2a). $K_{12} = \frac{kT}{h} \exp[-\frac{1}{RT}(\Delta G_1 + \frac{zFV}{4})]$.

$$K_{21} = \frac{kT}{h} \exp[-\frac{1}{RT}(\Delta G_1 + \frac{zFV}{4} - \Delta G_2 - \frac{zFV}{2})].$$

$$= \frac{kT}{h} \exp[-\frac{1}{RT}(\Delta G_1 - \Delta G_2 - \frac{zFV}{4})].$$

$$K_{23} = \frac{kT}{h} \exp[-\frac{1}{RT}(\Delta G_3 + \frac{3zFV}{4} - \Delta G_2 - \frac{zFV}{2})]$$

$$= \frac{kT}{h} \exp[-\frac{1}{RT}(\Delta G_3 - \Delta G_2 + \frac{zFV}{4})].$$

$$K_{32} = \frac{kT}{h} \exp[-\frac{1}{RT}(\Delta G_3 + \frac{3zFV}{4} - zFV)].$$

$$= \frac{kT}{h} \exp[-\frac{1}{RT}(\Delta G_3 - \frac{zFV}{4})].$$

2b) using the rate constant above, the differential equations for the conformations can be written as,

$$\frac{\partial P_1}{\partial t} = -K_{12}P_1 + K_{21}P_2$$

$$\frac{\partial P_2}{\partial t} = K_{12}P_1 - (K_{21} + K_{23})P_2 + K_{32}P_3.$$

$$\frac{\partial P_3}{\partial t} = K_{23}P_2 - K_{32}P_3.$$

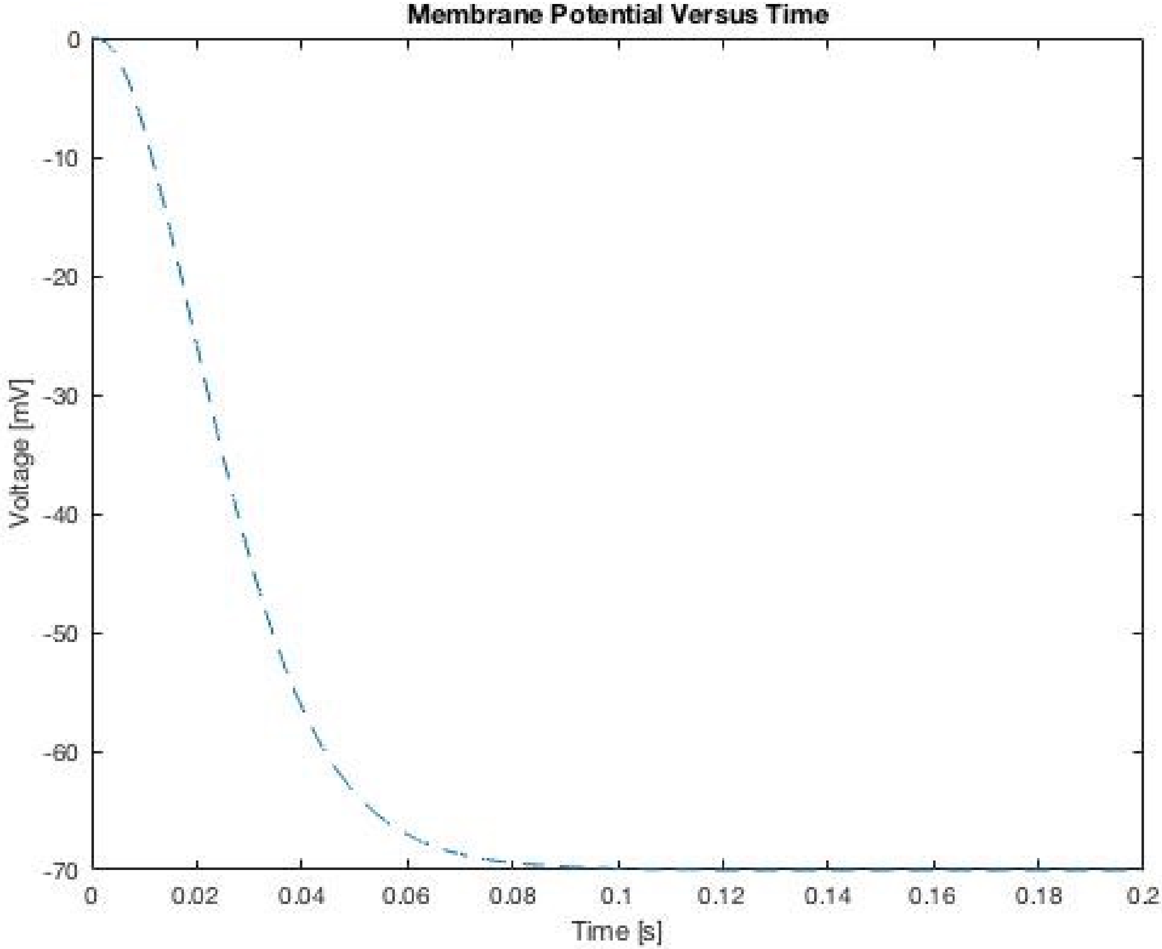
2(c) $C_m \frac{\partial V_t}{\partial t} + G_i(V_t)(V_t - E_i) = 0 \quad \forall t.$

