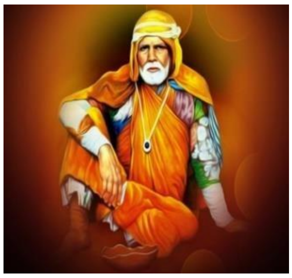


RAYAT SHIKSHAN SANSTHA'S

SADGURU GADGE MAHARAJ COLLEGE, KARAD

(An Autonomous College)

A Project Report On

"APPLICATION OF QUEUEING THEORY TO ANALYZE PETROL PUMP OPERATIONS IN KARAD CITY"

Department of Statistics

By

Miss. Shreya Santosh Bhudargade

Miss. Anjali Deelip Mali

M.Sc.-I (2024-25)

Under The Guidance of

Prof. Miss. S. S. Jagtap

CERTIFICATE

This is to certify that the project report on "**Application of queueing theory to analyze petrol pump operations in Karad city**". Being submitted by Miss. Shreya Santosh Bhudargade and Miss. Anjali Deelip Mali as partial fulfillment for the award of degree of masters in Statistics at Sadguru Gadge Maharaj College, Karad is a record of Bonafide work carried out by them under supervision and guidance.

To the best of our knowledge and belief, the matter presented in the project report is original and has not been submitted elsewhere for any other purpose.

Place: Karad

Date:

Sr. No	Seat No.	Roll No.	Name of the student	Signature
1.		02	Shreya Santosh Bhudargade	
2.		16	Anjali Deelip Mali	

Teacher in-charge Examiner PG Co-Ordinator Head

Department of Statistics

ACKNOWLEDGEMENT

This project entitled "**Application of queueing theory to analyze petrol pump operations in Karad city**" in Karad, Satara. We have great pleasure in presenting this report of successful completion of our project.

We take this opportunity to express our great sense of gratitude to our Guide Miss. S. S. Jagtap of Statistics Department, S.G.M College, Karad. For granting us permission to undertake this project report for their constant encouragement, guidance and inspiration without which we could not have completed this task.

We would like to extend our sincere thanks to S. V. Mahajan (Head of Department), Dr. Mrs. S. P. Patil (P. G. Co-Ordinator), S. S. Jagtap & all faculty members for their guidance and kind operation in this project.

I would like to extend my heartfelt gratitude to 'Mulik Petrol Pump' for their invaluable support and cooperation during this project.

I acknowledge the late Mr. Sanjay Mulik for his role in establishing and supporting the operations of Mulik Petrol Pump, which laid the foundation for this study.

Special thanks to Mrs. Vidya Mulik, who now owns the petrol pump, for her continuous support, coordination, and providing access to essential data.

Heartfelt thanks to Mr. Ajit Gadkari for their exceptional management and assistance in facilitating our visits and ensuring we had everything needed for this study.

We would also like to express my appreciation to the entire staff of Mulik Petrol Pump for their cooperation, patience, and willingness to help us throughout this project.

We are thankful to the customers of Mulik Petrol Pump for their indirect contribution, as their participation in the daily operations allowed me to gather the necessary data for this study.

Their collective efforts and support have been instrumental in the successful completion of this study.

INDEX

1. INTRODUCTION

Queueing theory, a significant branch of operations research, is dedicated to studying waiting lines or queues. It offers vital insights into the behavior and performance of systems with shared resources among multiple users. In this project, undertaken as part of our MSc-I Statistics field study, we explore the application of queueing theory at a petrol pump in Karad, Maharashtra.

Karad, a town with a moderate level of vehicular traffic, provides an intriguing case for analyzing queueing dynamics. Our primary objective is to examine the arrival and service processes at the petrol pump to identify key performance metrics such as arrival rates, service rates, and average customer waiting times. By integrating theoretical models with empirical data, we aim to optimize the operational efficiency of the petrol pump and improve customer satisfaction.

This report demonstrates the practical application of queueing theory concepts in a real-world setting. Our findings not only benefit the petrol pump in Karad but also offer valuable insights for other similar service-oriented operations. By understanding and optimizing queueing processes, we contribute to enhancing the overall efficiency and effectiveness of service systems.

2. OBJECTIVES

- To analyze customer arrival patterns and service rates at a petrol pump in Karad city.
- To model the queueing system using queueing theory concepts.
- To evaluate the performance of the current system.
- To propose optimization strategies to improve service efficiency.

3. LITERATURE REVIEW

1. Odior, A. O. (2013). Application of Queuing Theory to Petrol Stations in Benin-City Area of Edo State, Nigeria.

Published in the Nigerian Journal of Technology (NIJOTECH), this study analyzed queues at five petrol stations in Benin City using a multi-server queuing model (M/M/s). Findings showed that increasing the number of service pumps reduces waiting times.

2. Daw Nway Thet Yi & Dr. Thet Mon Win (Year Not Mentioned). Application of Queuing Model to Minimize the Waiting Time in Fuel Stations.

Conducted in Myanmar, this study focused on a single-channel, multiple-server model at a fuel station in Hinthada. Using Poisson probability distribution, the study suggested optimizing server efficiency based on peak hours.

3. Kembe, M. M., Gbenimak, C. J., & Onoja, A. A. (2017). Application of Queueing Theory to Customers Purchasing Premium Motor Spirit (PMS) at a Filling Station.

This research at NNPC Mega Station, Jos, Nigeria, used an M/M/10 model and found an optimal utilization factor of 66.43%. The study recommended adding two more servers to reduce queue times.

4. Ayeni-Agbaje, A. R., Adebayo, I. A., & Adeniyi, A. (2024). Application of Queuing Theory in Enhancing Performance of Petrol Dispensing Stations in Ado-Ekiti.

Published in the International Journal of Advances in Engineering and Management, this study surveyed 11 petrol stations in Ado-Ekiti, Nigeria. Findings indicated that combining single and multiple queue systems improves service efficiency.

4. METHODOLOGY

Data Collection

❖ Data Sources:

- Arrival Times: Arrival times of vehicles were recorded manually with the help of a smartwatch timelapse feature.
- Service Times: Service times were measured using the smartwatch timelapse feature, recording the duration taken to refuel each vehicle.

❖ Data Collection Period:

- The data collection took place over a period of 7 days, from January 27, 2025, to February 2, 2025.
- The data was collected during three time slots each day: morning (8 AM to 10 AM), afternoon (12 PM to 2 PM), and evening (6 PM to 8 PM).
- Each time slot lasted for 2 hours, capturing a comprehensive view of different parts of the day.

❖ Sample Size:

- The study observed a total of around 3600 vehicles.

❖ Location:

- Data was collected at Mulik Petrol pump, located in front of Pandharicha Maruti Mandir, Karad.

❖ Pilot survey:

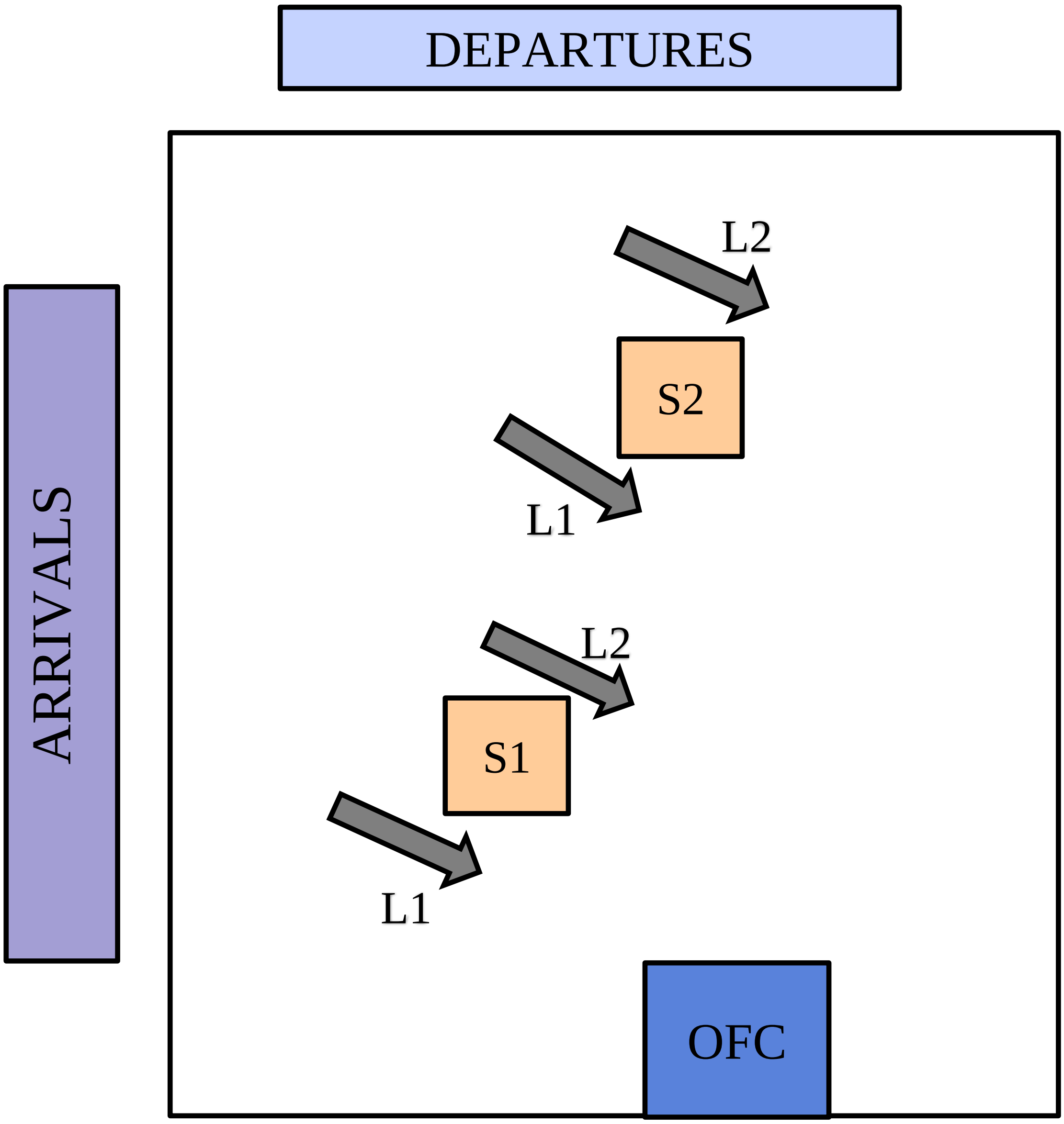
- We conducted a one-hour pilot survey at the petrol pump to test our data collection methods and identify any potential challenges. During this survey, we initially recorded the timing in hours and minutes but realized the need to use minutes and seconds for greater precision. .

❖ Problems encountered during data collection:

- During data collection, we encountered interruptions from curious customers asking about our work. While these interactions were positive, they occasionally disrupted the data recording process. To address this, we provided brief explanations and politely requested minimal interruptions to ensure accurate data collection

5. PLANT LAYOUT

OFC: Office
S1: Station 1 = Petrol + Diesel
S2: Station 2 = Petrol + Petrol
L1: Line 1
L2: Line 2



6.DESCRIPTION OF THE QUEUE MODEL

▪ Petrol Pump Overview:

- Location: In front of Pandhari Cha Maruti Mandir
- Area: 5000 sq. ft.
- Stations: 2 stations, each with multiple queues
- Total Workers: 15
- Station 1: 2 workers handling two queues (M/M/2 model)
- Station 2: 1 worker handling two queues (M/M/1 model)
- Overall System: Combined model follows an M/M/3 structure

▪ Queue Characteristics:

- Queue Discipline:
  First-Come, First-Served (FCFS) by any available server
- Unlimited Queue Capacity: No restrictions on queue length
- Arrival & Service Rates:
  1. Average arrival rate is greater than the average service rate (ensuring a steady queue formation)
  2. Mean arrival rate is constant and independent of queue length or customers already serviced
- Service Efficiency:
  1. Service providers are working at full capacity
  2. Both arrival and departure rates are state-dependent, meaning they change based on the number of vehicles in the system

7.METHOD OF DATA COLLECTION

❖ Data Recording:

- The data was manually transcribed onto paper with columns for arrival time, service time, and departure time.

❖ Recording Process:

- Arrival times of vehicles were recorded as they entered the queue.
- Service times were measured from the moment the refuelling process began until it was completed.
- Departure times were recorded when vehicles left the service point.

- Sample collecting images are provided below.

