Problem 1

(a). Write the analytical solution of the cable equation based on the assumption on G_{OUT} .

(i). $G_{out} = G_\infty$

$$\frac{V(X)}{V(0)} = \frac{\cosh(L-X)+\sinh(L-X)}{\cosh(L)+\sinh(L)} = e^{-X}$$

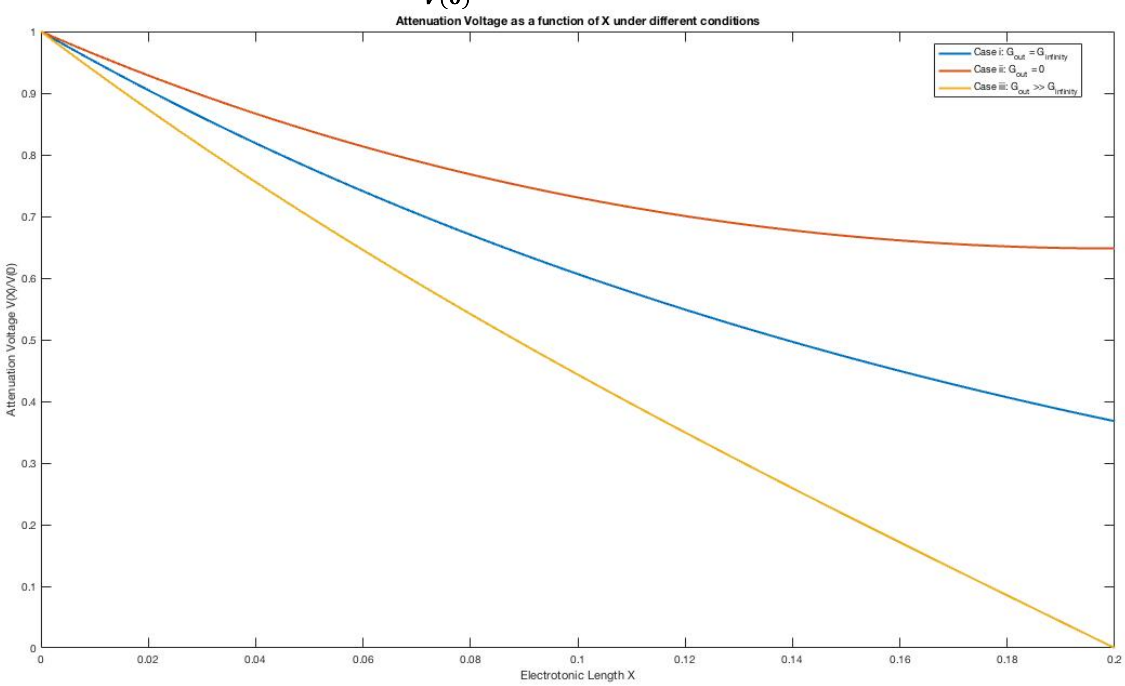
(ii). $G_{out} = 0$

$$\frac{V(X)}{V(0)} = \frac{\cosh(L-X)}{\cosh(L)}$$

(iii). $G_{out} >> G_\infty$

$$\frac{V(X)}{V(0)} = \frac{\sinh(L-X)}{\sinh(L)}$$

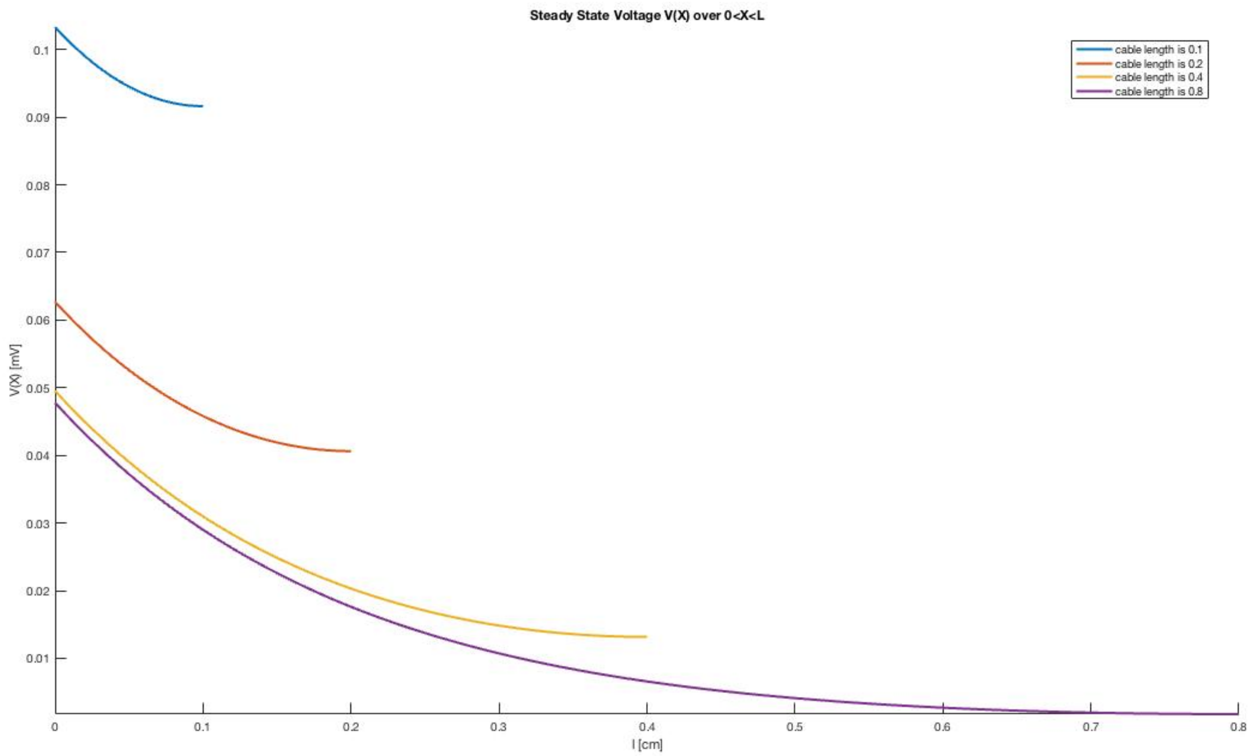
(a). Plot the attenuation voltage $\frac{\mathbf{V(X)}}{\boldsymbol{V(0)}}$ as a function of X over the range 0<X<L.



