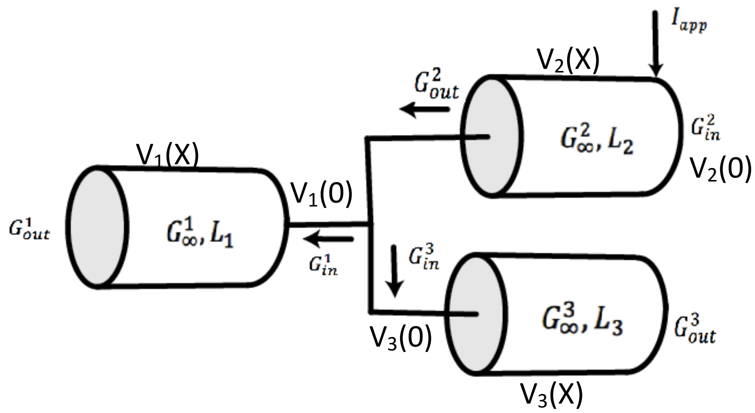


Problem 2

<u>When current is applied at point A:</u>



[a.i.]

$\boldsymbol{G_{out}^1} = 0$ (sealed-end boundary condition)

$\boldsymbol{G_{out}^2} = G_{in}^1 + G_{in}^3 = G_\infty^1 \tanh(L_1) + G_\infty^3 \tanh(L_3)$

$\boldsymbol{G_{out}^3} = 0$ (sealed-end boundary condition)

[a.ii.]

$$\boldsymbol{G_{in}^1} = G_\infty^1 \frac{\cancel{\frac{G_{out}^1}{G_\infty}} + \tanh(L_1)}{1 + \cancel{\frac{G_{out}^1}{G_\infty}} \tanh(L_1)} = G_\infty^1 \tanh(L_1)$$

$$\boldsymbol{G_{in}^2} = G_\infty^2 \frac{\frac{G_{out}^2}{G_\infty^2} + \tanh(L_2)}{1 + \frac{G_{out}^2}{G_\infty^2} \tanh(L_2)}$$

$$\boldsymbol{G_{in}^3} = G_\infty \frac{\cancel{\frac{G_{out}^3}{G_\infty}} + \tanh(L_3)}{1 + \cancel{\frac{G_{out}^3}{G_\infty}} \tanh(L_3)} = G_\infty^3 \tanh(L_3)$$

[a.iii.]

$$\boldsymbol{V_1(0)} = \boldsymbol{V_3(0)} = V_2(L_2) = V_2(0) * \frac{\cosh(0) + \frac{G_{out}^2}{G_\infty^2}\sinh(0)}{cosh(L_2) + \frac{G_{out}^2}{G_\infty^2}\sinh(L_2)}$$

$$= V_2(0) * \frac{1}{cosh(L_2) + \frac{G_{out}^2}{G_\infty^2}\sinh(L_2)}$$

$$\boldsymbol{V_2(0)} = \frac{I_{app}}{G_{in}^2}$$

[a.iv]

$$\boldsymbol{V_1(X)} = V_1(0) * \frac{\cosh(L_1 - X) + \cancel{\frac{G^1_{out}}{G^1_{\infty}} \sinh\,(L_1 - X)}}{\cosh(L_1) + \cancel{\frac{G^1_{out}}{G_{\infty}} \sinh\,(L_1)}}$$