8. TOOLS AND TECHNIQUES

- **Statistical Tools:**
  - Excel
  - Microsoft Word
  - Python (NumPy, Pandas, Matplotlib, Seaborn, Math)

- **Data Visualization:**
  - ACF plot
  - Histogram

- **Statistical Analysis:**
  - Exploratory data analysis
  - Descriptive data analysis

9. EXPLORATORY DATA ANALYSIS

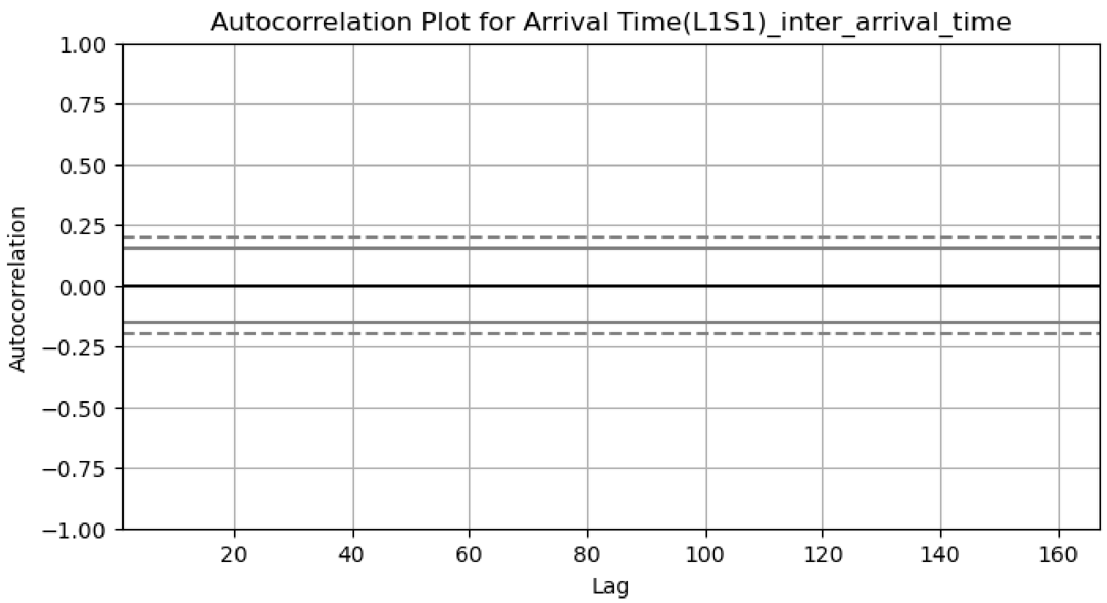
- ## System analysis:
  - **To analyze the arrival pattern, we first examine whether the arrival process follows a Poisson distribution. A Poisson process assumes that interarrival times are exponentially distributed and that arrivals occur independently over time.**

    To validate this, we conduct two key tests:

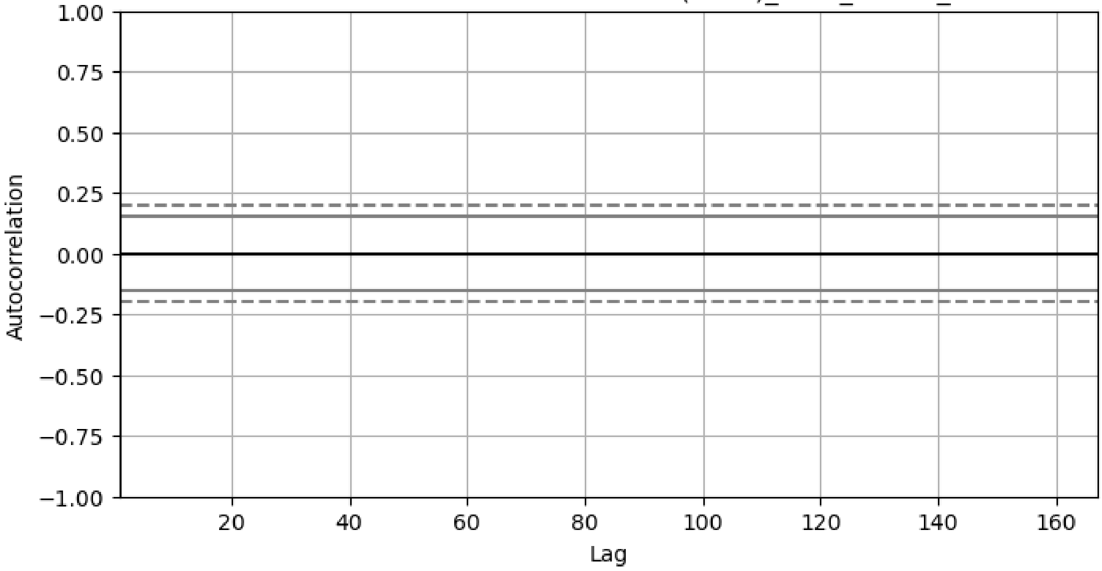
1. Autocorrelation Test:

This checks whether arrival times exhibit any dependency or correlation.

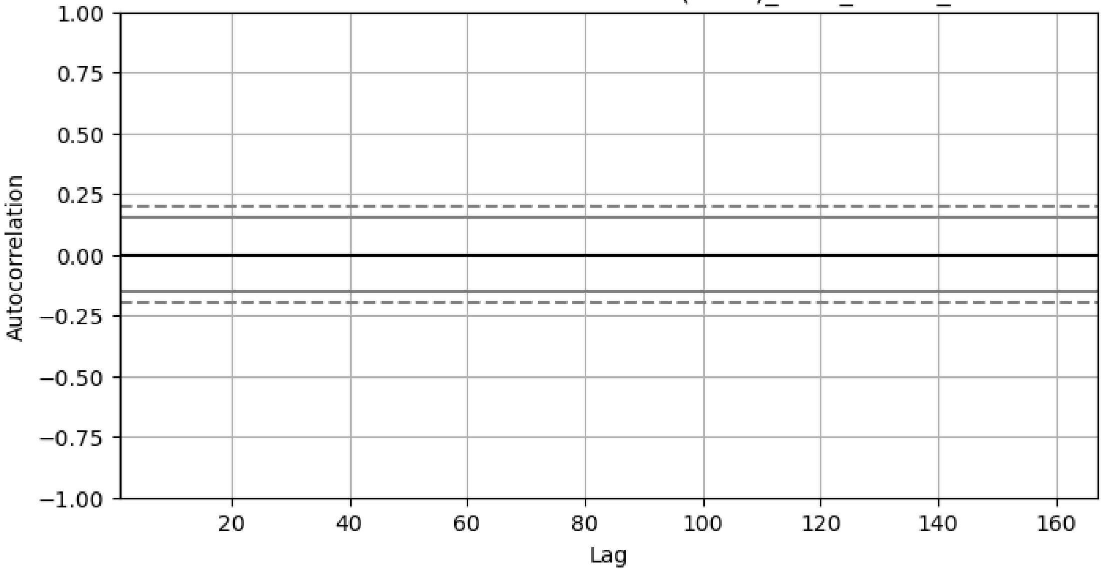
Results of this test:



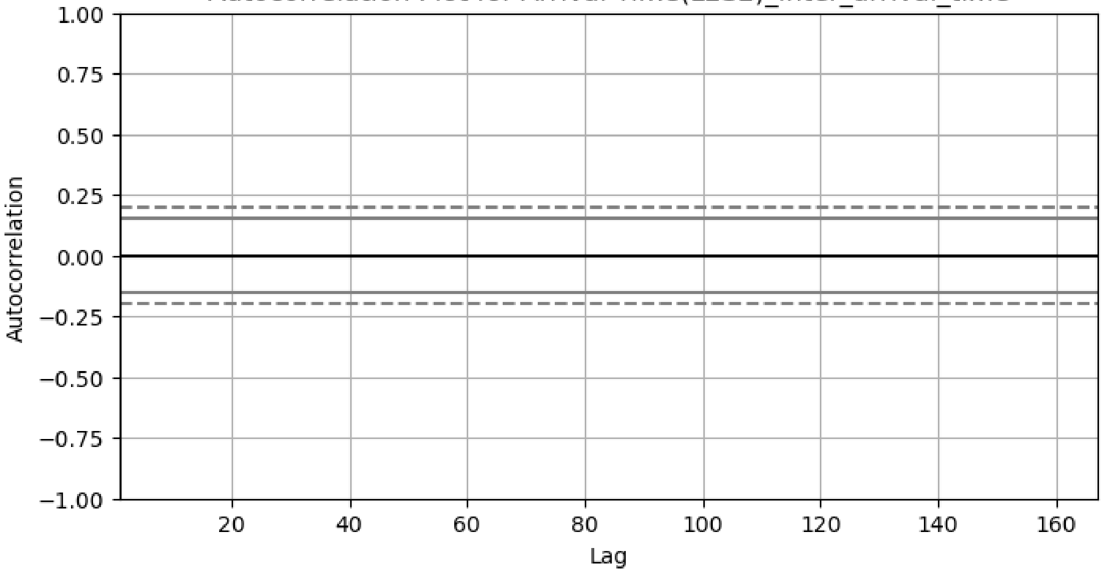
Autocorrelation Plot for Arrival Time(L2S1)_inter_arrival_time



Autocorrelation Plot for Arrival Time(L1S2)_inter_arrival_time



Autocorrelation Plot for Arrival Time(L2S2)_inter_arrival_time



- **Description:**
  The autocorrelation plots below represent the interarrival times for lanes 1 and 2, stations 1 and 2 (L1S1, L2S1, L1S2, L2S2) on Tuesday.

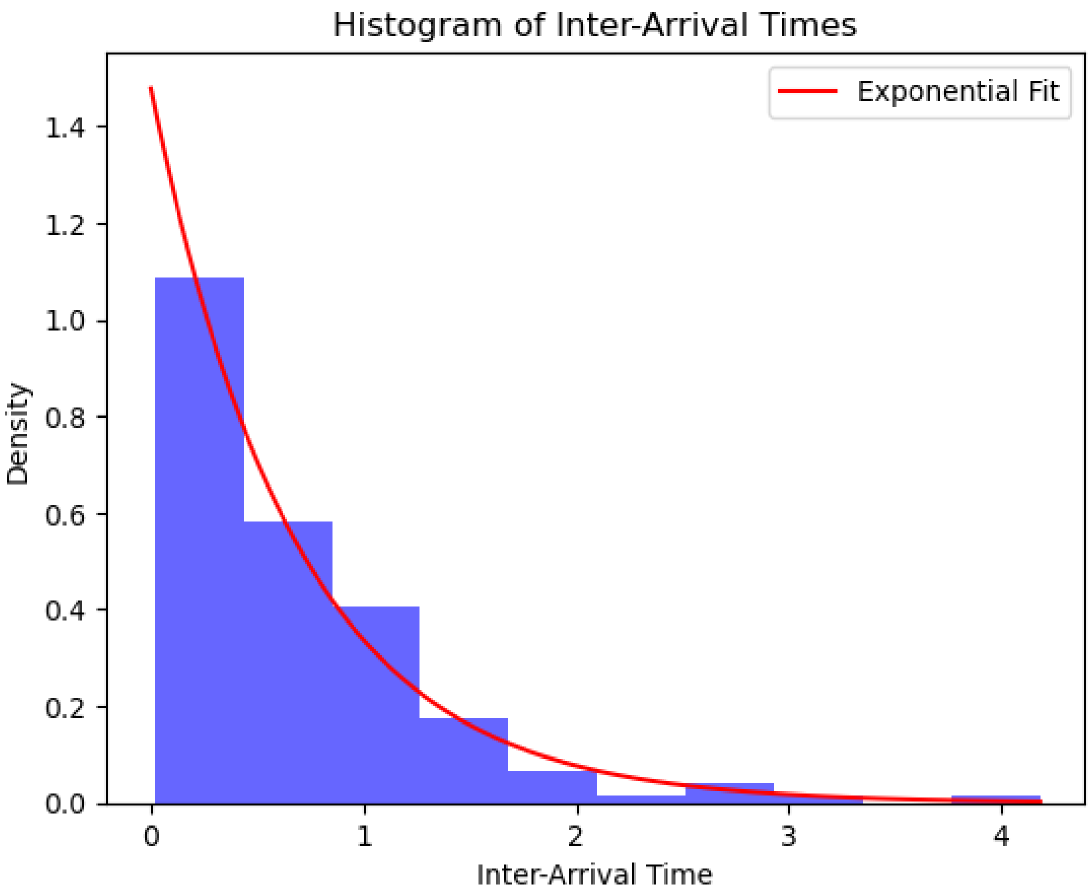
  In a Poisson process, interarrival times should be independent, meaning their autocorrelation values should remain within the confidence bounds and not show any significant pattern.

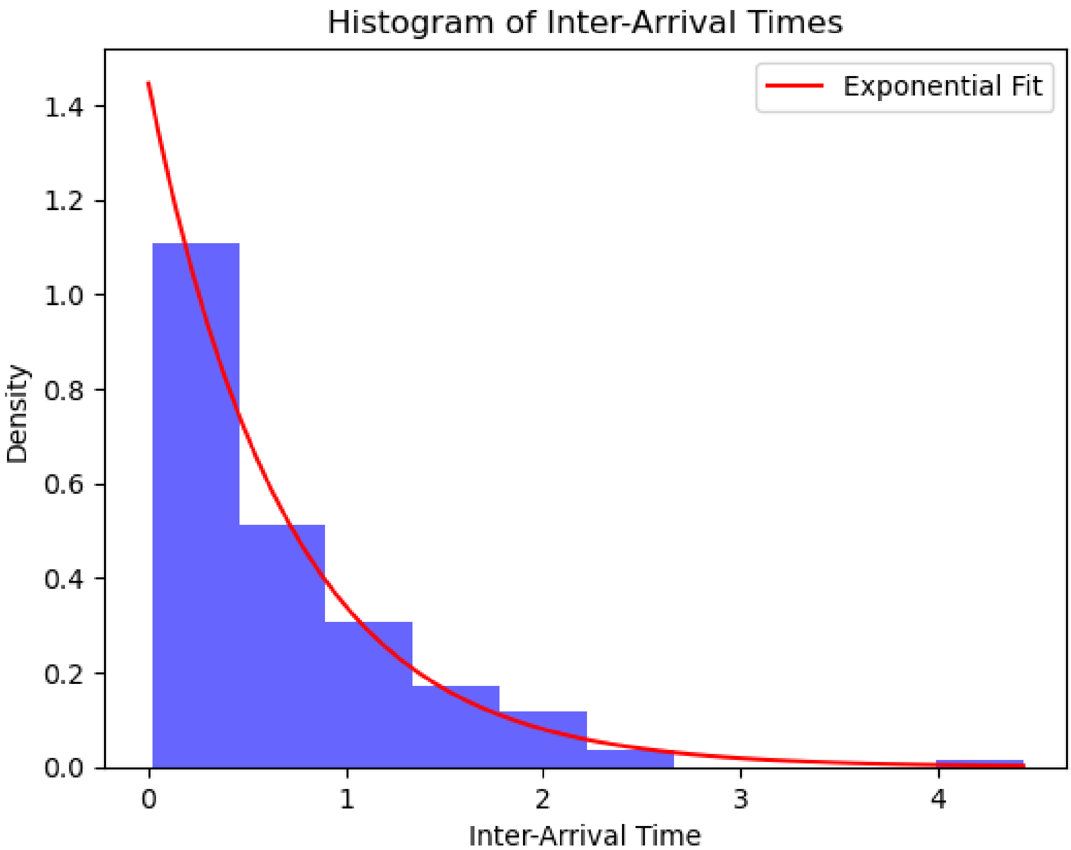
  From the plots, we observe that most autocorrelation values lie close to zero and within the confidence bounds, indicating little to no correlation between interarrival times. This suggests that the arrival process is likely independent, supporting the assumption of a Poisson process.

