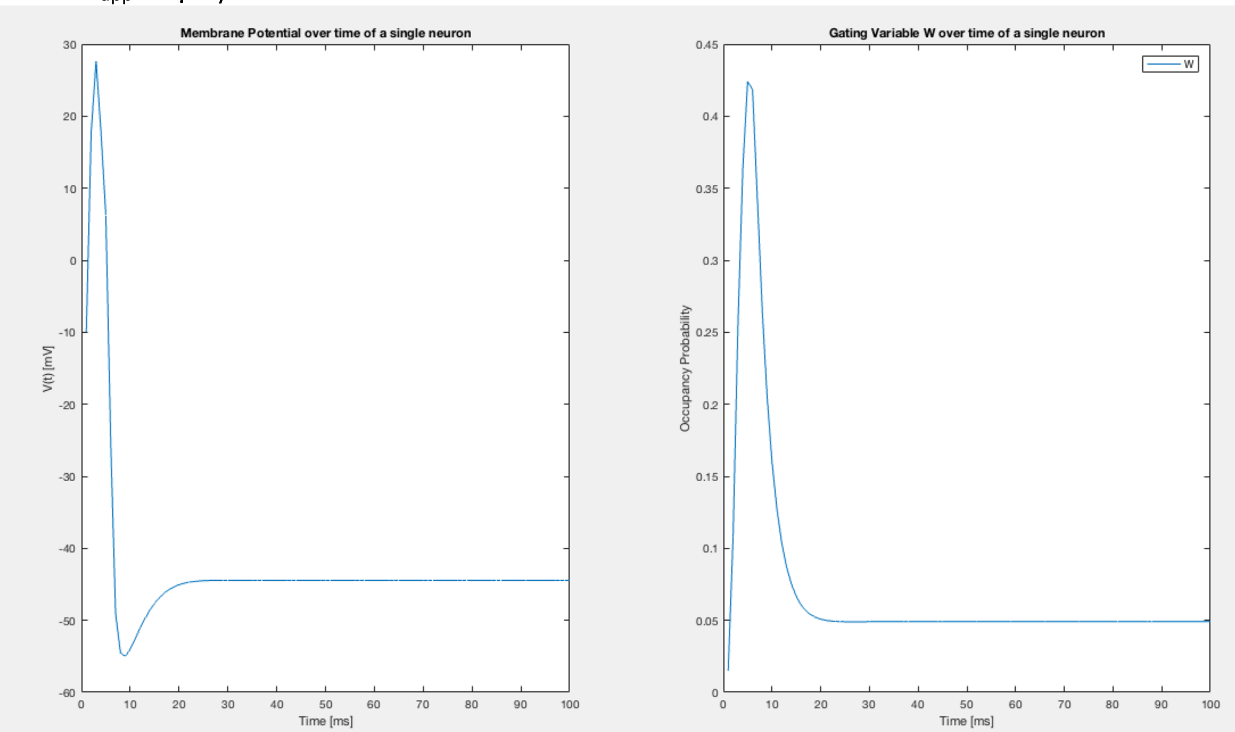
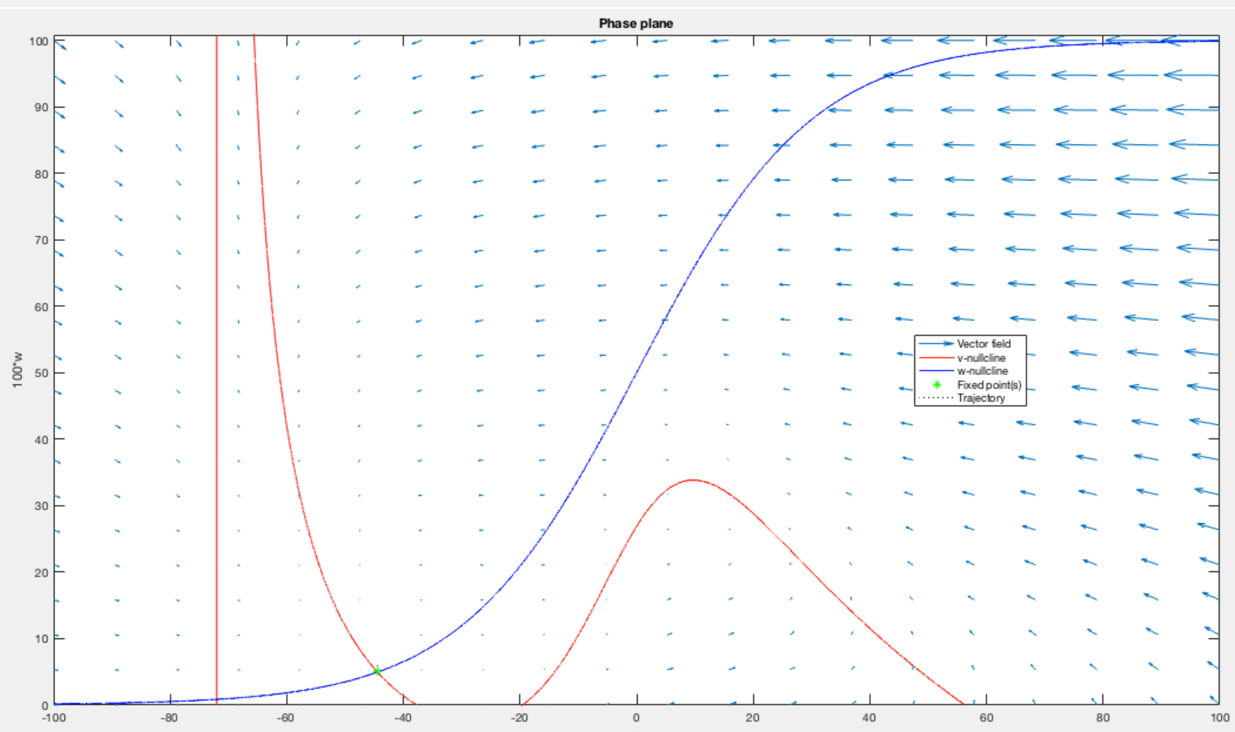
Problem 2.