$$= V_1(0)\,\frac{\cosh(L_1 - X)}{\cosh(L_1)}$$

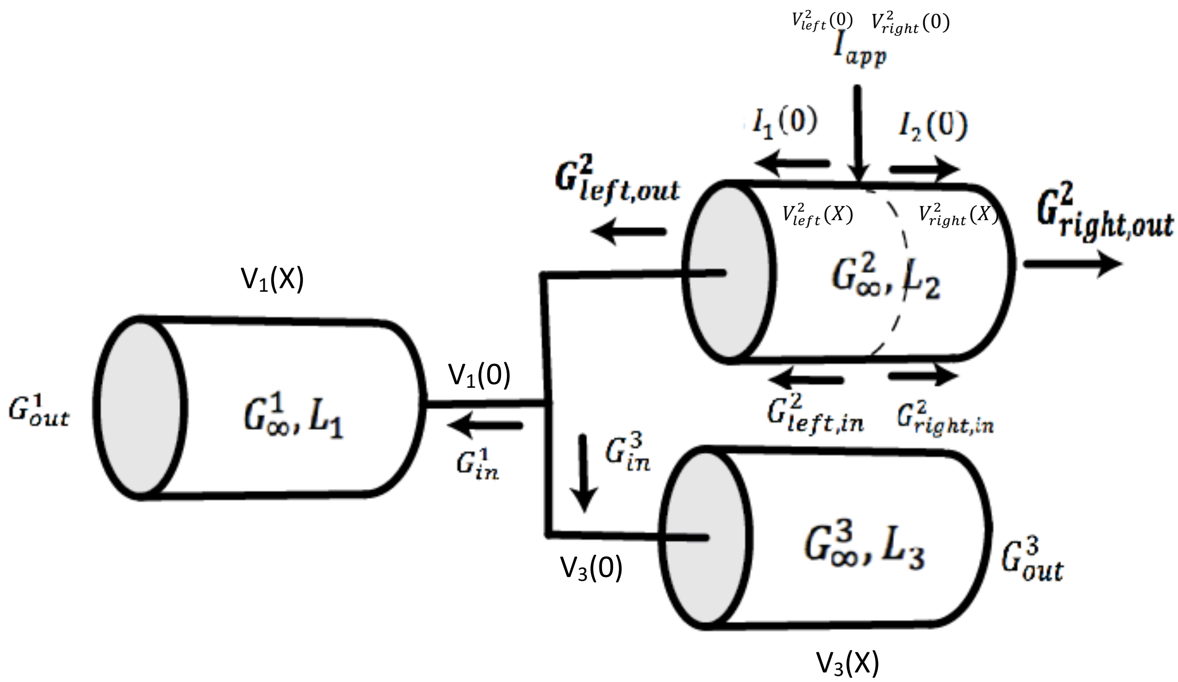
$$\boldsymbol{V_2(X)} = V_2(0) * \frac{\cosh(L_2 - X) + \frac{G^2_{out}}{G^2_{\infty}} \sinh\,(L_2 - X)}{\cosh(L_2) + \frac{G^2_{out}}{G^2_{\infty}} \sinh\,(L_2)}$$

$$\boldsymbol{V_3(X)} = V_3(0) * \frac{\cosh(L_3 - X) + \cancel{\frac{G^3_{out}}{G^3_{\infty}} \sinh\,(L_3 - X)}}{\cosh(L_3) + \cancel{\frac{G^3_{out}}{G^3_{\infty}} \sinh\,(L_3)}}$$

$$= V_3(0) * \frac{\cosh(L_3 - X)}{\cosh(L_3)}$$

<u>When the current is applied at point B:</u>

I_{app} bifurcates into two sub-branches $I_1(0)$ and $I_2(0)$. Similarly, we can cut the cable 2 into 2 sections, the left cable with entering conductance $G^2_{left,in}$ and the right cable with entering conductance $G^2_{right,in}$. According to Kirchhoff's current law, since the left cable and the right cable are in parallel with each other, $I_{app} = I_1(0) + I_2(0)$. We define the length of left part of cable 2 as $L_{2,left}$ and the length of the right part as $L_{2,right}$.



[a.i.]

$\boldsymbol{G^1_{out}} = 0$ (sealed-end boundary condition)

$\boldsymbol{G^2_{right,out}} = 0$ (sealed-end boundary condition)

$\boldsymbol{G^2_{left,out}} = G^1_{in} + G^3_{in} = G^1_\infty \tanh(L_1) + G^3_\infty \tanh(L_3)$

$\boldsymbol{G^3_{out}} = 0$ (sealed-end boundary condition)

[a.ii.]

