1. **Ice Cream** The "icecream.csv" file contains data on ice cream consumption measured over 30 four-week periods from March 18, 1951 to July 11, 1953.

   Variable Names:

   Date: Time period (1-30) of the study (from 3/18/51 to 7/11/53)

   IC: Ice cream consumption in pints per capita

   Price: Price of ice cream per pint in dollars

   Income: Weekly family income in dollars

   Temp: Mean temperature in degrees F.

   (a) Use all of the ice cream data EXCEPT for the last time point (date=30) to model ice cream consumption as a function of time, price, income and mean temperature. Consider transformations, outliers, nonlinear trends, temporal autocorrelation, and anything else that is relevant. Clearly write out your final model and provide evidence that modeling assumptions are met, or comment on assumptions that are not met. Only include terms in your final model that are statistically significant. Interpret your model and the parameter estimates.

   (b) Clearly show how to obtain the p-value of the hypothesis test that $\beta_k = 0$ for one parameter in your model. If you have correlated errors, you may assume that the correlation matrix is fixed and known (that is, you do not need to account for any uncertainty in the estimate you have made of $\mathbf{W}$). Reproduce the p-value in R.

   (c) Use your model to predict ice cream consumption in the last four-week time period (date=30). Provide a 95% prediction interval of ice cream consumption that incorporates all modeling assumptions. Clearly describe any assumptions you are making. Compare your prediction interval with the observed ice cream consumption for "date=30".

2. **Exponential Family Random Variables**. Show that the following random variables are exponential family random variables. In each case, find the mean and variance of the random variable, the natural parameter, the canonical link function and canonical response function. Write down the form of a generalized linear model with canonical link function.

   (a) Poisson. $y \sim \text{Pois}(\lambda)$

   $$P(y = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k = 0, 1, 2, \ldots$$

   (b) Rayleigh. $y \sim R(\sigma^2)$

   $$f(y; \sigma^2) = \frac{y}{\sigma^2} e^{-y^2/(2\sigma^2)}, \quad y \in (0, \infty)$$

   (c) Geometric. $y \sim G(p)$

   $$P(y = k) = (1 - p)^k p, \quad k = 0, 1, 2, \ldots$$