(Case i)

When I_{app} =-5$\mu A/cm^2$





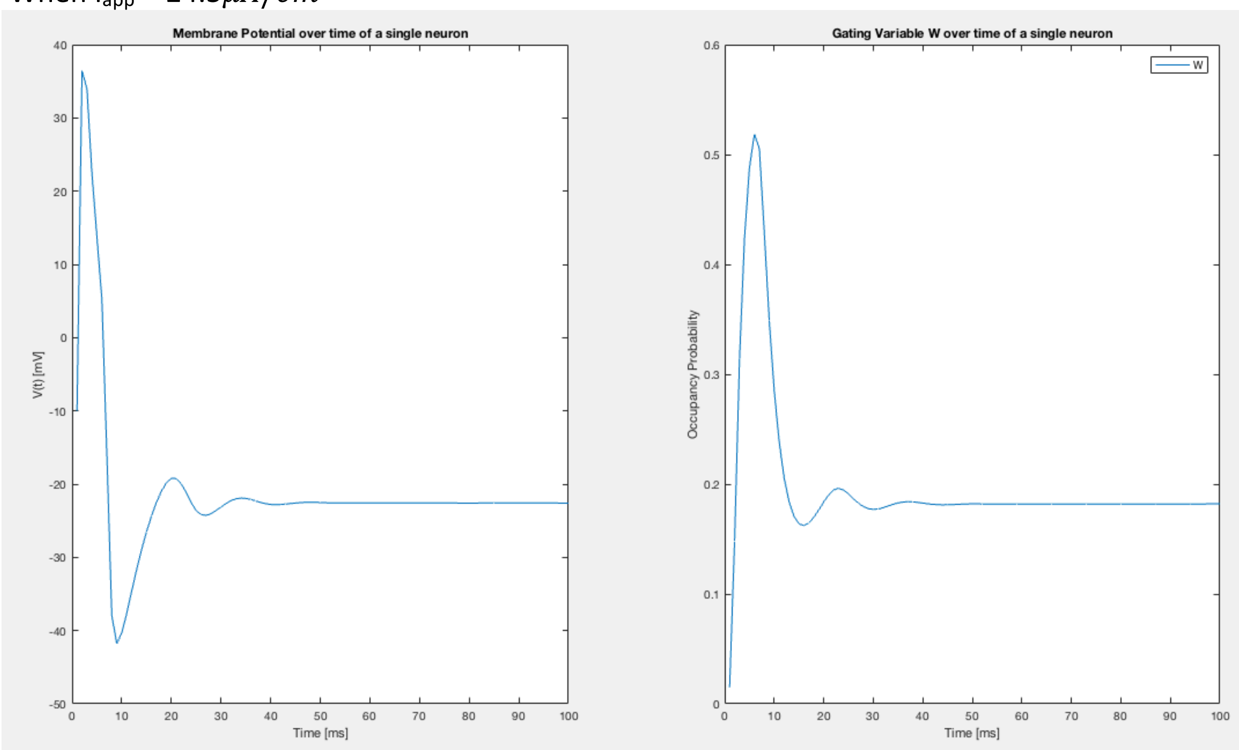
$$J = \begin{bmatrix} \left(\frac{11 * \tanh\left(\frac{v}{15} + \frac{1}{15}\right)^2}{300} - \frac{11}{300}\right) * (v - 100) - \frac{11 * \tanh\left(\frac{v}{15} + \frac{1}{15}\right)}{20} - 2 * w - \frac{21}{20} & -2 * v - 144 \\ \frac{\sinh\left(\frac{v}{60}\right) * \left(\frac{\tanh\left(\frac{v}{30}\right)}{2} - w + \frac{1}{2}\right)}{300} - \frac{\cosh\left(\frac{v}{60}\right) * \left(\frac{\tanh\left(\frac{v}{30}\right)^2}{60} - \frac{1}{60}\right)}{5} & -\frac{\cosh\left(\frac{v}{60}\right)}{5} \end{bmatrix}$$