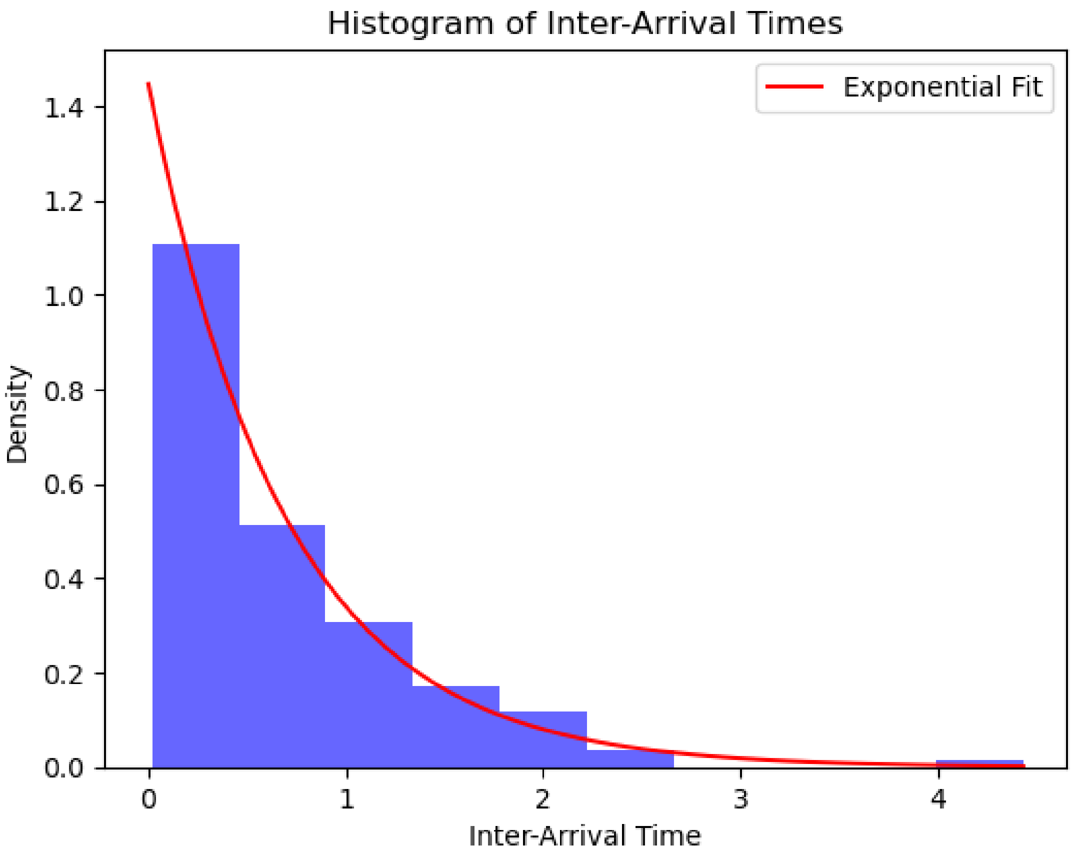
  Since the autocorrelation plots for Monday, Wednesday, Thursday, Friday, Saturday, and Sunday exhibit a similar pattern, we present only these representative plots.

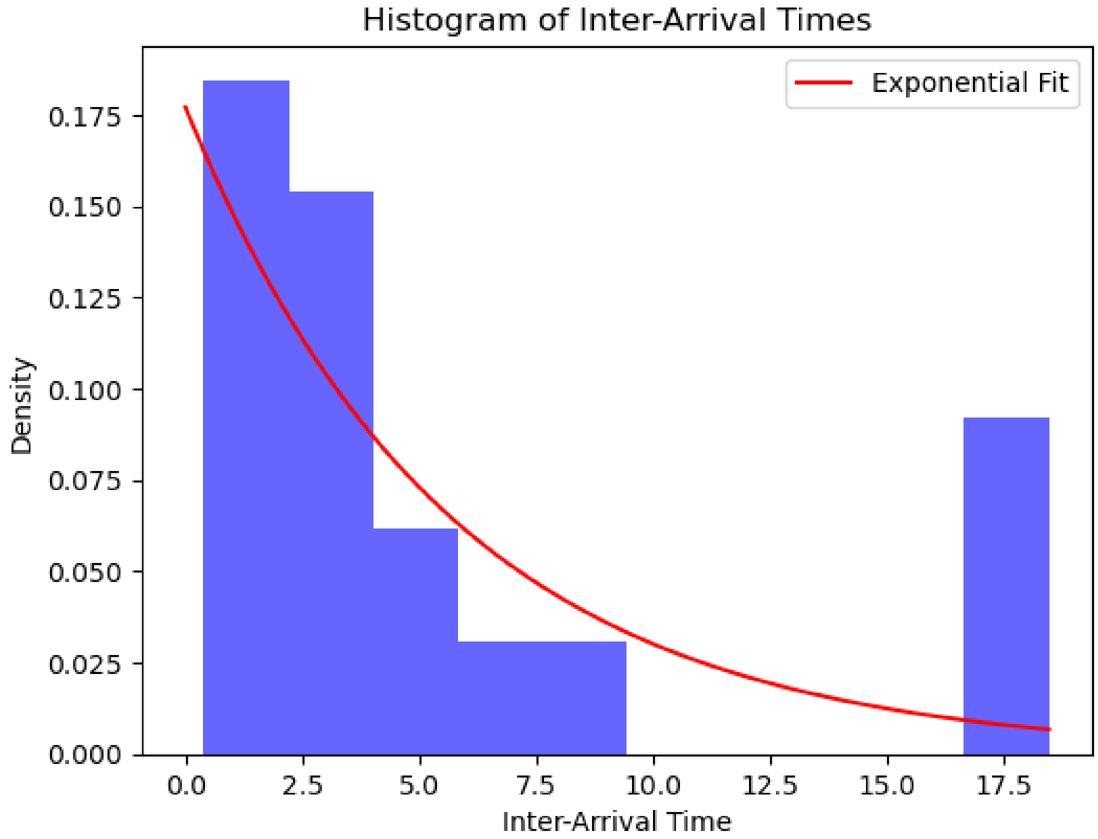
2. Exponential Fit:

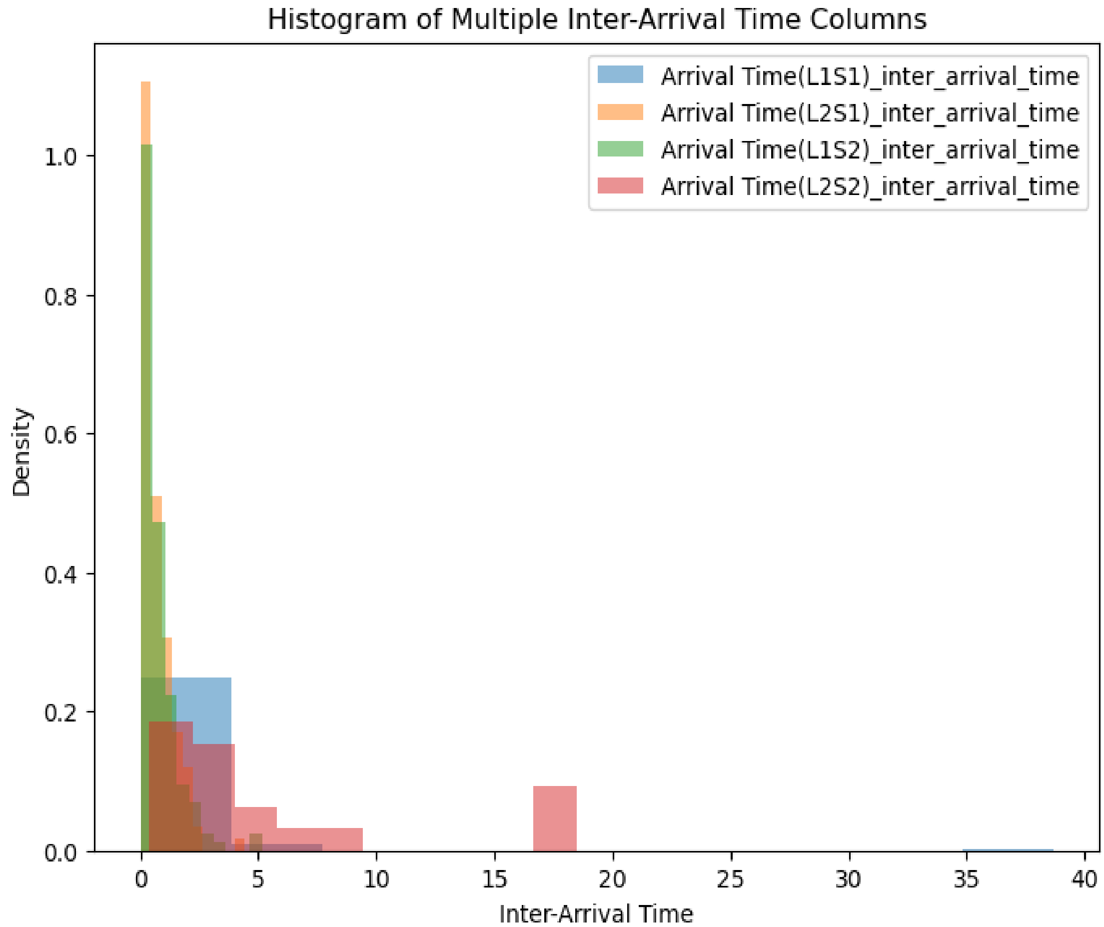
This examines whether the interarrival times follow an exponential distribution.




Histogram of Inter-Arrival Times
Exponential Fit
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Density
0
1
2
3
4
Inter-Arrival Time


Histogram of Inter-Arrival Times
Exponential Fit
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Density
0
1
2
3
4
Inter-Arrival Time


Histogram of Inter-Arrival Times
Exponential Fit
Density
0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
Inter-Arrival Time


Histogram of Multiple Inter-Arrival Time Columns
Arrival Time(L1S1)_inter_arrival_time
Arrival Time(L2S1)_inter_arrival_time
Arrival Time(L1S2)_inter_arrival_time
Arrival Time(L2S2)_inter_arrival_time
Density
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
25
30
35
40
Inter-Arrival Time

- **Description:**

  The histograms below represent the distribution of interarrival times observed on Tuesday for lanes 1 and 2, stations 1 and 2 (L1S1, L2S1, L1S2, L2S2). To test whether the interarrival times follow an exponential distribution, an exponential probability density function (red curve) is fitted to the data.

  In a Poisson process, interarrival times should follow an exponential distribution, characterized by a rapid decline in frequency as time increases.

  From the plots, we observe that the histograms closely follow the shape of the exponential curves, suggesting a good fit. This supports that vehicle arrivals occur randomly over time.

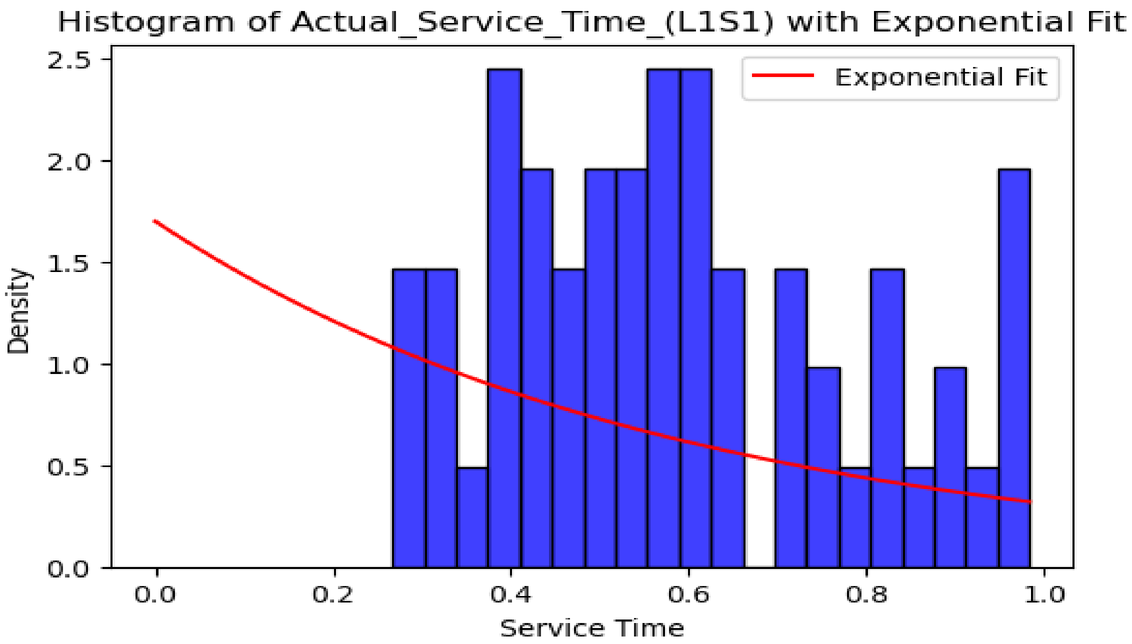
  Since the histograms for Monday, Wednesday, Thursday, Friday, Saturday, and Sunday exhibit a similar pattern, we present only these representative plots.