(c). Describe how voltage attenuation changes for each G_{out}? What condition is this G_{out} simulating?

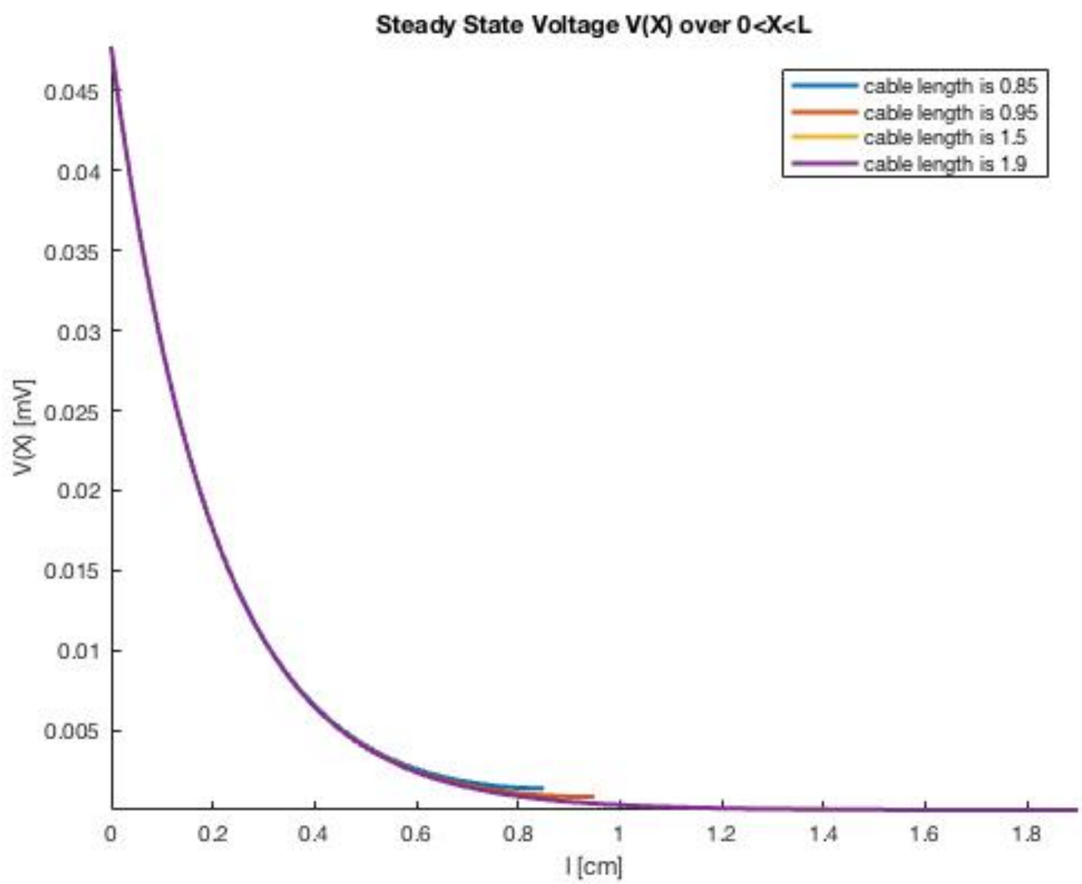
They are all decaying as electrotonic length increases. Among these three, the case iii decays the fastest and the ii case decays the slowest. Case i simulates the case when the conductance is the same as the steady state. Case ii simulates the sealed-end boundary condition. Case iii simulates the end conductance is really huge compared to the intrinsic G_∞.

