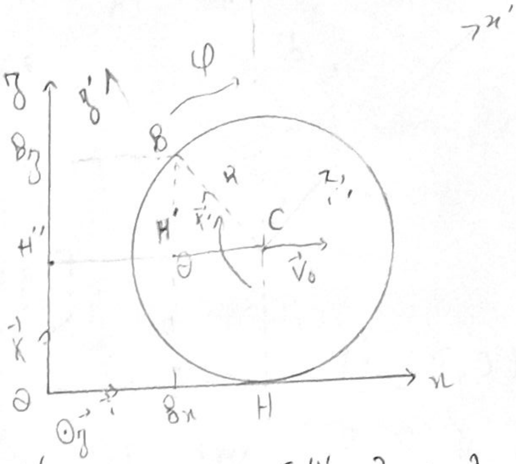
Serie N° 2

Exercice 1 :

1)



$R(O, \vec{i}, \vec{j}, \vec{K})$

R absolu

à $t = 0$ $\quad \theta = 0$

- $x(t) = OB_x = OH - B_x H$

$OH = \widehat{HB} = R\theta$

$\cos\left(\theta - \frac{\pi}{2}\right) = \frac{CH'}{R} = \sin\theta \Rightarrow CH' = R\sin\theta$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$\sin(a-b) = \sin a \cancel{\cos} b - \cos a \sin b$

$B_x H = CH' = R\sin\theta$

$x(t) = R\theta - \sin\theta\, R = R(\theta - \sin\theta)$

- $z(t) = OH'' + H''B_z$

$\sin\left(\theta - \frac{\pi}{2}\right) = \frac{H''B_z}{R} = \frac{H'B}{R} = -\cos\theta \Rightarrow H'B = -R\cos\theta$

$z(t) = R(1 - \cos\theta)$

$\vec{OB} = x(t)\vec{i} + z(t)\vec{K}$

$= R(\theta - \sin\theta)\vec{i} + R(1-\cos\theta)\vec{K}$

2) $\vec{V}(B) = \vec{V}(B/R) = \left.\frac{d\vec{OB}}{dt}\right)_R = \frac{d}{dt}\left(R(\theta - \sin\theta)\vec{i} + R(1-\cos\theta)\vec{K}\right)$

$= R(\dot\theta - \dot\theta\cos\theta)\vec{i} + R\dot\theta\sin\theta\vec{K}$

$\vec{V_B}(\theta) = \vec{V_a}(B) = R\dot\theta\left[(1-\cos\theta)\vec{i} + \sin\theta\vec{K}\right]$

$\vec{V_B}(0) = \vec{0}$

$V_B\left(\frac{\pi}{2}\right) = R\dot\theta(\vec{i} + \vec{K})$

$V_B(\pi) = 2R\dot\theta\vec{i}$

$\vec{V_B}\left(\frac{3\pi}{2}\right) = R\dot\theta(\vec{i} - \vec{K})$

$\vec{V_B}(2\pi) = \vec{0}$

3) Accélération,

$$\vec{\gamma}_a(B) = \vec{\gamma}_{(B/M)} = \left.\frac{d(\vec{V}(B/M))}{dt}\right)_R = \frac{d}{dt}\left(R\dot{\theta}\left[(1-\cos\theta)\vec{i} + \sin\theta\vec{K}\right]\right)$$

$$= R\ddot{\theta}\left((1-\cos\theta)\vec{i} + \sin\theta\vec{K}\right)$$

$$+ R\dot{\theta}\left[\dot{\theta}\sin\theta\vec{i} + \dot{\theta}\cos\theta\vec{K}\right]$$

$$= \left[R\ddot{\theta}(1-\cos\theta) + R\dot{\theta}^2\sin\theta\right]\vec{i} + \left[R\ddot{\theta}\sin\theta + R\dot{\theta}^2\cos\theta\right]\vec{K}$$

$R'(C, \vec{i}', \vec{j}, \vec{K}')$ $\quad \varphi = (\widehat{\vec{j}, \vec{j}}) = (\widehat{\vec{i}', \vec{i}})$

