

7-13. Let's find roots of some non-real non-imaginary complex numbers!

- Find all the square roots of the complex number given by the point on the unit circle with an angle of 30° .
- Find all the square roots of $\cos(\pi/4) + i \sin(\pi/4)$.
- Find all the square roots of $\frac{1}{2} + \frac{\sqrt{3}}{2}i$.
- How are these square roots different from the square roots of 1?
- Now find the cube roots of all three of those points.
- Find the quartic roots of those three points.
- In general, how do we find the n th roots of some point on the unit circle?

7-14. Finally, find every root of every complex number! One formula for everyyyyyyything.

$$\sqrt[n]{z} = \underbrace{|z|^{1/n}}_{\text{real part of complex number "z" raised to the } 1/n \text{ power to obtain radius}} \underbrace{\left(\cos \frac{\theta + 2\pi k}{n} + i \sin \frac{\theta + 2\pi k}{n} \right)}_{\text{Each root of complex number z will be evenly spaced on the graph, } \left(\frac{2\pi}{n} \right). \text{ However, in order for the formula to be able to differentiate between complex numbers, we must add the argument of z } \left(\frac{\theta + 2\pi}{n} \right). \text{ Now, in order to get n roots (which must occur according to the fundamental theorem of algebra) we must include variable k to ensure that giving: } \frac{\theta + 2\pi k}{n} \text{ (k=0, 1, 2, \dots, n-1)}$$

real part of complex number "z" raised to the $1/n$ power to obtain radius

Each root of complex number z will be evenly spaced on the graph, $\left(\frac{2\pi}{n} \right)$. However, in order for the formula to be able to differentiate between complex numbers, we must add the argument of z $\left(\frac{\theta + 2\pi}{n} \right)$. Now, in order to get n roots (which must occur according to the fundamental theorem of algebra) we must include variable k to ensure that giving: $\frac{\theta + 2\pi k}{n}$ ($k=0, 1, 2, \dots, n-1$)

When it's put all together:

$$\sqrt[n]{z} = |z|^{1/n} \left(\cos \frac{\theta + 2\pi k}{n} + i \sin \frac{\theta + 2\pi k}{n} \right)$$