Module - 3

2-D Transformations

Transformation

Transformation one the operations applied to geometrical description of an object to change its position, orientation or size and called geometric transformation

Basic geometric transformations:

Translation

Scaling

Rotation

Other transformations:

Reflection

Shear.

Translation

It is applied to the object by representing it along a straigent line parter from one co-ordinate

location to another.

A translation moves all points in an object along the same straight-line path to new positions.

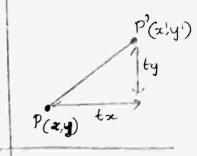
The path is represented by a vector called the translation vector or shift vector.

Translation of point

In point p the coordinates are (2,4).

In point P' the coordinates are (x', y')

$$x' = x + tx$$
 In matrix form,



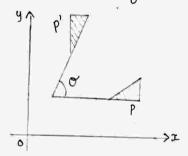
Translation of polygon:

Here translation vectors we is added to all vertices.

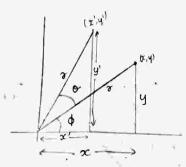
Toanslation of circle/curve:Here, move the center of original circle/curve, then
redraw the circle using radius.

Rotation

It is applied to along a circular path in on an xy plane. For rotation, we specify an angle or called notation angle.



First position of Δ is P. We are rotating (move in a circular path in X-Y plane) then it moves to P' with coordinate (x',y'). It is moved with angle or



Rotation of a point from position (x,y) to position (x',y') through an angle o relative to the coordinate

origin. The original angular displacement of the point from the x axis is ϕ

$$x' = r \cos (\phi + \phi)$$

$$= r \cos \phi - r \sin \phi \sin \phi$$

$$y' = r \sin(\phi + \phi)$$

$$= r \cos \phi \sin \phi + r \sin \phi \cos \phi$$

$$x = r \cos \phi$$

 $y = r \sin \phi$
 $x' = x \cos \phi - y \sin \phi$
 $y' = x \sin \phi + y \cos \phi$
 $P' = R + P$ where,

$$R = \begin{cases} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{cases}, P = \begin{bmatrix} x \\ y \end{bmatrix}, P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} x \cos \theta + -y \sin \theta \\ x \sin \theta + y \cos \theta \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \cos \theta - y \sin \theta \\ x \sin \theta + y \cos \theta \end{bmatrix}$$

Scaling

It alters the size of an object and involves two size factors Sx and Sy for the x- and y- coordinates.

It is usually carried out by multipujing inc Vertex values by scaling factors.

$$x' = x \cdot S_x$$
 $S_x - Scales in x direction$
 $y' = y \cdot S_y$ $S_y - Scales in y direction$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow$$
 As $(Sx, Sy) < 1$,

reduce the size of object & move the object close to the coordinate origin

$$(x,y) = (2,5)$$

 $x' = x \cdot Sx$

$$= a \times 0.5 = 1$$
 = $5 \times 0.5 = 2.5$

P(2,5)

· P(a,5),

Sx = 2

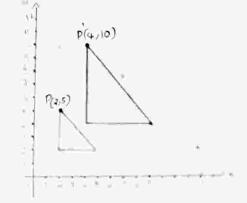
Incuase the size of object \$ moves object l'arthur from the origin.

$$(\alpha, y) = (a, s)$$

 $\alpha' = \alpha \times Sx \quad y' = y \times Sy$

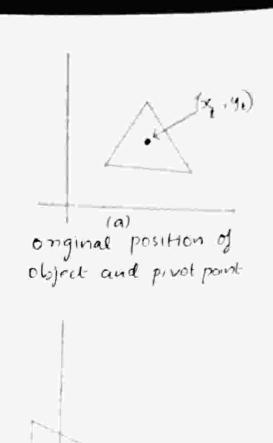
$$= 3.43 = 5.65$$

= 4 = 10

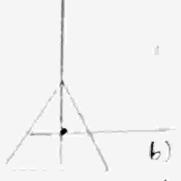




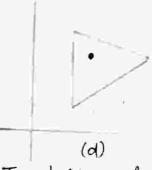
General pivot point Rotation



Rotation was about Origin



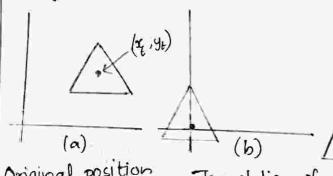
Translation of object so tent pivot point (set, y.) is at origin



Translation of the object so that the pivot point is returned to position (x,140)