$$\boldsymbol{G^1_{in}} = G^1_\infty \frac{\cancel{\frac{G^1_{out}}{G_\infty}} + \tanh(L_1)}{1 + \cancel{\frac{G^1_{out}}{G_\infty}\tanh(L_1)}} = G^1_\infty \tanh(L_1)$$

$$\boldsymbol{G^2_{right,in}} = G^2_\infty \tanh(L_{2,right})$$

$$\boldsymbol{G^2_{left,in}} = G^2_\infty \frac{\frac{G^2_{left,out}}{G^2_\infty} + \tanh(L_{2,left})}{1 + \frac{G^2_{left,out}}{G^2_\infty}\tanh(L_{2,left})}$$

$$\boldsymbol{G_{in}^3} = G_\infty^3 \frac{\cancel{\frac{G_{out}^3}{G_\infty}} + \tanh(L_3)}{1 + \cancel{\frac{G_{out}^3}{G_\infty}}\tanh(L_3)} = G_\infty^3 \tanh(L_3)$$

[a.iii.]

$\cancel{I_1(0) + I_2(0) = I_{app}}$

$\cancel{\boldsymbol{V_{left}^2(0)} = \frac{I_1(0)}{\boldsymbol{G_{left,in}^2}} = \boldsymbol{V_{right}^2(0)} = \frac{I_2(0)}{\boldsymbol{G_{right,in}^2}}}$

~~--- The equality of the voltages holds because the 2 sub-cables are in parallel with each other.~~

$G_0 = \boldsymbol{G_{left,in}^2} + \boldsymbol{G_{right,in}^2}$

$\boldsymbol{V_{left}^2(0)} = \boldsymbol{V_{right}^2(0)} = \frac{\boldsymbol{I_{app}}}{G_0}$

$$\boldsymbol{V_3(0)} = \boldsymbol{V_1(0)} = \boldsymbol{V_{left}^2(L_{2,left})} = V_{left}^2(0)\frac{\cosh(L_{2,left} - X) + \frac{G_{left,out}^2}{G_\infty^2}\sinh(L_{2,left} - X)}{\cosh(L_{2,left}) + \frac{G_{left,out}^2}{G_\infty^2}\sinh(L_{2,left})}$$

$$= V_{left}^2(0)\frac{1}{\cosh(L_{2,left}) + \frac{G_{left,out}^2}{G_\infty^2}\sinh(L_{2,left})}$$

[a.iv.]

$$\boldsymbol{V_{left}^2(X)} = V_{left}^2(0)\frac{\cosh(L_{2,left} - X) + \frac{G_{left,out}^2}{G_\infty^2}\sinh(L_{2,left} - X)}{\cosh(L_{2,left}) + \frac{G_{left,out}^2}{G_\infty^2}\sinh(L_{2,left})}$$

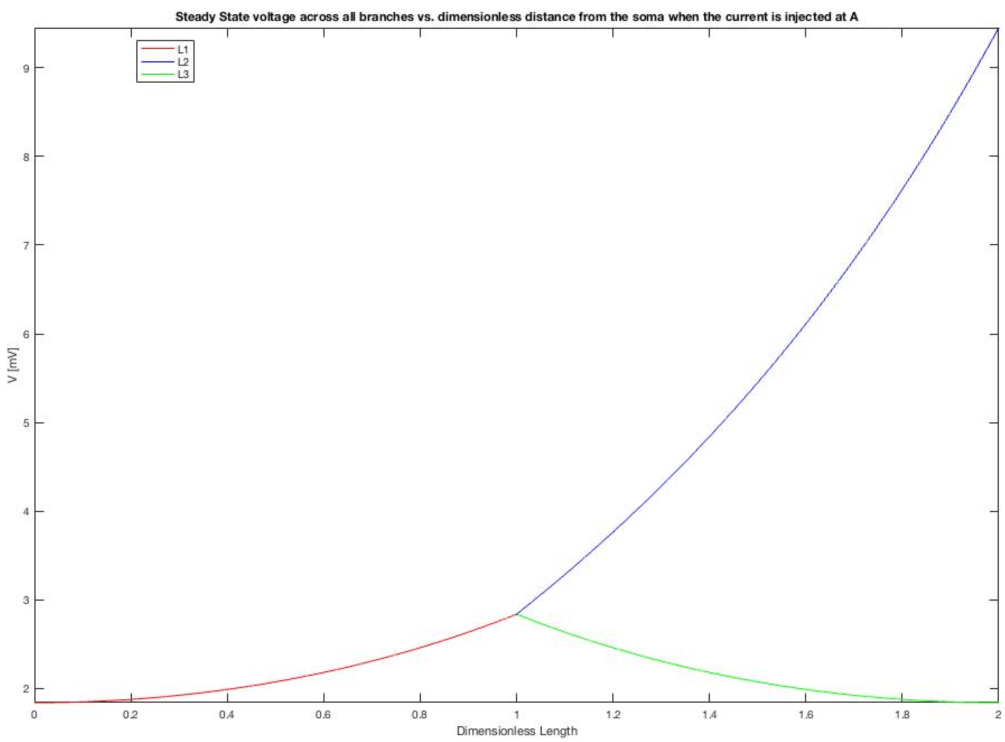
$$\boldsymbol{V_{right}^2(X)} = V_{right}^2(0)\frac{\cosh(L_{2,right} - X) + \cancel{\frac{G_{right,out}^2}{G_\infty^2}}\sinh(L_{2,right} - X)}{\cosh(L_{2,right}) + \cancel{\frac{G_{right,out}^2}{G_\infty^2}}\sinh(L_{2,right})}$$