$\varphi = \pi - \theta$ $\quad$ avec $\vec{\Omega}_{R'/R} = \dot{\varphi}\vec{j}$

$$\left.\frac{d\vec{i}'}{dt}\right)_R = \left.\frac{d\vec{i}'}{dt}\right)_{R'}^{=0} + \vec{\Omega}_{R'/R} \wedge \vec{i}'$$

$$\vec{i}' = \cos\varphi\,\vec{i} + \sin\varphi\,\vec{K}$$

$$\left.\frac{d\vec{i}'}{dt}\right)_R = \dot{\varphi}(-\sin\varphi + \cos\varphi\,\vec{K}) = \dot{\varphi}\vec{K}'$$

$$= \dot{\varphi}\,\vec{i}\wedge\vec{j} = -\dot{\varphi}\,\vec{j}\wedge\vec{i}$$

$$\vec{V}_a(B) = \vec{V}_r(B) + \vec{V}_e(B)$$

$$\vec{V}_r(B) = \left.\frac{d\vec{CB}}{dt}\right)_{R'} = \left.\frac{dR\vec{K}'}{dt}\right)_{R'} = \vec{0}$$

$$\vec{V}_e(B) = \vec{V}_a(C) + \Omega_{R'/R} \wedge \vec{CB}$$

$$= \left.\frac{d\vec{OC}}{dt}\right)_R + \dot{\varphi}\vec{j} \wedge \vec{RK'}$$

$$\vec{OC} = OH\,\vec{i} + HC\,\vec{K}$$

$$= R\theta\,\vec{i} + R\vec{K}$$

$$\left.\frac{d\vec{OC}}{dt}\right)_R = R\dot{\theta}\,\vec{i}$$

$$\vec{j}\wedge\vec{CB} = \vec{j}\wedge(-\sin\varphi\,\vec{i} + \cos\vec{K})R$$

$$\vec{CB} = -R\sin\varphi\,\vec{i} + R\cos\varphi\,\vec{K}$$

$$\vec{j}\wedge\vec{CB} = R\sin\varphi\,\vec{K} + R\cos\varphi\,\vec{i}$$

$$\dot{\varphi} = \frac{d(\pi-\theta)}{dt} = -\dot{\theta}$$

$$\dot\varphi \vec{j} \wedge \vec{CB} = -R\dot\theta\left(\cos\varphi\, \vec{i} + \sin\varphi\, \vec{K}\right)$$

$$= -R\dot\theta\left(\cos(\pi-\theta)\,\vec{i} + \sin(\pi-\theta)\,\vec{K}\right)$$

$$= -R\dot\theta\left(-\cos\theta\,\vec{i} + \sin\theta\,\vec{j}\right)$$

$$-\dot\varphi \vec{j} \wedge \vec{CB} = R\dot\theta\left(-\cos\theta\,\vec{i} + \sin\theta\,\vec{K}\right)$$

$$\vec{V_a}(B) = R\dot\theta\,\vec{j} + R\dot\theta\left(-\cos\theta\,\vec{i} + \sin\theta\,\vec{K}\right)$$

$$\vec{V_a}(B) = R\dot\theta\left((1-\cos\theta)\,\vec{i} + \sin\theta\,\vec{K}\right)$$

Exercice 2 :

1) $R(O, \vec{i}, \vec{j}, \vec{K})$ absolu

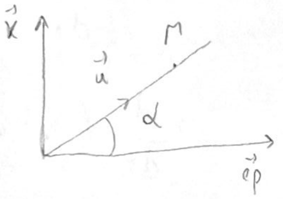
$R_1(O_1, \vec{i_1}, \vec{j_1}, \vec{K})$ relatif
$\vec{e_\rho}, \vec{e_\varphi}, \vec{K}$

$$\vec{OO_1} = \rho\,\vec{e_\rho}$$

$$\vec{O_1M} = V_0 t\,\vec{u}$$

I) $V_0 =$ cte

1) $\vec{u} = \cos\alpha\,\vec{e_\rho} + \sin\alpha\,\vec{K}$



2) $\vec{OM} = \vec{OO_1} + \vec{O_1M}$

$$= \rho\,\vec{e_\rho} + V_0 t\,\vec{u}$$

$$= \rho\,\vec{e_\rho} + V_0 t\left(\cos\alpha\,\vec{e_\rho} + \sin\alpha\,\vec{K}\right)$$

$$\vec{OM} = (\rho + V_0 t\cos\alpha)\,\vec{e_\rho} + V_0 t\sin\alpha\,\vec{K}$$

3) $\vec{V}(M/R) = \vec{V_a}(M) = \left.\frac{d\vec{OM}}{dt}\right)_R$ calcul direct