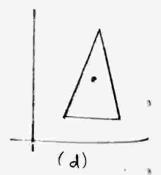
Greneral fixed point scaling

- Translate object so that the fixed point coincides with the coordinate origin.
- · Scale lue object with respect to the coordinate origin.
- · Use the inverse translation for of step 1 to return the object to its original position



Original position of object and fixed point Translation of object so that fixed point (2, y) at oxigin

Scaling was about to origin



Translation of the object so that the fixed point is netword to position

Composite Transformations (A) Translations

If two successive translation vectors (t_{x_1}, t_{y_1}) and (t_{x_2}, t_{y_2}) are applied to a coordinate position P, the final transformed location P' is calculated as: -

$$P'=T(t_{x2},t_{y2}) . \{T(t_{x1},t_{y1}) .P\}$$

= $\{T(t_{x2},t_{y2}) . T(t_{x1},t_{y1})\} .P$

Where P and P' are represented as homogeneous-coordinate column vectors. We can verify this result by calculating the matrix product for the two associative groupings. Also, the composite transformation matrix for this sequence of transformations is: -

Or,
$$T(t_{x2},t_{y2}) \cdot T(t_{x1},t_{y1}) = T(t_{x1}+t_{x2},t_{y1}+t_{y2})$$

Which demonstrate that two successive translations are additive

(B) Rotations

Two successive rotations applied to point P produce the transformed position: -

$$P' = R(\Theta_2) \cdot \{R(\Theta_1) \cdot P\}$$

= \{R(\Omega_2) \cdot R(\Omega_1)\} \cdot P

By multiplication the two rotation matrices, we can verify that two successive rotations are additive:

$$R(\Theta_2)$$
. $R(\Theta_1) = R(\Theta_1 + \Theta_2)$

So that the final rotated coordinates can be calculated with the composite rotation matrix as: -

$$P' = R(\Theta_1 + \Theta_2) \cdot P$$

(C) Scaling

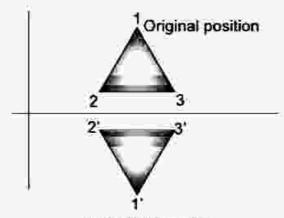
Concatenating transformation matrices for two successive scaling operations produces the following composite scaling matrix: -

Or,
$$S(S_{x2}, S_{y2})$$
, $S(S_{x1}, S_{y1}) = S(S_{x1}, S_{x2}, S_{y1}, S_{y2})$

The resulting matrix in this case indicates that successive scaling operations are multiplicative.

Other transformations

 <u>Reflection</u> is a transformation that produces a mirror image of an object. It is obtained by rotating the object by 180 deg about the reflection axis



Reflected position

Reflection about the line y=0, the X- axis, is accomplished with the transformation matrix

Reflection

Original position

2

1

3

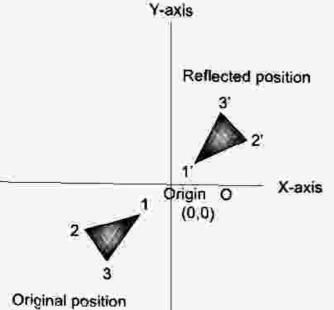
Reflected position



Reflection about the line x=0, the Y- axis, is accomplished with the transformation matrix

Reflection

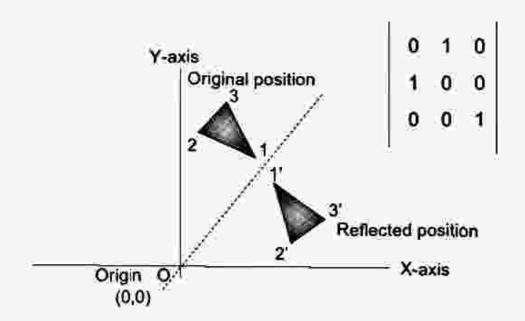
Reflection of an object relative to an axis perpendicular to the xy plane and passing through the coordinate origin



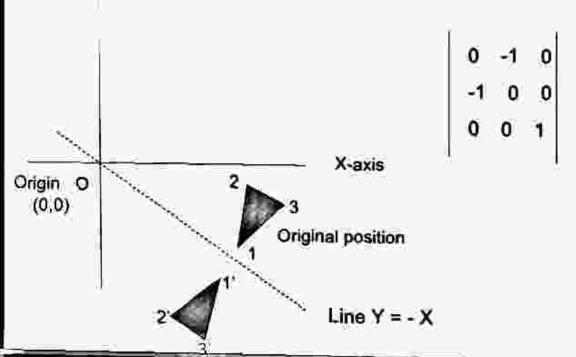
The above reflection matrix is the rotation matrix with angle=180 degree.