(d). Now assume a sealed-end boundary condition for this cable. The current I(0) is still being applied at X=0.



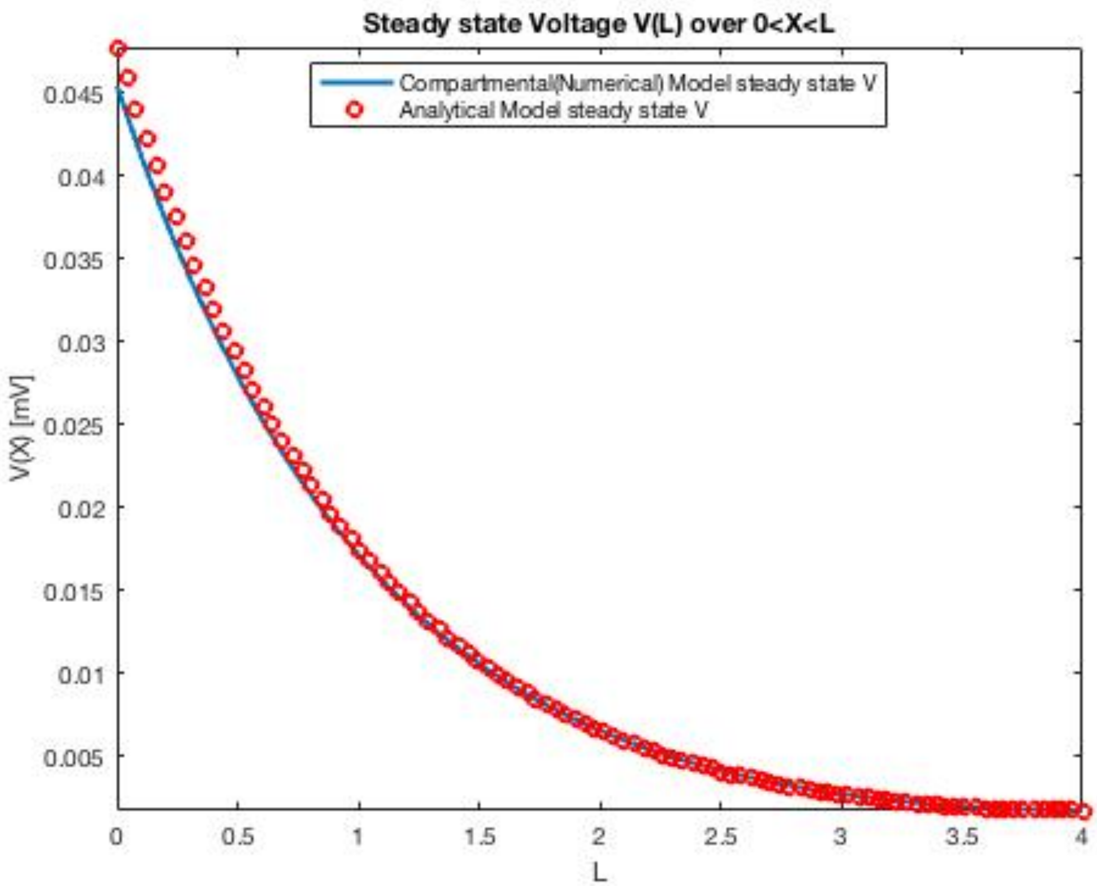
The shorter cable has higher voltage and the longer cable has smaller voltage value. The longest cable would converge to zero in the end. This is because the conductance of a shorter cable is smaller compared to a longer cable, therefore according to Ohm's law, the cable carry less conductance (shorter cable) would result in a bigger voltage while the longer one with more conductance becomes smaller. The conductance is a function of distance, so the longer the cable, the bigger the conductance at the further distance and the less voltage would be carry at the further site.