$$= V_{right}^2(0)\frac{\cosh(L_{2,right} - X)}{\cosh(L_{2,right})}$$

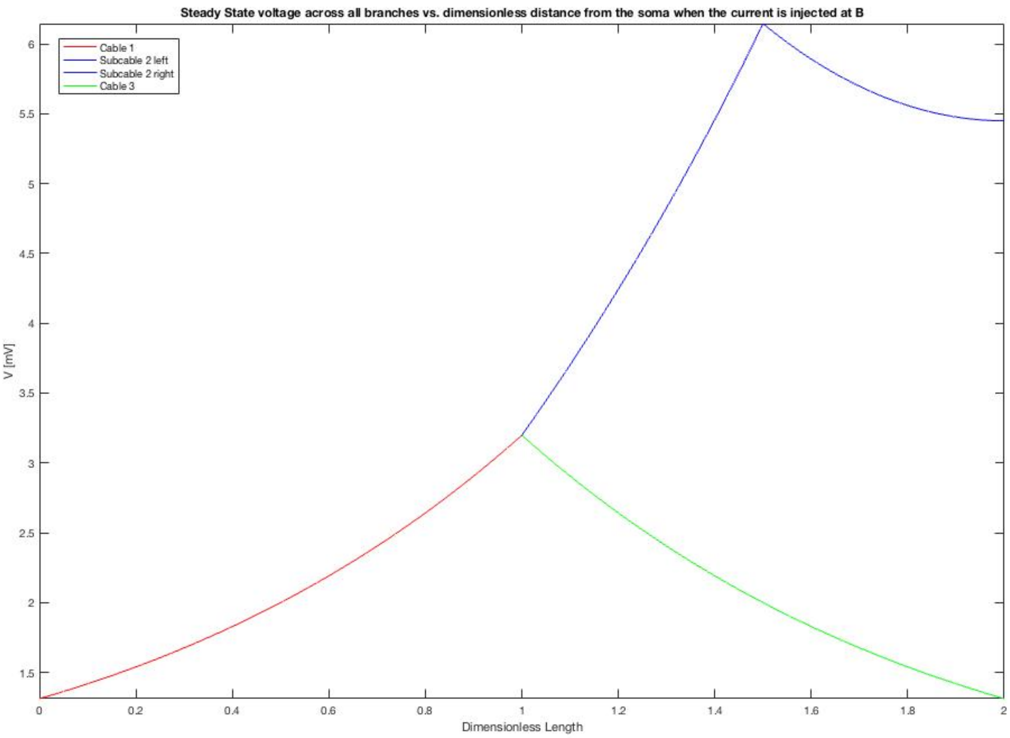
$$\boldsymbol{V_1(X)} = V_1(0)\frac{\cosh(L_1 - X) + \dfrac{G_{in}^1}{G_\infty^1}\sinh\,(L_1 - X)}{\cosh(L_1) + \dfrac{G_{in}^1}{G_\infty^1}\sinh\,(L_1)}$$

$$\boldsymbol{V_3(X)} = V_1(0)\frac{\cosh(L_3 - X) + \dfrac{G_{in}^3}{G_\infty^1}\sinh\,(L_3 - X)}{\cosh(L_3) + \dfrac{G_{in}^3}{G_\infty^3}\sinh\,(L_3)}$$