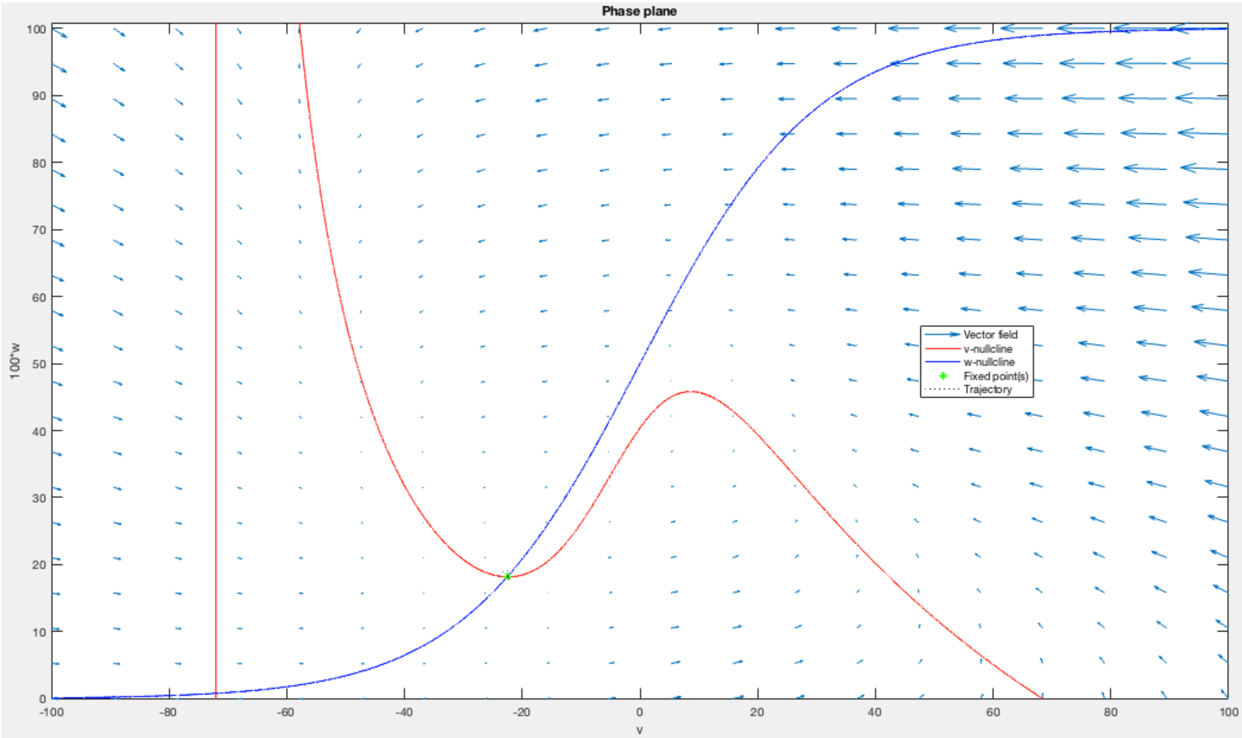
$$\lambda = -0.3974 \pm 0.1568i$$

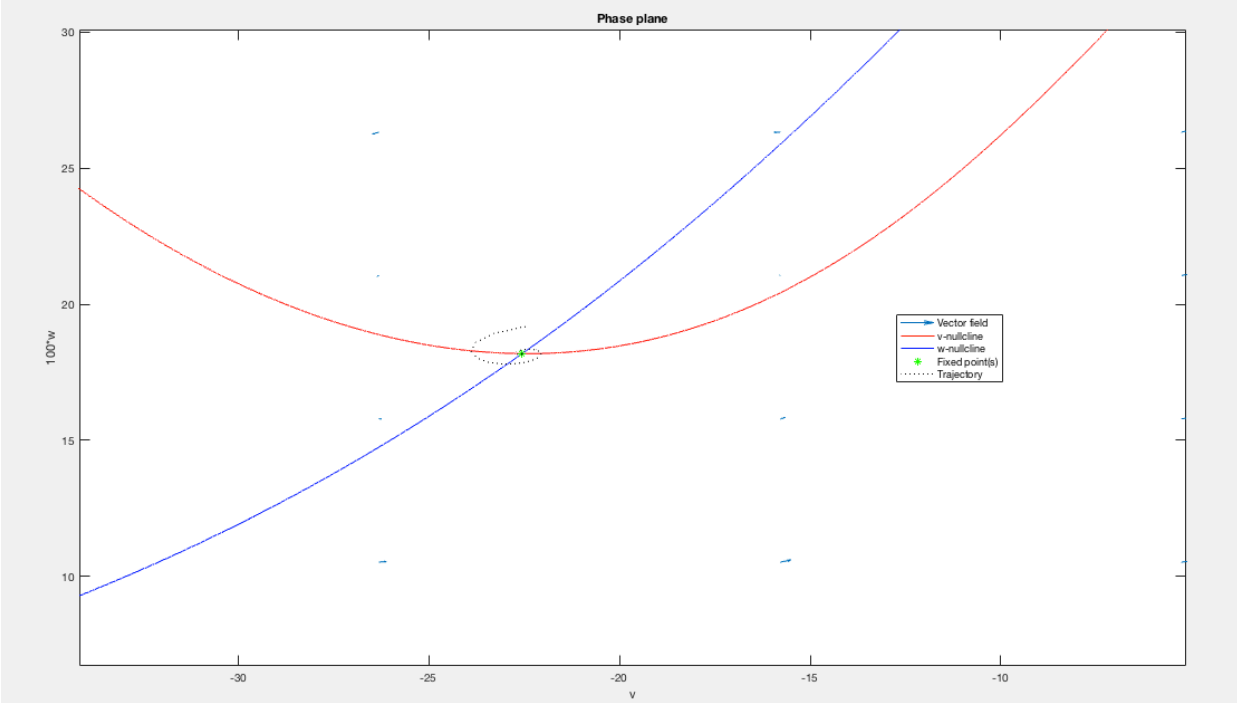
The fixed point at (-44.4474,0.049118) is stable because the real part of the eigenvalue is negative

(Case ii)

When I_{app} =-24.5$\mu A/cm^2$







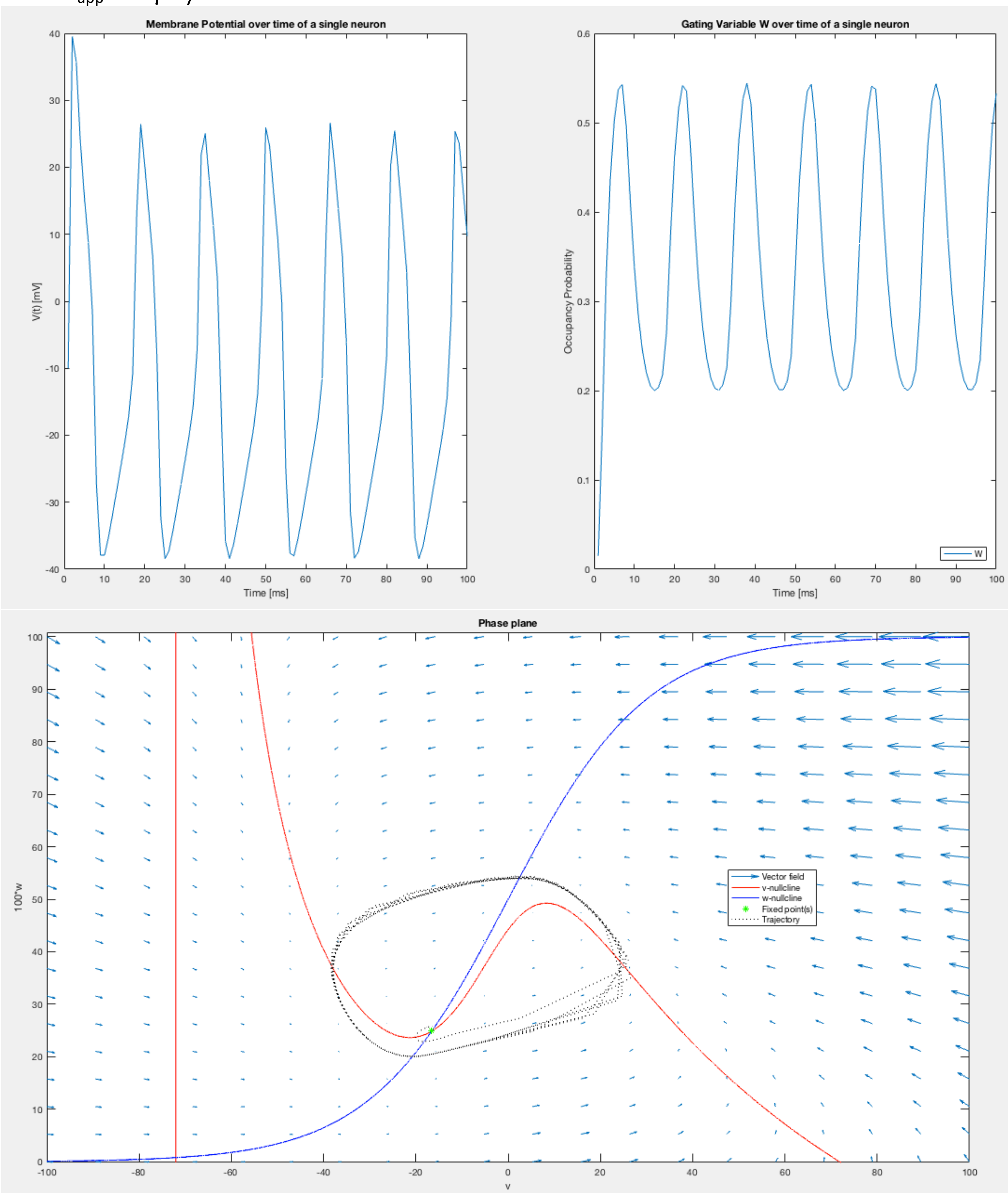
$$J = \begin{bmatrix} \left(\frac{11 * \tanh\left(\frac{v}{15} + \frac{1}{15}\right)^2}{300} - \frac{11}{300}\right) * (v - 100) - \frac{11 * \tanh\left(\frac{v}{15} + \frac{1}{15}\right)}{20} - 2 * w - 21/20 & -2 * v - 144 \\ \frac{\sinh\left(\frac{v}{60}\right) * \left(\frac{\tanh\left(\frac{v}{30}\right)}{2} - w + \frac{1}{2}\right)}{300} - \frac{\cosh\left(\frac{v}{60}\right) * \left(\frac{\tanh\left(\frac{v}{30}\right)^2}{60} - \frac{1}{60}\right)}{5} & -\frac{\cosh\left(\frac{v}{60}\right)}{5} \end{bmatrix}$$