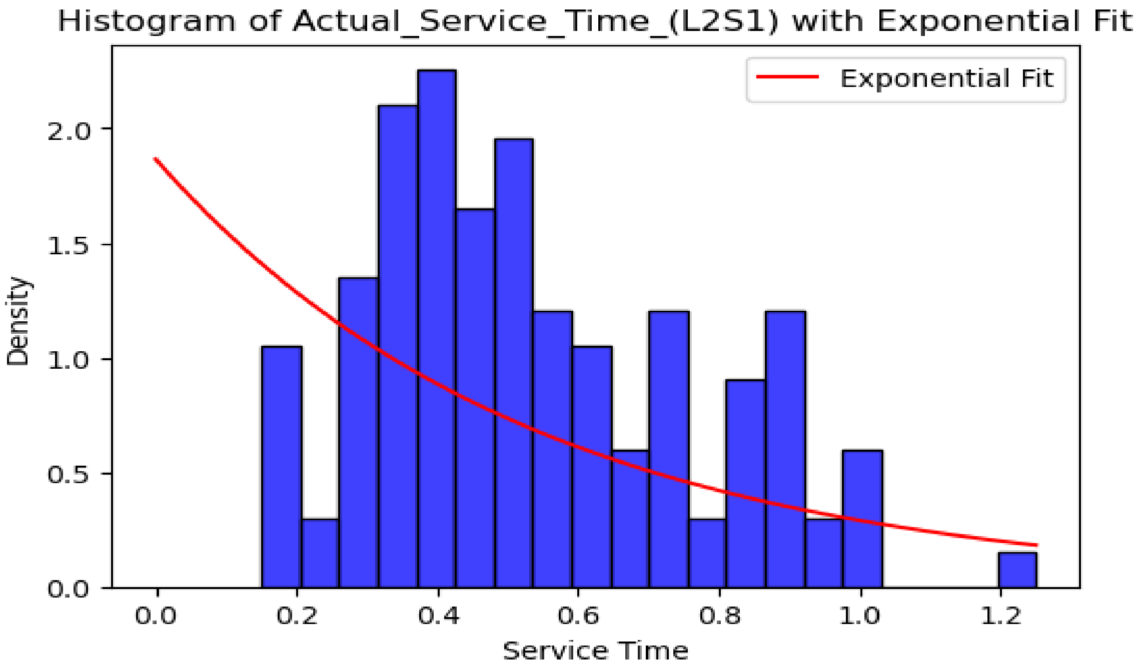
- **To analyze the service pattern at the petrol pump, we examine whether the service times follow an exponential distribution**

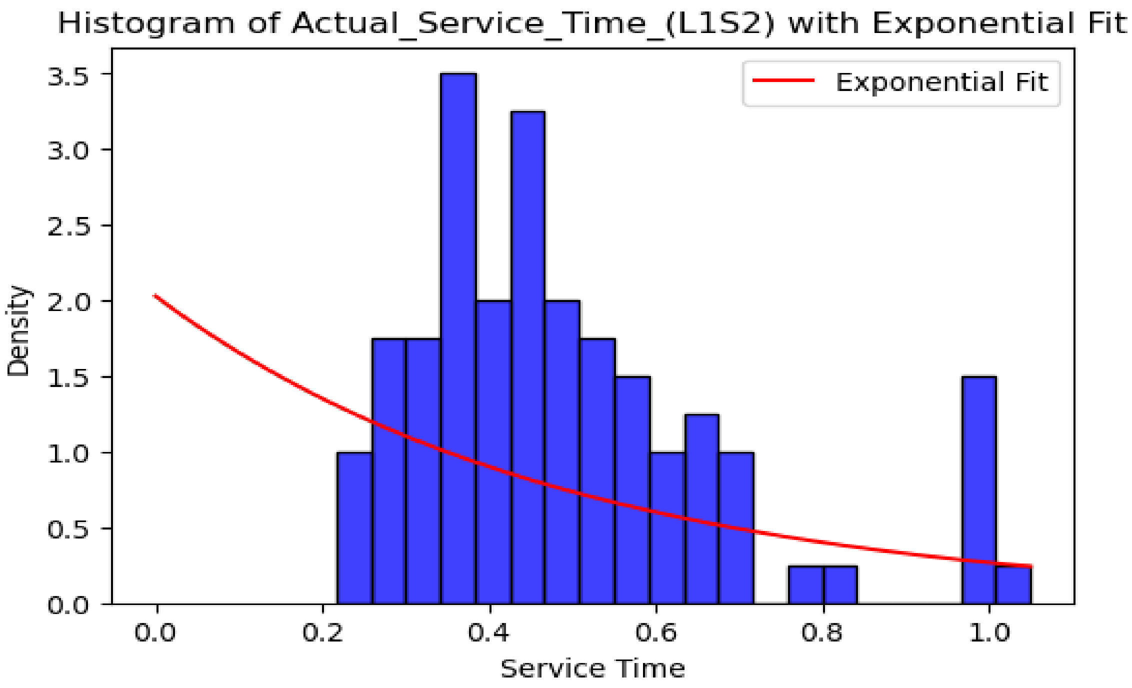
To validate this, we conduct key test:

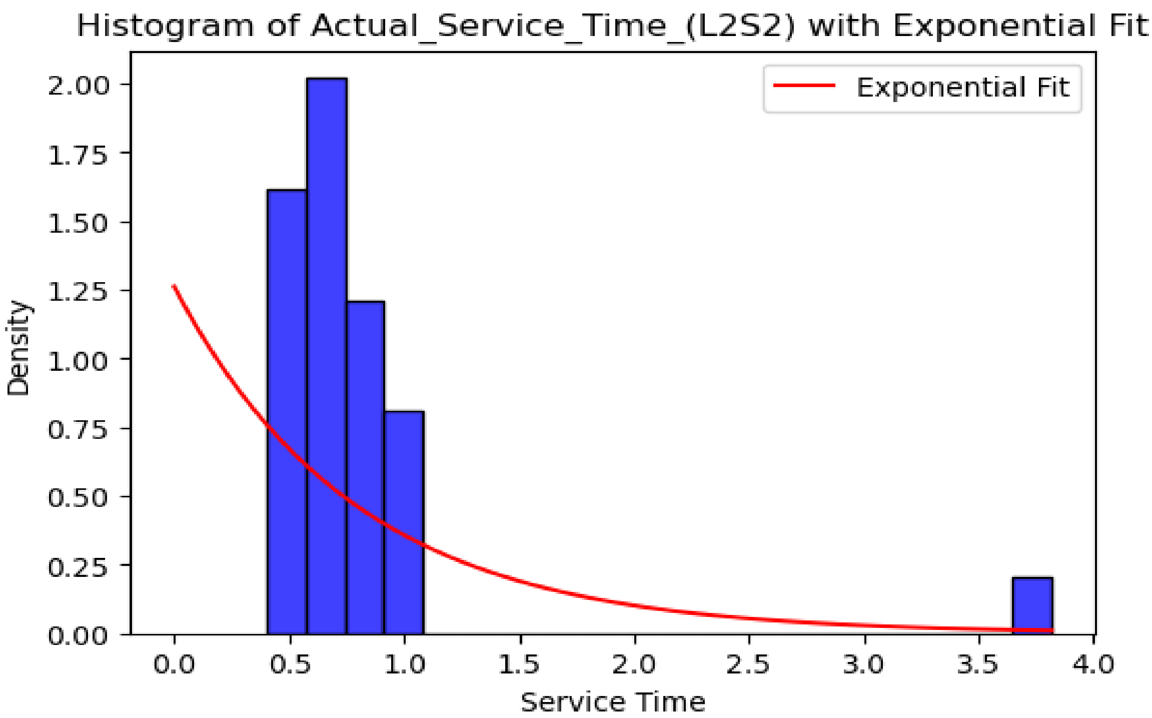
1.Exponential Fit:

An exponential distribution of service times indicates that the process is memoryless, meaning the likelihood of service completion remains constant over time.






Histogram of Actual_Service_Time_(L1S2) with Exponential Fit
Exponential Fit
Density
Service Time
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
0.0
0.2
0.4
0.6
0.8
1.0


Histogram of Actual_Service_Time_(L2S2) with Exponential Fit
Exponential Fit
Density
Service Time
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

- ## Description:

The histograms below represent the distribution of observed service times on Tuesday for lanes 1 and 2, stations 1 and 2 (L1S1, L2S1, L1S2, L2S2). To test whether the service times follow an exponential distribution, an exponential probability density function is fitted to the data.

In a queuing system, service times are often assumed to follow an exponential distribution, indicating a memoryless property where the probability of service completion remains constant over time.

From the plots, we observe that the histograms closely align with the shape of the exponential curves, suggesting a reasonable fit. This supports that service times are exponentially distributed.

Since the histograms for Monday, Wednesday, Thursday, Friday, Saturday, and Sunday exhibit a similar pattern, we present only these representative plot

10. DESCRIPTIVE DATA ANALYSIS

- ## Monday:

Metrics	Values	Description
Arrival Rate(λ)	**4.6916** customers per minute	Number of customers arriving per minute.
Service Rate(μ)	**1.8400** per server per minute	Customers served by one server per minute.
Total Service Rate(μ_{total})	**5.52** per minute	Combined service rate of all servers
Utilization Factor (ρ)	**0.8551**	System load; should be < 1 for stability
Probability of Idle System(P_0)	**0.0381**	Chance of no customers in the system
Average Queue Length (L_q)	**4.3592** customers	Expected number of waiting customers.
Average Waiting Time in Queue (W_q)	**0.9291** minutes	Average time a customer waits in the queue.
Average no. of customers in the system (L_s)	**6.9244** customers	Average no. of customers waiting in the system
Total Time Spent in System (W_s)	**1.4759** minutes	Time from arrival to service completion.

• Comparison of Station 1 and Station 2 Analysis

Metrics	Station 1	Station 2
Arrival Rate (λ)	**3.125** customers per minute	**1.5667** customers per minute
Service Rate (μ)	**1.8462** per server per minute	**1.8277** per Server per minute
Total Service Rate (μ_{total})	**3.6924** per minute	**1.8277** per minute
Utilization Factor (ρ)	**0.8540**	**0.8572**
Probability of Idle System(P_0)	**0.0788**	**0.1428**
Average Queue Length (L_q)	**4.6004** customers	**5.1446** customers
Average Waiting Time in Queue (W_q)	**1.4721** minutes	**3.2838** minutes
Average no. of customers in the system (L_s)	**6.3083** customers	**6.0018** customers
Total Time Spent in System (W_s)	**2.0187** minutes	**3.8309** minutes

- **Description:**

The system has an arrival rate (λ) of 4.6916 customers per minute, indicating how quickly customers enter the system. Each server operates at a service rate (μ) of 1.8400 customers per minute, and when all servers are combined, the total service rate (μ_{total}) is 5.52 customers per minute. The utilization factor (p) is 0.8551, showing that the system is operating close to its capacity, as a value below 1 is required for stability. The probability of the system being idle (P_o) is 0.0381, meaning there is a very low chance of no customers being in the system at any given time.

On average, there are 4.3592 customers waiting in the queue (L_q), with an average waiting time in the queue (W_q) of 0.9291 minutes. The average number of customers in the entire system (L_s) is 6.9244, and the total time a customer spends in the system (W_s) is 1.4759 minutes, from arrival to service completion.

When examining the two stations within the system, differences in performance become apparent. The first station has an arrival rate (λ) of 3.125 customers per minute, while the second station has a lower arrival rate of 1.5667 customers per minute. The service rates (μ) for the first and second stations are 1.8462 and 1.8277 customers per server per minute, respectively. The total service rates (μ_{total}) are 3.6924 for the first station and 1.8277 for the second station.

Both stations have high utilization factors (p), with the first station at 0.8540 and the second station at 0.8572, indicating they are operating near their capacities. However, the second station has a higher probability of being idle (P_o = 0.1428) compared to the first station (P_o = 0.0788), suggesting it experiences more downtime.

The first station has an average queue length (L_q) of 4.6004 customers, while the second station has a slightly higher average queue length of 5.1446 customers. The average waiting time in the queue (W_q) is 1.4721 minutes for the first station and significantly higher at 3.2838 minutes for the second station. The average number of customers in the system (L_s) is 6.3083 for the first station and 6.0018 for the second station. The total time spent in the system (W_s) is 2.0187 minutes for the first station and 3.8309 minutes for the second station.

Overall, the system is operating efficiently but is close to its capacity, as indicated by the high utilization factors. The first station handles a higher arrival rate with slightly better efficiency in terms of waiting times and queue lengths compared to the second station. However, both stations exhibit high utilization and significant queue lengths, suggesting potential areas for optimization to improve customer experience and system performance.

- ## Tuesday:

Metrics	Values	Description
Arrival Rate(λ)	**3.4333** customers per minute	Number of customers arriving per minute.
Service Rate(μ)	**1.4106** per server per minute	Customers served by one server per minute.
Total Service Rate(μ_{total})	**4.2318** per minute	Combined service rate of all servers
Utilization Factor (ρ)	**0.8113**	System load; should be < 1 for stability
Probability of Idle System(P_0)	**0.0523**	Chance of no customers in the system
Average Queue Length (L_q)	**2.8625** customers	Expected number of waiting customers.
Average Waiting Time in Queue (W_q)	**0.8337** minutes	Average time a customer waits in the queue.
Average no. of customers in the system (L_s)	**5.2964** customers	Average no. of customers waiting in the system
Total Time Spent in System (W_s)	**1.5426** minutes	Time from arrival to service completion.