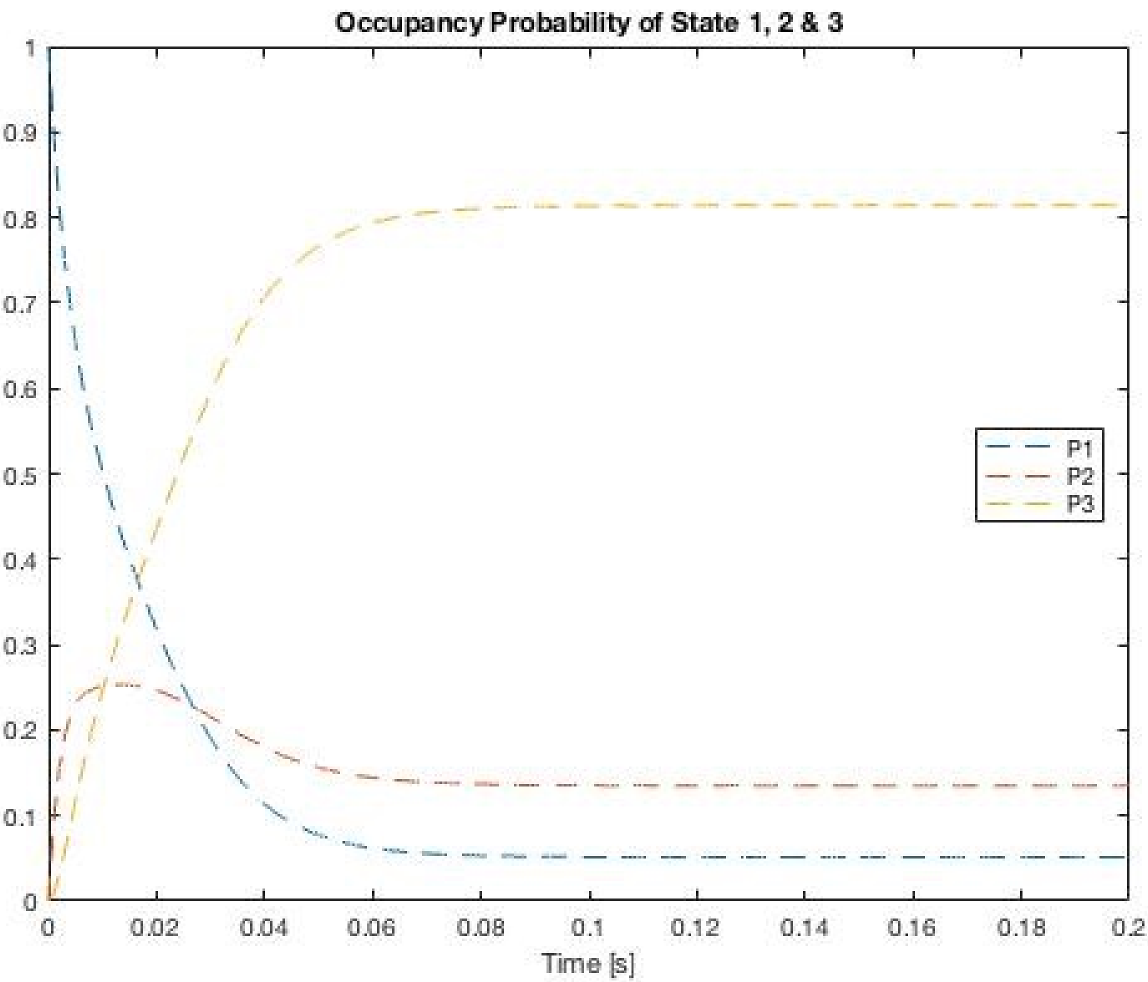
where $C_m = 100\text{pF}$ $N = 100$ $E = -70\text{mV}$. $G_{single} = 100\text{pS}$.

