4^{th} Order Pohlhausen Method on Boundary Layer Approximation Analysis of a NACA 2410 Airfoil

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Abstract

The goal of the present research is to approximate the boundary layer of a NACA 2410 airfoil at several angle of attacks at Reynolds numbers of 700.000. This boundary layer is investigated numerically and compared with the results from JavaFoil. First, the boundary layer equations were derived into boundary layer integral equations. Then, we calculate the flow velocity around airfoil by using vortex panel method, an inviscid solver. To solve the boundary layer, next we use the 4^{th} order Pohlhausen parameters to guess the velocity profile thus solve the boundary layer equations. At last, we compare the aerodynamic results such as lift, drag, and pressure coefficient between our code and JavaFoil results.

Keywords: Pohlhausen Approximation, Laminar, Boundary Layer Approximation, Viscous-Inviscid Interaction

1. Introduction

When a viscous fluid flows along a fixed impermeable wall, an essential condition is that the velocity at any point on the wall or other fixed surface is zero. The extent to which this condition modifies the general character of flow depends upon the value of the viscosity. If the body is streamlined shaped and if the viscosity is small without being negligible, the modifying effect appears to be confined within narrow regions adjacent to the solid surfaces, these are called boundary layers.

In this research, we want to approximate boundary layer on NACA 2410 airfoil at Reynolds number of 700.000. We develop codes that coupled inviscid and viscous solver to calculate the boundary layer, separation/transition points, and other aerodynamic aspects (such as lift, drag, and pressure coefficients). We use the vortex panel method as the inviscid solver and use the 4^{th} order Pohlhausen method to guess velocity profile to solve the boundary layer.

2. Basic Theories

Inviscid Solver – Vortex Panel Method

The inviscid solver of the calculation is based on the vortex panel method where it is one of the variations of the panel method that utilize the concept of distributing vortex along panels represented as vortex-panels. The vortex panels modelled in an airfoil problem varies linearly by strength and is continuous across the corner as could be illustrated in the figure shown below.

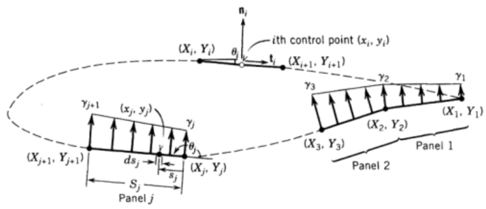

Figure 1 Airfoil Vortex Panel

The panels along the airfoil would be planar and defined consecutively in the clockwise direction starting from the trailing edge. At the mid-point of each panels, there are control points where it satisfies the zero normal velocity component condition due to the streamline condition of the airfoil. Based on the velocity potential theory, for a uniform flow of velocity V_∞ and at an angle of attack α with m number of vortex panels defined towards the airfoil, the velocity potential ϕ at the i^{th} control point with coordinate (x_i, y_i) is defined by the equation

$$\phi(x_i, y_i) = \phi_{\infty,i} + \phi_{\gamma,i} \tag{1}$$

$$\phi_{\infty,i} = V_\infty(x_i \cos\alpha_i + y_i \sin\alpha_i) \tag{2}$$

$$\phi_{\gamma,i} = -\sum_{j=1}^{m} \int_j \frac{\gamma(s_j)}{2\pi} \tan^{-1}\left(\frac{y_i - y_j}{x_i - x_j}\right) ds_j \tag{3}$$

$$\gamma(s_j) = \gamma_j + (\gamma_{j+1} - \gamma_j)\frac{s_j}{S_j} \tag{4}$$

as γ represents the vortex strength of the j^{th} panel of length S_j, with a distance s_j from the leading edge of the panel. As the boundary condition requires that the velocity in the direction of the unit outward normal vector $\vec{n}_i$ vanish at the i^{th} control point

$$\frac{\partial}{\partial n_i}\phi(x_i, y_i) = 0 \tag{5}$$

Then, after performing differentiation and integration, the following equation is obtained.