This can be generalized to any reflection point in the xy plane. This reflection is the same as a 180 degree rotation in the xy plane using the reflection point

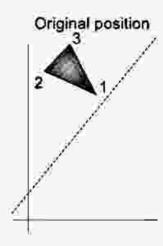
Reflection of an object w.r.t the straight line y=x

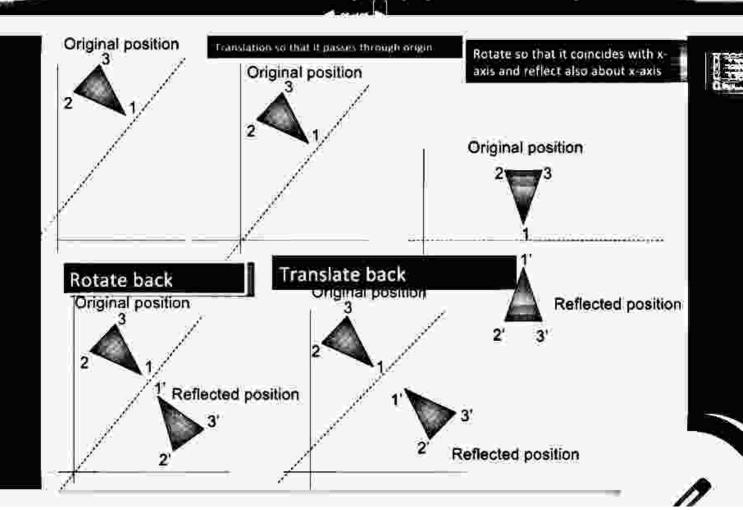


Reflection of an object w.r.t the Y-axis straight line y=-x



Reflection of an arbitrary axis y=mx+b

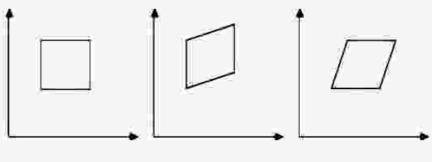




Shear Transformations

- Shear is a transformation that distorts the shape of an object such that the transformed shape appears as if the object were composed of internal layers that had been caused to slide over each other
- Two common shearing transformations are those that shift coordinate x values and those that shift y values

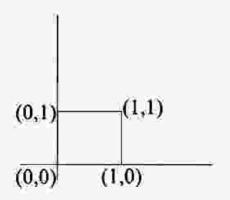


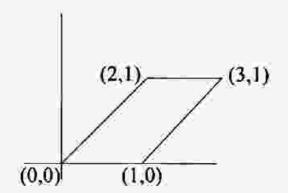


y Shear	x Shear
1 0 0	1 sh, 0
sh 1 0	0 1 0
0 0 1	0 0 1
	1 0 0 sh 1 0

An X- direction Shear

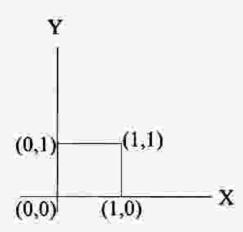
For example, Sh_x=2

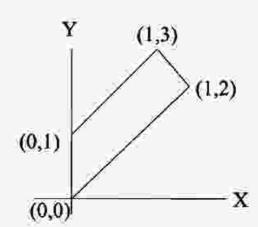




An Y- direction Shear

For example, Sh_y=2

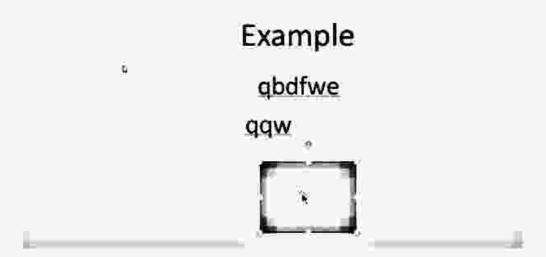




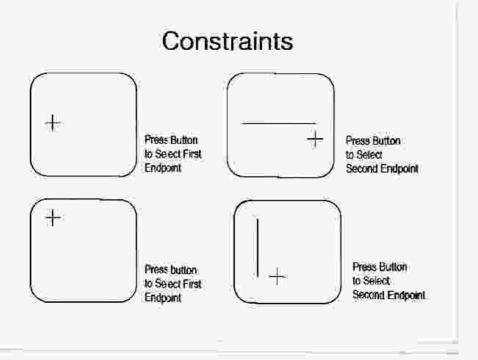
Interactive picture construction techniques, Computer Graphics

