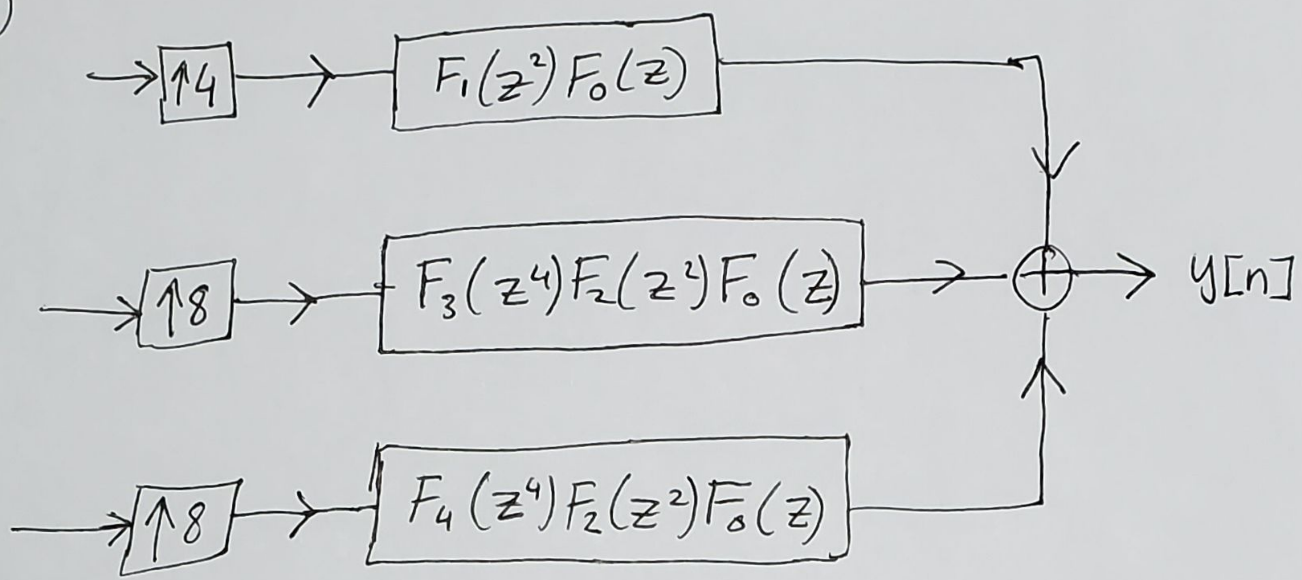
Nicholas Singh and Ahmad Malik ECE310 HW #3

1)



2)

a)

$$\begin{bmatrix} H_0(z) \\ H_1(z) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} E_0(z^2) & 0 \\ 0 & E_1(z^2) \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} E_0(z^2) \\ E_1(z^2) z^{-1} \end{bmatrix} = \begin{bmatrix} E_0(z^2) + E_1(z^2) z^{-1} \\ E_0(z^2) - E_1(z^2) z^{-1} \end{bmatrix}$$

so,

$$\boxed{\begin{aligned} H_0(z) &= E_0(z^2) + E_1(z^2) z^{-1} \\ H_1(z) &= E_0(z^2) - E_1(z^2) z^{-1} \end{aligned}}$$

$$\begin{bmatrix} F_0(z) \\ F_1(z) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} E_1(z^2) & 0 \\ 0 & E_0(z^2) \end{bmatrix} \begin{bmatrix} z^{-1} \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} E_1(z^2) z^{-1} \\ E_0(z^2) \end{bmatrix} = \begin{bmatrix} E_1(z^2) z^{-1} + E_0(z^2) \\ E_1(z^2) z^{-1} - E_0(z^2) \end{bmatrix}$$

so,

$$\boxed{\begin{aligned} F_0(z) &= E_1(z^2) z^{-1} + E_0(z^2) \\ F_1(z) &= E_1(z^2) z^{-1} - E_0(z^2) \end{aligned}}$$

b)

verify $H_1(z) = H_0(-z)$,

$$H_0(-z) = E_0((-z)^2) + E_1((-z)^2)(-z)^{-1}$$

$$H_0(-z) = E_0(z^2) - E_1(z^2)(z)^{-1}$$

$$H_0(-z) = H_1(z) \checkmark$$

Verify $H_0(z) = F_0(z)$,

$$F_0(z) = E_1(z^2) z^{-1} + E_0(z^2)$$

$$H_0(z) = E_0(z^2) + E_1(z^2) z^{-1}$$

$$H_0(z) = F_0(z) \checkmark$$

Verify $F_1(z) = -H_1(z)$,

$$-H_1(z) = -1\left(E_0(z^2) - E_1(z^2) z^{-1} \right)$$

$$= E_1(z^2) z^{-1} - E_0(z^2)$$

$$= F_1(z)$$

$$F_1(z) = -H_1(z) \checkmark$$

c)

