Machine Learning and Computational Statistics, Spring 2016 Homework 2: Lasso

Due: Monday, February 15, 2016, at 6pm (Submit via NYU Classes)

Instructions: Your answers to the questions below, including plots and mathematical work, should be submitted as a single PDF file. You may include your code inline or submit it as a separate file. You may either scan hand-written work or, preferably, write your answers using software that typesets mathematics (e.g. Langer, Langer,

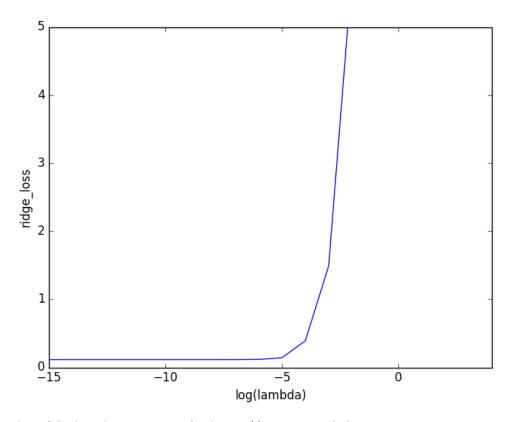
1 Preliminaries

1.1 Dataset construction

Done as part of createdataset function in the code that is submitted along.

1.2 Experiments with Ridge Regression

1. The λ value choosen is 10**-10. It can be seen from the graph that the validation loss is minimum at around that value of λ .



The value of θ where loss converges for $\lambda = 10^{**}-10$ is given below

[-9.97078811e+00-9.94896606e+00-9.91693187e+00-9.75743181e+00-9.85992999e+00-1.01373407e+01-9.79856957e+00-1.01281901e+01-9.80381646e+00-1.00731125e+01-2.72097142e-01-6.48934430e-02-2.78305362e-01-1.80346366e-01-4.34798615e-02-1.42873962e-01-4.65720912e-02-2.83894078e-02-3.39961435e-02-1.41480673e-01-2.29230737e-01-1.30172965e-01-4.73448622e-02-1.40895080e-01-3.04825570e-01-1.81369197e-01-1.63071622e-01-2.55285980e-01-8.64921547e-02-1.48544968e-01-3.85508105e-01-1.87926893e-02-2.43938673e-01-4.44195422e-01-2.76351680e-01-9.36075273e-02-4.97978168e-03-1.80403702e-01-1.28208184e-01-1.47787388e-01-1.81369987e-01-3.94292494e-02-1.19387104e-01-1.14322493e-01-1.56063159e-01-1.27033886e-01-1.62222500e-01-2.53772872e-02-1.49366859e-01-5.28729829e-02-9.93249974e-02-5.73802138e-02-1.25475286e-01-3.05958118e-01-5.14486295e-02-2.99610761e-02-1.75485414e-01-2.44970598e-01-1.38078673e-01-5.43552090e-03-3.43687433e-02-1.22835705e-01-3.21331952e-01-5.71236960e-03-1.73771826e-02-2.05816282e-02-2.43973139e-01-4.30598523e-02-3.01921036e-01-9.63561659e-02-2.08044204e-01-5.88348587e-03-5.48503830e-02-3.62098325e-01-3.16021485e-02]

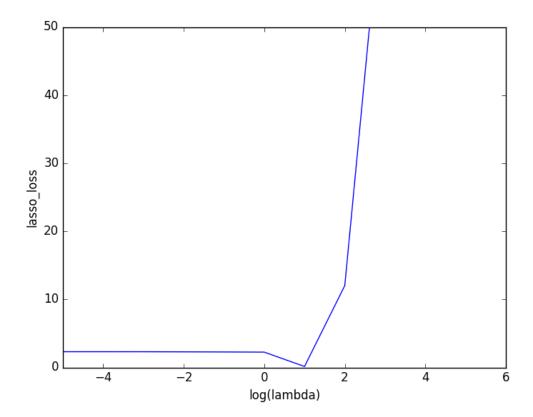
Here, there are no components with true value=0. However, even if we consider a threshold of 10e-3 zero coefficients remain at zero count.

2 Coordinate Descent for Lasso (a.k.a. The Shooting algorithm)

2.1 Experiments with the Shooting Algorithm

1. The function that computes lasso solution in code is compute_lasso_shooting_algorithm. The plot of validation loss vs λ is shown below. Note: Calculated validation loss using compute_lasso_shooting_algorithm_vectorized. This is just to speed up the calculation (since vectorized and non vectorized give same values). For comparison of non vectorized and vectorized calls to both are made in code.

 λ value choosen is 10.

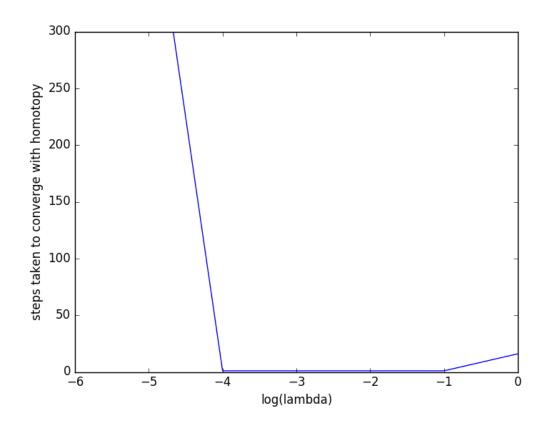


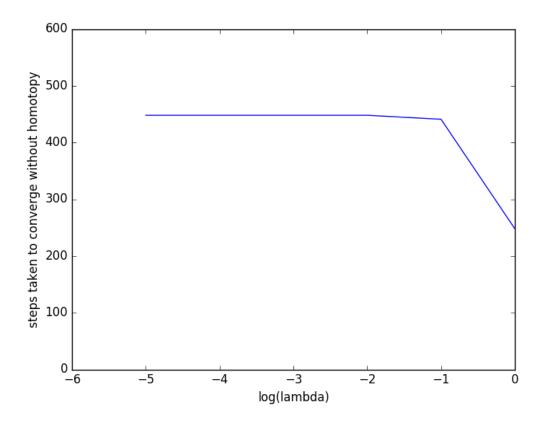
2. The value of coefficients with $\lambda = 10$ is as below:

Note that unlike ridge regression, the solution is sparse. All the values with which we had

generated our data set as zero is zero in the solution as well. This confirms the spares nature of lasso solution.

3. The time taken by homotopy solution is 0.902 units and without homotopy for same set of λ the time taken is 3.851 units. Homotopy solution gives faster results. This can be seen in the form of graphs also. In homotopy solution, for first value of λ the steps taken is same as that of without homotopy. However for next set of λ , for homotopy solution the steps taken is almost close to 1 and for without homotopy steps remain almost same.





. Lasso regularization without vectorization took 3.96 units of time. Lasso regularization with vectorization took 0.24 units of time. This shows the power of vectorization.

Vectorization form:

$$a_j = 2X^T X[j, j]$$

$$c_j = 2X[j, j](y - Xw + w_j X[j, j])$$