Interactive picture- construction methods are commonly used in variety of applications, including design and painting packages. These methods provide user with the capability to position objects, to constrain lig. to predefined orientations or alignments, to sketch fig., and to drag objects around the screen. Grids, gravity fields, and rubber band methods are used to aid in positioning and other picture construction operations. The several techniques used for interactive picture construction that are incorporated into graphics packages are:

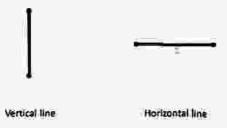
(1) Basic positioning methods:- coordinate values supplied by locator input are often used with positioning methods to specify a location for displaying an object or a character string. Coordinate positions are selected interactively with a pointing device, usually by positioning the screen cursor.



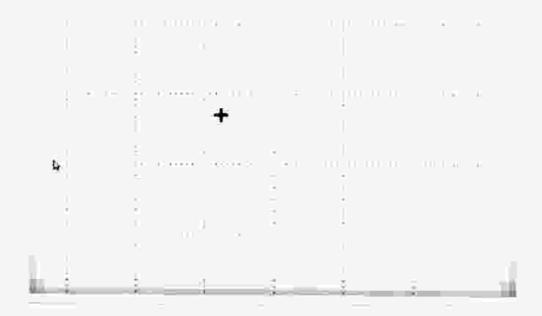
(2) constraints:-A constraint is a rule for altering input coordinates values to produce a specified orientation or alignment of the displayed coordinates, the most common constraint is a horizontal or vertical alignment of straight lines.



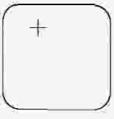
Example:



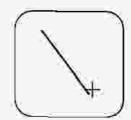
(3) Grids:- Another kind of constraint is a grid of rectangular lines displayed in some part of the screen area. When a grid is used, any input coordinate position is rounded to the nearest intersection of two grid lines.



Grids



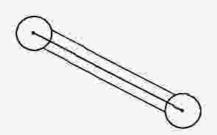
Press Button to Select First Endpoint



Press Button to Select Second Endpoint

(4) Gravity field:- When it is needed to connect lines at positions between endpoints, the graphics packages convert any input position near a line to a position on the line. The conversion is accomplished by creating a gravity area around the line. Any related position within the gravity field of line is moved to the nearest position on the line. It illustrated with a shaded boundary around the line.

Gravity



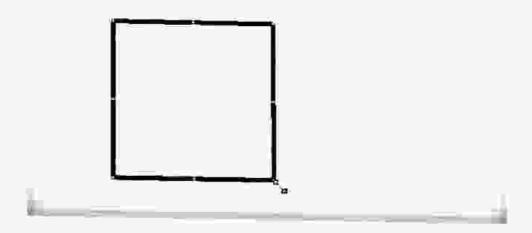
(5) Rubber Band Methods:- Straight lines can be constructed and positioned using rubber band methods which stretch out a line from a starting position as the screen cursor.

Rubber-Band

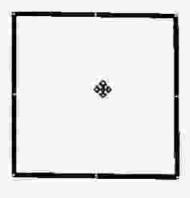


(6) Dragging: This methods move object into position by dragging them with the screen cursor.

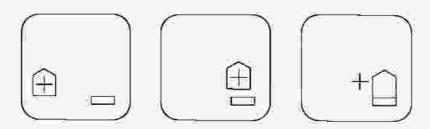
Example:

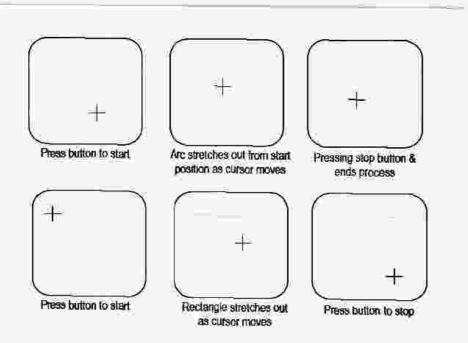


Example:



Dragging

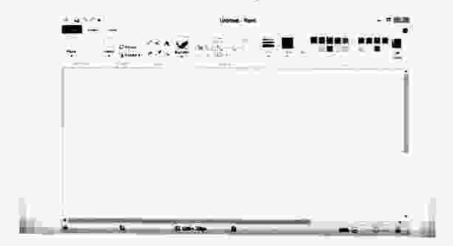




(7) Painting and Drawing:- Cursor drawing options can be provided using standard curve shapes such as circular arcs and splices, or with freehand sketching procedures. Line widths,

