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Attitude Dynamics ASE 372K Final Project

Introduction

For this project, the attitude control system of an Earth pointing cubesat was simulated for one full orbit. It was in an inclined circular orbit of 6657 km from the center of the Earth, or 300 km above the reference surface using an Earth radius of 6357 km. The orbit inclination was 45 degrees. The inertial frame was approximated to be the Earth centered inertial frame and the attitude profile was the satellite body fixed frame. The body x-axis pointed to the center of the Earth, the body y-axis pointed along the velocity vector and the body z-axis completed the right handed system. The attitude control mechanism was three orthogonal reaction wheels assumed to be acting at the center of mass of the satellite. Two perturbation torques were included in this mission: aerodynamic torques and gravity gradient torques.

Reference Mission

For the mission, the satellite was to follow a perfectly circular orbit inclined at 45 degrees following the inertial path with position and velocity as:

$$\boldsymbol{r}^i(t) = \frac{\rho}{\sqrt{2}} \begin{bmatrix} -\cos\omega t \\ \sqrt{2}\sin\omega t \\ \cos\omega t \end{bmatrix}$$

$$\dot{\boldsymbol{r}}^i(t) = \frac{\omega\rho}{\sqrt{2}} \begin{bmatrix} \sin\omega t \\ \sqrt{2}\cos\omega t \\ -\sin\omega t \end{bmatrix}$$

where

$$\rho = 6657km$$
$$\omega = \sqrt{\mu/\rho^3}$$

Where μ is Earth's gravitational constant.

The inertial matrix of the cubesat in the body frame was given by

$$J = \begin{bmatrix} .0067 & 0 & 0 \\ 0 & .03333 & 0 \\ 0 & 0 & .03333 \end{bmatrix} kg\, m^2$$

The nominal attitude profile of the satellite was such that the body x-axis was always pointing to the center of the Earth, the body y-axis was pointing along the velocity vector, and teh body z-axis completed the right handed system.

Dynamics Model

The satellite angular velocity relative to the inertial frame parameterized in the body frame and the attitude given by the inertial to body quaternion were integrated over time from the following equations

$$\dot{\bar{\omega}}^b_{b/i}(t) = \left(\mathbf{J}^b_{cg}\right)^{-1} \left(-\bar{\omega}^b_{b/i}(t) \times (\mathbf{J}^b_{cg}\, \bar{\omega}^b_{b/i}(t)) + \mathbf{M}^b_{cg,nom}(t) \right)$$

$$\dot{\bar{\boldsymbol{q}}}^b_i(t) = \frac{1}{2} \begin{bmatrix} \bar{\omega}^b_{b/i}(t) \\ 0 \end{bmatrix} \otimes \bar{\boldsymbol{q}}^b_i(t)$$

where $\mathbf{M}^b_{cg,nom}(t)$ is the sum of all torques acting on the center of gravity of the satellite, which in this simulation was gravity gradient torques, aerodynamic torques, and reaction wheel torques when the controller was turned on. An initial pointing error of 2 degrees around the body y-axis and an initial angular velocity error of .005 rad/s around the body x-axis were used in this simulation.

Aerodynamic force was calculated using a flat plate model of the spacecraft assuming that the force acted at the center of the flat plate. The force was given by:

$$F_d = -.5\rho|V|^2 C_d A \hat{V} dot(\hat{V}, \hat{N})$$

The aerodynamic model was less accurate during the uncontrolled phase of the simulation because the satellite was a three dimensional object and did not maintain small angle pointing. The torque resulting from the drag force was given by

$$M_d = r\ x\ F_d$$

where r was assumed to be a fixed vector distance on the model 2D shape from the satellite center of gravity. The distance was approximated to be

$$r = [\ .01\ .15\ .01]\ m$$

assuming the cubesat was a 3u cubesat with the center of gravity at the center along the long axis.

The other constants were assumed to be

$$C_d = 4$$
$$A = .01\ m^2$$

Gravity gradient torque was calculated using the equation

$$M_{gg} = 3\mu(r\ x\ Jr)/|r|^5$$

where r was the satellites inertial position.