$$\sum_{j=1}^{m}\left(c_{n1_{ij}}\gamma'_j + c_{n2_{ij}}\gamma'_{j+1}\right) = \sin(\theta_i - \alpha) \tag{6}$$

where γ' is a dimensionless circulation density while θ_i is the orientation angle of the i^{th} panel. Then, after determining the unknown circulation densities, the velocity and pressure at control points V_i and C_{pi} respectively could be calculated by the equations as below.

$$V_i = \cos(\theta_i - \alpha) + \sum_{j=1}^{m}\left(A_{t_{ij}}\gamma'_j\right) \tag{7}$$

$$C_{pi} = 1 - {V_i}^2 \tag{8}$$

The total lift (L) of the airfoil could be calculated by the Kutta-Joukowski theorem.

$$L = \rho_\infty U \sum_{j=1}^{n}\gamma_j S_j \tag{9}$$

where ρ_∞ is the free stream air density, U is the freestream velocity and S_j as defined before is the length of the j^{th} panel.[2]

<u>Viscous Solver – One-Parameter Integral Methods: 4th Order Pohlhausen</u>

In solving the boundary layer and shear stress at the airfoil's surface, the Karman-Pohlhausen is one of the methods where the basic equation for this method is obtained by integrating the x direction momentum equation with respect to y from the wall at $y = 0$ to a distance $\delta(x)$ which is assumed to be outside the boundary layer. By assuming the velocity profile as a 4$^{\text{th}}$ order polynomial represented in the form of the similarity variable η the velocity profile polynomial is as shown below. [4]

$$\frac{u}{U} = 2\eta - 2\eta^3 + \eta^4 + \frac{\lambda}{6}[\eta(1-\eta)^3] \tag{10}$$

$$\eta = \frac{y}{\delta} \tag{11}$$

$$\lambda = \frac{\delta^2}{\nu}\frac{dU}{dx} \tag{12}$$

Then, the polynomial could be simplified by defining several dimensionless parameters in terms of the momentum thickness (θ) and shape factor (λ) by the equation as shown below.

$$L = \frac{\tau_w \theta}{\mu U} = \frac{\theta}{\delta}\left(2 + \frac{\lambda}{6}\right) \tag{13}$$

$$K = \frac{\theta^2}{\nu}\frac{dU}{dx} = \left(\frac{\theta}{\delta}\right)^2 \lambda \tag{14}$$

$$H = \frac{\delta^*}{\theta} \tag{15}$$

Then, by using the Walz's approximation the function could be approximated with a good degree of accuracy by a linear function of K that yields the equation for the momentum thickness as shown below.

$$\theta^2 = \frac{0.47\,\nu}{U^6}\int_0^x U^5\,dx \tag{16}$$

From the equations above, the boundary layer thickness and the shear stress at the surface of the airfoil could be obtained and the skin friction coefficient (C_f) could be calculated by the equation as shown below.

$$C_f = \frac{\tau_w}{0.5\rho_\infty U^2} \tag{17}$$

3. Workflow – Viscous-Inviscid Interaction

Initially, we start it by performing inviscid analysis. The inviscid flow appears around the airfoil by solving the vortex panel method. The velocities are located on each panel. Next, we perform the viscous analysis. Due to viscosity, the flow is having a deficit in its velocity and make the location of inviscid flow will be deflected outward as high as the boundary layer thickness (in term of displacement thickness). We can see this as if the airfoil changing its shape. As consequence, the inviscid flow would give new panel velocities values. We can iterate this process until the changing of boundary layer thickness can be neglected. To sum up, the coupling strategy are:

1. The potential flow around airfoil is calculated with boundary condition $u_n = 0$.
2. Then, the boundary layer is calculated. We need the displacement thickness distribution $\delta^*(x)$.
3. Next, the potential flow (from step 1) is recalculated with a new boundary condition $u_n = \frac{d}{dx}(u_e\delta^*)$.
4. Iterate step 2-3 until the displacement thickness is convergence.

We set $u_e = 0.03$ as proposed by Yousefi [3].