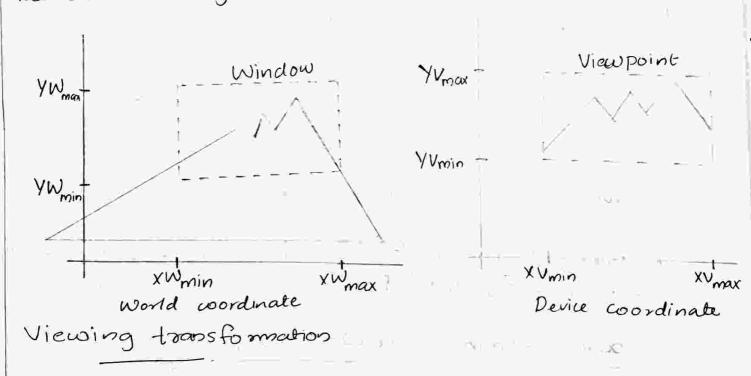
line styles and other attribute options are also commonly found in painting and drawing packages.

Example



Viewing pipeline

- * Window: An acrea on a display device / what is to be displayed.
- 1 Viewpost: where it is to be displayed
- * A world-coordinate area selected for display is called a window. An area on a display device to which a window is mapped is called a viewport.
- * The mapping of a part of a world coordinate scene to device coordinates is called viewing transformation
- * The 2-Dimensional Viewing transformation is also called window- two-viewport transformations or the windowing transformation.



* We construct the scene in world coordinate using the output primitives and attributes.

Next, to exobtain a particular orientation for the window we can set up a 2-Dimensional viewing coordinating system in the world coordinate plate & define a window in the viewing coordinate system.

- The viewing coordinate reference frame is used to provide a method for setting up arbitrary orientations for rectangular windows.
- * Once the viewing reference frame is established we can transform descriptions in world coordinates to viewing coordinates.
- Then, define a viewport in normalized coordinates.
- * At the final step, all parts of the picture that lie outside the viewport are dipped, and the content of the viewport are transferred to device coordinate.

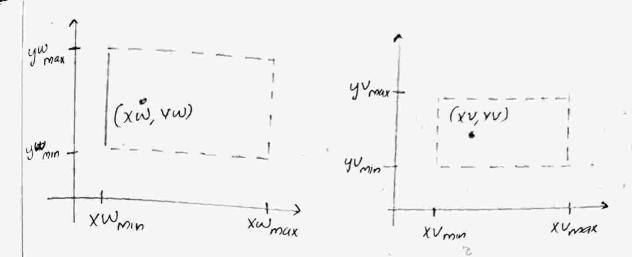
Map Viewing Convert Map construct world coordinates to normalized World-Mc coordinate scene wing modelling-VC Normalized viewpost Coordinates > to device > Viewing to Coordinate Coordinale viewing coordinates Transformation using Window coordinates Viewport Specifination

Window to viewport coordinate transformation

A point at position (ocw, yw) in the window is mapped into position (xv, yv) into the associated viewpost

$$\frac{xv - xv_{min}}{xv_{max} - xv_{min}} = \frac{xw - xw_{min}}{xw_{max} - xw_{min}} = 0$$

yv - yvmin ywax ywmin yw -ywmin



tig: A point at position (xw, yw) in a designated window is mapped to view post woordinate (xv, yv) so that relative positions in the two areas are the same.

$$\frac{XV - XVmin}{XV_{max} - XVmin} = \frac{XW - XWmin}{XW_{max} - XWmin}$$

$$XV = \left[\frac{XW - XW_{min}}{XW_{max}} \times V_{min} \times V_{min}\right] + XV_{min}$$

Solving 3

$$XV = XV_{min} + (XW - XW_{min}) SX$$

$$\frac{1}{XV_{max}} \times V_{min} = SX$$

$$\frac{1}{XW_{max}} \times V_{min} = SX$$

$$\frac{1}{XW_{max}} \times V_{min} = SX$$

where the scaling factors are,

This conversion is performed with the following sequence of transformation.