Figure (1) displays the results of the uncontrolled attitude evolution over time

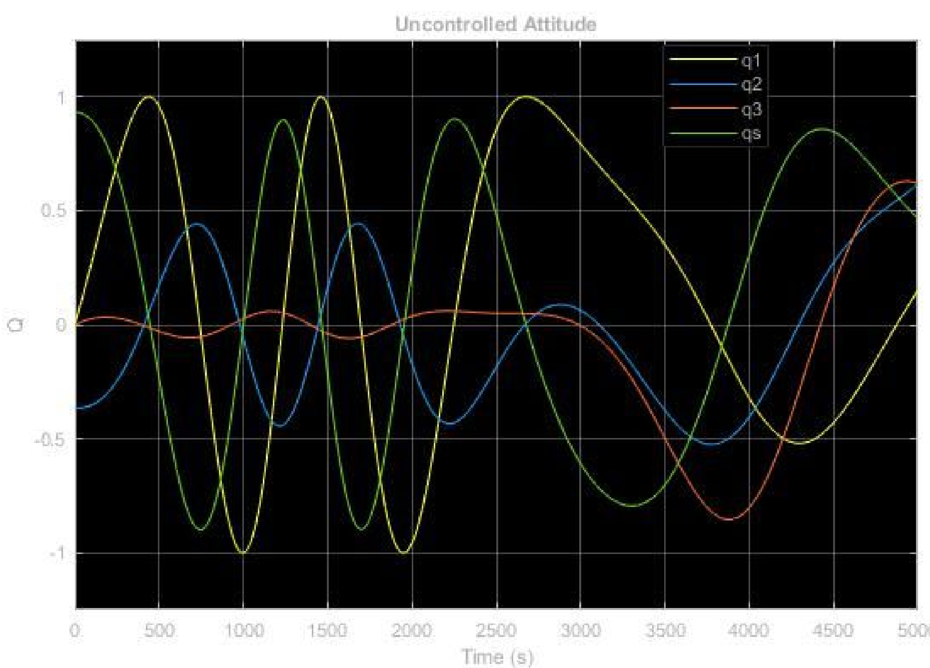

Figure 1: Uncontrolled Attitude Evolution with Perturbations

Sensors and Actuators

There were three different sensors simulated in this simulation: an IMU with a gyroscope, a magnetometer, and a horizon sensor.

IMU bias, *b*, and angular random walk, *v(t)*, were given by

$$b = 1\ deg/hr$$
$$\nu_{gyro} = .22\ deg/\sqrt{hr}$$

and measured angular velocity was given by

$$\tilde{\boldsymbol{\omega}}^b_{b/i}(t) = \boldsymbol{\omega}^b_{b/i}(t) + \boldsymbol{b} + \boldsymbol{\nu}_{gyro}(t)$$

where walk was scaled in each of the three body directions by randomly generated MATLAB constants using rand.

Horizon sensor error, *v*, was given by

$$\nu_{hor} = +/-1\ deg$$

and was randomly modulated using Simulink's random signal generator; the measurement from the horizon sensor was given by

$$\tilde{\boldsymbol{d}}^b(t) = \boldsymbol{T}(\boldsymbol{\nu}_{Hor})\,\boldsymbol{T}^b_i(t)\,\frac{-\boldsymbol{r}^i(t)}{\|\boldsymbol{r}^i(t)\|}$$

Lastly, the magnetometer error, *v*, was given by

$$\nu_{mag} < 8\ nT$$

and the magnetometer measurement was given by

$$\tilde{\boldsymbol{B}}^b(t) = \boldsymbol{T}^b_i(t)\,\boldsymbol{B}^i(t) + \boldsymbol{\nu}_{mag}(t)$$

where the error was modulated, again, by a randomly generated Simulink signal.