$G_i(V_t) = P_3(t) N G_{single}$,

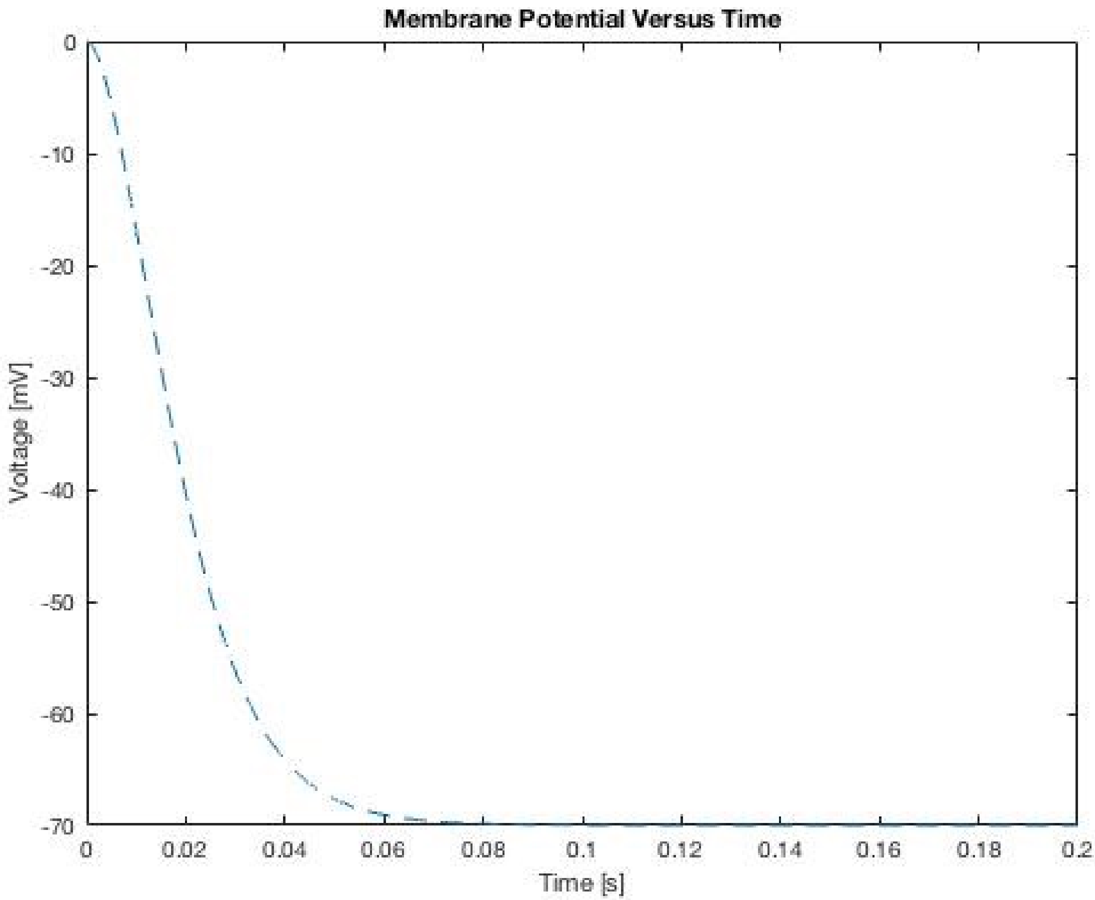
$$\boxed{\frac{\partial V_t}{\partial t} = \frac{-P_3(t) N G_{single}(V_t - E_i)}{C_m}}$$