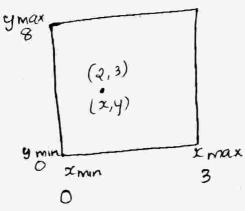
- 1) Perform a scaling transformation using a fixed point position of (xwmin, Ywmin) that scales the window are to the size of the viewport
- @ Translate the scaled window area to the position of the view port.
 - * Relative proportion of objects are maintained, if the scaling factors are the same (sx = sy)
 - * Character strings can be handled in a ways:
- 1) Maintain a constant character size, even though the viewport area may be enlarged or reduced relative to the window.
- Character fromed with line segments, the mapping to the viewport can be carried out as a sequence of line transformation.
- * Any number of output devices can be opened in a particular application and another window to viewport toans formation can be performed for each open output device. This mapping is called workstation transformation.

Clipping Operation

* Any procedure that identifies those position of a picture that are either inside or outside of a specified region that are either inside a dippiner alamin of space is called a clipping algorithm or clipping.

* The region against which an object is to be clipped is called a clip window

Primitive types + Point clipping * Line dipping / straight line segments * Area clipping / polygons * Curve Clipping * Text alipping Point elipping Clip window is a rectangle Consider a point, P = (x,y) for display, if the following inequalities are satisfied: xwmin < x < xwmax ywmin = y = ywmox When the edges of the clip window (xwmin, xwmax, ywmin, Ywmax) com be either the word coordinate window boundaries coordinates or viewport boundagies. If any one of these inequalities is not satisfied, the point is dipped (not satisfied saved for display)



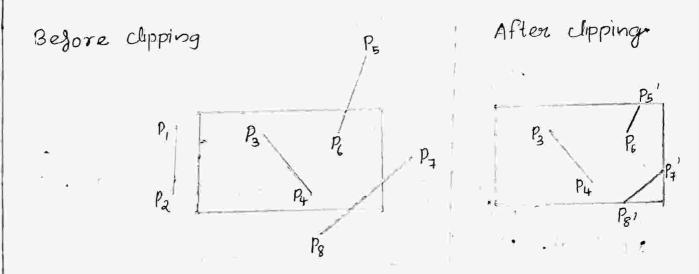
Line clipping

A line clipping procedure involves:

O Test a given line segment to delermine, whether it lies completely inside the clipping window.

If it does not determine whether it lies completely.

- 3 If we cannot identify a line as completely inside or completely outside, perform intersection calculation with one or more clipping.
- A line with both the end points inside all dipping boundary is saved.
- A line with both on the end points outside, any of the clip boundaries is outside the window. All other lines cross one or more dipping boundaries, may require calculation of multiple intersection points.



Clipping algorithms are used to efficiently identify outside lines and reduce intersection calculations,

. For a line segment with end points (x, y) and (x, y) and (x, y) and one one or both end points outside the dipping rectangle, the parametric representation.

$$x = x_1 + \mathcal{M}(x_2 - x_1)$$

$$y = y_1 + \mathcal{M}(y_2 - y_1) \qquad 0 \le \mathcal{M} \le 1$$

If the value of u is outside the range o to I then the line does not enter the interior of the window of the window of the value of the window u' is within the

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range from 0 to 1, the line segment does cross into the dipping area.

(ohen - Sutherland line dipping

This is one of the oldest and most popular line-clipping procedures. Every line ending point in a picture is assigned a 4-digit binary code, called a region code, that identifies the location of the point relative to the boundaries of the clipping reclangle. Regions are the boundaries of the clipping reclangle. Regions are set up in reference to the boundaries as shown in the figure below

0001 0000 0010 0000 0010

fig: "Binary region codes assigned to line endpoints according binary region codes assigned to line endpoints according to relative positions with respect to the clipping rectangle".

Regions are set up in segerose to the boundaries of

Each goit position in the region code is used to indicate Each goit position in the region code is used to indicate one of the four relative co-ordinate positions of the point with respect to the dip window, to the left, right,

top or bottom.

bit 1: left

bita: gright

bit 3: below

lf a point is within the clipping rectangle, the region lf a point is within that is below and to the left of code is 0000. A point that is below and to the left of the rectangle has a region code of 0101.

produce of a state of a party

Region-bicode bit values can be determined with the following a sleps: (1) (alculate differences between endpoint coordinates and clipping boundaries. (a) Use the resultant sign bit of each difference calculation to set the corresponding value in the region code.

Any lines that are completely contained within the window bound aries have a region code of 0000 for both end points. Any lines that have a 1 in the same bit positions in the region codes for each endpoints and a code of 0101 for the other end-pink A method that can be used to test lines for total clipping is to perform the logical and operation with both region codes. If the result is not 0000, the line is completely outside the clipping region.