(e).

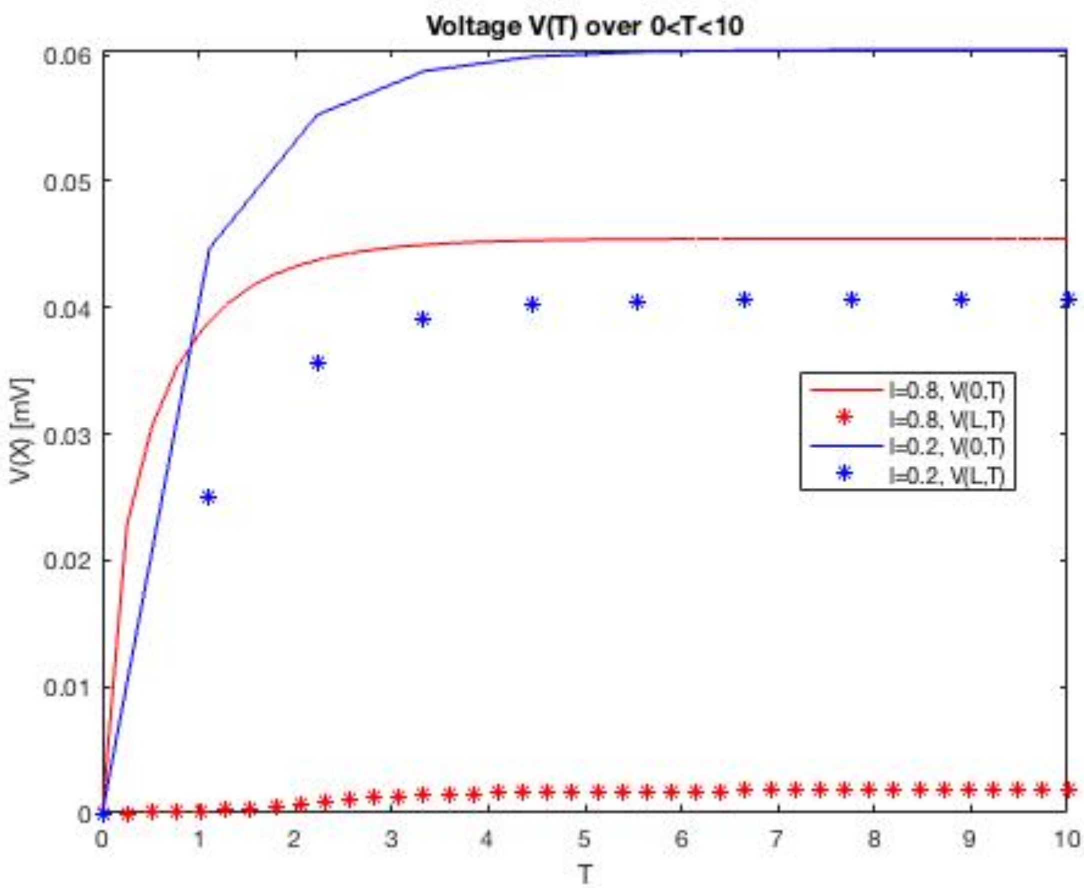


V(X) is going to be a fixed curve as the cable length increases to a certain length because from the plot we observe that as length increases, the curves overlap more and more closely. The longer cable is going to decay at zero at the end point. The length of the curve continues on for the longer cable.

(f). Since the minimum compartment has to fulfill the constrain that $\Delta L = \frac{l}{N} = 0.1\lambda$, the minimum N is 40. Therefore, we cut the cable into 40 smaller pieces. The $\Delta x = 0.02cm$ or 0.1 (dimensionless) is the minimum for the solution to agree. From our comparison of the numerical solution and the analytical solution, they fit closely together.



(g). For the $l = 0.2cm$ cable, the minimum compartments is 10 according to the l/N =0.1*lambda rule. So we only cut the cable into 10 pieces.



For the two solutions, we observe that when the cable is longer, the steady state voltage at the injection site (L=0) is much smaller than the shorter cable. The site which is further away from current injection is also much smaller than the shorter cable and the longer the cable is, the closer to zero the steady state voltage. This is because of the conductance of the cable will gradually reduce the voltage. We also observe that the difference between X=L and X=0 is bigger for longer cable than the shorter cable. The reasoning behind this phenomenon is also because the longer length means the cable has more conductance to reduce the voltage.