4. Results and Analysis

<u>Test Case</u>

In this present research, we approximate the boundary layer of a NACA 2410 airfoil. The airfoil coordinates were obtained by using JavaFoil. We performed analysis at Reynolds number 700.000 and angle of attacks from -4 to 10 degrees.

Iteration Convergence

In this research, the change of displacement thickness (δ^*) is being evaluated for every iteration. We stop the iteration when the error is below than 1.0x10^{-5}. Figure below is one of the error histories when calculating the boundary layer at Re 700.000 and 0.0° AoA. The error at the end of calculation is 3.05x10^{-7}.

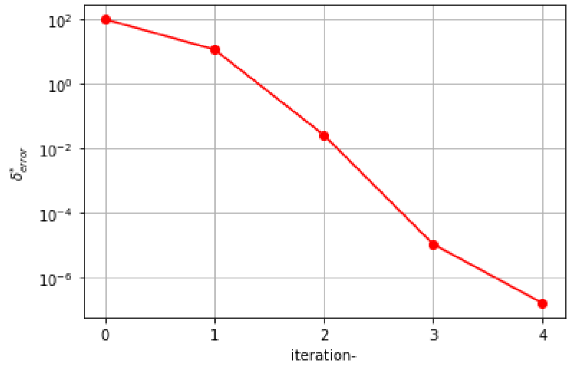

Figure 2 Convergence of Sum of Displacement Thickness

Boundary Layer Thickness

Figure 3 shows the generated boundary layer at AoA 0°. The boundary layer is successfully generated and bounds the airfoil. The maximum boundary layer thickness is around 0.005 m which located at the trailing edge of the airfoil.

Further, we put two signs which represent the transition points at the upper and lower part of airfoil. From leading edge to this sign, the boundary layer is on laminar condition. Continuing to upstream, its flow condition is changing from laminar to turbulent. Our solver is limited only for laminar condition, thus there might be an error of boundary layer thickness.

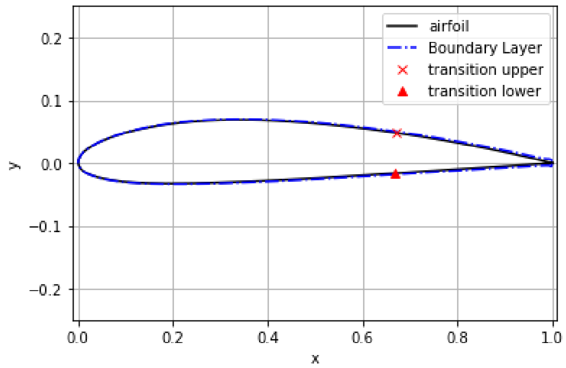

Figure 3 Boundary Layer Thickness at AoA 0°

Figure 4 and Figure 5 shows the boundary layer at specific location, at the leading edge and the trailing edge, respectively. As we discussed on Section 2, the boundary layer thickness is set to 0 at the stagnation point. The stagnation point is located on the leading edge for AoA 0°. Along the downstream, the boundary layer thickness is developed in both upper and lower part of the airfoil. Our boundary layer develops smoothly until the second last panel. We face a continuity issues at the last panel thus create a non-smooth development of boundary layer thickness.

