Ahmad Malik
ECE302
Project 5

$$s[n] \longrightarrow \boxed{c[n]} \longrightarrow \underset{\uparrow \atop d[n]}{\oplus} \xrightarrow{r[n]} \boxed{h[n]} \longrightarrow \hat{s}[n]$$

Part I:

Cross-Correlation:

$$R_{sr}[n] = E\big[s[m]\,r[m-n]\big]$$

$$= E\Big[s[m]\Big\{\sum_{k=0}^{2} c[k]\,s[m-n-k] + d[m-n]\Big\}\Big]$$

$$= E\Big[s[m]\Big\{c[0]\,s[m-n] + c[1]\,s[m-n-1] + c[2]\,s[m-n-2] + d[m-n]\Big\}\Big]$$

$$= E\Big[s[m]\,s[m-n] + 0.2\,s[m]\,s[m-n-1] + 0.4\,s[m]\,s[m-n-2] + s[m]\,d[m-n]\Big]$$

$$= R_{ss}[n] + 0.2\,R_{ss}[n+1] + 0.4\,R_{ss}[n+2] + \underbrace{R_{sd}[n]}_{=0}$$

signal/noise are uncorrelated

$$\boxed{R_{sr}[n] = R_{ss}[n] + 0.2\,R_{ss}[n+1] + 0.4\,R_{ss}[n+2]}$$

Auto - Correlation:

$$R_{rr}[n] = E[r[m]r[m-n]]$$

$$= E\left[\left\{\sum_{k=0}^{2} C[k]S[m-k] + d[m]\right\} * \left\{\sum_{k=0}^{2} C[k]S[m-n-k] + d[m-n]\right\}\right]$$

$+d[m]$

$$= E\Big[\big(C[0]S[m] + C[1]S[m-1] + C[2]S[m-2]\big) *$$

$+d[m-n]$

$$\big(C[0]S[m-n] + C[1]S[m-n-1] + C[2]S[m-n-2]\big)\Big]$$

$$= E\Big[\big(S[m] + 0.2S[m-1] + 0.4S[m-2] + d[m]\big) * \underbrace{\big(S[m-n] + 0.2S[m-n-1] + 0.4S[m-n-2]} _{\text{multiply this out}} + d[m-n]\big)\Big]$$

[illegible]

$$= R_{ss}[n] + 0.2R_{ss}[n+1] + 0.4R_{ss}[n+2] + R_{sd}[n] +$$

$+$ [illegible]

$$0.4R_{ss}[n] + 0.2R_{ss}[n-1] + 0.08R_{ss}[n+1] + 0.2R_{sd}[n-1]$$

$$+ 0.4R_{ss}[n-2] + 0.08R_{ss}[n-1] + 0.4R_{sd}[n-2] + 0.16R_{ss}[n]$$

$$+ R_{ds}[n] + 0.2R_{ds}[n+1] + 0.4R_{ds}[n+2] + R_{dd}[n]$$

$$\boxed{R_{rr}[n] = 1.2R_{ss}[n] + 0.28R_{ss}[n+1] + 0.4R_{ss}[n+2] + 0.28R_{ss}[n-1] + 0.4R_{ss}[n-2] + R_{dd}[n]}$$

Normal Equations

For $N=4$:

$$\begin{bmatrix} R_{rr}[0] & R_{rr}[-1] & R[-2] & R_{rr}[-3] \\ R_{rr}[1] & R_{rr}[0] & R[-1] & R_{rr}[-2] \\ R_{rr}[2] & R_{rr}[1] & R[0] & R_{rr}[-1] \\ R_{rr}[3] & R_{rr}[2] & R[1] & R_{rr}[0] \end{bmatrix} \begin{bmatrix} h[0] \\ h[1] \\ h[2] \\ h[3] \end{bmatrix} = \begin{bmatrix} R_{sr}[0] \\ R_{sr}[1] \\ R_{sr}[2] \\ R_{sr}[3] \end{bmatrix}$$

$$\begin{bmatrix} 2.2 & 0.28 & 0.4 & 0 \\ 0.28 & 2.2 & 0.28 & 0.4 \\ 0.4 & 0.28 & 2.2 & 0.28 \\ 0 & 0.4 & 0.28 & 2.2 \end{bmatrix} \begin{bmatrix} h[0] \\ h[1] \\ h[2] \\ h[3] \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$h = \begin{bmatrix} 0.48 \\ -0.054 \\ -0.082 \\ 0.02 \end{bmatrix}$$

```
%Ahmad Malik
%ECE302-1
%Project 5

clc; clear; close all;


%{
In this project, an iid signal s[n] is sent through a filter c[n]
 which is
then combined with zero mean gaussian noise d[n]. Thus we recieve a
 signal
r[n]. To recover signal s[n], we pass r[n] through a wiener filter
 h[n] to
get s^[n] which is an estimate of the original signal s[n]. To find
 the
best wiener filter given it's length, we must compute the normal
 equations
that require finding the cross and auto correlations of the sent and
recieved signal. We can measure how well the filter performs by
 computing
the MSE.
%}
```

Part 2

```
C = [1, 0.2, 0.4]; % C[n]
N = [4, 6, 10]; %Length of filter
variance = .5; %variance
sigma = sqrt(variance); %covariance
mu = 0;

%discrete random signal +/- 1
s = randi(2, [1,1e6]);
s = -1*double(s==2) + double(s==1);

%output of first filter:  r = filter{s} + d
r = filter(C, 1, s) + normrnd(mu, sigma , 1, 1e6);

%MSE for N = 4,6,10
MSE = zeros(1,3);
MSE(1) = wienerMSE(s,r,N(1));
MSE(2) = wienerMSE(s,r,N(2));
MSE(3) = wienerMSE(s,r,N(3));

T = table([MSE(1); MSE(2); MSE(3)], 'RowNames',
 {'N=4', 'N=6', 'N=10'});
T.Properties.VariableNames = ("MSE")

%MSE function
function MSE = wienerMSE(s,r,N)
    Rsr = zeros(N, 1);
```

```
    Rrr = zeros(N, 1);
    for i = 1:N
        %cross correlation
        Rsr(i) = mean(s(i:end) .* r(1:end + 1 - i));
        %auto correlation
        Rrr(i) = mean(r(i:end) .* r(1:end + 1 - i));
    end
    %generating matrix for normal equations
    Rrr_Matrix = toeplitz(Rrr);

    %solve for h
    h = inv(Rrr_Matrix)* Rsr;

    %computing s^[n]
    s_hat = filter(h , 1, r);

    %computing MSE
    MSE = mean((s - s_hat) .^ 2);
end
```

```
T =

  3×1 table

               MSE
             _______

    N=4      0.36296
    N=6      0.36131
    N=10     0.36123
```

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