$$\lambda = -0.1138 \pm 0.4473i$$

The fixed point at (-22.5574,0.18186) is stable.

(Case iii)

When $I_{app} = -30 \mu A/cm^2$

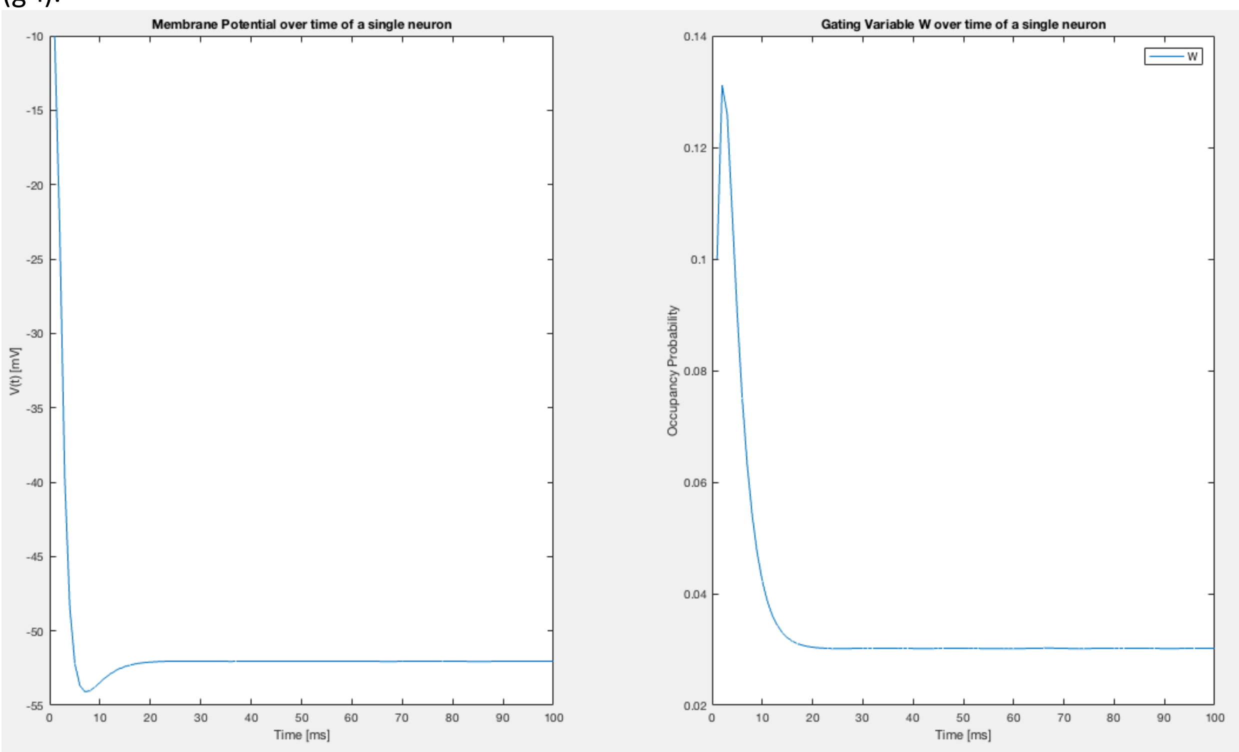


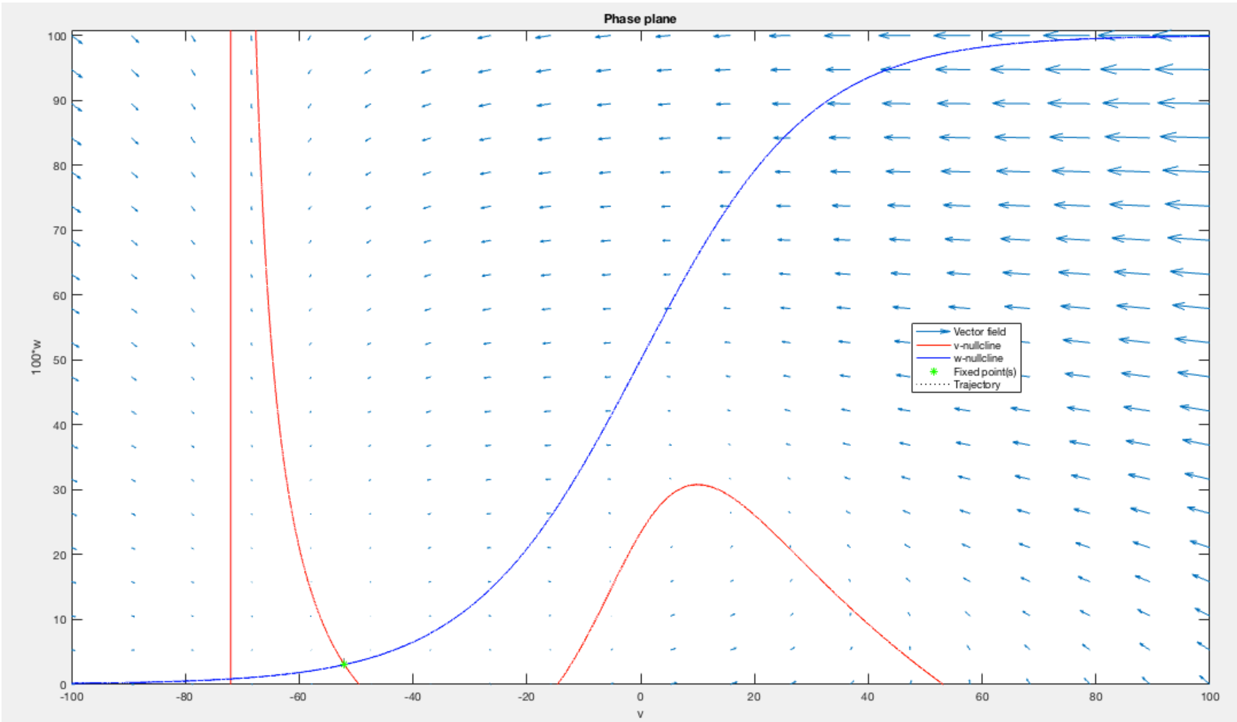
$$J = \begin{bmatrix} \left(\frac{11 * \tanh\left(\frac{v}{15} + \frac{1}{15}\right)^2}{300} - \frac{11}{300}\right) * (v - 100) - \frac{11 * \tanh\left(\frac{v}{15} + \frac{1}{15}\right)}{20} - 2 * w - \frac{21}{20} & -2 * v - 144 \\ \frac{\sinh\left(\frac{v}{60}\right) * \left(\frac{\tanh\left(\frac{v}{30}\right)}{2} - w + \frac{1}{2}\right)}{300} - \frac{\cosh\left(\frac{v}{60}\right) * \left(\frac{\tanh\left(\frac{v}{30}\right)^2}{60} - \frac{1}{60}\right)}{5} & -\frac{\cosh\left(\frac{v}{60}\right)}{5} \end{bmatrix}$$

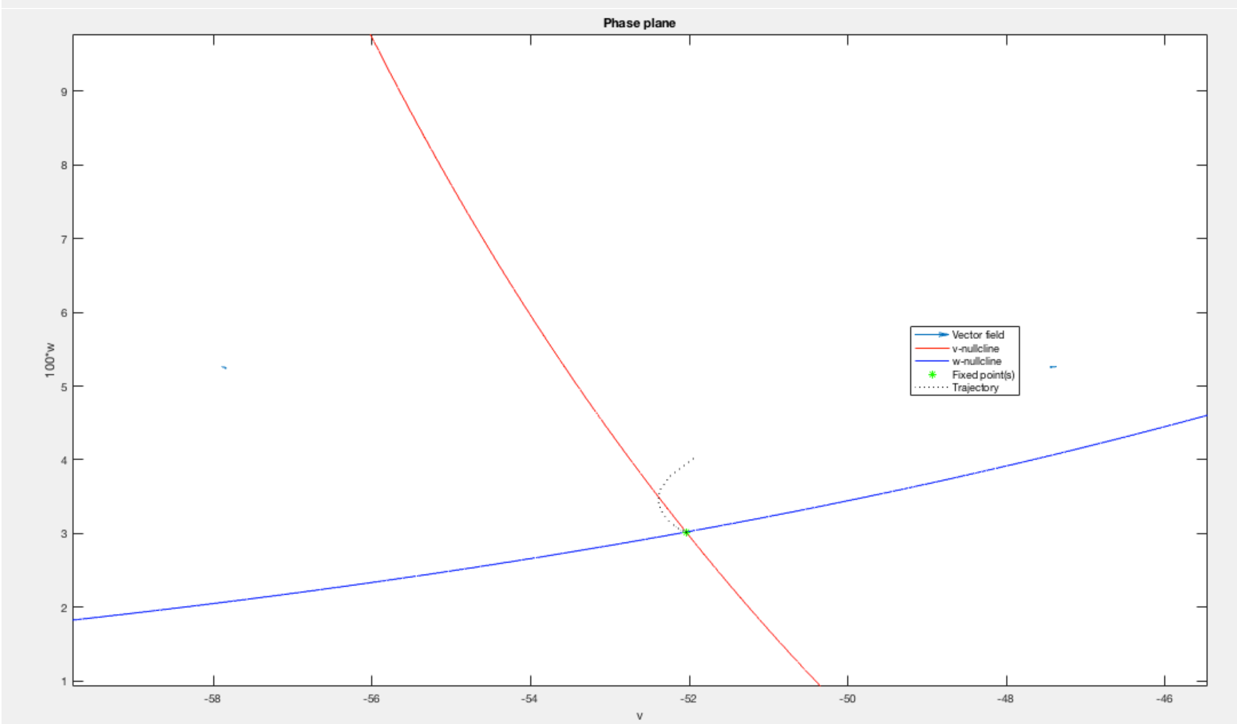
$$\lambda = 0.1816 \pm 0.3677i$$

The fixed point at (-16.5864,0.24866) is unstable.

(g-i).