For the magnetometer and horizon sensor measurements, the transformation DCM, *T*, was given by

$$\boldsymbol{T}_i^b = \boldsymbol{I}_{3\times3} - 2q_s[\boldsymbol{q}_v\times] + 2[\boldsymbol{q}_v\times]^2$$

The actuators used were reaction wheels assumed to be acting at the center of gravity and perfectly orthogonal. The moment of inertia about the spin axis of the wheels is given by

$$C_w = .00001057\,kg\,m^2$$

Actuators were found that had a maximum rpm of 10000 rpm with a resolution of <5 rpm, so error in the wheels was approximated using the following equation, assuming that resolution error scaled with angular velocity

$$^b\dot{\boldsymbol{\omega}}_W^b(t) = \, ^b\dot{\boldsymbol{\omega}}_{W,comm}^b(t) + \delta\boldsymbol{\alpha}_W^b(t)$$

$$\delta\alpha_w^b(t) = 5w_{actual}/10000$$

where $\delta\alpha_w^b(t)$ is the resolution error and is modulated using a random signal generation vector in Simulink. The reaction wheel angular velocity and momentum were then found thusly

$$\boldsymbol{\omega}_W^b(t) = \begin{bmatrix}\tilde{\omega}_1\\ \tilde{\omega}_2\\ \tilde{\omega}_3\end{bmatrix} = \int \dot{\boldsymbol{\omega}}_W^b(t)\,dt$$

$$^b\dot{\boldsymbol{H}}_W^b(t) = C_W\, ^b\dot{\boldsymbol{\omega}}_W^b(t)$$

$$\boldsymbol{H}_W^b(t) = C_W\,\boldsymbol{\omega}_W^b(t)$$

Attitude Determination and Control System

The attitude determination system used the magnetometer sensor data and the horizon sensor measurement. The orientation DCM was calculated using the TRIAD algorithm and was estimated with the following equation

$$\hat{\boldsymbol{T}}_i^b = \begin{bmatrix} \mathbf{x}^b & \mathbf{y}^b & \mathbf{z}^b \end{bmatrix} \begin{bmatrix} \mathbf{x}^i & \mathbf{y}^i & \mathbf{z}^i \end{bmatrix}^{\mathrm{T}}$$

Where

$$x^b = d_1^b$$
$$z^b = d_1^b \; x \; d_2^b / norm(d_1^b \; x \; d_2^b)$$
$$y^b = z^b \; x \; x^b$$
$$x^i = d_1^i$$
$$z^i = d_1^i \; x \; d_2^i / norm(d_1^i \; x \; d_2^i)$$
$$y^i = z^i \; x \; x^i$$

and d_1^b was acquired from the horizon sensor measurement.

$$d_1^i = -r/|r|$$
$$d_2^i = B^i/|B^i|$$
$$d_2^b = B^b/|B^b|$$

For the controller portion of the simulation, a PD controller was designed using the following gains values

$$K_p = .0005; K_d = .1$$

These gains values were used to calculate the reaction wheel rates to produce torques necessary to keep the satellite close to the nominal attitude profile. The reaction wheel rates were determined from the following equation

$${}^b\dot{\boldsymbol{\omega}}^b_{W,comm} = \begin{bmatrix} \dot{\omega}_{1,comm} \\ \dot{\omega}_{2,comm} \\ \dot{\omega}_{3,comm} \end{bmatrix} = \frac{\boldsymbol{J}^b_{sat}\boldsymbol{K}_d}{C_W}\boldsymbol{\epsilon}^b + \frac{\boldsymbol{J}^b_{sat}\boldsymbol{K}_p}{C_W}\dot{\boldsymbol{\epsilon}}^b - \hat{\boldsymbol{\omega}}^b_{b/i} \times \begin{bmatrix} \bar{\omega}_1 \\ \bar{\omega}_2 \\ \bar{\omega}_3 \end{bmatrix}$$

where $\begin{bmatrix}\tilde{\omega}_1\\ \tilde{\omega}_2\\ \tilde{\omega}_3\end{bmatrix}$ is the reaction wheel angular momentum, $\hat{\omega}^b_{b/i}$ is the measured satellite angular velocity with respect to the inertial frame, and *Cw* is the reaction wheel moment of inertia. The following values are calculated as

$$\delta \boldsymbol{T}(t) = \hat{\boldsymbol{T}}^b_i(t)\, \boldsymbol{T}^b_i(t)^{\mathrm{T}}$$

$$\boldsymbol{\epsilon}^b(t) \simeq -0.5 \begin{bmatrix}\delta T_{32} - \delta T_{23}\\ \delta T_{13} - \delta T_{31}\\ \delta T_{21} - \delta T_{12}\end{bmatrix}$$

$$dw = w_{meas} - w_{nom}$$

$$\dot{\epsilon} = -[w_{nom}\ x]\epsilon + dw$$

where the transformation matrices are the measured and nominal DCMs, respectively.

