Every Root of Every Complex Number

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Formula: $\sqrt[n]{a \angle b} = \sqrt[n]{a} \angle \frac{b}{n} + \frac{2\pi x}{n}$ $x: \{0, 1, 2, \ldots, n-2, n-1\}$

First we have that the radii for the n^{th} roots of a complex number with radius a is $\sqrt[n]{a}$ because multiplying a complex number with a radius $\sqrt[n]{a}$ with itself n times gives a complex number with radius a. This will be the radius for all n^{th} roots. Now we need to find the angles.

The first angle will be $\frac{b}{n}$ because multiply a complex number with angle $\frac{b}{n}$ with itself n times gives a complex number with angle b. Each other angle will be a rotation of $\frac{b}{n}$ around the origin. To be exact n rotation that together make up 2π radians or 360 degrees. So n rotations totalling to 2π radians would be $\frac{2\pi}{n}$ radians. We would rotate $\frac{b}{n}$ by $\frac{2\pi}{n}$ radians $n-1$ times because if we did it n times we would get $\frac{b}{n}$ again. So our angles are $\frac{b}{n} + \frac{2\pi x}{n}$ where x is all integers from 0 to $n-1$. This is how we get our formula.