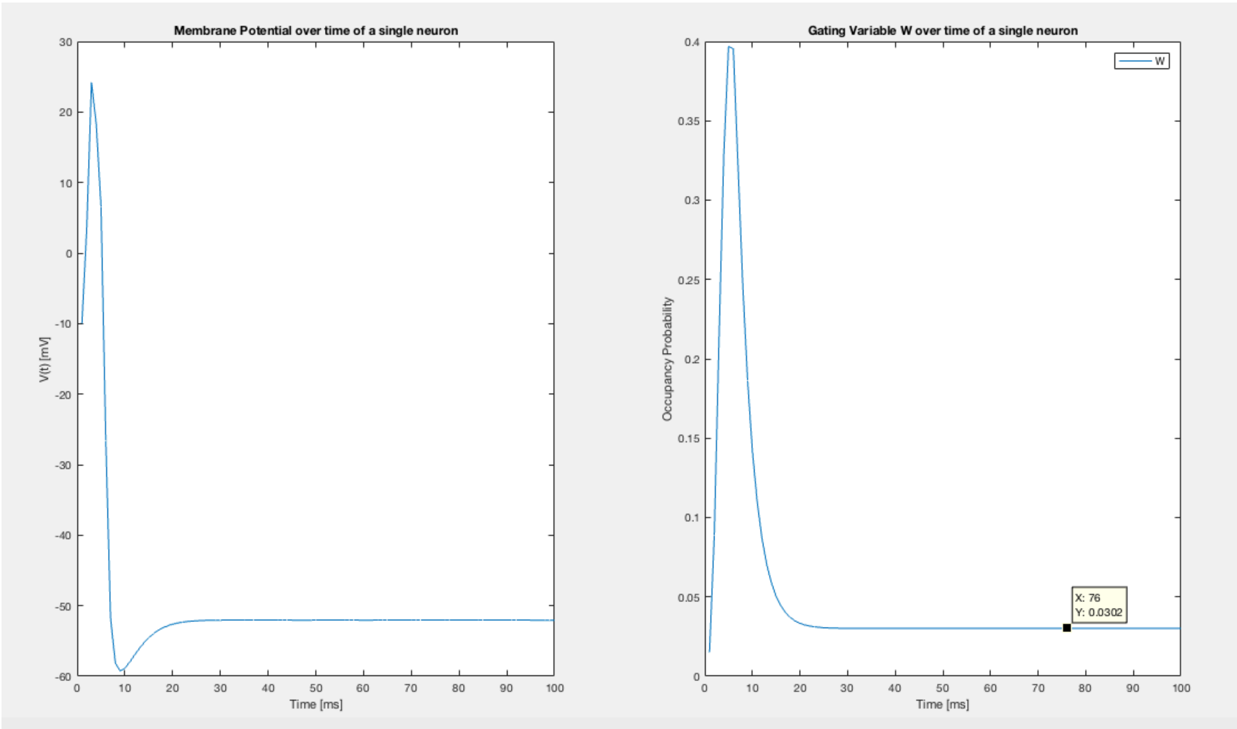




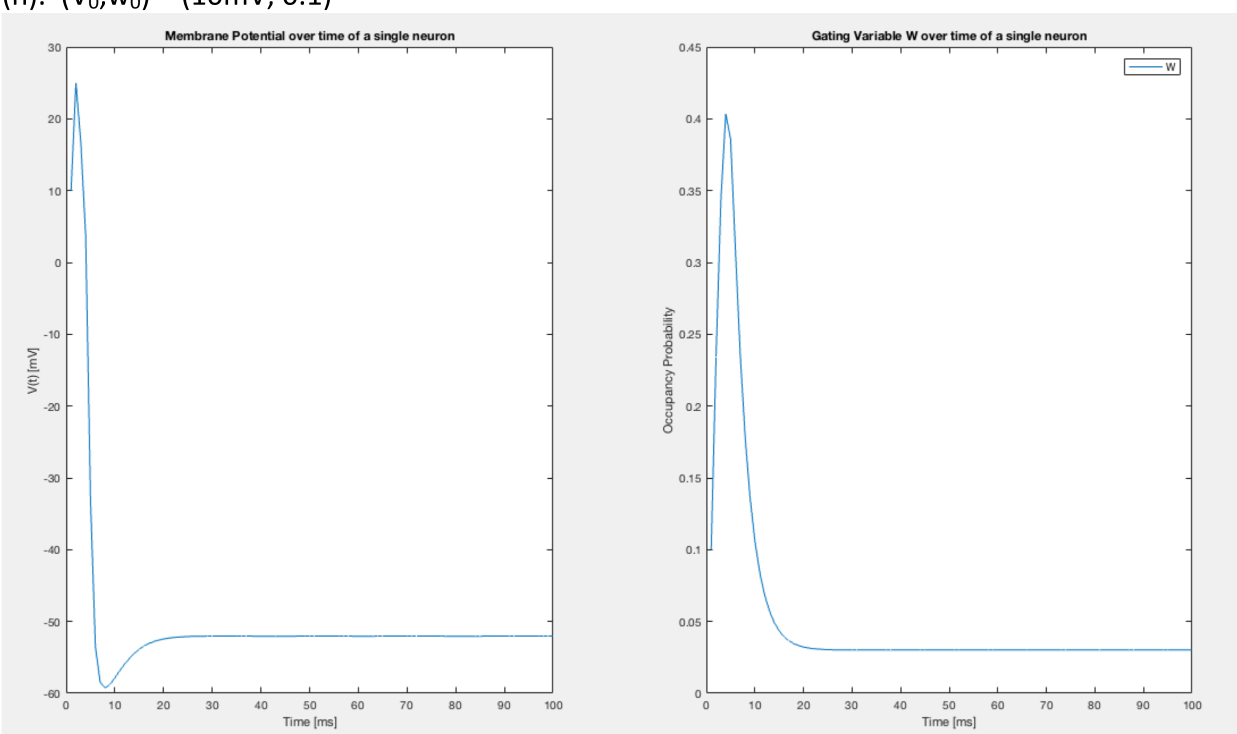


The fixed point at (-52.0407,0.030196) is stable.