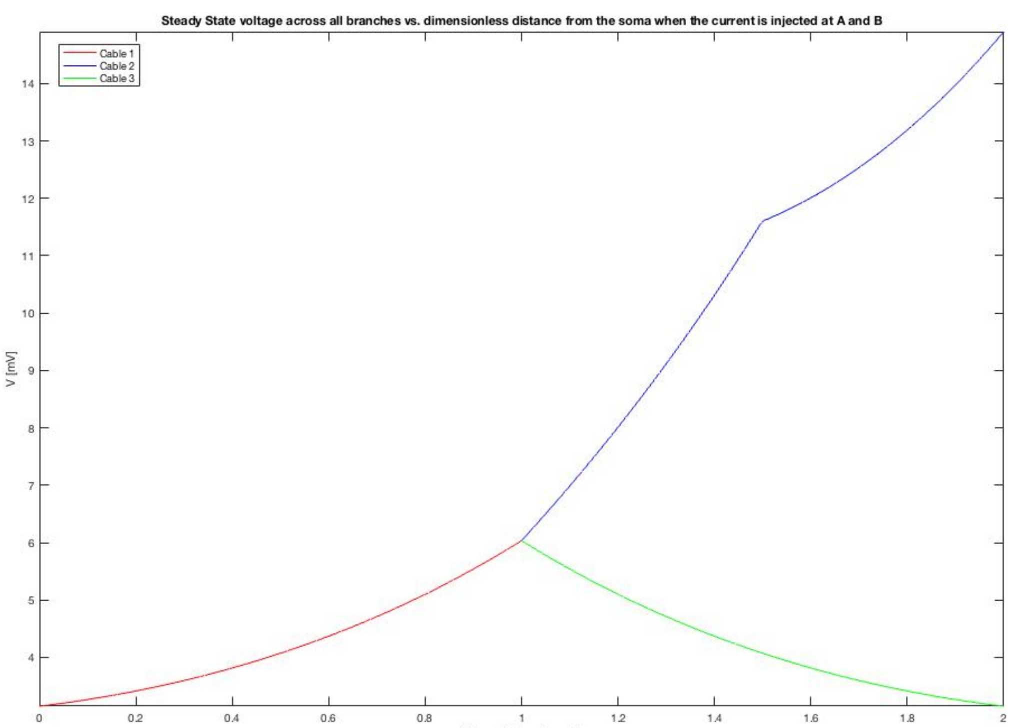
(b). Plot the steady state voltage across all branches as a function of dimensionless distance from the soma. Be sure to label each segment.



(c). Repeat when current is applied at point B.

