLEX ANALYSIS
AND PROBABILITY THEORY**

(2021 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **all** questions. Each question carries **1** mark.

1. State Dirichlet conditions for the convergence of a fourier series of a function $f(x)$ of period 2π .
2. State linearity property of Fourier Transform.
3. State the formula for the Fourier transform of $f(x)$.
4. State De Moivre's theorem.
5. Find the residue of $\frac{\sin z}{z}$ at $z = 0$.
6. Define residue.
7. Find the real part of $\frac{1}{z}$.

8. A number selected from numbers 1 to 10 at random. What is the probability of choosing an odd number?
9. Find the Binomial distribution whose mean is 9 and variance is 6.
10. Define Sample space.

(10 × 1 = 10 Marks)

PART – B

Answer any **eight** questions. Each question carries **2** marks.

11. Write down the Euler formula for calculating the Fourier coefficient.
12. Find the Fourier series of the function $f(x) = x, -\pi < x < \pi$.
13. Find the Fourier Cosine transform of $f(x) = e^{-ax}$, where $a > 0$.
14. If $F(s)$ is the complex Fourier transform of $f(x)$, then show that $F(f(x - a)) = e^{isa} F(s)$.
15. Calculate the residue of $\frac{z+1}{z^2-2z}$ at its poles.
16. Distinguish between pole and essential singular points.
17. Let $f(z) = z^2 + 3z$. Find U and V and Calculate the value of f at $z = 1 + 4i$.
18. State Cauchy's Integral formula.
19. What is the probability of showing two heads and one tail. When three coins are tossed?
20. Given $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{1}{3}$, evaluate $P(A/B)$ and $P(A \cap B')$.
21. What is the chance that a leap year selected at random will contain 53 Sundays?
22. A letter of English alphabet is chosen at random. What is the probability that it is one of the letter in the word probability.

(8 × 2 = 16 Marks)

PART – C

Answer any **six** questions. Each question carries **4** marks.

23. Find the Fourier series of the function $f(x) = \begin{cases} 0 & \text{if } -2 < x < -1 \\ k & \text{if } -1 < x < 1 \\ 0 & \text{if } 1 < x < 2 \end{cases}$.
24. Prove that $U = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate.
25. If 3% of electric bulbs manufactured by a company are defective. Find the probability that in a sample of 100 bulbs exactly five bulbs are defective.
26. Evaluate $\int_C \frac{4z-3}{z(z-1)} dz$. Where C is the circle $|z|=2$.
27. A random variable x has the following probability function
- | | | | | | | |
|-----------------|-----|-----|-----|------|-----|-----|
| Values of x : | -2 | -1 | 0 | 1 | 2 | 3 |
| $p(x)$: | 0.1 | k | 0.2 | $2k$ | 0.3 | k |
- Find the value of k and calculate $P(X < 0)$ and $P(X > 1)$.
28. Find the Fourier transform of $f(x)$ given by
- $$f(x) = \begin{cases} 1-x^2 & \text{if } |x| < 1 \\ 0 & \text{otherwise} \end{cases}$$
29. Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent's series valid for $1 < |z| < 3$.
30. Suppose two dice are thrown. If X denote the sum of the numbers on the dice, find the probability density function of X .
31. Find the Fourier sine transform of $e^{-|x|}$.

(6 × 4 = 24 Marks)

PART – D

Answer any **two** questions. Each question carries **15** marks.

32. (a) Find the Fourier transform of e^{-ax^2} where $a > 0$.

(b) Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2 & \text{for } |x| \leq a \\ 0 & \text{for } |x| > a \end{cases}$

Hence deduce that $\int_0^\infty \frac{\sin t - t \cos t}{t^3} dt = \frac{\pi}{4}$.

33. (a) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where C is $|z| = 3$.

(b) Prove that the function $f(z) = e^x (\cos y - i \sin y)$ is nowhere differentiable.

34. (a) Evaluate $\int_0^{2\pi} \frac{1}{5 + 3 \cos \theta} d\theta$.

(b) X is a normal variate with mean 30 and S.D 5, find the probabilities that

(i) $26 \leq X \leq 40$,

(ii) $X \geq 45$ and

(iii) $|X - 30| > 5$.

35. (a) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D of the distribution.

(b) Out of 800 families with 5 children each, how many would you expect to have

(i) 3 boys,

(ii) 5 girls and

(iii) Either 2 or 3 boys?

Assume equal probabilities for boys and girls.

(2 × 15 = 30 Marks)