Models of the Neuron — Hui Shi

1. (a) • V_m represents the membrane potential and is the difference between the inner membrane potential and the outer membrane potential.

• $G_i = 1/R_i$ represents the channel conductance for ionic specie i or the ease of ion to flow through the channel.

• E_i is the electric potential that drives concentration gradient, specific for ionic specie i.

• C_m is the membrane capacitance or the ability for a membrane to separate charge.

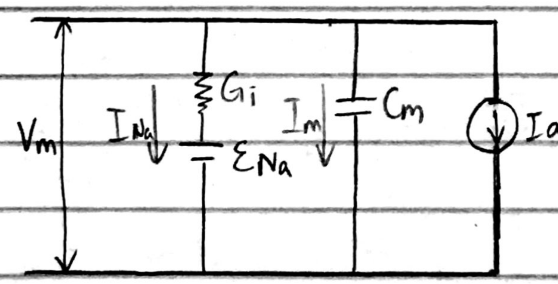
• I_i is the current existed because of the charged ion move into or out of the membrane.

(b). Apply the Nernst equation, we learn that the equilibrium potential is

$$E_i = \frac{RT}{ZF}\ln\frac{[Na^+]_{out}}{[Na^+]_{in}} = \frac{[8.314472J/(k*mol)]*(273.15+20)K}{96485C/mol} * \ln\left(\frac{440*10^{-3}M}{50*10^{-3}M}\right)$$

$$= -0.055V = \boxed{55mV}$$

(C)



Apply KCL: $I_m + I_{app} + I_{Na} = 0$.

where $I_m = C_m * \frac{\delta(V_m - E_{Na})}{\delta t} = C_m * \frac{\delta V_m}{\delta t}$

$I_{Na} = G_{Na} * (V_m - E_{Na})$

Therefore, $G_{Na} * V_m + C_m \frac{\delta V_m}{\delta t} = G_{Na} * E_{Na} - I_{app}$

$$\frac{G_{Na}*V_m}{C_m} + \frac{\delta V_m}{\delta t} = \frac{G_{Na}*E_{Na}}{C_m} - \frac{I_{app}}{C_m}$$

$$\frac{\delta V_m}{\delta t} = \frac{G_{Na}*E_{Na}}{C_m} - \frac{I_{app}}{C_m} - \frac{G_{Na}*V_m}{C_m} = \frac{G_{Na}(E_{Na}-V_m) - I_{app}}{C}$$

$$\frac{C_m}{G_{Na}(E_{Na}-V_m) - I_{app}}\delta V_m = \delta t$$

$$\int_{E_{Na}}^{V_m} \frac{C_m}{G_{Na}(E_{Na}-V_m)\pm I_{app}} \delta V_m = \int_{t_0}^{t} \delta t$$

$$-\frac{C_m}{G_{Na}} \ln(G_{Na}(E_{Na}-V_m)-I_{app})\Big|_{E_{Na}}^{V_m} = t-t_0$$

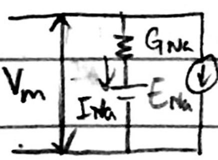
$$\ln(G_{Na}(E_{Na}-V_m)-I_{app})\Big|_{E_{Na}}^{V_m} = -\frac{G_{Na}}{C_m}(t-t_0)$$

$$\ln\left(\frac{G_{Na}(E_{Na}-V_m)-I_{app}}{-I_{app}}\right) = -\frac{G_{Na}}{C_m}(t-t_0)$$

$$\frac{G_{Na}(-E_{Na}+V_m)+I_{app}}{+I_{app}} = e^{-\frac{G_{Na}}{C_m}(t-t_0)}$$

$$V_m = \frac{I_{app}}{G_{Na}}\left(e^{-\frac{G_{Na}}{C_m}(t-t_0)}-1\right)+E_{Na}.$$

d) (i) $C_m=0$, $G_{Na}>0$.

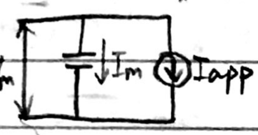


$I_{Na}+I_{app}=0$.

$G_{Na}\times(V_m-E_{Na})+I_{app}=0$.

$V_m = \frac{-I_{app}+E_{Na}\times G_{Na}}{G_{Na}}$.

(ii) $C_m>0$, $G_{Na}=0$



$I_m+I_{app}=0$.