$$\left.\frac{d\vec{e_\rho}}{dt}\right)_R = \dot\varphi\,\vec{e_\varphi}$$

$$\left.\frac{d\vec{e_\varphi}}{dt}\right)_R = -\dot\varphi\,\vec{e_\rho}$$

$$\left.\frac{d\vec{e_\rho}}{dt}\right)_R = \left.\frac{d\vec{e_\rho}}{dt}\right)_{R'} + \vec\Omega_{R'/R} \wedge \vec{e_\rho}$$

$$= \vec{0} + \dot\varphi\,\vec{K}\wedge\vec{e_\rho} = \dot\varphi\,\vec{e_\varphi}$$

$$\left.\frac{d\vec{e_\varphi}}{dt}\right)_R = \left.\frac{d\vec{e_\varphi}}{dt}\right)_{R'} + \Omega_{R'/R}\wedge\vec{e_\varphi} = \dot\varphi\,\vec{K}\wedge\vec{e_\varphi}$$
$$= -\dot\varphi\,\vec{e_\rho}$$

$$\vec{V_a}(M) = \frac{d}{dt}\left((\rho + V_0 t\cos\alpha)\,\vec{e_\rho} + V_0 t\sin\alpha\,\vec{K})\right)\Big)_R$$

$$= \frac{d\rho}{dt}\,\vec{e_\rho} + \frac{d}{dt}(V_0 t\cos\alpha)\,\vec{e_\rho}$$

$$+ (\rho + V_0 t\cos\alpha)\left.\frac{d\vec{e_\rho}}{dt}\right)_R \quad = 0$$

$$+ \left.\frac{dV_0 t\sin\alpha}{dt}\right)_R \vec{K} + V_0 t\sin\alpha \left.\frac{d\vec{K}}{dt}\right)_R$$

$$= \dot\rho\,\vec{e_\rho} + V_0\cos\alpha\,\vec{e_\rho} + (\rho + V_0 t\cos\alpha)\,\dot\varphi\,\vec{e_\varphi}$$
$$+ V_0\sin\alpha\,\vec{K}$$

$$= (\dot\rho + V_0\cos\alpha)\,\vec{e_\rho} + (\rho + V_0 t\cos\alpha)\,\dot\varphi\,\vec{e_\varphi}$$
$$+ V_0\sin\alpha\,\vec{R}$$

5) Accélération absolue :

$$\vec\gamma_a(M) = \left.\frac{d\vec{V_a}(M)}{dt}\right)_R$$

$$= \ddot\rho\,\vec{e_\rho} + (\dot\rho + V_0\cos\alpha)\,\dot\varphi\,\vec{e_\varphi}$$

$$+ (\dot\rho + V_0\cos\alpha)\,\dot\varphi\,\vec{e_\varphi}$$

$$+ (\rho + V_0 t\cos\alpha)\,\ddot\varphi\,\vec{e_\varphi}$$

$$- (\rho + V_0 t\cos\alpha)\,\dot\varphi^2\,\vec{e_\rho} + 0$$

$$= \left(\ddot\rho - (\rho + V_0 t\cos\alpha)\,\dot\varphi^2\right)\vec{e_\rho} +$$

$$\left[2(\dot\rho + V_0 t\cos\alpha)\,\dot\varphi + (\rho + V_0 t\cos\alpha)\,\ddot\varphi\right]\vec{e_\varphi}$$

II)

1)

$\vec{\Omega}(R_1/R) = \dot{\varphi}\vec{K}$?

Formule de Bour

$$\left.\frac{d\vec{e\rho}}{dt}\right)_R = \left.\frac{d\vec{e\rho}}{dt}\right)_{R'} + \Omega_{R'/R} \wedge \vec{e\rho}$$

$\dot{\varphi}\vec{e\varphi}$

$\dot{\varphi}\vec{e\varphi} = \vec{\Omega}_{R'/R} \wedge \vec{e\rho}$ et $\vec{e\varphi} = \vec{K} \wedge \vec{e\rho}$