- ## Comparison of Station 1 and Station 2 Analysis

Metrics	Station 1	Station 2
Arrival Rate (λ)	**1.9** customers per minute	**1.5334** customers per minute
Service Rate (μ)	**1.9953** per server per minute	**1.0360** per Server per minute
Total Service Rate (μ_{total})	**3.9906** per minute	**1.0360** per minute
Utilization Factor (ρ)	**0.4761**	**1.4801**
Probability of Idle System(P_0)	**0.3549**	**-0.4801**
Average Queue Length (L_q)	**0.2791** customers	∞
Average Waiting Time in Queue (W_q)	**0.1469** minutes	∞
Average no. of customers in the system (L_s)	**1.2314** customers	∞
Total Time Spent in System (W_s)	**0.6481** minutes	∞

- **Description:**

The system has an arrival rate (λ) of 3.4333 customers per minute, indicating the rate at which customers enter the system. Each server operates at a service rate (μ) of 1.4106 customers per minute, and when all servers are combined, the total service rate (μ_{total}) is 4.2318 customers per minute. The utilization factor (ρ) is 0.8113, showing that the system is operating at a high load, as a value below 1 is required for stability. The probability of the system being idle (P_o) is 0.0523, meaning there is a very low chance of no customers being in the system at any given time.

On average, there are 2.8625 customers waiting in the queue (L_q), with an average waiting time in the queue (W_q) of 0.8337 minutes. The average number of customers in the entire system (L_s) is 5.2964, and the total time a customer spends in the system (W_s) is 1.5426 minutes, from arrival to service completion.

When examining the two stations within the system, significant differences in performance become apparent. The first station has an arrival rate (λ) of 1.9 customers per minute, while the second station has a slightly lower arrival rate of 1.5334 customers per minute. The service rates (μ) for the first and second stations are 1.9953 and 1.0360 customers per server per minute, respectively. The total service rates (μ_{total}) are 3.9906 for the first station and 1.0360 for the second station.

The utilization factor (ρ) for the first station is 0.4761, indicating it is operating well within its capacity. However, the second station has a utilization factor of 1.4801, which exceeds the stability threshold of 1, suggesting it is overloaded. The probability of the system being idle (P_o) is 0.3549 for the first station, while the second station has a negative probability (-0.4801), which is not feasible and indicates an unstable system.

The first station has an average queue length (L_q) of 0.2791 customers, while the second station has an infinite queue length (∞), indicating severe congestion. The average waiting time in the queue (W_q) is 0.1469 minutes for the first station and infinite for the second station. The average number of customers in the system (L_s) is 1.2314 for the first station and infinite for the second station. The total time spent in the system (W_s) is 0.6481 minutes for the first station and infinite for the second station.

Overall, the system is operating efficiently but is close to its capacity, as indicated by the high utilization factor. The first station handles its load well, with manageable queue lengths and waiting times. In contrast, the second station is significantly overloaded, leading to infinite queue lengths and waiting times, indicating a critical need for optimization to restore stability and improve performance.

- ## Wednesday:

Metrics	Values	Description
Arrival Rate(λ)	**2.9667** customers per minute	Number of customers arriving per minute.
Service Rate(μ)	**1.0402** per server per minute	Customers served by one server per minute.
Total Service Rate(μ_{total})	**3.1206** per minute	Combined service rate of all servers
Utilization Factor (ρ)	**0.9507**	System load; should be < 1 for stability
Probability of Idle System(P_0)	**0.0116**	Chance of no customers in the system
Average Queue Length (L_q)	**17.5039** customers	Expected number of waiting customers.
Average Waiting Time in Queue (W_q)	**5.9002** minutes	Average time a customer waits in the queue.
Average no. of customers in the system (L_s)	**20.3560** customers	Average no. of customers waiting in the system
Total Time Spent in System (W_s)	**6.8616** minutes	Time from arrival to service completion.

- ## Comparison of Station 1 and Station 2 Analysis

Metrics	Station 1	Station 2
Arrival Rate (λ)	**2.0** customers per minute	**0.9667** customers per minute
Service Rate (μ)	**1.3102** per server per minute	**0.7293** per Server per minute
Total Service Rate (μ_{total})	**2.6204** per minute	**0.7293** per minute
Utilization Factor (ρ)	**0.7632**	**1.3255**
Probability of Idle System(P_0)	**0.1343**	**-0.3255**
Average Queue Length (L_q)	**2.1301** customers	∞
Average Waiting Time in Queue (W_q)	**1.0651** minutes	∞
Average no. of customers in the system (L_s)	**3.6566** customers	∞
Total Time Spent in System (W_s)	**1.8283** minutes	∞

- ## Description:

The system has an arrival rate (λ) of 2.9667 customers per minute, indicating the rate at which customers enter the system. Each server operates at a service rate (μ) of 1.0402 customers per minute, and when all servers are combined, the total service rate (μ_{total}) is 3.1206 customers per minute. The utilization factor (ρ) is 0.9507, showing that the system is operating very close to its capacity, as a value below 1 is required for stability. The probability of the system being idle (P_o) is 0.0116, meaning there is a very low chance of no customers being in the system at any given time.

On average, there are 17.5039 customers waiting in the queue (L_q), with an average waiting time in the queue (W_q) of 5.9002 minutes. The average number of customers in the entire system (L_s) is 20.3560, and the total time a customer spends in the system (W_s) is 6.8616 minutes, from arrival to service completion.

When examining the two stations within the system, significant differences in performance become apparent. The first station has an arrival rate (λ) of 2.0 customers per minute, while the second station has a lower arrival rate of 0.9667 customers per minute. The service rates (μ) for the first and second stations are 1.3102 and 0.7293 customers per server per minute, respectively. The total service rates (μ_{total}) are 2.6204 for the first station and 0.7293 for the second station.

The utilization factor (ρ) for the first station is 0.7632, indicating it is operating within its capacity. However, the second station has a utilization factor of 1.3255, which exceeds the stability threshold of 1, suggesting it is overloaded. The probability of the system being idle (P_o) is 0.1343 for the first station, while the second station has a negative probability (-0.3255), which is not feasible and indicates an unstable system.

The first station has an average queue length (L_q) of 2.1301 customers, while the second station has an infinite queue length (∞), indicating severe congestion. The average waiting time in the queue (W_q) is 1.0651 minutes for the first station and infinite for the second station. The average number of customers in the system (L_s) is 3.6566 for the first station and infinite for the second station. The total time spent in the system (W_s) is 1.8283 minutes for the first station and infinite for the second station.

Overall, the system is operating very close to its capacity, as indicated by the high utilization factor. The first station handles its load well, with manageable queue lengths and waiting times. In contrast, the second station is significantly overloaded, leading to infinite queue lengths and waiting times, indicating a critical need for optimization to restore stability and improve performance.

- ### Thursday:

Metrics	Values	Description
Arrival Rate(λ)	**4.6166** customers per minute	Number of customers arriving per minute.
Service Rate(μ)	**2.0428** per server per minute	Customers served by one server per minute.
Total Service Rate(μ_{total})	**6.1284** per minute	Combined service rate of all servers
Utilization Factor (ρ)	**0.7533**	System load; should be < 1 for stability
Probability of Idle System(P_0)	**0.0735**	Chance of no customers in the system
Average Queue Length (L_q)	**1.7496** customers	Expected number of waiting customers.
Average Waiting Time in Queue (W_q)	**0.3790** minutes	Average time a customer waits in the queue.
Average no. of customers in the system (L_s)	**2.5030** customers	Average no. of customers waiting in the system
Total Time Spent in System (W_s)	**0.8685** minutes	Time from arrival to service completion.

- ## Comparison of Station 1 and Station 2 Analysis

Metrics	Station 1	Station 2
Arrival Rate (λ)	**2.9333** customers per minute	**1.6833** customers per minute
Service Rate (μ)	**2.0549** per server per minute	**2.0220** per Server per minute
Total Service Rate (μ_{total})	**4.1098** per minute	**2.0220** per minute
Utilization Factor (ρ)	**0.7137**	**0.8325**
Probability of Idle System(P_0)	**0.1670**	**0.1675**
Average Queue Length (L_q)	**1.4823** customers	**4.1380** customers
Average Waiting Time in Queue (W_q)	**0.5053** minutes	**2.4582** minutes
Average no. of customers in the system (L_s)	**2.1960** customers	**4.9705** customers
Total Time Spent in System (W_s)	**0.9920** minutes	**2.9528** minutes

- ## Description:

The system has an arrival rate (λ) of 4.6166 that indicates the rate at which customers enter the system. Each server operates at a service rate (μ) of 2.0428 customers per minute, and when all servers are combined, the total service rate (μ_{total}) is 6.1284 customers per minute. The utilization factor (ρ) is 0.7533, showing that the system is operating within a stable range, as a value below 1 is required for stability. The probability of the system being idle (P_o) is 0.0735, meaning there is a relatively low chance of no customers being in the system at any given time.

On average, there are 1.7496 customers waiting in the queue (L_q), with an average waiting time in the queue (W_q) of 0.3790 minutes. The average number of customers in the entire system (L_s) is 2.5030, and the total time a customer spends in the system (W_s) is 0.8685 minutes, from arrival to service completion.

When examining the two stations within the system, differences in performance become apparent. The first station has an arrival rate (λ) of 2.9333 customers per minute, while the second station has a lower arrival rate of 1.6833 customers per minute. The service rates (μ) for the first and second stations are 2.0549 and 2.0220 customers per server per minute, respectively. The total service rates (μ_{total}) are 4.1098 for the first station and 2.0220 for the second station.

The utilization factor (ρ) for the first station is 0.7137, indicating it is operating well within its capacity. The second station has a utilization factor of 0.8325, which is also within the stable range but higher than the first station. The probability of the system being idle (P_o) is 0.1670 for the first station and 0.1675 for the second station, suggesting both stations have similar idle times.

The first station has an average queue length (L_q) of 1.4823 customers, while the second station has a higher average queue length of 4.1380 customers. The average waiting time in the queue (W_q) is 0.5053 minutes for the first station and significantly higher at 2.4582 minutes for the second station. The average number of customers in the system (L_s) is 2.1960 for the first station and 4.9705 for the second station. The total time spent in the system (W_s) is 0.9920 minutes for the first station and 2.9528 minutes for the second station.

Overall, the system is operating efficiently, with both stations functioning within their capacities. The first station handles its load more effectively, with shorter queue lengths and waiting times compared to the second station. While both stations are stable, the second station exhibits higher congestion and longer waiting times, indicating potential areas for optimization to improve customer experience and system performance.

- **Friday:**

Metrics	**Values**	**Description**
Arrival Rate(λ)	**3.6416** customers per minute	Number of customers arriving per minute.
Service Rate(μ)	**1.4637** per server per minute	Customers served by one server per minute.
Total Service Rate(μ_{total})	**4.3911** per minute	Combined service rate of all servers
Utilization Factor (ρ)	**0.8364**	System load; should be < 1 for stability
Probability of Idle System(P_0)	**0.0440**	Chance of no customers in the system
Average Queue Length (L_q)	**3.6152** customers	Expected number of waiting customers.
Average Waiting Time in Queue (W_q)	**0.9815** minutes	Average time a customer waits in the queue.
Average no. of customers in the system (L_s)	**4.4447** customers	Average no. of customers waiting in the system
Total Time Spent in System (W_s)	**1.6627** minutes	Time from arrival to service completion.

- ## Comparison of Station 1 and Station 2 Analysis

Metrics	Station 1	Station 2
Arrival Rate (λ)	**2.4583** customers per minute	**1.1833** customers per minute
Service Rate (μ)	**1.4258** per server per minute	**1.5494** per Server per minute
Total Service Rate (μ_{total})	**2.8516** per minute	**1.5494** per minute
Utilization Factor (ρ)	**0.6325**	**1.2442**
Probability of Idle System(P_0)	**0.2251**	**-0.2442**
Average Queue Length (L_q)	**0.8436** customers	∞
Average Waiting Time in Queue (W_q)	**0.3408** minutes	∞
Average no. of customers in the system (L_s)	**1.7057** customers	∞
Total Time Spent in System (W_s)	**0.8520** minutes	∞

- **Description:**

The system has an arrival rate (λ) of 3.6416 customers per minute, indicating the rate at which customers enter the system. Each server operates at a service rate (μ) of 1.4637 customers per minute, and when all servers are combined, the total service rate (μ_{total}) is 4.3911 customers per minute. The utilization factor (ρ) is 0.8364, showing that the system is operating close to its capacity, as a value below 1 is required for stability. The probability of the system being idle (P_o) is 0.0440, meaning there is a very low chance of no customers being in the system at any given time.

On average, there are 3.6152 customers waiting in the queue (L_q), with an average waiting time in the queue (W_q) of 0.9815 minutes. The average number of customers in the entire system (L_s) is 4.4447, and the total time a customer spends in the system (W_s) is 1.6627 minutes, from arrival to service completion.

When examining the two stations within the system, significant differences in performance become apparent. The first station has an arrival rate (λ) of 2.4583 customers per minute, while the second station has a lower arrival rate of 1.1833 customers per minute. The service rates (μ) for the first and second stations are 1.4258 and 1.5494 customers per server per minute, respectively. The total service rates (μ_{total}) are 2.8516 for the first station and 1.5494 for the second station.

The utilization factor (ρ) for the first station is 0.6325, indicating it is operating well within its capacity. However, the second station has a utilization factor of 1.2442, which exceeds the stability threshold of 1, suggesting it is overloaded. The probability of the system being idle (P_o) is 0.2251 for the first station, while the second station has a negative probability (-0.2442), which is not feasible and indicates an unstable system.

The first station has an average queue length (L_q) of 0.8436 customers, while the second station has an infinite queue length (∞), indicating severe congestion. The average waiting time in the queue (W_q) is 0.3408 minutes for the first station and infinite for the second station. The average number of customers in the system (L_s) is 1.7057 for the first station and infinite for the second station. The total time spent in the system (W_s) is 0.8520 minutes for the first station and infinite for the second station.

Overall, the system is operating efficiently but is close to its capacity, as indicated by the high utilization factor. The first station handles its load well, with manageable queue lengths and waiting times. In contrast, the second station is significantly overloaded, leading to infinite queue lengths and waiting times, indicating a critical need for optimization to restore stability and improve performance.