Simulation Results

This simulation, using the equations listed throughout, was able to produce a “measured” attitude that followed the nominal attitude profile. Figure (2) displays the simulated attitude profile, while Figure (3) displays the nominal attitude profile.

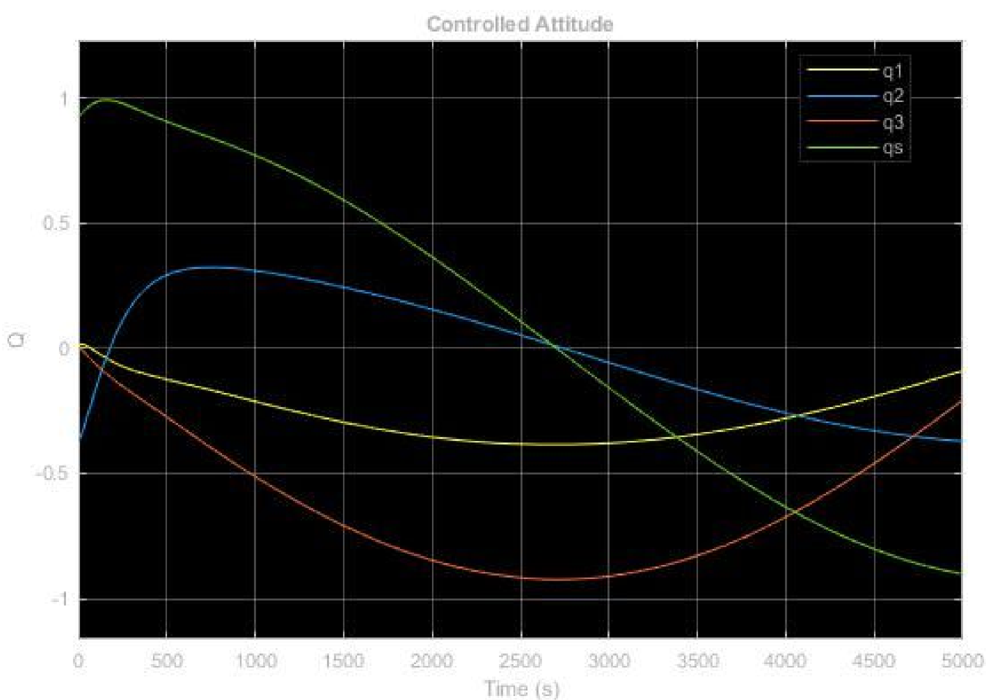

Figure 2: displays the controlled inertial to body quaternion

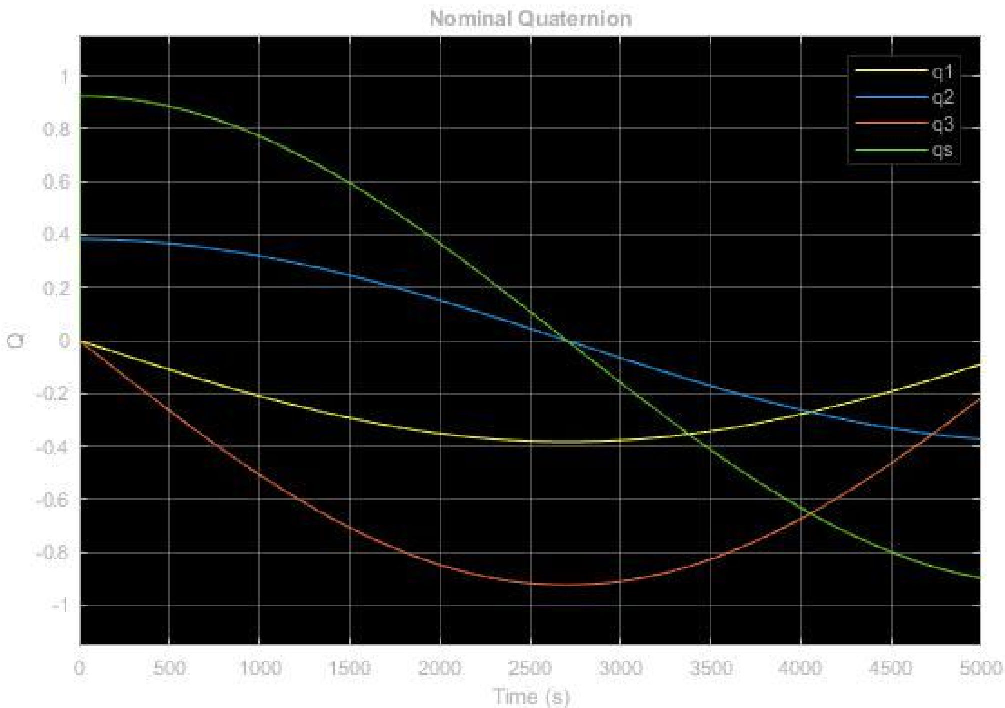

Figure 3: displays the nominal inertial to body quaternion