$\dot{\varphi}\vec{K} \wedge \vec{e\rho} = \Omega_{R'/R} \wedge \vec{e\rho}$

$\vec{e\rho} \neq 0 \Rightarrow \Omega_{R'/R} = \dot{\varphi}\vec{K}$

2) vitesse relative

$$\vec{V}_r(M) = \vec{V}(M/R') = \left.\frac{d\vec{O_1M}}{dt}\right)_{R'}$$

$\vec{O_1M} = V_0 t \vec{u} = V_0 t \cos\alpha \vec{e\rho} + V_0 t \sin\alpha \vec{K}$

$$\left.\frac{d\vec{O_1M}}{dt}\right)_{R_1} = V_0 \cos\alpha \vec{e\rho} + V_0 \sin\alpha \vec{K}$$

$$\left.\frac{d\vec{e\rho}}{dt}\right)_{R'} = \vec{0}$$

3) Vitesse d'entrainement

$\vec{V_e}(M) = \vec{V_a}(O_1) + \vec{\Omega}_{R_1/R} \wedge \vec{O_1M}$

* $\vec{V_a}(O_1) = \left.\frac{d\vec{OO_1}}{dt}\right)_R = \left.\frac{d\rho\vec{e\rho}}{dt}\right)_R$

$= \dot{\rho}\vec{e\rho} + \rho\dot{\varphi}\vec{e\varphi}$

* $\vec{\Omega}_{R_1/R} \wedge \vec{O_1M} = \dot{\varphi}\vec{K} \wedge (V_0 t \cos\alpha \vec{e\rho} + V_0 t \sin\alpha \vec{K})$

$= \dot{\varphi} V_0 t \cos\alpha \vec{e\varphi}$

$$\begin{cases} \vec{K} \wedge \vec{e\rho} = \vec{e\varphi} \\ \vec{K} \wedge \vec{K} = 0 \end{cases}$$

$\vec{V_e}(M) = \dot{\rho}\vec{e\rho} + (\rho + V_0 t \cos\alpha)\dot{\varphi}\vec{e\varphi}$

4) Déduire la vitesse absolue

On utilise la loi de composition des vitesses.

$\vec{V_a}(M) = \vec{V_r}(M) + \vec{V_e}(M)$

$= V_0 \cos\alpha \vec{e\rho} + V_0 \sin\alpha \vec{K} + \dot{\rho}\vec{e\rho}$

$+ (\rho + V_0 t \cos\alpha)\dot{\varphi}\vec{e\varphi}$

$= (\dot{\rho} + V_0 \cos)\vec{e\rho} + (\rho + V_0 t \cos\alpha)\dot{\varphi}\vec{e\varphi}$

$+ V_0 \sin\alpha \vec{K}$

La loi de composition des vitesses est bien vérifiée

5) Accélération relatives

$$\vec{\gamma_r}(M) = \left.\frac{d\vec{V_r}(M)}{dt}\right)_{R'}$$

$$= \frac{d}{dt}\left(V_0 t \cos\alpha \vec{e\rho} + V_0 \sin\alpha \vec{K}\right)_{R_1}$$

$= \vec{0}$

6) Accélération d'entrainement,

$$\vec{\gamma_e}(M) = \vec{\gamma_a}(O_1) + \frac{d\vec{\Omega}_{R'/R}}{dt} \wedge \vec{O_1M}$$

$$+ \vec{\Omega}_{R'/R} \wedge (\vec{\Omega}_{R'/R} \wedge \vec{O_1M})$$

$$\vec{\gamma_a}(O_1) = \left.\frac{d\vec{V_a}(O_1)}{dt}\right)_R$$

$$= \frac{d}{dt}\left(\dot{\rho}\vec{e\rho} + \rho\dot{\varphi}\vec{e\varphi}\right)\Big)_R$$

$= \ddot{\rho}\vec{e\rho} + \dot{\rho}\dot{\varphi}\vec{e\varphi} + \dot{\rho}\dot{\varphi}\vec{e\varphi} + \rho\ddot{\varphi}\vec{e\varphi}$