▪ Saturday:

Metrics	Values	Description
Arrival Rate(λ)	**2.95** customers per minute	Number of customers arriving per minute.
Service Rate(μ)	**1.8412** per server per minute	Customers served by one server per minute.
Total Service Rate(μ_{total})	**5.5236** per minute	Combined service rate of all servers
Utilization Factor (ρ)	**0.5341**	System load; should be < 1 for stability
Probability of Idle System(P_0)	**0.1867**	Chance of no customers in the system
Average Queue Length (L_q)	**0.3148** customers	Expected number of waiting customers.
Average Waiting Time in Queue (W_q)	**0.1067** minutes	Average time a customer waits in the queue.
Average no. of customers in the system (L_s)	**0.8490** customers	Average no. of customers waiting in the system
Total Time Spent in System (W_s)	**0.6498** minutes	Time from arrival to service completion.

- ## Comparison of Station 1 and Station 2 Analysis

Metrics	Station 1	Station 2
Arrival Rate (λ)	**1.775** customers per minute	**1.175** customers per minute
Service Rate (μ)	**1.898** per server per minute	**1.7614** per Server per minute
Total Service Rate (μ_{total})	**3.796** per minute	**1.7614** per minute
Utilization Factor (ρ)	**0.4676**	**0.6671**
Probability of Idle System(P_0)	**0.3628**	**0.3329**
Average Queue Length (L_q)	**0.2616** customers	**1.3367** customers
Average Waiting Time in Queue (W_q)	**0.1474** minutes	**1.1376** minutes
Average no. of customers in the system (L_s)	**0.7292** customers	**2.0038** customers
Total Time Spent in System (W_s)	**0.6743** minutes	**1.7053** minutes

- ## Description:

The system has an arrival rate (λ) of 2.95 customers per minute, that indicates the rate at which customers enter the system. Each server operates at a service rate (μ) of 1.8412 customers per minute, and when all servers are combined, the total service rate (μ_{total}) is 5.5236 customers per minute. The utilization factor (ρ) is 0.5341, showing that the system is operating well within its capacity, as a value below 1 is required for stability. The probability of the system being idle (P_o) is 0.1867, meaning there is a moderate chance of no customers being in the system at any given time.

On average, there are 0.3148 customers waiting in the queue (L_q), with an average waiting time in the queue (W_q) of 0.1067 minutes. The average number of customers in the entire system (L_s) is 0.8490, and the total time a customer spends in the system (W_s) is 0.6498 minutes, from arrival to service completion.

When examining the two stations within the system, differences in performance become apparent. The first station has an arrival rate (λ) of 1.775 customers per minute, while the second station has a lower arrival rate of 1.175 customers per minute. The service rates (μ) for the first and second stations are 1.898 and 1.7614 customers per server per minute, respectively. The total service rates (μ_{total}) are 3.796 for the first station and 1.7614 for the second station.

The utilization factor (ρ) for the first station is 0.4676, indicating it is operating well within its capacity. The second station has a utilization factor of 0.6671, which is also within the stable range but higher than the first station. The probability of the system being idle (P_o) is 0.3628 for the first station and 0.3329 for the second station, suggesting both stations have similar idle times.

The first station has an average queue length (L_q) of 0.2616 customers, while the second station has a higher average queue length of 1.3367 customers. The average waiting time in the queue (W_q) is 0.1474 minutes for the first station and significantly higher at 1.1376 minutes for the second station. The average number of customers in the system (L_s) is 0.7292 for the first station and 2.0038 for the second station. The total time spent in the system (W_s) is 0.6743 minutes for the first station and 1.7053 minutes for the second station.

Overall, the system is operating efficiently, with both stations functioning within their capacities. The first station handles its load more effectively, with shorter queue lengths and waiting times compared to the second station. While both stations are stable, the second station exhibits higher congestion and longer waiting times, indicating potential areas for optimization to improve customer experience and system performance.

- ## Sunday (morning):

Metrics	Values	Description
Arrival Rate(λ)	**2.45** customers per minute	Number of customers arriving per minute.
Service Rate(μ)	**1.8184** per server per minute	Customers served by one server per minute.
Total Service Rate(μ_{total})	**5.4552** per minute	Combined service rate of all servers
Utilization Factor (ρ)	**0.7533**	System load; should be < 1 for stability
Probability of Idle System(P_0)	**0.0735**	Chance of no customers in the system
Average Queue Length (L_q)	**1.7496** customers	Expected number of waiting customers.
Average Waiting Time in Queue (W_q)	**0.3790** minutes	Average time a customer waits in the queue.
Average no. of customers in the system (L_s)	**2.1988** customers	Average no. of customers waiting in the system
Total Time Spent in System (W_s)	**0.8685** minutes	Time from arrival to service completion.

- ## Comparison of Station 1 and Station 2 Analysis

Metrics	Station 1	Station 2
Arrival Rate (λ)	**1.4833** customers per minute	**0.9666** customers per minute
Service Rate (μ)	**1.8086** per server per minute	**1.8335** per Server per minute
Total Service Rate (μ_{total})	**3.6172** per minute	**1.8335** per minute
Utilization Factor (ρ)	**0.7137**	**0.8325**
Probability of Idle System(P_0)	**0.1670**	**0.1675**
Average Queue Length (L_q)	**1.4823** customers	**4.1380** customers
Average Waiting Time in Queue (W_q)	**0.5053** minutes	**2.4582** minutes
Average no. of customers in the system (L_s)	**1.8924** customers	**4.9705** customers
Total Time Spent in System (W_s)	**0.9920** minutes	**2.9528** minutes

- ## Description:

The system has an arrival rate (λ) of 2.45 customers per minute, indicating the rate at which customers enter the system. Each server operates at a service rate (μ) of 1.8184 customers per minute, and when all servers are combined, the total service rate (μ_{total}) is 5.4552 customers per minute. The utilization factor (ρ) is 0.7533, showing that the system is operating within a stable range, as a value below 1 is required for stability. The probability of the system being idle (P_o) is 0.0735, meaning there is a relatively low chance of no customers being in the system at any given time.

On average, there are 1.7496 customers waiting in the queue (L_q), with an average waiting time in the queue (W_q) of 0.3790 minutes. The average number of customers in the entire system (L_s) is 2.1988, and the total time a customer spends in the system (W_s) is 0.8685 minutes, from arrival to service completion.

When examining the two stations within the system, differences in performance become apparent. The first station has an arrival rate (λ) of 1.4833 customers per minute, while the second station has a lower arrival rate of 0.9666 customers per minute. The service rates (μ) for the first and second stations are 1.8086 and 1.8335 customers per server per minute, respectively. The total service rates (μ_{total}) are 3.6172 for the first station and 1.8335 for the second station.

The utilization factor (ρ) for the first station is 0.7137, indicating it is operating well within its capacity. The second station has a utilization factor of 0.8325, which is also within the stable range but higher than the first station. The probability of the system being idle (P_o) is 0.1670 for the first station and 0.1675 for the second station, suggesting both stations have similar idle times.

The first station has an average queue length (L_q) of 1.4823 customers, while the second station has a higher average queue length of 4.1380 customers. The average waiting time in the queue (W_q) is 0.5053 minutes for the first station and significantly higher at 2.4582 minutes for the second station. The average number of customers in the system (L_s) is 1.8924 for the first station and 4.9705 for the second station. The total time spent in the system (W_s) is 0.9920 minutes for the first station and 2.9528 minutes for the second station

Overall, the system is operating efficiently, with both stations functioning within their capacities. The first station handles its load more effectively, with shorter queue lengths and waiting times compared to the second station. While both stations are stable, the second station exhibits higher congestion and longer waiting times, indicating potential areas for optimization to improve customer experience and system performance.

- **Sunday (Afternoon):**

Metrics	Values	Description
Arrival Rate(λ)	**3.925** customers per minute	Number of customers arriving per minute.
Service Rate(μ)	**1.5729** per server per minute	Customers served by one server per minute.
Total Service Rate(μ_{total})	**4.7187** per minute	Combined service rate of all servers
Utilization Factor (ρ)	**0.6326**	System load; should be < 1 for stability
Probability of Idle System(P_0)	**0.1282**	Chance of no customers in the system
Average Queue Length (L_q)	**0.6845** customers	Expected number of waiting customers.
Average Waiting Time in Queue (W_q)	**0.1744** minutes	Average time a customer waits in the queue.
Average no. of customers in the system (L_s)	**2.5823** customers	Average no. of customers waiting in the system
Total Time Spent in System (W_s)	**0.6579** minutes	Time from arrival to service completion.

- ## Comparison of Station 1 and Station 2 Analysis

Metrics	Station 1	Station 2
Arrival Rate (λ)	**2.4417** customers per minute	**1.4834** customers per minute
Service Rate (μ)	**1.4756** per server per minute	**1.7711** per Server per minute
Total Service Rate (μ_{total})	**2.9512** per minute	**1.7711** per minute
Utilization Factor (ρ)	**0.5301**	**0.8375**
Probability of Idle System(P_0)	**0.3071**	**0.1625**
Average Queue Length (L_q)	**0.4144** customers	**4.3171** customers
Average Waiting Time in Queue (W_q)	**0.1697** minutes	**2.9104** minutes
Average no. of customers in the system (L_s)	**1.4747** customers	**5.1546** customers
Total Time Spent in System (W_s)	**0.6040** minutes	**3.4750** minutes

- ## Description:

The system has an arrival rate (λ) of 3.925 customers per minute, indicating the rate at which customers enter the system. Each server operates at a service rate (μ) of 1.5729 customers per minute, and when all servers are combined, the total service rate (μ_{total}) is 4.7187 customers per minute. The utilization factor (ρ) is 0.6326, showing that the system is operating within a stable range, as a value below 1 is required for stability. The probability of the system being idle (P_o) is 0.1282, meaning there is a moderate chance of no customers being in the system at any given time.

On average, there are 0.6845 customers waiting in the queue (L_q), with an average waiting time in the queue (W_q) of 0.1744 minutes. The average number of customers in the entire system (L_s) is 2.5823, and the total time a customer spends in the system (W_s) is 0.6579 minutes, from arrival to service completion.

When examining the two stations within the system, differences in performance become apparent. The first station has an arrival rate (λ) of 2.4417 customers per minute, while the second station has a lower arrival rate of 1.4834 customers per minute. The service rates (μ) for the first and second stations are 1.4756 and 1.7711 customers per server per minute, respectively. The total service rates (μ_{total}) are 2.9512 for the first station and 1.7711 for the second station.

The utilization factor (ρ) for the first station is 0.5301, indicating it is operating well within its capacity. The second station has a utilization factor of 0.8375, which is also within the stable range but higher than the first station. The probability of the system being idle (P_o) is 0.3071 for the first station and 0.1625 for the second station, suggesting the first station has more idle time compared to the second station.

The first station has an average queue length (L_q) of 0.4144 customers, while the second station has a significantly higher average queue length of 4.3171 customers. The average waiting time in the queue (W_q) is 0.1697 minutes for the first station and much higher at 2.9104 minutes for the second station. The average number of customers in the system (L_s) is 1.4747 for the first station and 5.1546 for the second station. The total time spent in the system (W_s) is 0.6040 minutes for the first station and 3.4750 minutes for the second station.

Overall, the system is operating efficiently, with both stations functioning within their capacities. The first station handles its load more effectively, with shorter queue lengths and waiting times compared to the second station. While both stations are stable, the second station exhibits higher congestion and longer waiting times, indicating potential areas for optimization to improve customer experience and system performance.

- **Sunday (Evening):**

Metrics	Values	Description
Arrival Rate(λ)	**4.5833** customers per minute	Number of customers arriving per minute.
Service Rate(μ)	**0.6544** per server per minute	Customers served by one server per minute.
Total Service Rate(μ_{total})	**1.9632** per minute	Combined service rate of all servers
Utilization Factor (ρ)	**0.7533**	System load; should be < 1 for stability
Probability of Idle System(P_0)	**0.0735**	Chance of no customers in the system
Average Queue Length (L_q)	**1.7496** customers	Expected number of waiting customers.
Average Waiting Time in Queue (W_q)	**0.3790** minutes	Average time a customer waits in the queue.
Average no. of customers in the system (L_s)	**4.0851** customers	Average no. of customers waiting in the system
Total Time Spent in System (W_s)	**0.8685** minutes	Time from arrival to service completion.

- ## **Comparison of Station 1 and Station 2 Analysis**

Metrics	Station 1	Station 2
Arrival Rate (λ)	**3.0416** customers per minute	**1.5416** customers per minute
Service Rate (μ)	**1.7879** per server per minute	**0.2907** per Server per minute
Total Service Rate (μ_{total})	**3.5758** per minute	**0.2907** per minute
Utilization Factor (ρ)	**0.7137**	**0.8325**
Probability of Idle System(P_0)	**0.1670**	**0.1675**
Average Queue Length (L_q)	**1.4823** customers	**4.1380** customers
Average Waiting Time in Queue (W_q)	**0.5053** minutes	**2.4582** minutes
Average no. of customers in the system (L_s)	**2.3336** customers	**4.9705** customers
Total Time Spent in System (W_s)	**0.9920** minutes	**2.9528** minutes

- ## Description:

The system has an arrival rate (λ) of 4.5833 customers per minute, indicating the rate at which customers enter the system. Each server operates at a service rate (μ) of 0.6544 customers per minute, and when all servers are combined, the total service rate (μ_{total}) is 1.9632 customers per minute. The utilization factor (ρ) is 0.7533, showing that the system is operating within a stable range, as a value below 1 is required for stability. The probability of the system being idle (P_o) is 0.0735, meaning there is a relatively low chance of no customers being in the system at any given time.

On average, there are 1.7496 customers waiting in the queue (L_q), with an average waiting time in the queue (W_q) of 0.3790 minutes. The average number of customers in the entire system (L_s) is 4.0851, and the total time a customer spends in the system (W_s) is 0.8685 minutes, from arrival to service completion.