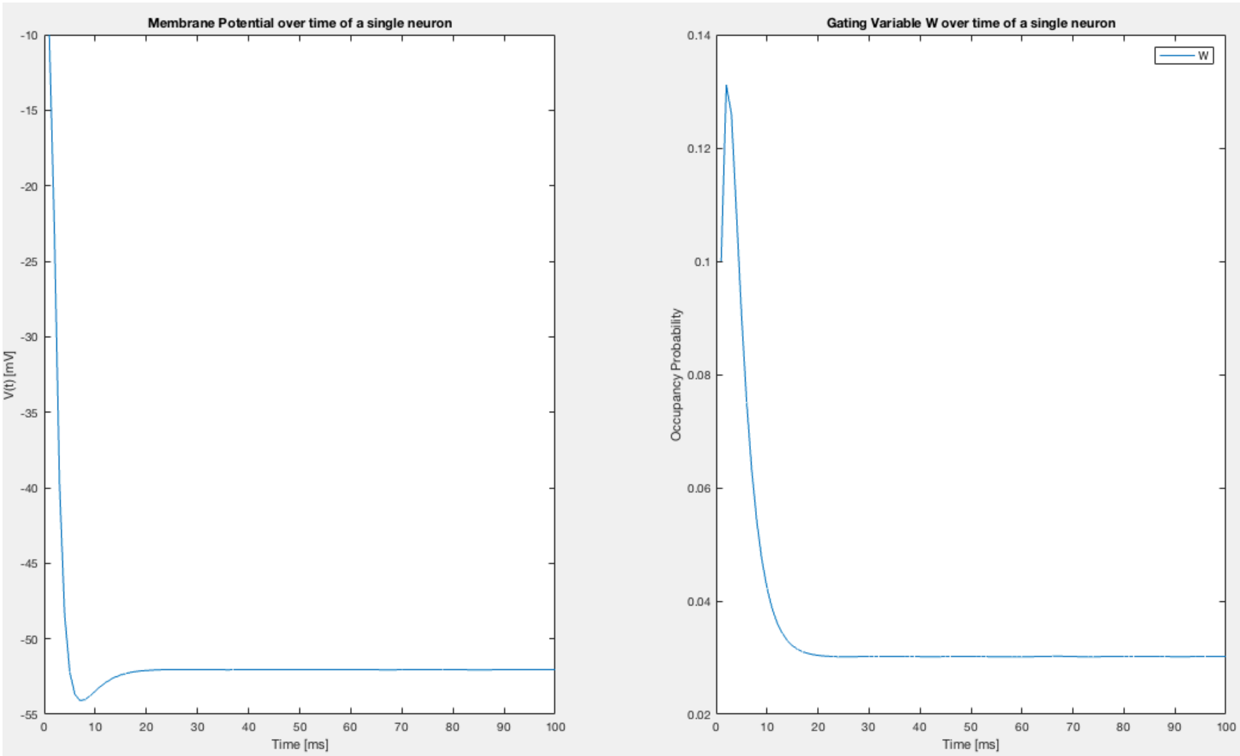
(g). The steady state value of w at resting potential is 0.0302;



(h). (V_0,w_0) = (10mV, 0.1)



(V_0,w_0) = (-10mV, 0.1)



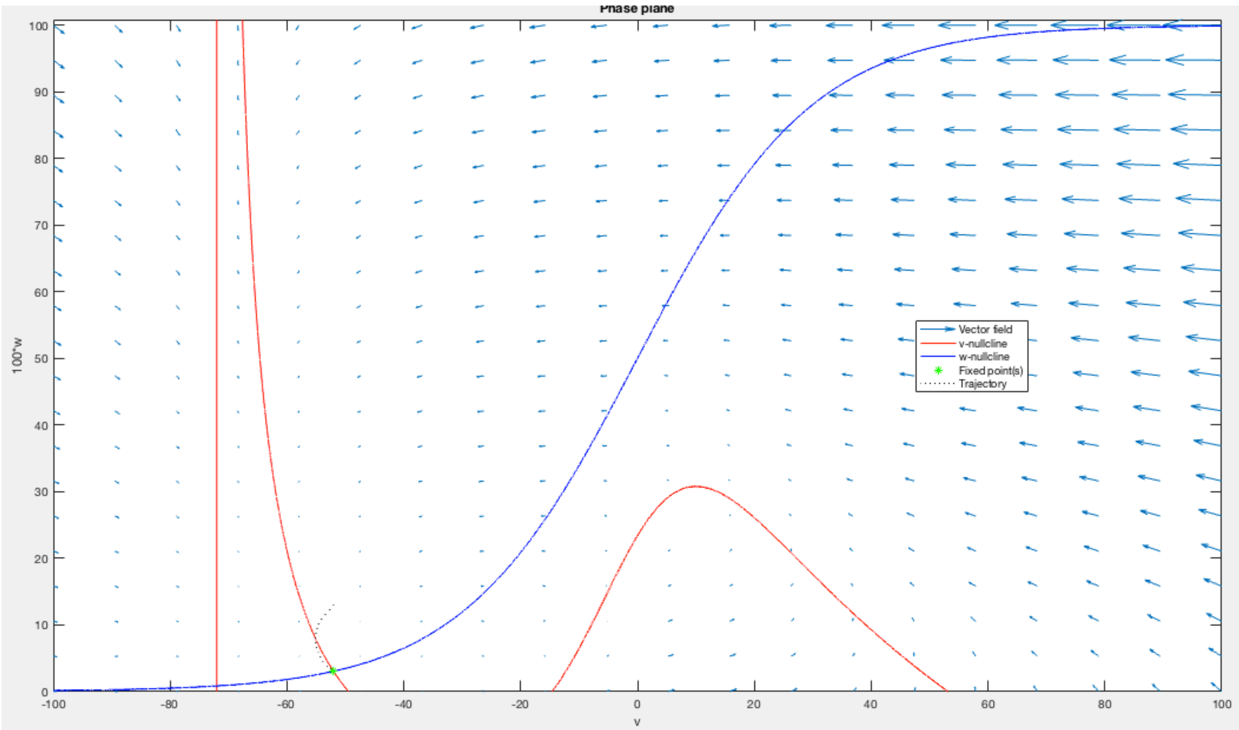
Comparing the membrane potential, we can find out that both membrane potentials share a general trend of first dropping down to a value and then rebound a little and reach a steady state in the end. The difference is that in the 10 mV model the cell jumps to a higher membrane potential at around 24 V and then drops down while the other one only decreases; The 10 mV model also drops down to a lower value and reaches a lower steady state value.

Comparing the w over time, we observe that the 10 mV, in comparison to the -10 mV, reaches a higher occupancy probability and over time has a higher occupancy value at its steady states;

The biological relevance of the stimulation is that the initial membrane potential can possible be a positive feedback system for a higher maximum value and a lower minimum value. The gating variable would also be more open up in the 10 mV model.

