Stochastic intervention: Brystsmerter and ambulance

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1 Notation

We consider data as follows: $X \in \mathbb{R}^d$ are general covariates, $W \in \{0,1\}$ is an indicator of brystsmerter, $A \in \{0,1\}$ is an indicator of ambulance, $Y \in \{0,1\}$ is an outcome indicator (30 day survival).

Let $\pi(W, X) = P(A = 1 \mid W, X)$ denote the conditional distribution of ambulance. Let $\hat{\pi}_n$ denote its estimator (super learner).

All data are used for initial estimation.

Note that the population in the following sections consists only of those i for which $W_i = 0$. Thus, n are the number of subjects with $W_i = 0$.

2 Target parameter

Our target parameter is:

$$\Psi(P) = \Psi_1(P) - \Psi_2(P),$$

where

$$\Psi_1(P) = \mathbb{E}\bigg[\sum_{a=0,1} \mathbb{E}[Y \mid A = a, W = 0, X]\hat{\pi}_n(W = 1, X)\bigg], \tag{1}$$

with the outer expectation taken over the observed distribution of X, and,

$$\Psi_2(P) = \mathbb{E}\big[\mathbb{E}[Y \mid A, W = 0, X]\big], \tag{2}$$

with the outer expectation taken over both A and X. Note that Ψ_1 defined by (1) is the risk among those with no brystsmerter ($W = 0$) had they had an ambulance sent for them with the same probability that an ambulance was sent to someone in the same population with same characteristics X but *with* brystsmerter. Ψ_2 defined by (2) is the observed risk among those who did not report brystsmerter ($W = 0$).

2.1 Efficient influence function

The efficient influence function for Ψ_1 is given by

$$\begin{aligned}\phi_1(P)(O) = &\frac{\hat{\pi}_n(W = 1, X)^A\big(1 - \hat{\pi}_n(W = 1, X)\big)^{1-A}}{\pi(W = 0, X)^A\big(1 - \pi(W = 0, X)\big)^{1-A}}\big(Y - \mathbb{E}[Y \mid A, W = 0, X]\big) \\ &+ \sum_{a=0,1} \mathbb{E}[Y \mid A = a, W = 0, X]\hat{\pi}_n(W = 1, X) - \Psi_1(P)(O),\end{aligned}$$

and the efficient influence function for Ψ_2 is given by

$$\phi_2(P)(O) = \big(Y - \mathbb{E}[Y \mid A, W=0, X]\big) + \mathbb{E}[Y \mid W=0, X] - \Psi_2(P)(O).$$

Thus, the efficient influence function for $\Psi = \Psi_1 - \Psi_2$ is

$$\phi(P)(O) = \phi_1(P)(O) - \phi_2(P)(O).$$

3 Asymptotic distribution of estimator

Under regularity conditions, bla bla, we can compute standard errors for a TMLE estimator $\hat{\psi}_n^*$ for $\Psi(P_0)$ simply by:

$$\sqrt{\frac{\hat{\sigma}_n}{n}}, \qquad \text{where} \quad \hat{\sigma}_n^2 = \frac{1}{n}\sum_{i=1}^{n}\big(\phi(\hat{P}_n^*)(O_i)\big)^2,$$

the latter being an estimate of the variance of the efficient influence function. With estimators $\hat{\pi}_n(w, X)$ for $P(A=1 \mid W=w, X)$ and $\hat{Q}_n(a, L, w)$ for $Q(a, L, w) = \mathbb{E}[Y \mid A=a, W=w, L]$ the summand of the formula above reads:

$$\begin{aligned}\phi(\hat{P}_n^*)(O_i) = \frac{\hat{\pi}_n(W=1, X_i)^{A_i}\big(1-\hat{\pi}_n(W=1,X_i)\big)^{1-A_i}}{\hat{\pi}_n(W=0, X_i)^{A_i}\big(1-\hat{\pi}_n(W=0,X_i)\big)^{1-A_i}}\big(Y - \hat{Q}_n(A, W=0, X_i)\big)\\ + \sum_{a=0,1}\hat{Q}_n(A=a, W=0, X_i)\hat{\pi}_n(W=1, X_i) - \hat{\psi}_1\\ + \big(Y_i - \hat{Q}_n(A_i, W=0, X_i)\big) + \sum_{a=0,1}\hat{Q}_n(A=a, W=0, X_i)\hat{\pi}_n(W=0, X_i) - \hat{\psi}_2.\end{aligned}$$

References