Intersection points with a dipping boundary can be calculated using the slope-intercept form of the line equation for a line with endpoint coordinates (x, y,) and (x, yz) the y coordinate of the intersection point with a vertical boundary can be obtained with the calculation

 $y = y_i + m(x - x_i)$

when the x value is set either to xwmin or to xwmax and the slope of the line is calculated as m=(y2-y)/(x2-x1). Similarly, the intersection with a nonzontal boundary, the oc coordinate can be calculated as,

with y set either to your or to Ywman

Example. P, 8 P2 P, - 0001 (non-zero value) Pa-0001 (non-zero value) AND-0001 (non zero) Our output of AND operation is also non-zero, so it means that the line is completely outside the window: so we have to clip it (reject). 1 · · · P3 \$ P4 P3-0000 (3ero) P4-0000 (zero) AND-0000 (zero value) Our output of AND operation is zero and the 2 input values are also zero that means the line is completely inside the window so we accept it. Imple on a second · P5 \$ P6 Pg- 1000 (non-zero) P₆ - 0000 (zero) AND- 0000 If one is zero & other is a non-zero value, we perform pastial dipping ie, the line have some portion inside the

window and some portion outside the window. Then we consider the intersection of points of the line ie, here it is P5" P5 and P6 P5'- 0000 P6 - 0000 AND - 0000 Now (P5', P6) is inside the window. So we take it P7, P8 p,' -0000 P7 - 0010 r a j l m t P8'-0000 P8 - 0100 AND - 0000 P8-0100 P7 - 0010 18-0000 P7' - 0000 AND-0000 AND - 0000 Here also we apple Here we apply pastial clipping partial dipping 15 18 - T P_7 and P_8 . Now consider } We accept the line (P7, P8') bz all the 3 values are zero. P1 - 0000 AND - 0000 . After line clipping we got the lines inside the window. They are, → (**P**3 ,P4) -) (P51,P6) $\rightarrow (P_7', P_8)$

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Sutherland-Hodgeman polygon dipping

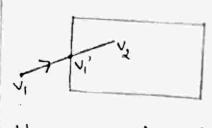
Begining with the initial set of polygon vertices, first the left dratangled Issundary new sequence of vertices. The new set of vertices could then be successively passed to a right boundary dipper, a bottom boundary clipper can a top boundary dipper at each step, a new sequence of output vertices is generated and passed to the next window boundary dipper. There are 4 possible cases! when processing vertices in sequence:

when processing vertices in suguence

- O If the first vertex is outside the window boundary and the and vertex is inside, both the intersection point of the polygon edge with the window boundary and the second vertex are added to the ofp vertex list.
- If both input vertices are inside the window boundary only the second vertex is added to the output vertex list
- 3 If the first vertex is inside the window boundary & the second vertex is outside, only the edge intersection with the window boundary is added to the output vertex list.
- If both input vertices are into outside the window boundary nothing is added to the output list.

Once all vertices have been processed for one clip window boundary the output list of vertices is clipped against the next window boundary.

1st Cases



Here the line is from outside to inside so we consider the intersection point as Vi, Va

is, out-in

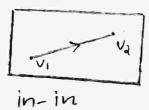
and case

in-out

then poin is,

√, '