If $A(z)=0$, the system is alias-free.

$$A(z) = \left(F_0(z)H_0(-z) + F_1(z)H_1(-z)\right)\frac{1}{2}$$

$$= \frac{1}{2}\Big[\left(E_1(z^2)z^{-1} + E_0(z^2)\right)\left(E_0((-z)^2) + E_1((-z)^2)(-z)^{-1}\right)$$
$$+\left(E_1(z^2)z^{-1} - E_0(z^2)\right)\left(E_0((-z)^2) - E_1((-z)^2)(-z)^{-1}\right)\Big]$$

$$= \frac{1}{2}\Big[\left(E_1(z^2)z^{-1} + E_0(z^2)\right)\left(E_0(z^2) - E_1(z^2)(z^{-1})\right)$$
$$+ \left(E_1(z^2)z^{-1} - E_0(z^2)\right)\left(E_0(z^2) + E_1(z^2)(z^{-1})\right)\Big]$$

$= \frac{1}{2}[0] = 0$, thus the system is Alia-free.

$$Y(z) = T(z)X(z)$$

$$= \frac{1}{2}\left[F_0(z)H_0(z) + F_1(z)H_1(z)\right]$$

$$= \frac{1}{2}\left[(H_0(z))^2 + -H_1(z)(H_1(z))\right]$$

$$= \frac{1}{2}\left[(H_0(z))^2 - (H_1(z))^2\right]$$

$$= \frac{1}{2}\left[(H_0(z))^2 - (H_0(-z))^2\right]$$

$$\boxed{T(x) = \frac{1}{2}\left[H_0(z)^2 - H_0(-z)^2\right]}$$

d)

$$= \begin{pmatrix} E_1(z) & 0 \\ 0 & E_0(z) \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} E_0(z) & 0 \\ 0 & E_1(z) \end{pmatrix}$$

$$= \begin{pmatrix} E_1(z) & 0 \\ 0 & E_0(z) \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} E_0(z) & 0 \\ 0 & E_1(z) \end{pmatrix}$$

$$= \begin{pmatrix} E_1(z) & 0 \\ 0 & E_0(z) \end{pmatrix} \begin{pmatrix} 2E_0(z) & 0 \\ 0 & 2E_1(z) \end{pmatrix}$$

$$= \begin{bmatrix} 2E_0(z)E_1(z) & 0 \\ 0 & 2E_1(z)E_0(z) \end{bmatrix}$$

d)

$$T(x) = \begin{bmatrix} z^{-1} & 1 \end{bmatrix} \begin{bmatrix} E_1(z^2) & 0 \\ 0 & E_0(z^2) \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} E_0(z^2) & 0 \\ 0 & E_1(z^2) \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \end{bmatrix}$$

$$T(x) = \begin{bmatrix} z^{-1}E_1(z^2) \cdot E_0(z^2) \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} E_0(z^2) \\ z^{-1}E_1(z^2) \end{bmatrix}$$

$$T(x) = \begin{bmatrix} z^{-1}E_1(z^2) + E_0(z^2) & z^{-1}E_1(z^2) - E_0(z^2) \end{bmatrix} \begin{bmatrix} E_0(z^2) + z^{-1}E_1(z^2) \\ E_0(z^2) - z^{-1}E_1(z^2) \end{bmatrix}$$

$$T(x) = \begin{bmatrix} F_0[z] & F_1[z] \end{bmatrix} \begin{bmatrix} H_0(z) \\ H_1(z) \end{bmatrix}$$

$$T(x) = \tfrac{1}{2}\left[F_0(z) H_0(z) + F_1(z) H_1(z)\right]$$

3) $$\tilde{H}(z) = z^{(L-1)} \sum_{n=0}^{L-1} h[n] z^{n} z^{-(L-1)}$$

$$= z^{(L-1)} \sum_{n=0}^{L-1} h[n] z^{n-(L-1)}$$

$$= z^{(L-1)} H'(z)$$

$$\tilde{H}(z) = z^{(L-1)} H'(z)$$

4)

b)

1. $H_1(z) = H_0(-z) z^{-1}$

$F_0(z) = \tilde{H}_0(z) z^{-1}$

$F_1(z) = H_0(-z)$