Penn State STAT 540
Homework #4, due Thursday, Nov 19, 2015

Submit: (i) `R` code in a file titled PSUemailidHW4.R (e.g. muh10HW4.R), (ii) pdf file that contains a clear writeup for the questions below named PSUemailidHW4.pdf Note that your code should be readily usable. It should also be well commented.

1. Assume that lightbulb lifetimes are independent and exponentially distributed with expectation θ. Suppose in an experiment, m bulbs are switched on at the same time, but are only completely observed up to time τ. Let the lifetimes of these bulbs be $A_1, A_2, \ldots, A_m$. However, since the bulbs are only observed till time τ, not all these lifetimes will be observed. Let the censoring time be τ. The bulbs that are still working have reported (censored) lifetime τ.

   Assume τ is 100 days and $m = 1000$. The censored data lifetimes are here: `http://www.stat.psu.edu/~mharan/540/hwdir/bulbA.dat`

   Write an EM algorithm to maximize the likelihood. Provide pseudocode for this algorithm.

2. Start your algorithm at θ=50. Plot the value of your estimate for θ versus the number of iterations of your algorithm. Now run the algorithm again but this time you are free to choose any starting value you like, but you must explain how you obtained the starting value. Also explain how you determined whether the algorithm had converged.

3. Provide your final estimate of θ.

4. Use the bootstrap to provide standard error estimates for $\hat{\theta}$. (Remember: this is not a Monte Carlo algorithm, so there is no Monte Carlo standard error! You should be reporting the standard error of your MLE.) Describe your bootstrap approach clearly along with any underlying assumptions.

5. Use the bootstrap t confidence interval as discussed in class to provide a bootstrap 95% interval for θ.

6. Describe clearly an alternative approach to obtaining standard error estimates. Is this more or less preferable to the bootstrap approach and why?