Eléctricité

Contrôle N°3 :

1) a) $\Phi = \iint \vec{E}\,\vec{dS}$

$= E \iint dS$

$= E \iint r^2 \sin\theta \, d\theta \, d\varphi$

$= E r^2 \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi$

$= E r^2 \left(-\cos\theta\right)_0^{\pi} \, 2\pi$

$\boxed{\Phi = 4\pi E r^2}$

b) pour $r < R_1$

on a $\Phi = E r^2 4\pi = \frac{q_{int}}{\varepsilon_0}$

et $q_{int} = \iiint \rho \, dv$

$= \iiint \rho r^2 \sin\theta \, d\theta \, dr \, d\varphi$

$= \rho \int_{R_1}^{r} r^2 dr \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi$

puisque $\rho = 0$ (car la charge est nulle dans la zone où $r < R_1$)

donc $q_{int} = 0 \Rightarrow \boxed{E = 0}$

c) pour $R_1 < r < R_2$

on a $dq = \rho \, dV$

donc $q_{int} = \rho \iiint dV$

$= \rho \iiint r^2 dr \sin\theta \, d\theta \, d\varphi$

$= \rho \int_{R_1}^{r} r^2 dr \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi$

$= \rho \left(\frac{r^3 - R_1^3}{3}\right) . 2 . 2\pi$

$\boxed{q_{int} = 4\pi \rho \frac{r^3 - R_1^3}{3}}$

$\Phi = \iiint \vec{E}\,\vec{dS} \Rightarrow \Phi = E \iiint dS$

$\frac{q_{int}}{\varepsilon_0} = E r^2 4\pi$

$\frac{4\pi \rho (r^3 - R_1^3)}{3\varepsilon_0} = E r^2 4\pi$

$E = \frac{\rho (r^3 - R_1^3)}{3\varepsilon_0 r^2}$

$\boxed{E = \frac{\rho}{3\varepsilon_0}\left(\cancel{r^3}_{3} - \frac{R_1^3}{r^2}\right)}$

d/ pour $r > R_2$

$$dq = \rho\, dV$$

$$q_{int} = \iiint \rho\, dV$$

$$= \rho \int_{R_1}^{R_2} r^2 dr \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\varphi$$

$$= \rho \left[\frac{r^3}{3}\right]_{R_1}^{R_2} [-\cos\theta]_0^{\pi} [\varphi]_0^{2\pi}$$

$$q_{int} = \frac{4\pi\rho}{3}(R_2^3 - R_1^3)$$

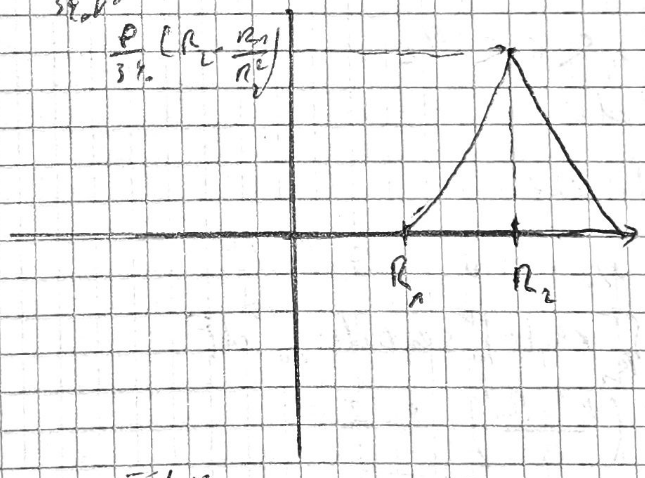
$$\phi = E r^2 4\pi = \frac{q_{int}}{\varepsilon_0}$$

$$\Leftrightarrow \quad E r^2 4\pi = \frac{4\pi\rho(R_2^3 - R_1^3)}{3\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\rho}{3\varepsilon_0 r^2}(R_2^3 - R_1^3)}$$

e) La courbe de $E(r)$

$$E(r) = \begin{cases} 0 & ; r < R_1 \\ \frac{\rho}{3\varepsilon_0}\left(r - \frac{R_1}{r^2}\right) & , R_1 < r < R_2 \\ \frac{\rho}{3\varepsilon_0 r^2}(R_2^3 - R_1^3) & , r > R_2 \end{cases}$$



2) on a $\vec{E} = -\vec{grad}\, V$

$$E = -\frac{dV}{dr}$$

$$dV = -E\, dr$$

$$V = -\int E\, dr$$

a) pour r ~~>~~ R_2

$$V = -\int_{R_1}^{R_2} \frac{\rho}{3\varepsilon_0 r^2}(R_2^3 - R_1^3)\, dr$$

$$V = -\frac{\rho(R_2^3 - R_1^3)}{3\varepsilon_0}\int_{R_1}^{R_2}\frac{1}{r^2}\,dr$$

$$\boxed{V = \frac{\rho(R_2^3 - R_1^3)}{3\varepsilon_0}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)}$$

b) pour $R_1 < r < R_2$

$$V = -\int E\,dr$$

$$= -\int_{R_1}^{r}\frac{\rho}{\varepsilon_0}\left(\frac{r}{3} - \frac{R_1^3}{3r^2}\right)dr$$

$$= -\frac{\rho}{\varepsilon_0}\int_{R_1}^{r} r^3\,dr + \frac{\rho}{3\varepsilon_0}R_1^3\int dr$$

$$V = -\int_{R_1}^{r}\frac{\rho}{\varepsilon_0}\left(\frac{r}{3} - \frac{R_1^3}{3r^2}\right)dr$$

$$= -\frac{\rho}{3\varepsilon_0}\left(\frac{r^2 - R_1^2}{2} + R_1^3\left(\frac{1}{r} - \frac{1}{R_1}\right)\right)$$

$$= -\frac{\rho(r^2 - R_1^2)}{6\varepsilon_0} + \frac{R_1^4 - R_1 r}{R_1}$$

$$\boxed{V = (r - R_1)\left(\frac{\rho(r + R_1)}{6\varepsilon_0} + \frac{1}{r}\right)}$$

c) pour $r < R_1$

$$V = -\int E\,dr$$

$$\boxed{V = cte}$$

3) a)

b) $\vec{E} = \frac{q_{int}}{4\pi\varepsilon_0 R_2^2}\vec{u_r}$

avec $dq = \sigma\,ds$

$$q_{int} = \iint \sigma\,ds$$

$$= \iint \sigma R_2^2 \sin\theta\,d\theta\,d\varphi$$

$$= \sigma R_2^2 \int_0^{\pi}\sin\theta\,d\theta\int_0^{2\pi}d\varphi$$

$$\boxed{q_{int} = \sigma R_2^2 4\pi}$$

d'où $\boxed{\vec{E} = \frac{\sigma}{\varepsilon_0}\vec{u_r}}$

c) on a $\vec{E} = -\vec{\text{grad}}\, V$

$$V = -\int_0^{R_2} E\, dr$$

$$= -\int_0^{R_2} \frac{\sigma}{\varepsilon_0}\, dr$$

$$\boxed{V = -\frac{\sigma}{\varepsilon_0} R_2}$$