2d (i)

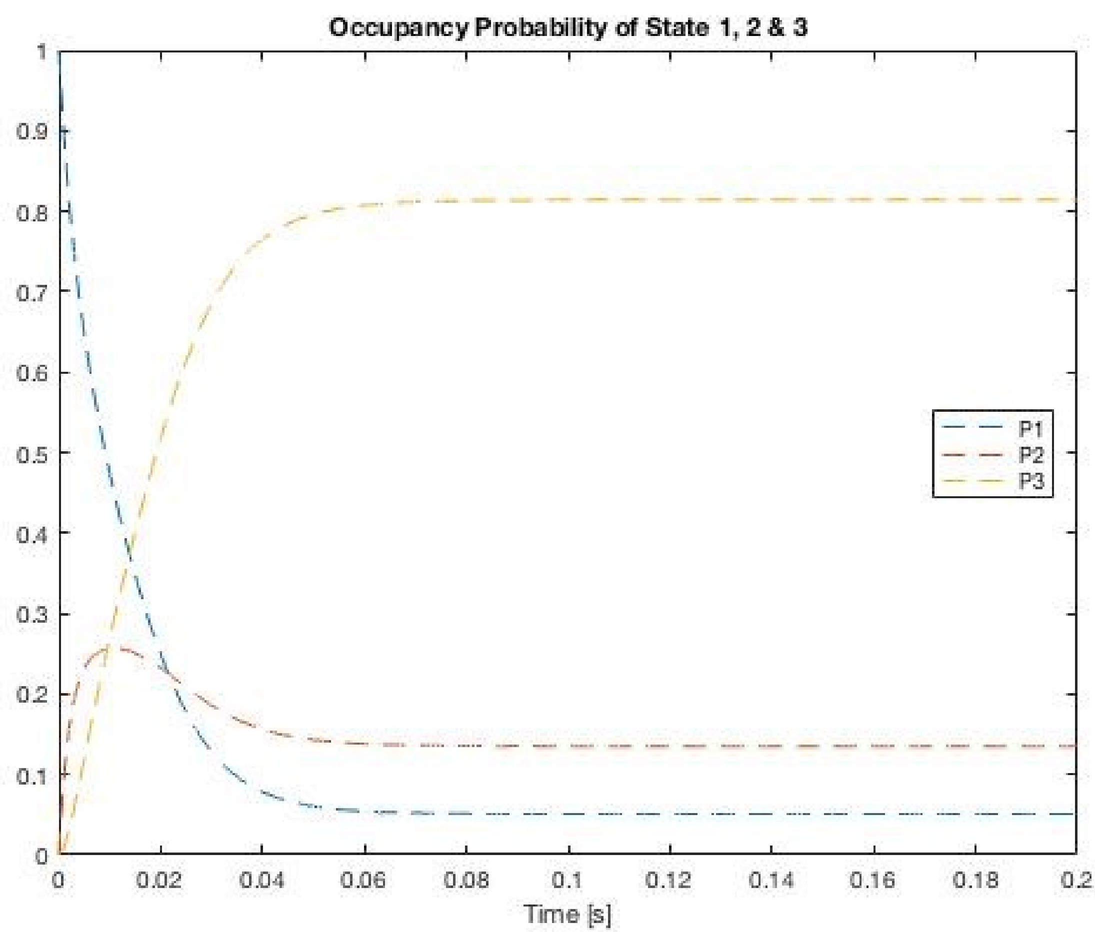

Membrane Potential Versus Time
Voltage [mV]
0
-10
-20
-30
-40
-50
-60
-70
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
Time [s]