As can be seen, the quaternion values for Figures (2) and (3) are nearly identical, signaling that the PD controller did work appropriately.

It was also found that the reaction wheel torques were well within their performance range, having a maximum torque of .635 nNm. Figure (4) shows the control torques over time.

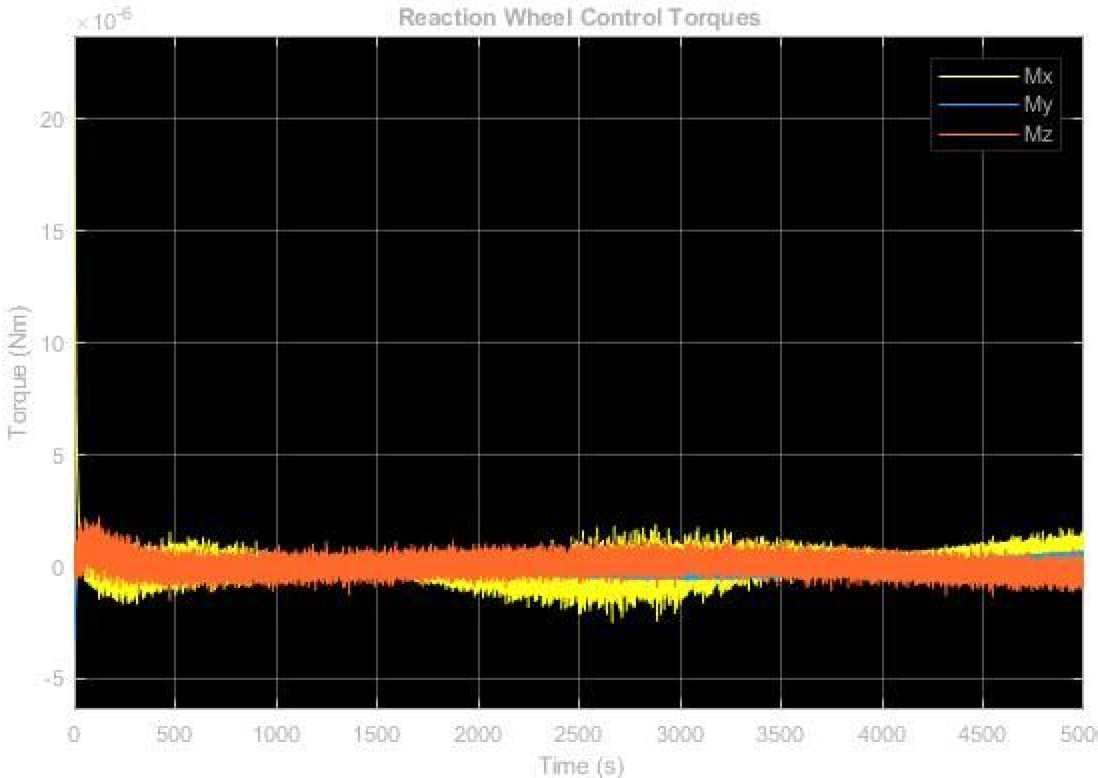

Figure 4: Displays the Control Torques to Keep the Nominal Attitude Profile

It was also seen that the satellite maintained the nominal angular velocity profile. The nominal angular velocities are shown in figures (5) and (6).