$C_m\frac{dV_m}{dt}+I_{app}=0$

$\frac{\partial V_m}{\partial t} = -\frac{I_{app}}{C_m}$

With initial condition $V_0=E_{Na}$

$V_m = -\frac{I_{app}}{C_m}(t-t_0)+E_{Na}$.

(iii) $C_m>0$, $G_{Na}>0$

The solution would be the same case as 1C.

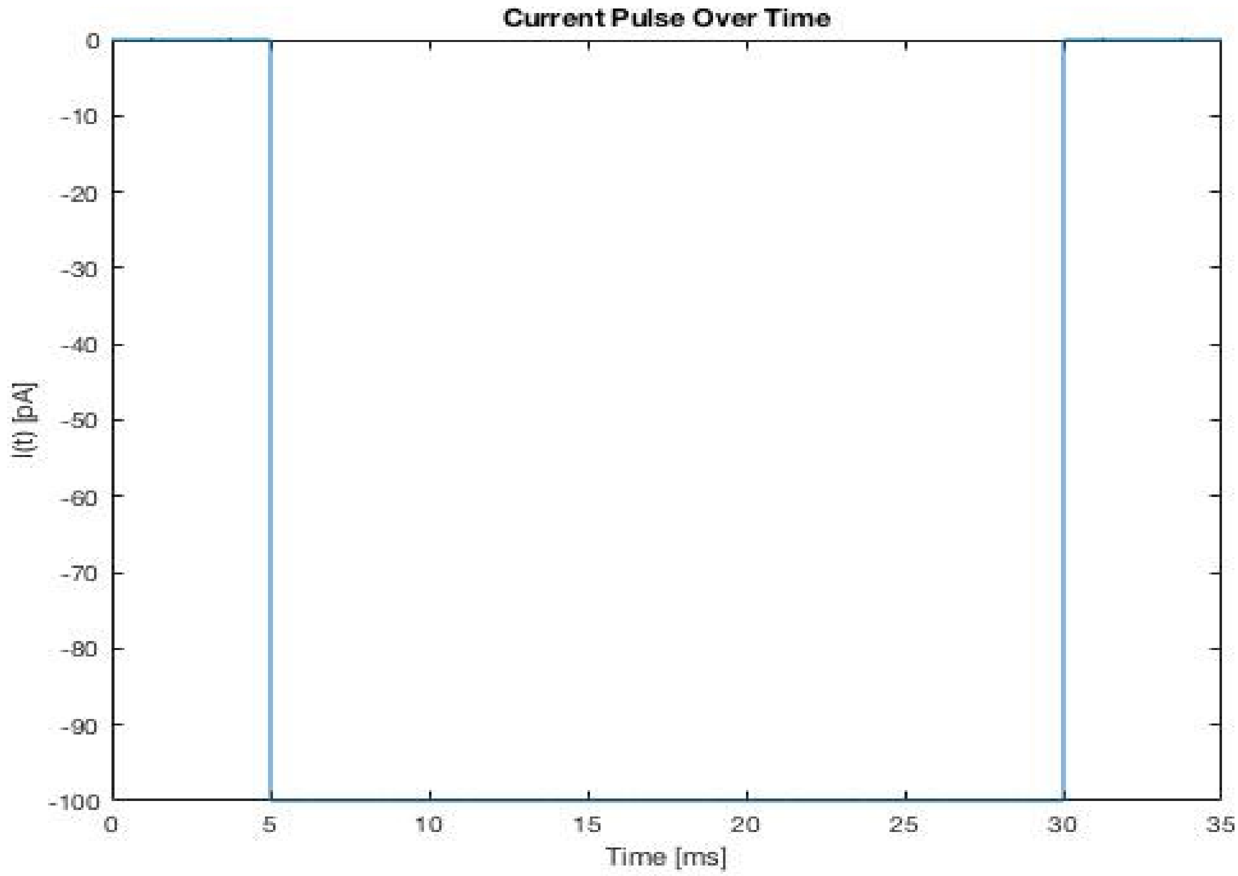
The answer is

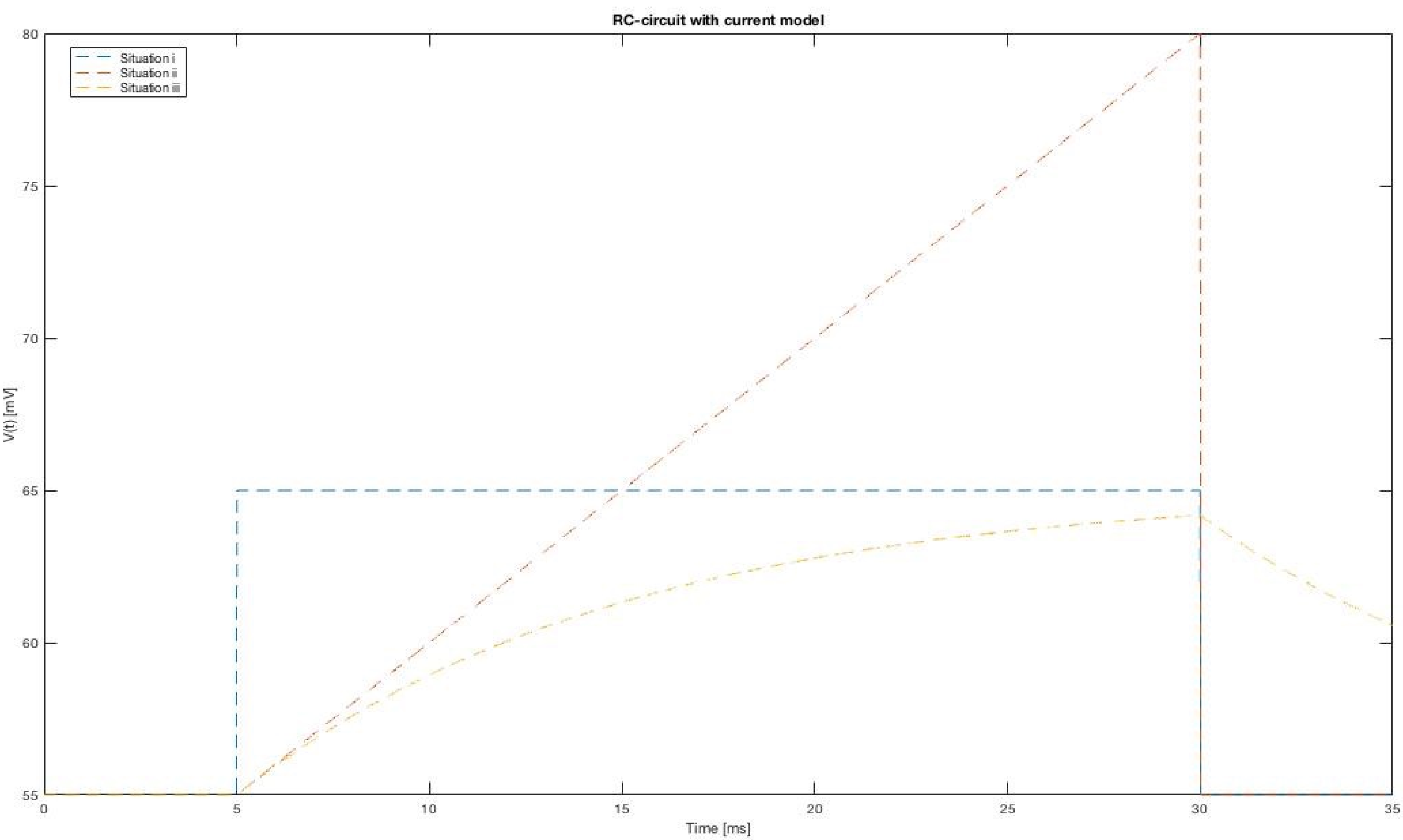
$V_m = \frac{I_{app}}{G_{Na}}\left(e^{-\frac{G_{Na}}{C_m}(t-t_0)}-1\right)+E_{Na}$

(f) ion cross membranes by 1) diffusing through the pore of ion-selective protein channels or 2) by being transported across the membrane ATP dependent pumps and carrier proteins.

By using the occupancy probability to account for the nonlinearity of the model.

1e


Current Pulse Over Time
I(t) [pA]
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
0
5
10
15
20
25
30
35
Time [ms]


RC-circuit with current model
Situation i
Situation ii
Situation iii
V(t) [mV]
80
75
70
65
60
55
0
5
10
15
20
25
30
35
Time [ms]

From the observation of the plot, we inspect that:

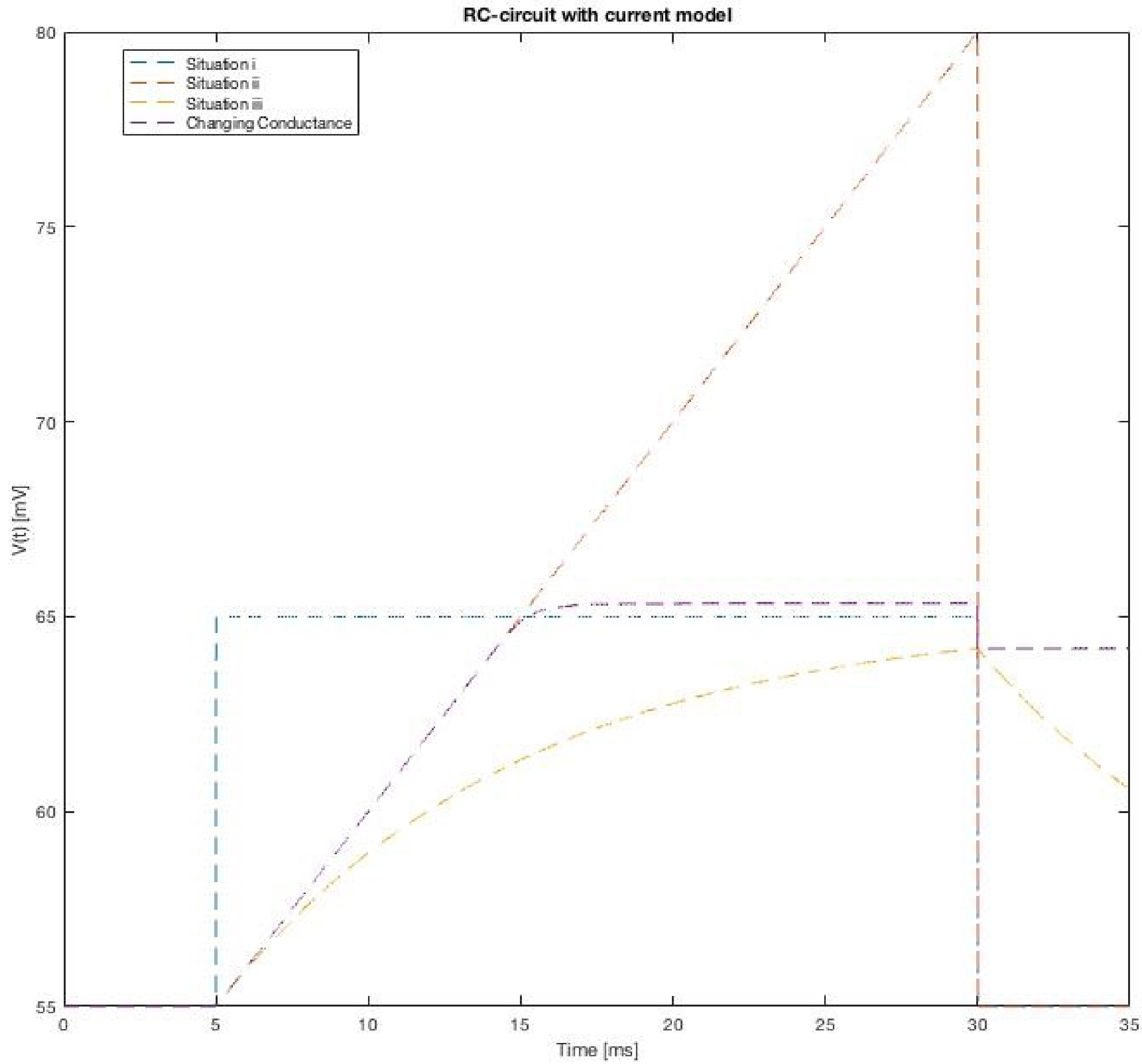
Situation i: Voltage increases like a step function and keeps at a constant value as current applied and the steady state is at 65mV

Situation ii: Voltage increases linearly with the applied current; Since the voltage is keep increasing, there is no steady state in this situation; When applied current is removed, the current immediately goes back to the resting potential;

Situation iii: Voltage increases exponentially and it gets closer to a plateau at 65mV (doesn't actually reach the steady state because the time window is too short)

Additional information about the code:
Name: problem1e.m
A script with the output of the two plots above;

1g



Notice that in the plot above, the purple curve is the nonlinear model

i). The steady state membrane potential is at 65.34 mV;

ii). The reason that the membrane potential remains depolarized even after Iapp is stopped is because of the capacitor in the R-C circuit. The model of the cell has a capacitor component and it can hold the voltage even after no current is applied;

Additional information about the MATLAB file:
Name: problem_1g.m
A script with 2 plots and figure 1 is the plot shown above;