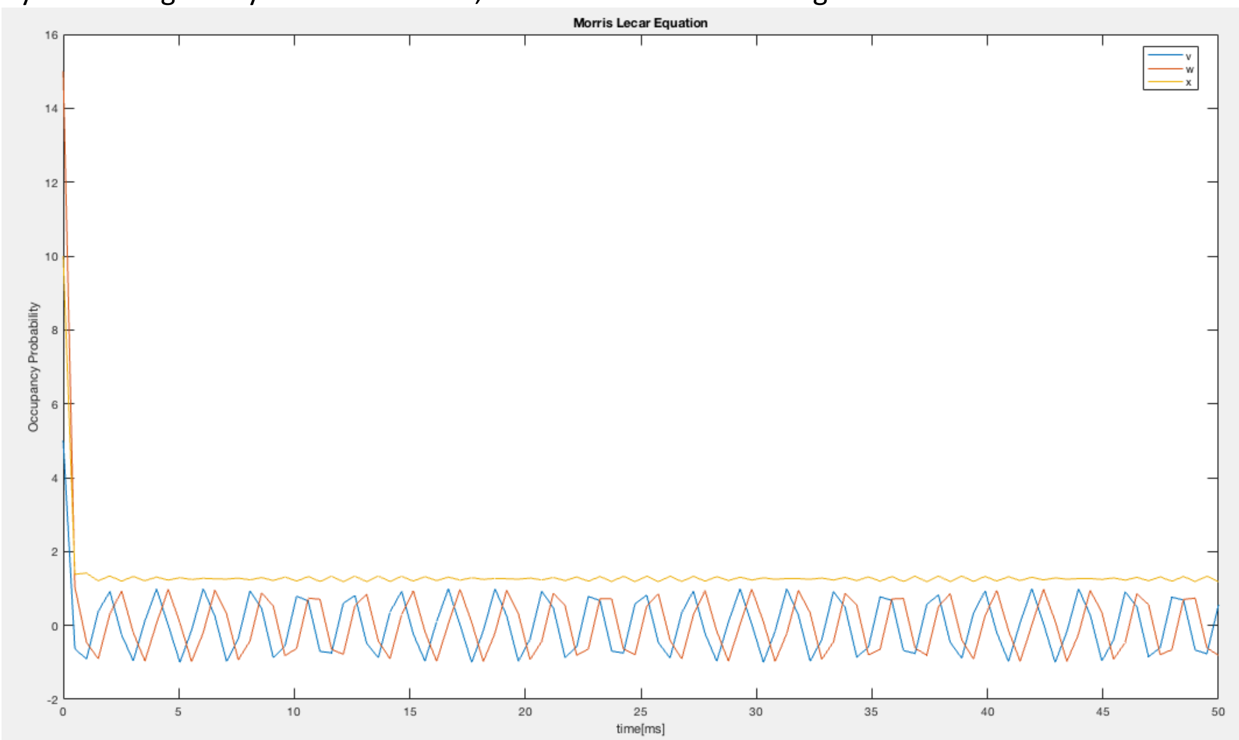
Problem 4

(a).

By observing the form, each derivative of a variable has the cubic form of the variable itself in the equation. Considering in an extreme case, for example, if v is approaching positive infinity, the derivative would be approaching to negative infinity, giving the next variable a number that is more reasonable. This would give rise to a oscillating curve for each variable as time goes on. By simulating the system in MATLAB, we observe the oscillating effect.



(b). c ensures that the dynamics of v and w are much faster than that of x. The reason is that both a, b are the coefficient of the variables in their first order differential equations. Only c doesn't exist in the dx/dt differential equation but in dv/dt and dw/dt.

(c). Finding the Jacobian Matrix of the differential equations:

$$J = \begin{pmatrix} -x - 3a^2 + b & -c & -v \\ c & -x - 3w^2a + b & -w \\ 2v & 0 & -3ax^2 + b \end{pmatrix}$$

Plugging in (v,w,x)=(0,0,q) into the Jacobian matrix

$$J = \begin{pmatrix} -q & -c & 0 \\ c & -q & 0 \\ 0 & 0 & -3aq^2 + b \end{pmatrix}$$

$$\det(J - I\lambda) = |A|$$

$$|A| = (-q-\lambda)[(-q-\lambda) * (-3aq^2 + b - \lambda)] + c[c * (-3aq^2 + b - \lambda)]$$

$$= (-q-\lambda)^2(-3aq^2 + b - \lambda) + c^2[-3aq^2 + b - \lambda]$$

$$= [(q+\lambda)^2 + c^2][-3aq^2 + b - \lambda] = 0$$

$$(1). \quad [(q+\lambda)^2 + c^2] = 0$$

$\lambda_{1,2} = \pm ic - q$; Therefore, q>0;

(2). $[-3aq^2 + b - \lambda] = 0$

$\lambda_3 = -3aq^2 + b$
Therefore, $-3aq^2 + b > 0$
In summary, $\begin{cases} \mathrm{q} > 0 \\ -3aq^2 + b > 0 \end{cases}$ does the equilibrium stable under the first order linearization.