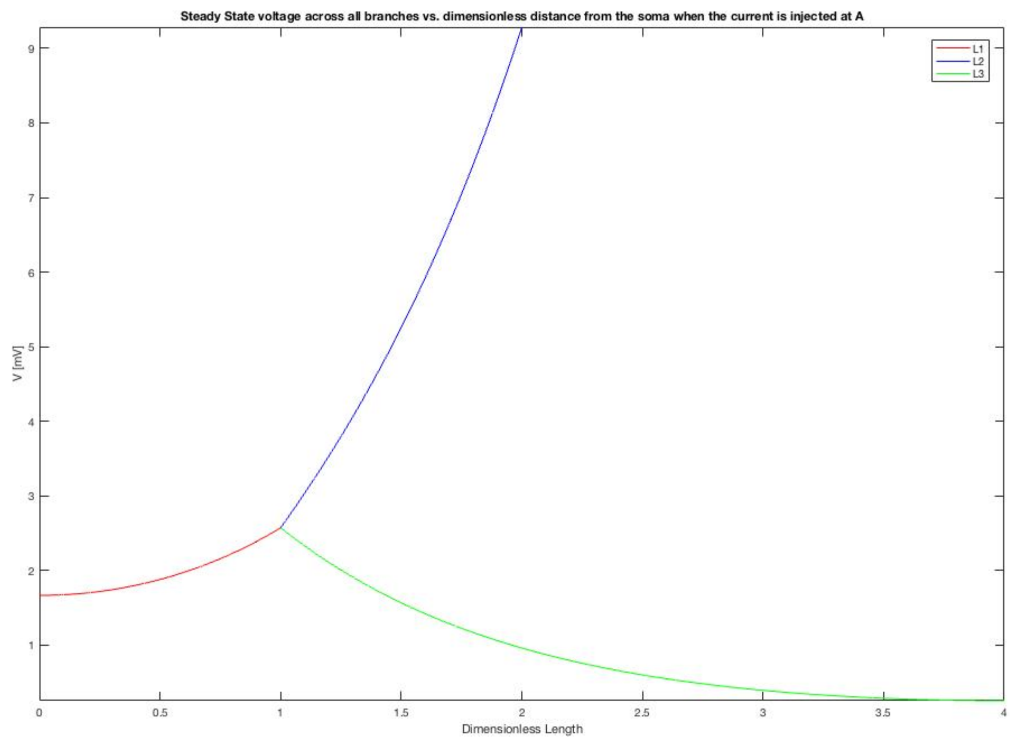


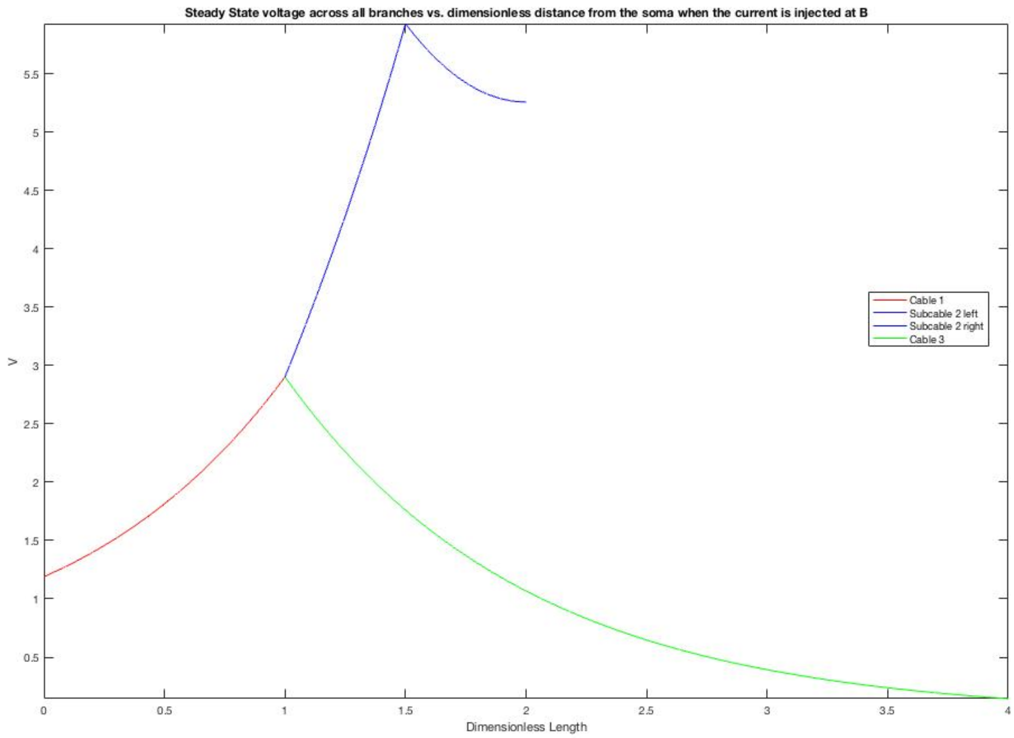
(d). When current is applied at point A and B simultaneously:



This plot 2d is the addition of plot b and c. This make sense because the system is a linear time invariant system so superposition makes sense.

(e). What happen when L_3=3?




Steady State voltage across all branches vs. dimensionless distance from the soma when the current is injected at B
V
Dimensionless Length
Cable 1
Subcable 2 left
Subcable 2 right
Cable 3
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5