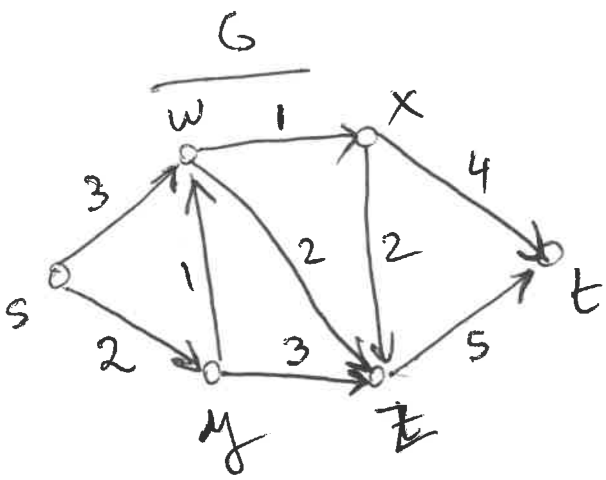
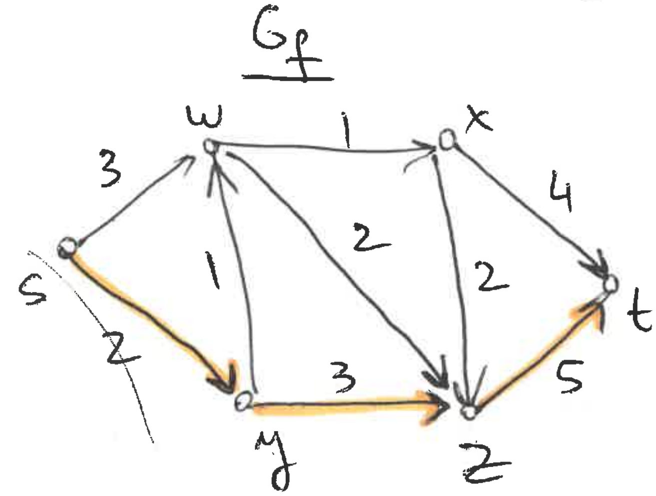
2.24.2017

• Use Ford-Fulkerson to compute the maximum flow:



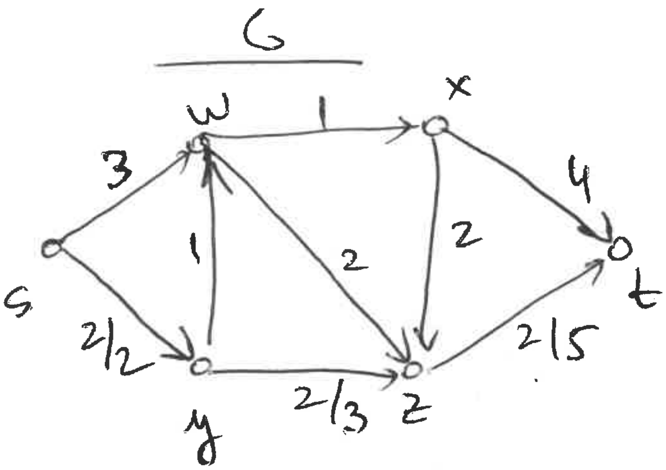
$|f|=0$



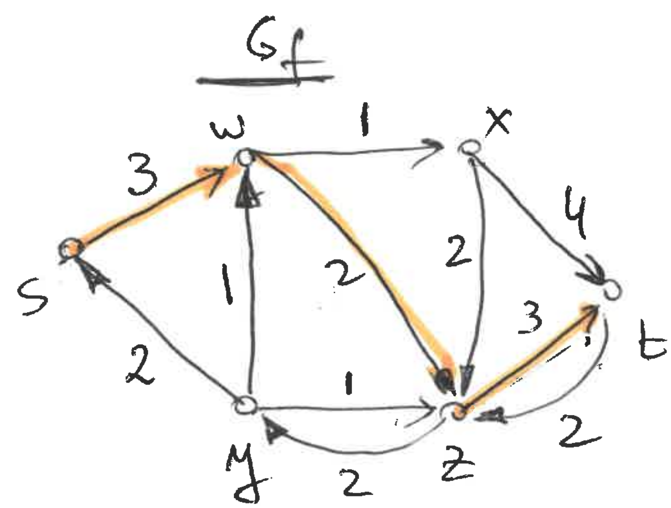
$p=\langle s,y,z,t\rangle$

$c_f(p)=2$

$|f_p|=2$



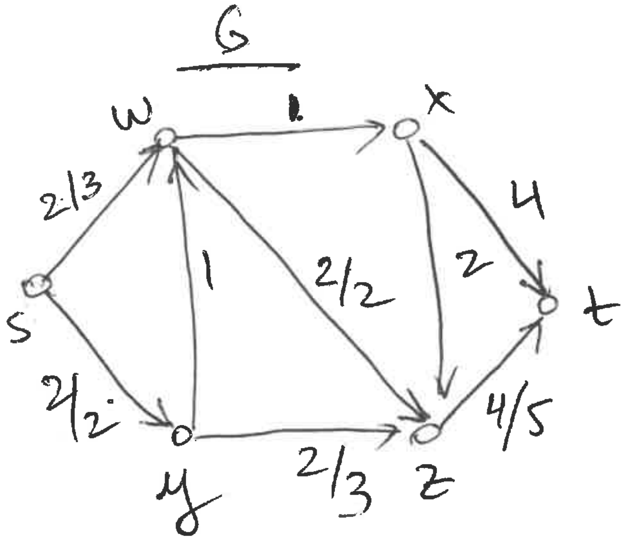
$|f|=2$



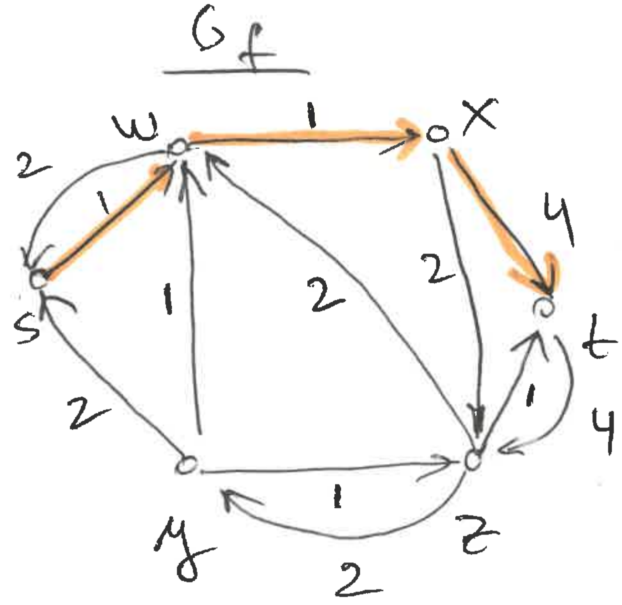
$p=\langle s,w,z,t\rangle$

$c_f(p)=2$

$|f_p|=2$



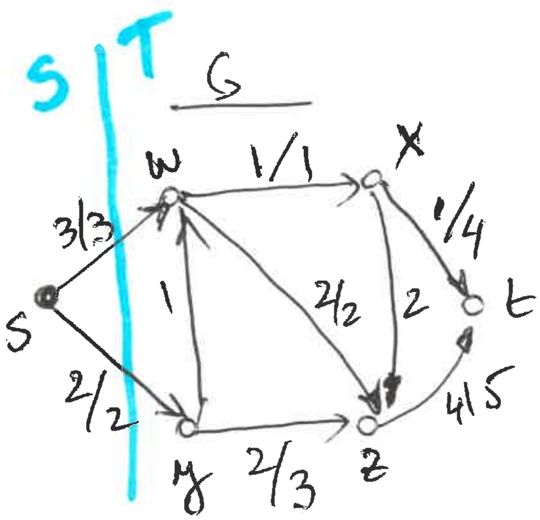
$|f| = 4$



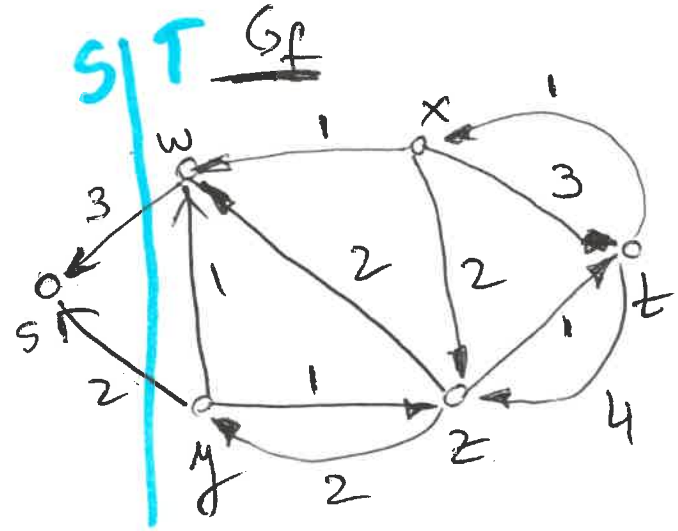
$p = \langle s, w, x, t \rangle$

$c_f(p) = 1$

$|f_p| = 1$



$|f| = 5$



- no augmenting path => max-flow reached

- find a cut (S,T) s.t. $c(S,T) = |f| = 5$ (for max-flow!)

$S = \{s\}$

$T = \{t, w, x, y, z\}$

$c(S,T) = 5$

flow-network

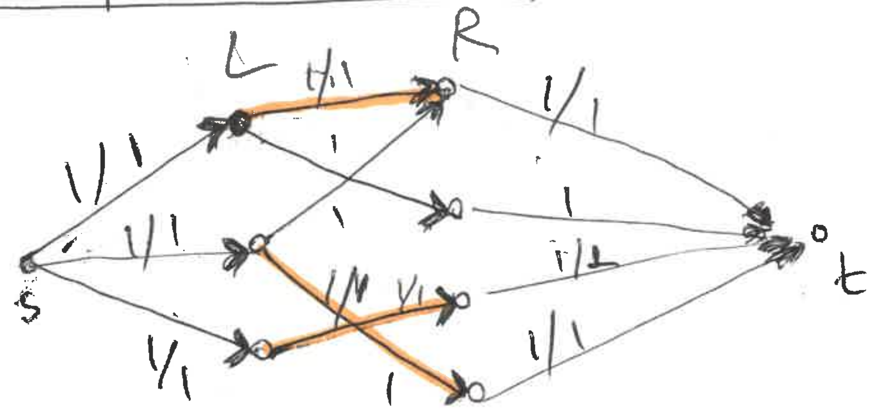
- graph $G(V,E)$ directed
- source s, sink t
- capacities c

$\longrightarrow$ **Max-flow solver** $\longrightarrow$ maximum-flow f

Ford-Fulkerson : $RT = O(|f^*| \cdot E)$

Edmonds-Karp : $RT = O(V \cdot E^2)$

Maximum bipartite matching



Input