$- \rho\dot{\varphi}^2\vec{e\rho}$

$$\vec{\gamma}_a(O_1) = (\ddot{\rho} - \rho\dot{\varphi}^2)\,\vec{e_\rho} + (2\dot{\rho}\dot{\varphi} + \rho\ddot{\varphi})\,\vec{e_\varphi}$$

$$\frac{d\vec{\Omega}}{dt} \wedge \vec{O_1 M} = \frac{d\dot{\varphi}\vec{K}}{dt} \wedge \vec{O_1 M}$$

$$= \ddot{\varphi}\vec{K} \wedge (V_0 t \cos\alpha\,\vec{e_\rho} + V_0 t \sin\alpha\,\vec{K})$$

$$= \ddot{\varphi}\, V_0 t \cos\alpha\,\vec{e_\varphi}$$

$$\vec{\Omega}_{R'/R} \wedge (\Omega_{R'/R} \wedge \vec{O_1 M}) = \dot{\varphi}\vec{K} \wedge (\dot{\varphi} V_0 t \cos\alpha\,\vec{e_\varphi})$$

$$= -\dot{\varphi}^2 V_0 t \cos\alpha\,\vec{e_\rho}$$

$$\vec{\gamma_e}(M) = (\ddot{\rho} - \rho\dot{\varphi}^2 - \dot{\varphi}^2 V_0 t \cos\alpha)\,\vec{e_\rho}$$

$$+ (2\dot{\rho}\dot{\varphi} + (\rho + V_0 t \cos\alpha)\ddot{\varphi})\,\vec{e_\varphi}$$

7) Accéleration de Coriolis

$$\vec{\gamma}_c(M) = 2\,\vec{\Omega}_{R'/R} \wedge \vec{V_r}(M)$$

$$= 2\dot{\varphi}\vec{K} \wedge (V_0 \cos\alpha\,\vec{e_\rho} + V_0 \sin\alpha\,\vec{K})$$

$$= 2\dot{\varphi} V_0 \cos\alpha\,\vec{e_\varphi}$$

8) $\vec{\gamma}_a(M) = \vec{\gamma_r}(M) + \vec{\gamma_e}(M) + \vec{\gamma_c}(M)$

$$= \vec{0} + (\ddot{\rho} - (\rho + V_0 t \cos\alpha)\dot{\varphi}^2)\,\vec{e_\rho}$$

$$+ [2(\dot{\rho} + V_0 \cos\alpha)\dot{\varphi} + (\rho + V_0 t \cos\alpha)\ddot{\varphi}]\,\vec{e_\varphi}$$

(La loi de composition des accélération est bien verifiée).

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Exercice 3^e Série II

1 : L'exprition de $\vec{i'}$, $\vec{j'}$ et $\vec{k'}$ du repère R' dans la base $(\vec{i}, \vec{j}, \vec{k})$

$$\vec{i'} = \cos\varphi \cdot \vec{i} + \sin\varphi \, \vec{j}$$
$$\vec{j'} = -\sin\varphi \, \vec{i} + \cos\varphi \, \vec{j}$$
$$\vec{k'} = 0\vec{i} + 0\vec{j} + \vec{k} = \vec{k}$$

en déduire $\frac{d\vec{i'}}{dt}$, $\frac{d\vec{j'}}{dt}$, $\frac{d\vec{k'}}{dt}$ en fonction de $\dot{\varphi}$, $\vec{i'}$, $\vec{j'}$ et $\vec{k'}$

$$\left.\frac{d\vec{i'}}{dt}\right)_R = -\sin\varphi \, \dot{\varphi} \, \vec{i} + \dot{\varphi} \cos\varphi \, \vec{j} = \dot{\varphi} \, \vec{j'}$$

$$\left.\frac{d\vec{j'}}{dt}\right)_R = -\cos\varphi \, \dot{\varphi} \, \vec{i} - \sin\varphi \, \dot{\varphi} \, \vec{j} = -\dot{\varphi} \, \vec{i'}$$
$$= -\dot{\varphi}(\cos\varphi \, \vec{i} + \sin\varphi \, \vec{j})$$

$$\left.\frac{d\vec{k'}}{dt}\right)_R = \left.\frac{d\vec{k}}{dt}\right)_R = 0$$

2 - l'exprition les vecteurs vitesse rotations $\vec{\Omega}(R'/R)$ et $\vec{\omega}$ dans la base $(\vec{i'}, \vec{j'}, \vec{k'})$

selon la formule de Bour $\left[\frac{d}{dt} \vec{U}\right]_{/R} = \left[\frac{d}{dt} \vec{U}\right]_{/R'} + \vec{\Omega}_{R'/R} \wedge \vec{U}$

on a

$$\left.\frac{d\vec{i'}}{dt}\right)_R = \cancel{\left.\frac{d\vec{i'}}{dt}\right)_{R'}}^{\,0} + \vec{\Omega}_{R'/R} \wedge \vec{i'}$$

$$\dot{\varphi} \cdot \vec{j'} = \vec{\Omega}_{R'/R} \wedge \vec{i'}$$

$$\dot{\varphi}(\vec{k'} \wedge \vec{i'}) = \Omega_{R/R} \wedge \vec{i'}$$

$$\boxed{\dot{\varphi} \, \vec{k'} = \Omega_{R'/R}}$$

puisque la relation ce fait au toure de l'axe O'Z selon l'angle θ donc $\vec{w} = \dot{\theta}\,\vec{k'}$