Occupancy Probability of State 1, 2 & 3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
Time [s]
P1
P2
P3

2f

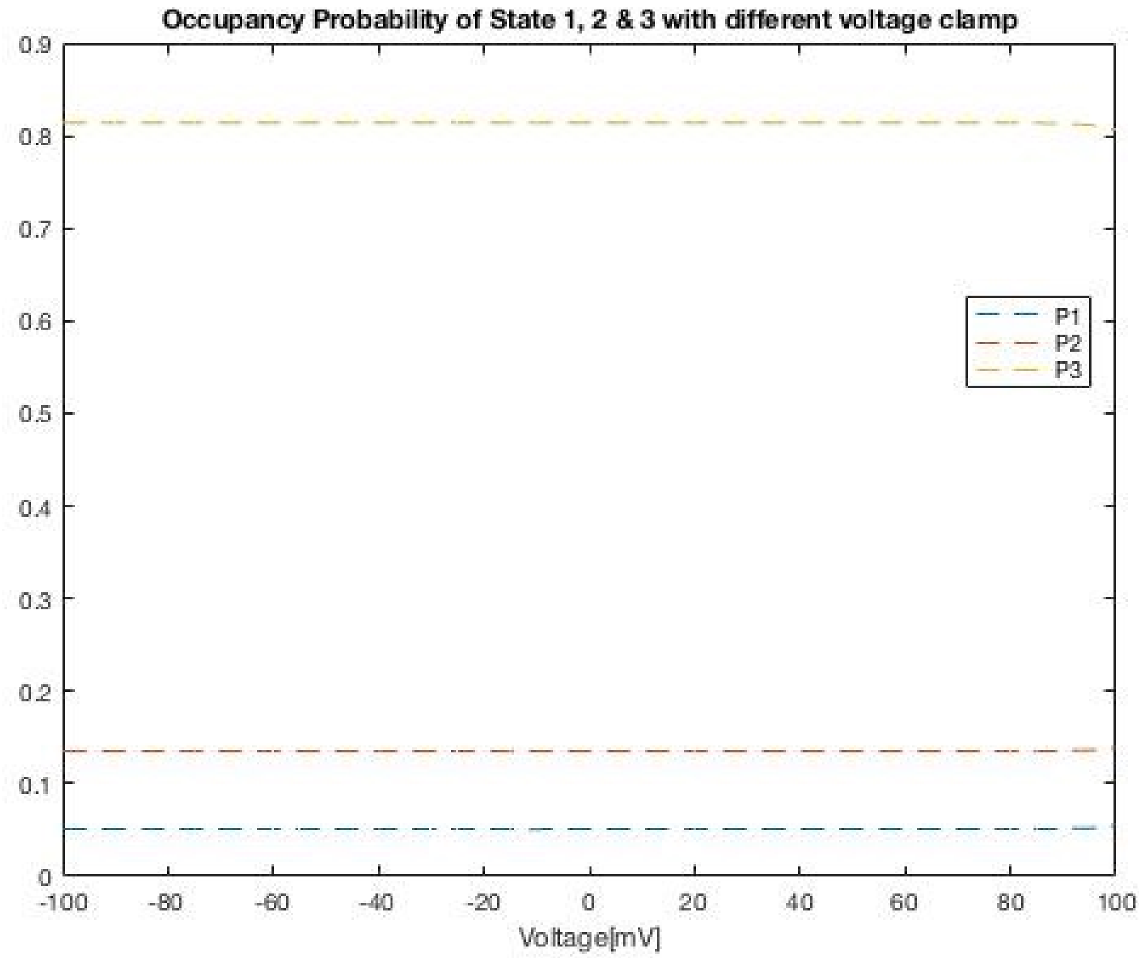


Membrane Potential Versus Time
Voltage [mV]
Time [s]
0
-10
-20
-30
-40
-50
-60
-70
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2