3rd case

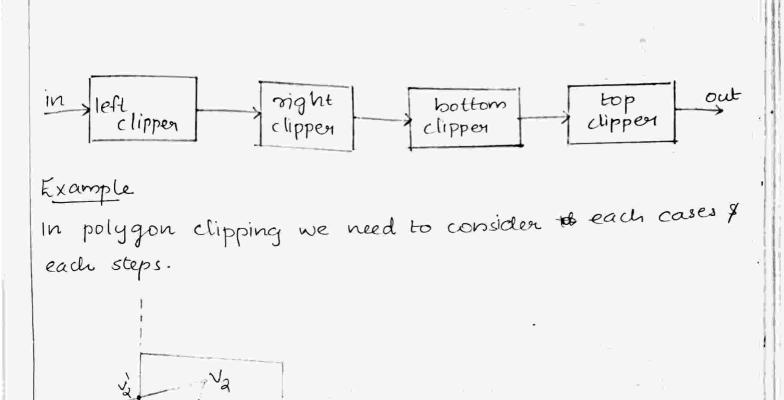


point - Va

4th case

out-out

then point is NIL



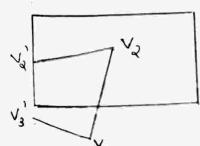
left clip o/p

V₁V₂ in-in V₃

V₃V₁ out-in V₃

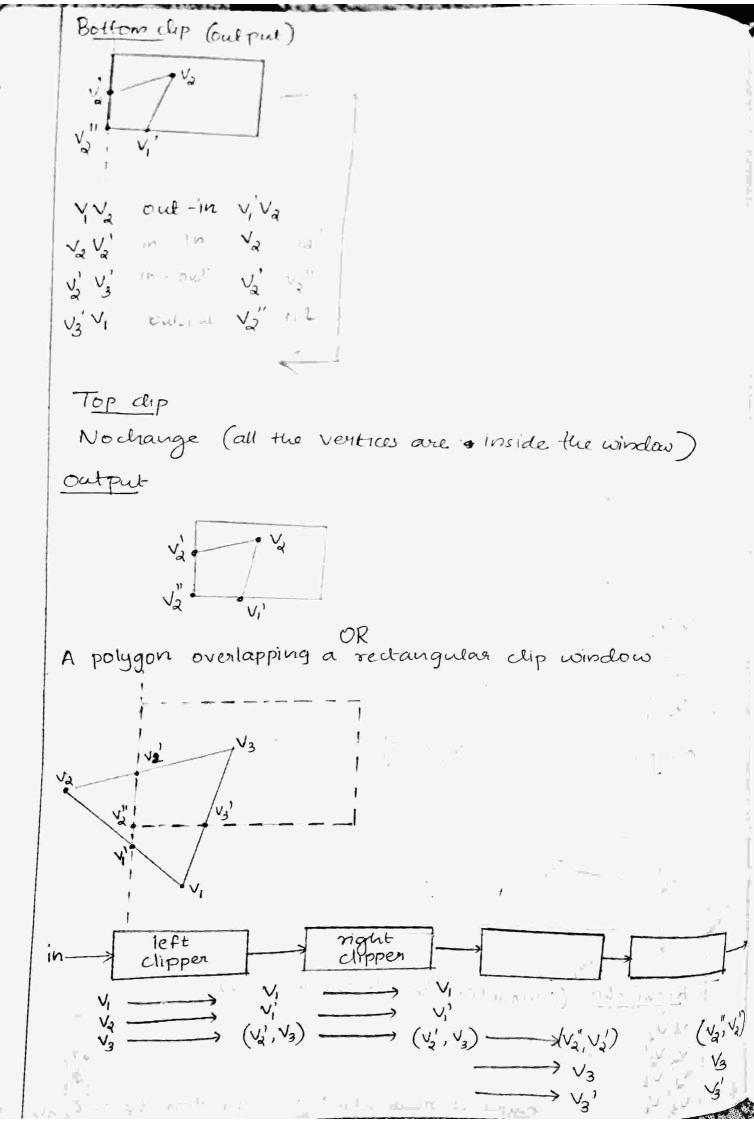
V₃V₁

output of left clip (input of right clip)



Right dip (completely inside the window)

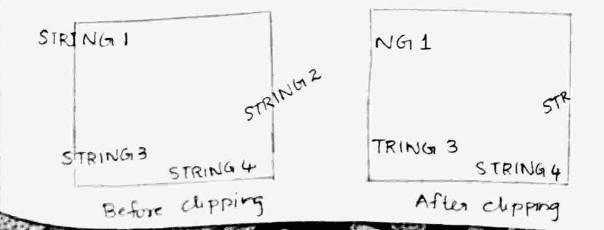
output of right clip & input of bottom clip is the above fig



The simplest method for processing the character strings relative to be a window boundary is to use "a all-or-none-string-clipping stratedgy. If all of the string is inside a clip window, we keep it. Otherwise the string is discarded. The boundary positions of the rectangle are compared to the window boundaries and the string is rejected if there is any overlap. This clipping method is considered as the fastest text clipping.



- An alternative method to rejecting an entire character string that overlaps a window boundary is to use all-or-none character-dipping stratedgy.
- In this method, discard only those characters that are not completely inside the window. Any character that either overlap or is outside the window boundary is dipped.



Next method for handling text dipping is to dip, the components of individual character.

If an individual character overlaps a clip window boundary, clip off the parts of the character that are outside the window.

Outline character fond, fromed with line segments can be processed in this way using a line dipping algorithm

Characters defined with bit maps would be clipped by comparing, the relative position of the individual pixels in the character grid patterns to the clipping boundaries.

