

You will need to use R for this homework. To install R on your computer, go to: <http://cran.r-project.org/> and download and install R. You should then install a text editor. The simplest approach is to install RStudio <http://www.rstudio.com/products/rstudio/download/>, which provides a Matlab-like interface for R. Other good editors include Emacs, Tinn-R, and many others.

For problems 1-4, provide written derivations for parts (a)-(b) and four plots made in R for parts (c)-(d). You can follow the template code in “Distributions.r”, which is available on Angel. Attach your R-code to the back of your homework.

1. Consider the Binomial distribution

$$y \sim \text{Binom}(n, p) \quad , \quad y \in \mathbb{N}$$

- (a) Derive the mean and variance of y .
 - (b) Find a set of parameters that make $E(y) = 3$ and if possible $\text{Var}(y) = 1$. In distributions with one parameter, it will not always be possible to make the variance equal to one. In two-parameter distributions this is almost always possible.
 - (c) Use R to simulate 1000 values of the random variable with mean=3 and (if possible) variance = 1. Plot the simulated random variables and a histogram of the simulated random variables.
 - (d) Find a power transformation of the simulated random variables such that the transformed variables are approximately normally-distributed (or as close as you can make it). Plot the QQ-plot of the transformed variables, and a histogram of the transformed variables.
2. Repeat Problem 1 for the Normal distribution.
 3. Repeat Problem 1 for the Poisson distribution.

$$P(y = k|\lambda) = \frac{\lambda^k}{k!} e^{-\lambda} \quad , \quad y \in \{0, 1, 2, \dots\}$$

4. Repeat Problem 1 for the Laplace distribution.

$$f(y|\mu, \sigma) = \frac{1}{2\sigma} \exp\left(-\frac{|x - \mu|}{\sigma}\right) \quad , \quad y \in \mathbb{R}$$

5. Download the “interarrival.Rdata” file from Angel and use the following code to read it into R. You will need to make sure your working directory is pointed to where the

```
> load("interarrival.Rdata")
> interarrival
```

The 100 numbers in “interarrival” are the times in minutes between arriving airplanes at an airport.

- (a) Propose a statistical distribution that you think is appropriate for this data. Specify all parameters in the distribution and explain how you chose or estimated the parameters. For example you could pick parameters by matching sample moments (mean and variance) to theoretical moments in your chosen distributional family.
- (b) Compare the support, sample mean, and sample variance of the data to the support, theoretical mean and theoretical variance under your proposed model. An appropriate model will show good agreement between theoretical and observed quantities.

- (c) Using your proposed model from (a), find the probability that the next interarrival time is greater than 15.
6. Examine the “cars” dataset included in R. The following will read in distance (ft) that 50 cars took to stop.

```
> dist=cars$dist  
> dist
```

Note that the “cars” dataset also includes the speed of the cars, but we will assume that is not known for now.

- (a) Propose a statistical distribution that you think is appropriate for this data. Specify all parameters in the distribution and explain how you chose or estimated the parameters. For example you could pick parameters by matching sample moments (mean and variance) to theoretical moments in your chosen distributional family.
- (b) Compare the support, sample mean, and sample variance of the data to the support, theoretical mean and theoretical variance under your proposed model. An appropriate model will show good agreement between theoretical and observed quantities.
- (c) Using your proposed model from (a), find the probability that the it will take a car longer than 80 feet to stop. Compare this theoretical probability with the observed frequency.
7. For the following matrices, find the desired quantities. You may either do this by hand or using R. See the “IntroToR.r” code for some examples.

$$\mathbf{A} = \begin{pmatrix} 1 & 5 & 2 \\ 3 & -2 & 2 \\ -1 & -1 & -3 \\ 9 & -1 & 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

- (a) \mathbf{Ab}
- (b) $\mathbf{b'b}$
- (c) $\mathbf{bb'}$
- (d) $\mathbf{A'A}$
- (e) $\text{trace}(\mathbf{A'A})$
- (f) $(\mathbf{A'A})^{-1}$
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