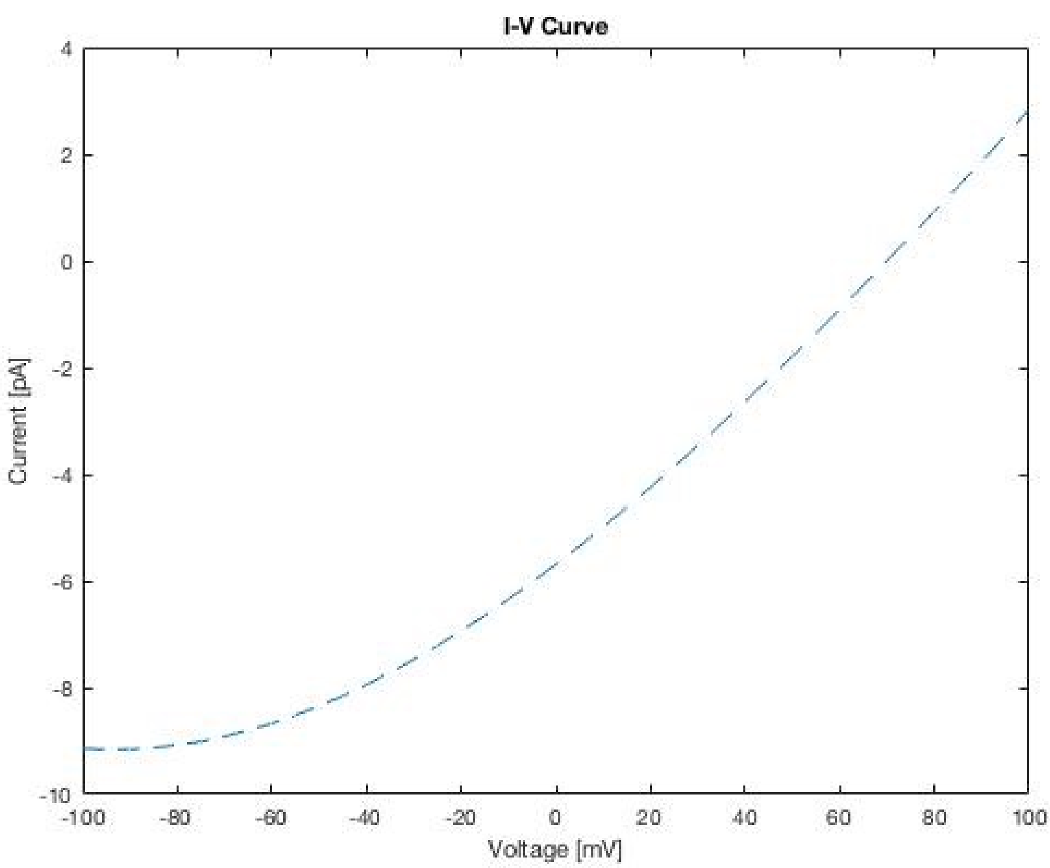
Occupancy Probability of State 1, 2 & 3
P1
P2
P3
Time [s]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2

2g (i)