3- Donner les vecteurs $\overrightarrow{O'M}$ et $\overrightarrow{OM}$

$$\boxed{\overrightarrow{O'M} = \cos\theta\,\vec{i'} + \sin\theta\,\vec{j'}}$$

$$\overrightarrow{OM} = \overrightarrow{OO'} + \overrightarrow{O'M}$$

$$= L.\vec{i'} + \cos\theta\,\vec{i'} + \sin\theta\,\vec{j'}$$

$$\boxed{\overrightarrow{OM} = (L + \cos\theta)\,\vec{i'} + \sin\theta\,\vec{j'}}$$

4- Le victeur vitesse

$$\left.\frac{d\overrightarrow{OM}}{dt}\right)_R = \frac{d\vec{i'}}{dt}(L + \cos\theta) + \vec{i'}\frac{d}{dt}(L + \cos\theta) + \vec{j'}\frac{d\sin\theta}{dt} + \frac{d\vec{j'}}{dt}.\sin\theta$$

$$= \dot{\varphi}\,\vec{j'}(L + \cos\theta) - \vec{i'}\,\dot{\theta}\sin\theta + \vec{j'}\,\dot{\theta}\cos\theta - \dot{\varphi}\,\vec{i'}\sin\theta$$

$$V = \vec{i'}\sin\theta(-\dot{\theta} - \dot{\varphi}) + \vec{j'}(\dot{\varphi}(L + \cos\theta) + \dot{\theta}\cos\theta)$$

Le victure accélération

$$\left.\frac{dV}{dt}\right)_R = \left.\frac{d\vec{i'}}{dt}\right)_R(-\dot{\theta}\sin\theta - \dot{\varphi}\sin\theta) + \frac{d}{dt}\left(\sin\theta(-\dot{\theta} - \dot{\varphi})\right)\vec{i'}$$

$$+ \left.\frac{d\vec{j'}}{dt}\right)_R\left(\dot{\varphi}(L + \cos\theta) + \dot{\theta}\cos\theta\right) + \vec{j'}\,\frac{d}{dt}\left(\dot{\varphi}(L + \cos\theta) + \dot{\theta}\cos\theta\right)$$

$$= \dot{\varphi}\,\vec{j'}(-\dot{\theta}\sin\theta - \dot{\varphi}\sin\theta) + \left(\dot{\theta}\cos\theta(-\dot{\theta} - \dot{\varphi}) + \sin\theta(-\ddot{\theta} - \ddot{\varphi})\right)\vec{i'}$$

$$- \dot{\varphi}\,\vec{i'}\left(\dot{\varphi}(L + \cos\theta) + \dot{\theta}\cos\theta\right) + \vec{j'}\left(\ddot{\varphi}(L + \cos\theta) - \right.$$

$$\left.\dot{\varphi}\,\dot{\theta}\cos\theta + \ddot{\theta}\cos\theta - \dot{\theta}\sin\theta\right)$$

$$= \vec{i'}\left[\dot{\theta}\cos\theta(-\dot{\theta} - \dot{\varphi}) + \sin\theta(-\ddot{\theta} - \ddot{\varphi}) - \dot{\varphi}(\dot{\varphi}(L + \cos\theta) + \dot{\theta}\cos\theta)\right]$$

$$+ \vec{j'}\left[\dot{\varphi}(-\dot{\theta}\sin\theta - \dot{\varphi}\sin\theta) + \ddot{\varphi}(L + \cos\theta) - \dot{\varphi}\dot{\theta}\cos\theta + \ddot{\theta}.\right.$$

$$\left.\cos\theta - \dot{\theta}\sin\theta\right]$$

5 -

a - Vitesse relative

$$V_r(M/R') = \frac{d\,O'M}{dt}\Big)_{R'} = \frac{d(\cos\theta\,\vec{i'} + \sin\theta\,\vec{j'})}{dt}$$