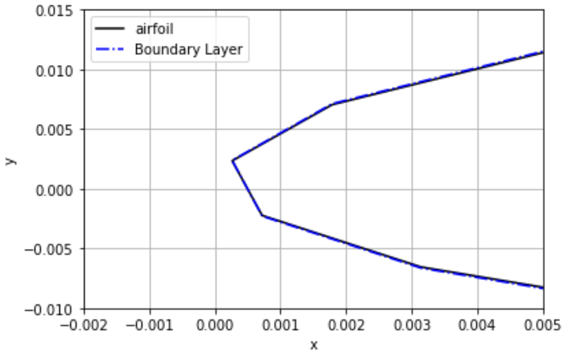

Figure 4 LE Boundary Layer Thickness at AoA 0°

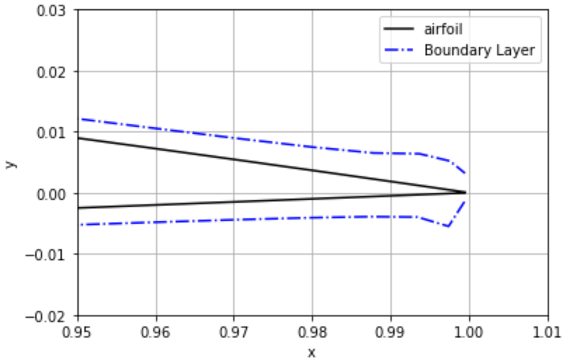

Figure 5 TE Boundary Layer Thickness at AoA 0°

Aerodynamic Coefficients

From the vortex panel method, we can calculate the lift coefficient from the obtained pressure coefficient. Next, to calculate the drag coefficient, we use the skin-friction coefficient calculated from the viscous solver.

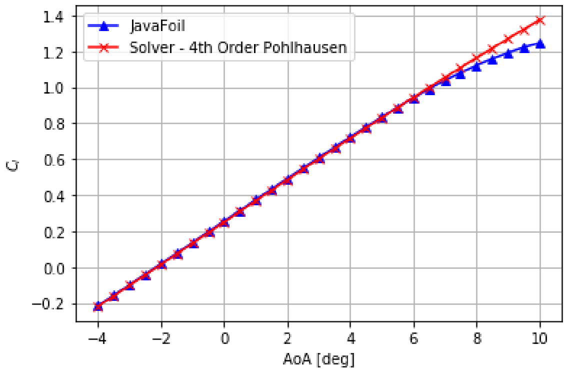

Figure 6 C_l-α Curve Comparison

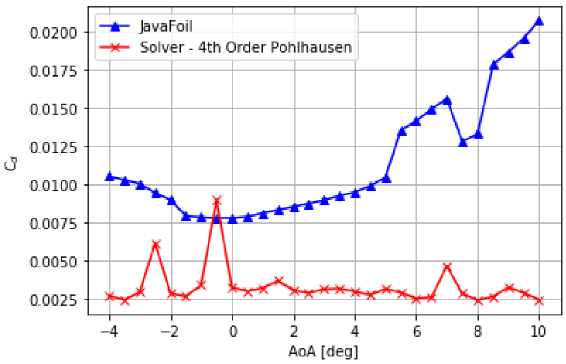

Figure 7 C_d-α Curve Comparison

Figure 6 and Figure 7 show the comparison of lift and drag coefficient from angle of attack -4° to 10°. From Figure 6, the lift coefficient lies on the same point at low angle of attacks. On the other hand, it differs at high angle of attacks. This phenomenon happens because of flow instability which is led to turbulent flow. Our vortex panel method solver does not evaluate this phenomenon.

Moreover, Figure 7 shows a big difference between our solver and JavaFoil in terms of drag coefficients. Our solver only considers skin-friction coefficient to calculate drag coefficient. On the other hand, JavaFoil drag coefficients are composed from skin-friction dan pressure drag. The pressure drag is a function of angle of attack which is make the drag increase along the angle of attack increment.

Figure 8 Pressure Coefficient Comparison at AoA 0°

Figure 8 shows the pressure coefficient comparison between JavaFoil and our solver. Excluding the trailing edge panel, the pressure coefficient distribution lies on the same points. This condition is the reason why the lift coefficient is similar between the JavaFoil and our solver.

Solver Code

The code is written with Python language on a Jupyter Notebook. The code is open for public use. Please visit the GitHub link below.

>> Go to the code <<

5. Conclusions and Recommendations

Based on the obtained results, it can be concluded that:

1. The viscous-inviscid interaction program can perform well to evaluate aerodynamic coefficient of airfoil.
2. The lift coefficient is similar between JavaFoil and our solver, except at high angle of attack. For low angle of attack.
3. The drag coefficient gives high error due to different way of calculating drag between JavaFoil and our solver.

For the future research, we recommend adding the pressure drag calculation.

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