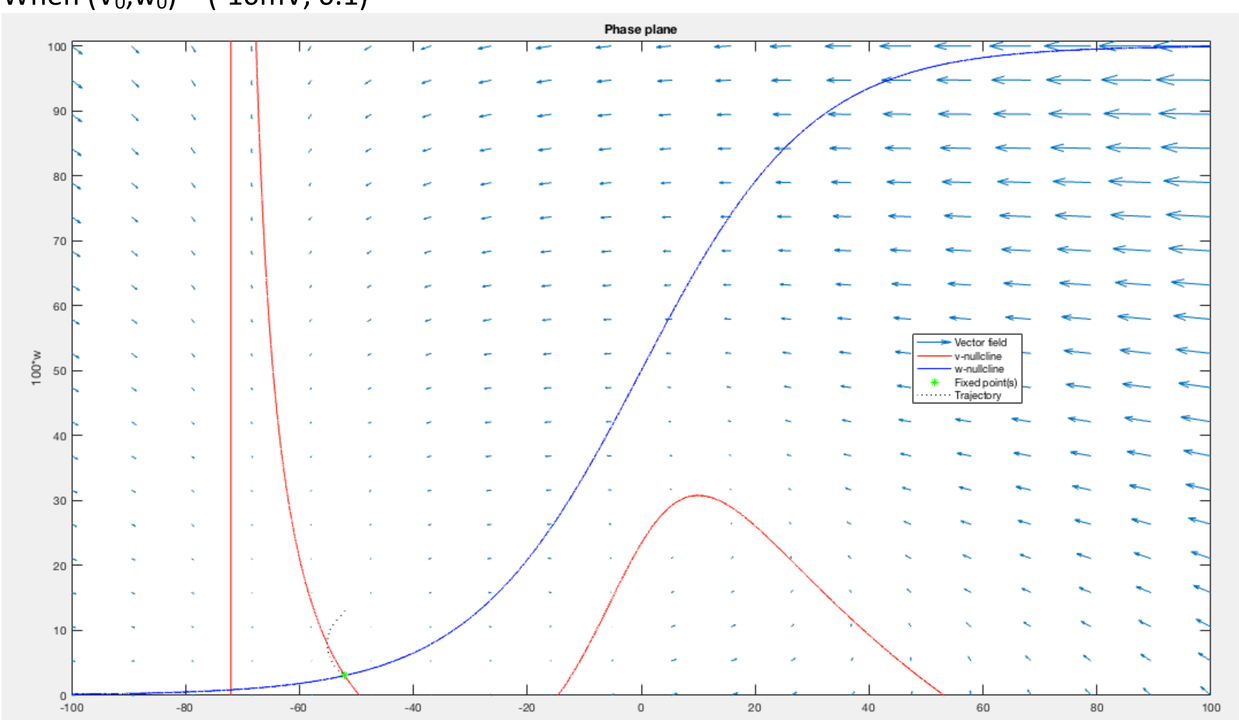
(i)

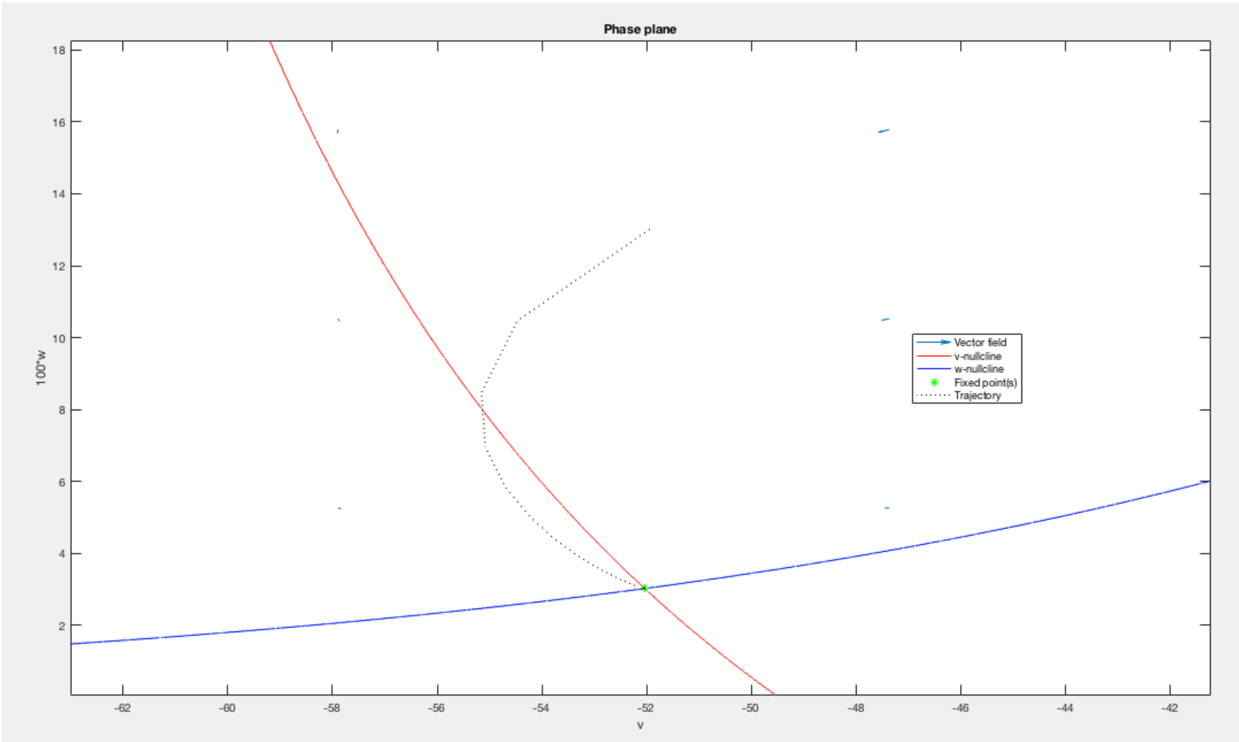
When (V_0,w_0) = (10mV, 0.1)



The fixed point at (-52.0407,0.030196) is stable.

When $(V_0,w_0) = (-10mV, 0.1)$





The fixed point at (-52.0407,0.030196) is stable.

From the simulation, we find that the two phase plane plots are the same; The mathematical origin is that the trajectory is based on the fixed point and the two cases have the same fixed point. Therefore, the result should be the same.