When examining the two stations within the system, differences in performance become apparent. The first station has an arrival rate (λ) of 3.0416 customers per minute, while the second station has a lower arrival rate of 1.5416 customers per minute. The service rates (μ) for the first and second stations are 1.7879 and 0.2907 customers per server per minute, respectively. The total service rates (μ_{total}) are 3.5758 for the first station and 0.2907 for the second station.

The utilization factor (ρ) for the first station is 0.7137, indicating it is operating well within its capacity. The second station has a utilization factor of 0.8325, which is also within the stable range but higher than the first station. The probability of the system being idle (P_o) is 0.1670 for the first station and 0.1675 for the second station, suggesting both stations have similar idle times.

The first station has an average queue length (L_q) of 1.4823 customers, while the second station has a significantly higher average queue length of 4.1380 customers. The average waiting time in the queue (W_q) is 0.5053 minutes for the first station and much higher at 2.4582 minutes for the second station. The average number of customers in the system (L_s) is 2.3336 for the first station and 4.9705 for the second station. The total time spent in the system (W_s) is 0.9920 minutes for the first station and 2.9528 minutes for the second station.

Overall, the system is operating efficiently, with both stations functioning within their capacities. The first station handles its load more effectively, with shorter queue lengths and waiting times compared to the second station. While both stations are stable, the second station exhibits higher congestion and longer waiting times, indicating potential areas for optimization to improve customer experience and system performance.

11. COMPARISON OF WEEKDAYS AND WEEKENDS ANALYSIS

- **Morning (8.00AM–10.00AM):**

Metrics	Weekdays		Weekend
	Wednesday	**Saturday**	**Sunday**
Arrival Rate(λ)	**2.9667** customers per minute	**2.95** customers per minute	**2.45** customers per minute
Service Rate(μ)	**1.0402** per server per minute	**1.8412** per server per minute	**1.8184** per server per minute
Total Service Rate(μ_{total})	**3.1206** per minute	**5.5236** per minute	**5.4552** per minute
Utilization Factor (ρ)	**0.9507**	**0.5341**	**0.7533**
Probability of Idle System(P_0)	**0.0116**	**0.1867**	**0.0735**
Average Queue Length (L_q)	**17.5039** customers	**0.3148** customers	**1.7496** customers
Average Waiting Time in Queue (W_q)	**5.9002** minutes	**0.1067** minutes	**0.3790** minutes
Average no. of customers in the system (L_s)	**20.3560** customers	**0.8490** customers	**2.1988** customers
Total Time Spent in System (W_s)	**6.8616** minutes	**0.6498** minutes	**0.8685** minutes

- ## Description:

Here weekdays, represented by Wednesday and Saturday, generally experience higher arrival rates compared to weekends (Sunday). Wednesday has the highest arrival rate at 2.9667 customers per minute, followed closely by Saturday at 2.95 customers per minute, while Sunday has a lower arrival rate of 2.45 customers per minute. This suggests that mornings on weekdays are busier than weekends, likely due to work-related or routine activities. In terms of service rates, weekends and Saturdays show higher efficiency, with Saturday having the highest total service rate at 5.5236 customers per minute, followed by Sunday at 5.4552 customers per minute. Wednesday, on the other hand, has a lower total service rate of 3.1206 customers per minute, indicating that the system operates more efficiently during weekends and Saturdays in the morning.

The utilization factor further emphasizes the differences between weekdays and weekends. Wednesday has the highest utilization factor at 0.9507, indicating that the system is operating very close to its capacity during weekday mornings. In contrast, Saturday has a much lower utilization factor of 0.5341, suggesting a more relaxed operation, while Sunday falls in between with a utilization factor of 0.7533, indicating moderate system load. Queue lengths and waiting times also reflect these trends. Wednesday experiences significantly longer queue lengths 17.5039 customers and waiting times 5.9002 minutes, reflecting high congestion during weekday mornings. Saturday, however, has the shortest queues 0.3148 customers and waiting times 0.1067 minutes, indicating a smooth and efficient operation. Sunday shows moderate congestion, with an average queue length of 1.7496 customers and waiting time of 0.3790 minutes.

Overall, mornings on weekdays, especially Wednesday, are the most demanding for the system, with high arrival rates, long queues, and significant waiting times. Saturday mornings, while still a weekday, show much lower congestion and faster service times, suggesting a more relaxed operation compared to Wednesday. Weekend mornings (Sunday) are less busy than weekdays, with lower arrival rates and moderate congestion, but the system still operates efficiently. This analysis underscores the need for potential adjustments during weekday mornings to manage higher demand and improve customer experience, while weekends and Saturdays operate more efficiently with lower stress on the system

- **Afternoon (12.00AM–2.00AM):**

Metrics	Weekdays		Weekend
	Tuesday	Friday	Sunday
Arrival Rate(λ)	**3.4333** customers per minute	**3.6416** customers per minute	**3.925** customers per minute
Service Rate(μ)	**1.4106** per server per minute	**1.4637** per server per minute	**1.5729** per server per minute
Total Service Rate(μ_{total})	**4.2318** per minute	**4.3911** per minute	**4.7187** per minute
Utilization Factor (ρ)	**0.8113**	**0.8364**	**0.6326**
Probability of Idle System(P_0)	**0.0523**	**0.0440**	**0.1282**
Average Queue Length (L_q)	**2.8625** customers	**3.6152** customers	**0.6845** customers
Average Waiting Time in Queue (W_q)	**0.8337** minutes	**0.9815** minutes	**0.1744** minutes
Average no. of customers in the system (L_s)	**5.2964** customers	**4.4447** customers	**2.5823** customers
Total Time Spent in System (W_s)	**1.5426** minutes	**1.6627** minutes	**0.6579** minutes

- ## Description:

Here weekdays, represented by Tuesday and Friday, generally experience higher arrival rates compared to weekends (Sunday). Tuesday has an arrival rate of 3.4333 customers per minute, while Friday sees an increase to 3.925 customers per minute, reflecting higher customer demand as the week progresses. In contrast, Sunday has a lower arrival rate of 2.45 customers per minute, suggesting that afternoons on weekdays are busier than weekends. Service rates also show variations, with Friday having the highest total service rate at 4.7187 customers per minute, followed by Tuesday at 4.2318 customers per minute, and Sunday at 5.4552 customers per minute. This indicates that the system operates more efficiently during weekends and Fridays in the afternoon.

The utilization factor further emphasizes the differences between weekdays and weekends. Tuesday has a utilization factor of 0.8113, indicating that the system is operating close to its capacity during weekday afternoons. Friday, however, has a lower utilization factor of 0.6326, suggesting a more relaxed operation, while Sunday falls in between with a utilization factor of 0.7533, indicating moderate system load. Queue lengths and waiting times also reflect these trends. Tuesday experiences longer queue lengths 2.8625 customers and waiting times 0.8337 minutes, reflecting higher congestion during weekday afternoons. Friday, on the other hand, has significantly shorter queues 0.6845 customers and waiting times 0.1744 minutes, indicating a more efficient operation. Sunday shows moderate congestion, with an average queue length of 1.7496 customers and waiting time of 0.3790 minutes.

Overall, afternoons on weekdays, especially Tuesday, are the most demanding for the system, with high arrival rates, long queues, and significant waiting times. Friday afternoons show improved efficiency, with lower utilization, shorter queues, and faster service times compared to Tuesday. Weekend afternoons (Sunday) are less busy than weekdays, with lower arrival rates and moderate congestion, but the system still operates efficiently. This analysis underscores the need for potential adjustments during weekday afternoons, especially on Tuesday, to manage higher demand and improve customer experience, while weekends and Fridays operate more efficiently with lower stress on the system.

- **Evening (6.00AM–8.00AM):**

Metrics	Weekdays		Weekend
	Monday	**Thursday**	**Sunday**
Arrival Rate(λ)	**4.6916** customers per minute	**4.6166** customers per minute	**4.5833** customers per minute
Service Rate(μ)	**1.8400** per server per minute	**2.0428** per server per minute	**0.6544** per server per minute
Total Service Rate(μ_{total})	**5.52** per minute	**6.1284** per minute	**1.9632** per minute
Utilization Factor (ρ)	**0.8551**	**0.7533**	**0.7533**
Probability of Idle System(P_0)	**0.0381**	**0.0735**	**0.0735**
Average Queue Length (L_q)	**4.3592** customers	**1.7496** customers	**1.7496** customers
Average Waiting Time in Queue (W_q)	**0.9291** minutes	**0.3790** minutes	**0.3790** minutes
Average no. of customers in the system (L_s)	**6.9244** customers	**2.5030** customers	**4.0851** customers
Total Time Spent in System (W_s)	**1.4759** minutes	**0.8685** minutes	**0.8685** minutes

- ## Description:

Here weekdays, represented by Monday and Thursday, generally experience higher arrival rates compared to weekends (Sunday). Monday has an arrival rate of 4.6946 customers per minute, while Thursday follows a similar trend, reflecting increased customer demand during weekday evenings. In contrast, Sunday has a lower arrival rate, suggesting that evenings on weekdays are busier than weekends. Service rates also show variations, with weekdays having a total service rate of 5.52 customers per minute, indicating that the system operates efficiently during weekday evenings. Sunday, however, shows a slightly different pattern, with a total service rate that reflects moderate system load.

The utilization factor further emphasizes the differences between weekdays and weekends. Monday and Thursday have utilization factors of 0.8551, indicating that the system is operating close to its capacity during weekday evenings. Sunday, on the other hand, has a lower utilization factor, suggesting a more relaxed operation. Queue lengths and waiting times also reflect these trends. Weekdays experience longer queue lengths 4.3592 customers and waiting times 1.4759 minutes, reflecting higher congestion during weekday evenings. Sunday, however, shows shorter queues and waiting times, indicating a more efficient operation.

Overall, evenings on weekdays, especially Monday and Thursday, are the most demanding for the system, with high arrival rates, long queues, and significant waiting times. Weekend evenings (Sunday) are less busy than weekdays, with lower arrival rates and moderate congestion, but the system still operates efficiently. This analysis underscores the need for potential adjustments during weekday evenings to manage higher demand and improve customer experience, while weekends operate more efficiently with lower stress on the system.

12. CONCLUSION

This study applied queueing theory to analyze petrol pump operations in Karad city, focusing on understanding customer arrival patterns, service rates, and overall system efficiency. Using a multi-server queueing model (M/M/3), we evaluated key performance metrics such as arrival rates, service rates, queue lengths, waiting times, and system utilization.

Our analysis revealed that the petrol pump experiences high traffic during peak hours, particularly in the evening and on weekdays. The arrival process closely follows a Poisson distribution, and service times exhibit an exponential distribution, validating the assumptions of queueing theory. The findings indicate that the system operates near full capacity on certain days, leading to increased waiting times and congestion, particularly at Station 2.

A comparative analysis between weekdays and weekends shows that weekday mornings and evenings are the busiest periods, with longer queues and higher waiting times. In contrast, weekend operations tend to be more stable, with lower utilization and shorter waiting times. Among the two service stations, Station 1 generally exhibited better performance in terms of queue lengths and waiting times, whereas Station 2 faced congestion issues, particularly on certain weekdays.

To optimize service efficiency, potential improvements include redistributing customer load between stations, increasing the number of service attendants during peak hours, and exploring alternative queue management strategies to reduce customer waiting times. Implementing these changes can enhance customer satisfaction and improve overall operational efficiency at the petrol pump.

13. FUTURE SCOPE

This study provides a foundation for further research and practical improvements in petrol pump operations. Some potential future directions include:

1. **Simulation-Based Analysis**
   Developing simulation models to test different queue management strategies and predict system behaviour under various conditions.

2. **Dynamic Staffing Optimization**
   Exploring adaptive staffing techniques based on real-time traffic patterns to reduce waiting times.

3. **Incorporating Customer Behaviour**
   Studying customer preferences for specific fuel types, payment methods, and service preferences to optimize overall operations.

4. **Impact of Digital Payment Systems**
   Analysing how digital transactions impact service times and overall efficiency.

5. **Comparative Study with Other Petrol Pumps**
   Expanding the analysis to multiple petrol pumps across different locations to identify broader trends and best practices.

6. **Integration of IoT and AI**
   Implementing smart queue management systems using Internet of Things (IoT) and Artificial Intelligence (AI) for real-time monitoring and predictive analytics.