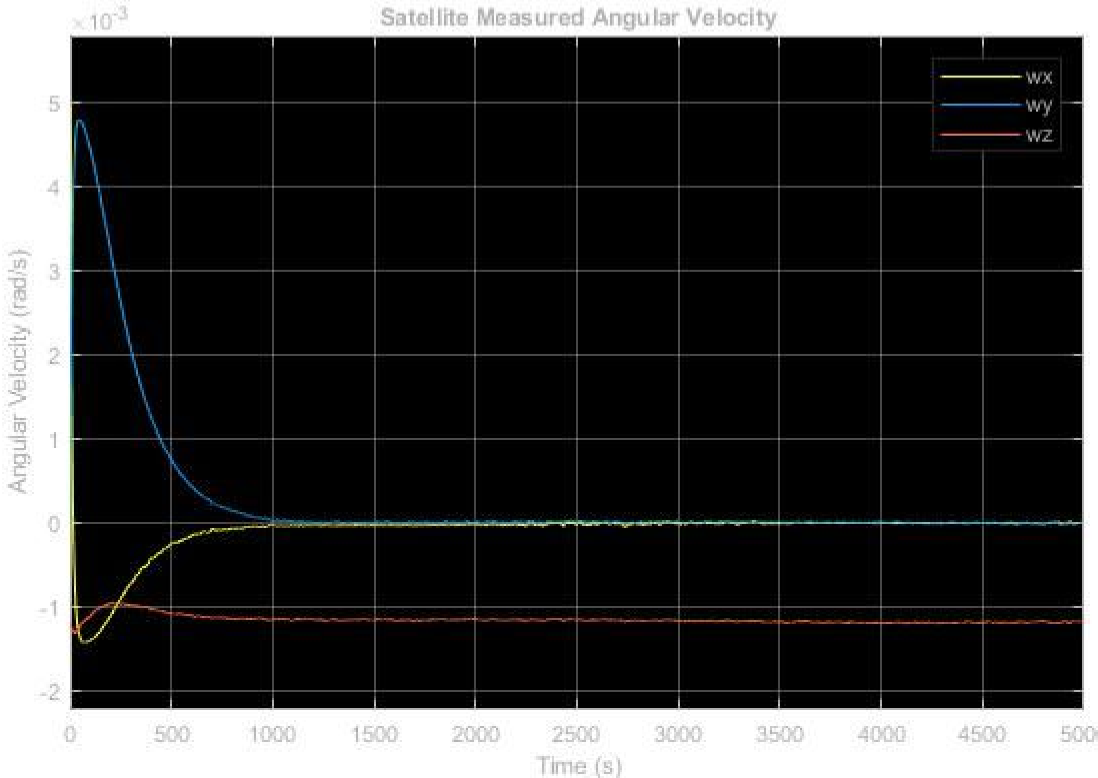

Figure 5: Displays the Measured Angular Velocity of the Satellite

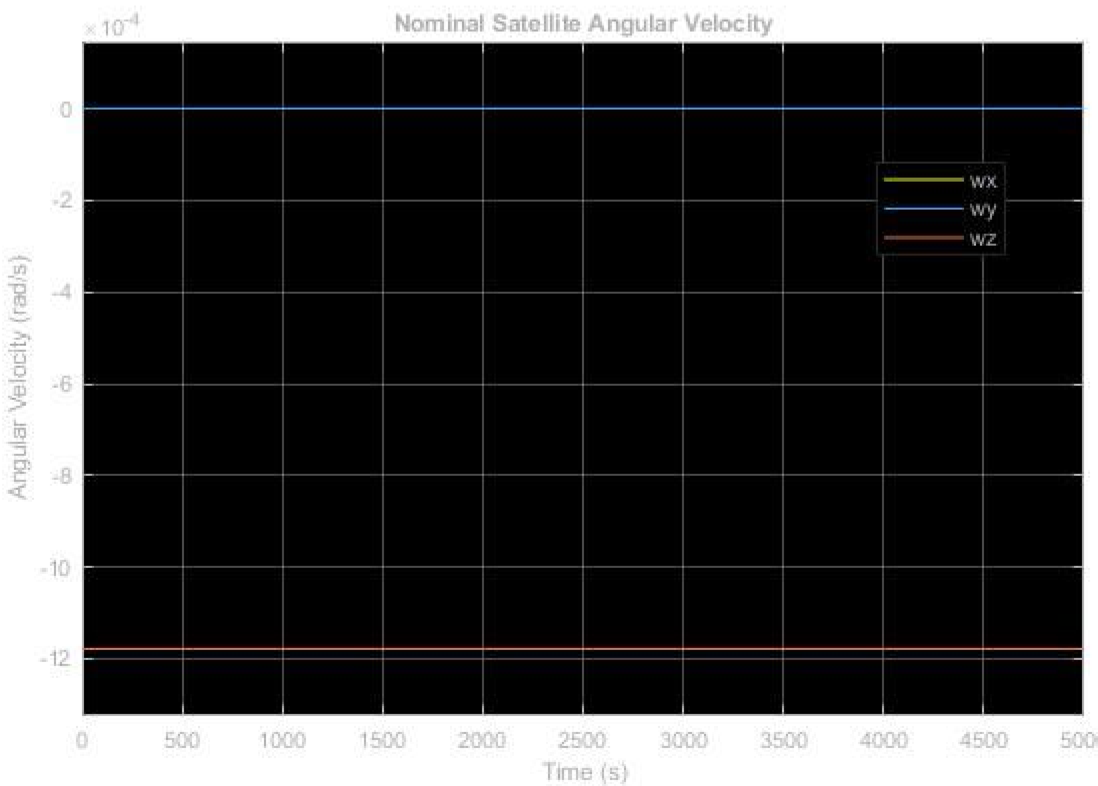

Figure 6: Displays the Nominal Angular Velocity of the Satellite

The figures show that the measured angular velocity of the satellite quickly converges to be the appropriate angular velocity.

Appendix: Sensor and Satellite Detail Sources

Cubesat- Moment of Inertia
A = .0067; B = C = .03333
https://pdfs.semanticscholar.org/706f/c0f32c1fdc437206f611bec2ad7f5fcf256c.pdf

Reaction Wheel
Cw = .00001057
Max Torque - .000635 Nm
https://www.cubesatshop.com/product/mai-400-reaction-wheel/
Resolution < 5 rpm - sort of high
https://www.cubesatshop.com/product/cubewheel-small/

Horizon Sensor
Error +- 1 deg
https://www.cubesatshop.com/wp-content/uploads/2016/06/MAI-SES-Specifications-20150928.pdf

Magnetometer
Error +- 1 deg
Resolution < 8 nT
https://www.cubesatshop.com/product/nss-magnetometer/

IMU
Bias = +- 1deg /hr %
Angular Random Walk - .22 deg/sqrt(hr)
https://www.memsense.com/assets/docs/uploads/ms-imu3020/MS-IMU3020_PSUG.pdf

For Aerodynamic Drag
Cd of a surface is 4 for a flat plate
Fd = -.5*p*|V|^2*dot(nhat, Vhat)*Vhat*Cd
M = cross(r/cg, Fd)
Earth radius - 6357 km, at 6657 km, alt is 300 km
At 300 km, density is 2e-11 kg/m^3
https://apps.dtic.mil/dtic/tr/fulltext/u2/a392479.pdf
http://mstl.atl.calpoly.edu/~bklofas/Presentations/DevelopersWorkshop2014/Omar_CubeSat_Formation_Control.pdf

Compare torques with
http://lasp.colorado.edu/home/csswe/files/2012/06/Gerhardt_SSC10_PMAC.pdf