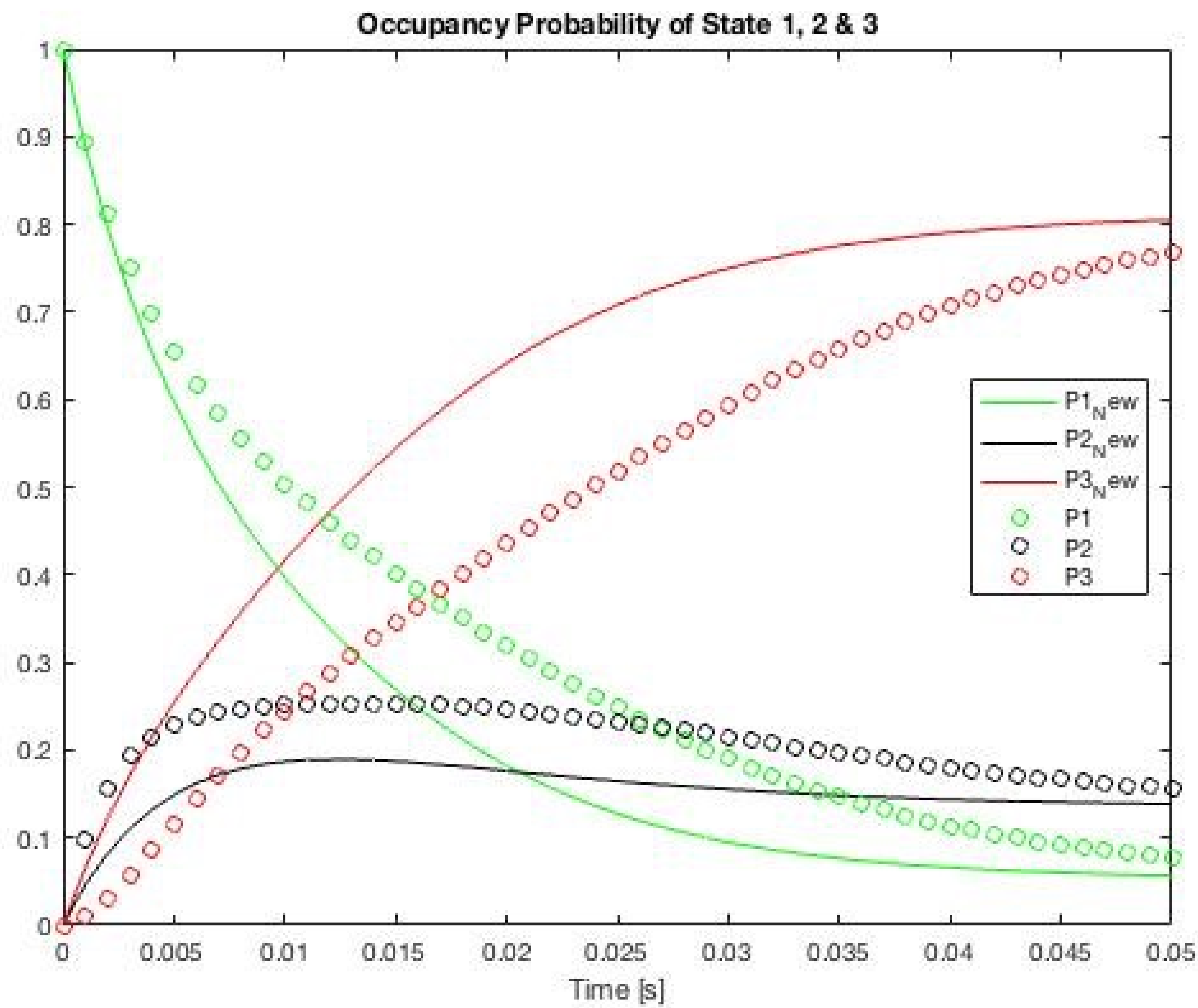
Occupancy Probability of State 1, 2 & 3 with different voltage clamp
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-100
-80
-60
-40
-20
0
20
40
60
80
100
Voltage[mV]
P1
P2
P3

2g (ii)



I-V Curve
4
2
0
-2
-4
-6
-8
-10
Current [pA]
-100
-80
-60
-40
-20
0
20
40
60
80
100
Voltage [mV]

2h (i)


Occupancy Probability of State 1, 2 & 3
P1_New
P2_New
P3_New
P1
P2
P3
Time [s]
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

(i). Plot is created by hw1_2d.m.

#2. (d) (ii) 0.1 s.

(iii) $P1 : P2 : P3 = 0.05088 : 0.135 : 0.8141$

Since there are 100 channels in total

there are 5 channels in State 1, 14 channels in State 2, 81 channels in state 3

(e). The sum of $P_1(t)$, $P_2(t)$, $P_3(t)$ must be One

i.e. $P_1(t) + P_2(t) + P_3(t) = 1$.