graph $G(V,E)$ bipartite
$V = L \cup R$

$\longrightarrow$

Flow Network

- build directed graph $G'(V', E')$
  - add edges in E with direction $L \rightarrow R$
  - add source s, sink t
  - add edges connecting s to each vertex in L
  - add edges from each vertex in R to t
  - add capacity 1 on each edge

$\longrightarrow$

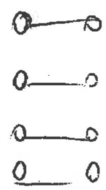
$\rightarrow$ $\boxed{\text{Max-flow solver}}$ $\longrightarrow$ Max-flow f $\rightarrow$ return the edges in G with flow >0 as being the edges in the maximum matching M

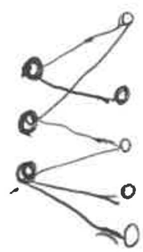
(Ford-Fulkerson or Edmonds-Karp)

RT analysis

Ford-Fulkerson: $RT = O(|f^*| \cdot E')$

$$|f^*| \leq \frac{|V|}{2}$$





$$|E'| = |E| + |V|$$

$|E| \geq \frac{|V|}{2}$ (each vertex is incident on at least one edge) $\Rightarrow |V| \leq 2 \cdot |E|$ $\Rightarrow$

$$\Rightarrow |E'| \leq |E| + 2|E| = 3|E|$$

$$\boxed{RT = O(V \cdot E)}$$