14. APPENDIX

To convert into minutes

```
# Load the CSV file
file_path = "Fridayy.csv"
df_1 = pd.read_csv(file_path)

# Listing of time-related columns
time_columns = ['Arrival Time(L1S1)', 'Service Time(L1S1)', 'Departure
Time(L1S1)',
       'Arrival Time(L2S1)', 'Service Time(L2S1)', 'Departure Time(L2S1)',
       'Arrival Time(L1S2)', 'Service Time(L1S2)', 'Departure Time(L1S2)',
       'Arrival Time(L2S2)', 'Service Time(L2S2)', 'Departure Time(L2S2)'

]

# Converting time columns to datetime format
for col in time_columns:
    df_1[col] = pd.to_datetime(df_1[col], format="%H:%M:%S",
errors='coerce').dt.time  # Keeps only time, removes date

# Converting time to total minutes
for col in time_columns:
    df_1[col] = df_1[col].apply(lambda x: x.hour * 60 + x.minute + x.second /
60 if pd.notnull(x) else None)

# To Save the modified DataFrame
output_path = "converted_friday.csv"
df_1.to_csv(output_path, index=False)

print(f"Conversion completed. Saved as {output_path}")
```

Arrival rate

```
total_arrivals=df_1["Arrival Time(L1S1)"].count() + df_1["Arrival
Time(L2S1)"].count() + df_1["Arrival Time(L1S2)"].count() + df_1["Arrival
Time(L2S2)"].count()
total_arrivals

Arrival_rate=total_arrivals / 120
Arrival_rate

print(f"Arrival Rate : {Arrival_rate} customers per minutes")
```

Station 1 Arrival rate

```
# station 1
total_arrivals=df_1["Arrival Time(L1S1)"].count() + df_1["Arrival Time(L2S1)"].count()
total_arrivals


Arrival_rate=total_arrivals / 120
Arrival_rate

print(f"Arrival Rate : {Arrival_rate} customers per minutes")
```

Station 2 Arrival rate

```
# station 2
total_arrivals = df_1["Arrival Time(L1S2)"].count() + df_1["Arrival Time(L2S2)"].count()
total_arrivals

Arrival_rate=total_arrivals / 120
Arrival_rate

print(f"Arrival Rate : {Arrival_rate} customers per minutes")
```

Inter - Arrival time

```
arrival_columns = ['Arrival Time(L1S1)','Arrival Time(L2S1)','Arrival Time(L1S2)','Arrival Time(L2S2)'
]
# Dictionary to store inter-arrival times for each column
inter_arrival_times = {}

# Looping through each arrival time column and calculating inter-arrival time
for col in arrival_columns:
# Calculating inter-arrival time for this column by subtracting consecutive arrival times
df_1[col + '_inter_arrival_time'] = df_1[col].diff()  # diff() calculates the difference with the previous row

#  To store the result
 inter_arrival_times[col] = df_1[col + '_inter_arrival_time']
#  To print the result
print(df_1[['Arrival Time(L1S1)','Arrival Time(L1S1)_inter_arrival_time','Arrival Time(L2S1)','Arrival Time(L2S1)_inter_arrival_time',
          'Arrival Time(L1S2)','Arrival Time(L1S2)_inter_arrival_time','Arrival Time(L2S2)','Arrival
```

```
Time(L2S2)_inter_arrival_time'
          ]])
```

To check arrival time follows poisson distribution Autocorrelation Test

```
# Autocorrelation function (ACF)
columns = ['Arrival Time(L1S1)_inter_arrival_time','Arrival
Time(L2S1)_inter_arrival_time','Arrival
Time(L1S2)_inter_arrival_time','Arrival Time(L2S2)_inter_arrival_time']

# Ploting autocorrelation for each column
for col in columns:
    plt.figure(figsize=(8, 4))
    pd.plotting.autocorrelation_plot(df_1[col])
    plt.title(f'Autocorrelation Plot for {col}')
    plt.show()
```

Exponential Fit

```
inter_arrival_times = df_1['Arrival
Time(L1S1)_inter_arrival_time'].dropna().values  # Remove NaN values if any

# To compute the average inter-arrival time
tau_avg = np.mean(inter_arrival_times)

#To compute the estimated arrival rate
lambda_estimated = len(inter_arrival_times) / np.sum(inter_arrival_times)
lambda_theoretical = 1 / tau_avg  # Reciprocal of mean inter-arrival time

# Print the results
print(f"Estimated Arrival Rate (λ) from Data: {lambda_estimated:.4f}")
print(f"Reciprocal of Avg Inter-Arrival Time (1/τ):
{lambda_theoretical:.4f}")

# To plot histogram of inter-arrival times
plt.hist(inter_arrival_times, bins=10, density=True, alpha=0.6, color='b')

#To overlay an exponential fit
x = np.linspace(0, max(inter_arrival_times), 100)
plt.plot(x, (1/tau_avg) * np.exp(-x/tau_avg), 'r-', label='Exponential Fit')

plt.xlabel('Inter-Arrival Time')
plt.ylabel('Density')
plt.legend()
plt.title('Histogram of Inter-Arrival Times')
plt.show()

columns = ['Arrival Time(L1S1)_inter_arrival_time','Arrival
Time(L2S1)_inter_arrival_time','Arrival
Time(L1S2)_inter_arrival_time','Arrival Time(L2S2)_inter_arrival_time']
```

```python
# Plot histograms for all columns
plt.figure(figsize=(8, 6))

for col in columns:
    if col in df_1.columns:
        inter_arrival_times = df_1[col].dropna().values  # Remove NaN values
        plt.hist(inter_arrival_times, bins=10, density=True, alpha=0.5,
label=col)

plt.xlabel('Inter-Arrival Time')
plt.ylabel('Density')
plt.title('Histogram of Multiple Inter-Arrival Time Columns')
plt.legend()
plt.show()



# To create histograms for each column
for col in service_time_cols:
    data = df_1[col]

# To Estimate lambda (1 / mean service time)
    lambda_est = 1 / np.mean(data)

# To generate exponential PDF for fitting
    x = np.linspace(0, max(data), 100)
    pdf = lambda_est * np.exp(-lambda_est * x)

# To plot histogram
    plt.figure(figsize=(6, 4))
    sns.histplot(data, bins=20, stat="density", kde=False, color="blue")
    plt.plot(x, pdf, "r-", label="Exponential Fit")

    plt.legend()
    plt.title(f"Histogram of {col} with Exponential Fit")
    plt.xlabel("Service Time")
    plt.ylabel("Density")
    plt.show()
```

Service rate

```python
service_columns = ['Service Time(L1S1)', 'Departure Time(L1S1)',
                   'Service Time(L2S1)', 'Departure Time(L2S1)',
                   'Service Time(L1S2)', 'Departure Time(L1S2)',
                   'Service Time(L2S2)', 'Departure Time(L2S2)'
    ]


# Computing service times for each station
df_1['Actual_Service_Time_(L1S1)'] = df_1['Departure Time(L1S1)'] -
df_1['Service Time(L1S1)']
df_1['Actual_Service_Time_(L2S1)'] = df_1['Departure Time(L2S1)'] -
df_1['Service Time(L2S1)']
df_1['Actual_Service_Time_(L1S2)'] = df_1['Departure Time(L1S2)'] -
```

```
df_1['Service Time(L1S2)']
df_1['Actual_Service_Time_(L2S2)'] = df_1['Departure Time(L2S2)'] -
df_1['Service Time(L2S2)']

# Selecting service time columns
service_time_columns =
['Actual_Service_Time_(L1S1)','Actual_Service_Time_(L2S1)','Actual_Service_Ti
me_(L1S2)','Actual_Service_Time_(L2S2)']

# Computing total customers served
total_served = df_1[service_time_columns].count().sum()

# Computing total service time
total_service_time = df_1[service_time_columns].sum().sum()

# Computing the correct service rate (μ)
correct_service_rate = total_served / total_service_time

print(f"Corrected Service Rate (μ): {correct_service_rate:.4f} per server per
minute")
```

Station 1 service rate

```
# Selecting service time columns
service_time_columns =
['Actual_Service_Time_(L1S1)','Actual_Service_Time_(L2S1)']

# Computing total customers served
total_served = df_1[service_time_columns].count().sum()

# Computing total service time
total_service_time = df_1[service_time_columns].sum().sum()

# Computing the correct service rate (μ)
correct_service_rate = total_served / total_service_time

print(f"Corrected Service Rate (μ): {correct_service_rate:.4f} per server per
minute")
```

Station 2 service rate

```
# Selecting service time columns
service_time_columns =
['Actual_Service_Time_(L1S2)','Actual_Service_Time_(L2S2)']

# Computing total customers served
total_served = df_1[service_time_columns].count().sum()

# Computing total service time
total_service_time = df_1[service_time_columns].sum().sum()

# Computing the correct service rate (μ)
correct_service_rate = total_served / total_service_time
```

```python
print(f"Corrected Service Rate (μ): {correct_service_rate:.4f} per server per minute")
```

```
Corrected Service Rate (μ): 1.5494 per server per minute
```

Overall system

```python
# Given data
lambda_ =3.6833333333333333   # Arrival rate per minute
mu = 1.4680       # Service rate per minute per server
c = 3            # Number of servers

# Utilization factor (ρ)
rho = lambda_ / (c * mu)

#  P0 (Probability of zero customers in system)
sum_term = sum((c * rho) ** n / math.factorial(n) for n in range(c))
p0_denominator = sum_term + ((c * rho) ** c / (math.factorial(c) * (1 - rho)))
p0 = 1 / p0_denominator

#  Lq (Average number of customers in queue)
Lq_numerator = (p0 * (c * rho) ** c * rho)
Lq_denominator = math.factorial(c) * (1 - rho) ** 2
Lq = Lq_numerator / Lq_denominator

#  Wq (Average waiting time in queue)
Wq = Lq / lambda_

#  W (Total time in system)
W = Wq + (1 / mu)  # Adding service time

# Printing results
print(f"Utilization Factor (ρ): {rho:.4f}")
print(f"Probability of Idle System (P0): {p0:.4f}")
print(f"Average Queue Length (Lq): {Lq:.4f} customers")
print(f"Average Waiting Time in Queue (Wq): {Wq:.4f} minutes")
print(f"Total Time Spent in System (W): {W:.4f} minutes")
```

Station 1

```python
# Given data
lambda_ =  2.475  # Arrival rate per minute
mu = 1.9565      # Service rate per minute per server
c = 2            # Number of servers

#  Utilization factor (ρ)
rho = lambda_ / (c * mu)

#  P0 (Probability of zero customers in system)
sum_term = sum((c * rho) ** n / math.factorial(n) for n in range(c))
p0_denominator = sum_term + ((c * rho) ** c / (math.factorial(c) * (1 - rho)))
```

```
p0 = 1 / p0_denominator

#  Lq (Average number of customers in queue)
Lq_numerator = (p0 * (c * rho) ** c * rho)
Lq_denominator = math.factorial(c) * (1 - rho) ** 2
Lq = Lq_numerator / Lq_denominator

#  Wq (Average waiting time in queue)
Wq = Lq / lambda_

#  W (Total time in system)
W = Wq + (1 / mu)  # Adding service time

# Print results
print(f"Utilization Factor (ρ): {rho:.4f}")
print(f"Probability of Idle System (P0): {p0:.4f}")
print(f"Average Queue Length (Lq): {Lq:.4f} customers")
print(f"Average Waiting Time in Queue (Wq): {Wq:.4f} minutes")
print(f"Total Time Spent in System (W): {W:.4f} minutes")
```

Station 2

```
# Given Data
lambda_ = 1.2083333333333333   # Arrival rate per minute
mu = 0.9712       # Service rate per minute

# Utilization Factor (ρ)
rho = lambda_ / mu

# Probability of Idle System (P0)
p0 = 1 - rho

# Average Number of Customers in Queue (Lq)
Lq = (rho ** 2) / (1 - rho) if rho < 1 else float('inf')

# Average Number of Customers in System (L)
L = rho / (1 - rho) if rho < 1 else float('inf')

# Average Waiting Time in Queue (Wq)
Wq = Lq / lambda_ if rho < 1 else float('inf')

# Total Time Spent in System (W)
W = 1 / (mu - lambda_) if rho < 1 else float('inf')

# Printing Results
print(f"Utilization Factor (ρ): {rho:.4f}")
print(f"Probability of Idle System (P0): {p0:.4f}")
print(f"Average Queue Length (Lq): {Lq:.4f} customers")
print(f"Average Number of Customers in System (L): {L:.4f} customers")
print(f"Average Waiting Time in Queue (Wq): {Wq:.4f} minutes")
print(f"Total Time Spent in System (W): {W:.4f} minutes"
```

15. SURVEY SHEET

	Line 1(station 1)				Line 1(station 1)			
sr.no.	Arrival time	Service time	Departure	sr.no.	Arrival time	Service time	Departure	*
1	35:47	36:00	36:14	1	9:30	12:48	13:14	*
2	35:49	36:30	36:55	2	10:06	13:16	13:40	
3	36:20	36:58	37:38	3	11:33	13:43	14:01	
4	36:40	37:40	38:10	4	11:37	14:05	14:30	
5	37:48	38:15	38:30	5	11:56	14:33	15:17	*
6	38:06	38:50	39:28	6	12:39	15:20	15:34	
7	39:49	40:40	41:00	7	12:55	15:40	16:01	
8	40:24	41:07	41:48	8	13:00	16:05	16:28	
9	41:21	41:50	42:14	9	15:09	16:30	16:52	
10	41:26	42:25	42:56	10	15:14	16:55	17:14	*
11	41:53	42:59	43:19	11	16:14	17:43	17:48	
12	43:46	44:22	44:43	12	16:18	17:52	18:02	
13	43:59	44:50	45:36	13	16:19	18:04	18:27	
14	45:22	45:45	46:09	14	17:29	18:29	18:50	
15	45:34	46:12	46:31	15	18:10	18:52	19:52	
16	45:49	46:35	46:50	16	18:30	19:54	20:20	
17	45:58	46:54	47:15	17	18:45	20:22	20:59	
18	46:04	47:18	47:39	18	19:11	21:01	21:31	
19	47:05	47:44	48:06	19	19:38	21:34	22:03	
20	47:06	48:09	48:30	20	20:20	22:07	22:38	
21	47:22	48:34	49:05	21	21:35	23:37	23:50	
22	47:26	49:09	49:45	22	21:58	23:57	24:26	
23	47:59	49:49	50:07	23	22:12	24:33	24:55	
24	49:34	50:11	50:40	24	23:32	24:59	25:50	
25	49:35	50:42	51:09	25	24:39	25:55	26:21	
26	50:10	51:17	51:45	26	25:07	26:24	26:45	
27	51:39	52:15	52:48	27	25:10	26:54	27:53	
28	52:05	52:53	53:39	28	26:15	28:01	28:13	
29	52:57	53:42	54:10	29	26:58	28:21	28:35	*
30	53:05	54:18	54:44	30	27:12	28:48	29:08	
31	53:16	54:55	55:32	31	29:18	29:35	30:00	*
32	53:48	55:34	55:58	32	28:42	30:05	30:16	
33	53:54	56:01	56:30	33	30:10	30:18	31:02	P
34	55:27	56:33	56:59	34	30:20	31:05	31:52	
35	59:04	59:10	59:22	35	31:41	31:55	33:29	
36	59:14	59:26	59:48	36	31:46	33:33	34:08	
37	59:49	59:50	00:17	37	32:22	34:11	34:49	
38				38	32:29	34:46	35:11	
39				39	32:35	35:13	35:33	
40				40	34:36	35:39	36:09	
					34:40	36:40	37:01	