$$= -\sin\theta\,\vec{i'} + \cos\theta\,\vec{j'}$$

accélération relative

$$\frac{dV_r}{dt}(M/R') = \frac{d}{dt}(-\sin\theta\,\vec{i'} + \cos\theta\,\vec{j'})$$

$$= -\dot\theta\cos\theta\,.\,\vec{i'} - \dot\theta\sin\theta\,\vec{j'}$$

b - Vitesse d'entraînement

$$V_e(M) = V_a(O') + \vec{\Omega}_{R'/R} \wedge O'M$$

$$= \frac{d\,\vec{OO'}}{dt}\Big)_R + \Omega_{R'/R} \wedge \vec{O'M}$$

$$= \frac{d\,L\,\vec{i'}}{dt}\Big)_R + \dot\varphi\,K \wedge (\cos\theta\,\vec{i'} + \sin\theta\,\vec{j'})$$

$$= L(\dot\varphi\,\vec{j'}) + \dot\varphi\,K \wedge (\cos\theta\,\vec{i'} + \sin\theta\,\vec{j'})$$

$$= L\dot\varphi\,\vec{j'} + \dot\varphi\cos\theta\,\vec{j'} - \dot\varphi\sin\theta\,\vec{i'})$$

accélération d'entraînement

$$\frac{dV_e}{dt}\Big)_R = \vec{\gamma}_a(O) + \frac{d\,\vec{\Omega}_{R'/R}}{dt} \wedge \vec{OM} + \vec{\Omega}_{R'/R} \wedge (\vec{\Omega}_{R'/R} \wedge \vec{OM})$$

$$= \frac{d^2(OO')}{dt}\Big)_R + \frac{d\,\dot\varphi\,\vec{K}}{dt} \wedge \vec{O'M} + \vec{\Omega}_{R'/R} \wedge (\vec{\Omega}_{R'/R} \wedge \vec{O'M})$$

$$= \frac{d\,L\,\dot\varphi\,\vec{j'}}{dt}\Big)_R + \ddot\varphi\,\vec{K} \wedge \vec{O'M} + \dot\varphi\,K \wedge (\dot\varphi\,K \wedge \vec{O'M})$$

$$= L\left(\frac{d\dot\varphi}{dt}\vec{j'} + \dot\varphi\frac{d\vec{j'}}{dt}\right) + \ddot\varphi\,\vec{K} \wedge (\cos\theta\,\vec{i'} + \sin\theta\,\vec{j'})$$

$$+ \dot\varphi\,\vec{K} \wedge (\dot\varphi\,K \wedge (\cos\theta\,\vec{i'} + \sin\theta\,\vec{j'})$$

$$= L(\ddot{\varphi}\,\vec{j'} - \dot{\varphi}\,\vec{i'}) + \ddot{\varphi}\cos\theta\,\vec{j'} - \ddot{\varphi}\sin\theta\,\vec{i'} + \dot{\varphi}\vec{K} \wedge$$

$$(\dot{\varphi}\cos\theta\,\vec{j'} - \ddot{\varphi}\sin\theta\,\vec{i'})$$

$$= L(\ddot{\varphi}\,\vec{j'} - \dot{\varphi}\,\vec{i'}) + \ddot{\varphi}\cos\theta\,\vec{j'} - \ddot{\varphi}\sin\theta\,\vec{i'} + \dot{\varphi}^2\cos\theta\,\vec{i'}$$

$$- \dot{\varphi}^2\sin\theta\,\vec{j'}$$

$$= \vec{i'}\left(-L\dot{\varphi} - \ddot{\varphi}\sin\theta - \dot{\varphi}^2\cos\theta\right) + \vec{j'}\left(L\ddot{\varphi} + \ddot{\varphi}\cos\theta - \dot{\varphi}^2\sin\theta\right)$$

accelération de Coriolis

$$\vec{\gamma_c}(M) = 2\,\vec{\Omega}_{R'/R} \wedge V_r(M)$$

$$= 2\dot{\varphi}\vec{K} \wedge (-\sin\theta\,\vec{i'} - \cos\theta\,\vec{j'})$$

$$= -2\dot{\varphi}\sin\theta\,\vec{j'} + 2\dot{\varphi}\cos\theta\,\vec{i'}$$