(f)(i) $C_m \frac{dV_m}{dt} + N \cdot P_3(t)\, G_{single3} (V_m - E) + N \cdot P_2(t)\, G_{single2} (V_m - E) = 0$.

$$\frac{dV_m}{dt} = \frac{-(P_3(t)\, G_{single3} + P_2(t)\, G_{single2})(V_m - E)\, N}{C_m}$$

(ii) plot in another page plotted with MATLAB code named hw1-2f.m.

(iii). From the plot we learnt that the time to get to steady state is approximately 0.083 s, which is faster than the situation in 2d. The reason is that conductance is the easiness for ions to flow through channel. The more conductance the faster the ions can get through the membrane and reaches a steady state

(iv). The distribution of channels among the 3 different states is the same with 2d(iii) Since the occupancy probability is voltage dependent. Therefore, since the voltage at steady state is the same for both cases, the distributions of both cases are the same.

(g)*(i) Plot in a different page ("2g.jpg")
Method: set the V_m constant to solve for $\frac{\partial P_i}{\partial t}$, then find the next value and the next until all values in V-clamp gets plotted.

(ii) Plot in a different page ("2g ii.jpg")

$$I(i) = G_i \cdot Y_\infty \cdot (V-E)$$
$$= P_3 \cdot G_{isingle} \cdot N (V-E)$$

go over all the values.
where $G_{isingle} = 100\ ps$. $N=100$. $E=-70mV$

(h) (i) plot in a different page ("2hi.jpg")

(ii)

$$1 \underset{k_{51}}{\overset{k_{15}}{\rightleftharpoons}} 5 \qquad \frac{\partial P_1}{\partial t} = P_5 \cdot k_{51} - P_1 \cdot k_{15}$$

$$1 \underset{k_{21}}{\overset{k_{12}}{\rightleftharpoons}} 2 \underset{k_{32}}{\overset{k_{23}}{\rightleftharpoons}} 3 \qquad \frac{\partial P_1}{\partial t} = P_2 \cdot k_{21} - P_1 k_{12}$$

Since k_{23} and k_{32} changes much faster than k_{12} and k_{21}, we can assume state 2 and state 3 are rapidly interchanging with each other, which results

$$P_2 \times k_{23} = P_3 \times k_{32}$$
$$P_3 = \frac{k_{23}}{k_{32}} \times P_2$$

(ii) Therefore $\frac{\partial P_1}{\partial t} = K_{S1} \cdot \left(1+\frac{K_{23}}{K_{32}}\right) P_2 - K_{1S} \cdot P_1.$

$$\frac{\partial P_1}{\partial t} = K_{21} \cdot P_2 - K_{12} \cdot P_1.$$

$$\therefore K_{S1} \cdot \left(1 + \frac{K_{23}}{K_{32}}\right) = K_{21}$$

$$K_{1S} = K_{12}.$$

$$\begin{cases} K_{S1} = \dfrac{K_{32} \cdot K_{21}}{K_{23} + K_{32}}. \\ K_{1S} = K_{12} \end{cases}$$

(iii) Unfortunately, I couldn't generate the plot because of lack of time

From the plot, we can see that $P_S(t) = P_2(t) + P_3(t)$

(iv) In steady state.

$$\begin{cases} P_2 \cdot K_{23} = P_3 \cdot K_{32}. \\ P_2 + P_3 = P_S. \end{cases}$$

$$\therefore P_3 = \frac{P_2 K_{23}}{K_{32}}$$

$$P^{\infty}_{(3|S)} = \frac{K_{32}}{K_{23}+K_{32}}$$

(v). $\hat{P}_3(t)$ is the same as $P_3(t)$ from part (i).

$$\hat{P}_3(t) = \frac{K_{32}}{K_{23}+K_{32}} P_3.$$

(vi). It is a good approximation because $P_S(t)$ is the same as the sum of $P_2(t)+P_3(t)$, and $\hat{P}_3(t)$ is the same as $P_3(t)$.