9.1) $T(N) = 2T(n-1)+3$

$T(N-1) = 2T(n-2)+3$

$T(n-2) = 2T(n-3)+3$

$T(n-3) = 2T(n-4)+3$

$T(n-\cancel{4}) = 2T(n-5)+3$

$$T(N) = 2T(n-1)+3$$
$$= 2(2T(N-2)+3)+3$$
$$= 2(2[2T(N-3)+3]+3)+3$$
$$= 2(2[2(T(N-4)+3)+3]+3)+3$$

$24 \quad 12 \quad 6 \quad 3$

$$= 2^3\,T(n-4) + 2^3\cdot 3 + 2^2\cdot 3 + 2^1\cdot 3 + 2^0\cdot 3$$
$$2^3\,T(n-4) + 3\cdot\sum_{i=0}^{3} 3\cdot 2^i$$
$$= 2^k\,T(n-k) + 3\cdot\sum_{i=0}^{k-1} 3\cdot 2^i$$
$$= 2^k\,T(n-k) + \frac{3(1-2^{k-1})}{1-2}$$
$$= 2^k\,T(n-k) + (3\cdot 2^{k-1} - 3)$$

$T(1) = 4$

$n-k=1 \Rightarrow k = n-1$

$$= 2^{n-1}\,T(1) + (3\cdot 2^n - 3)$$
$$= 2^{n-1}(4) + (3\cdot 2^n - 3)$$
$$= (2^{n-1})(2^2) + 3(2^n - 1)$$
$$T(n) = 2^{n+1} + 3(2^n - 1)$$

$$T(n) = \Theta(2^n)$$

9.2 $T(N) = T(\sqrt[4]{n^3}) + 5 \qquad m = \lg n \Rightarrow 2^m = n$

$T(2^m) = T(2^{\frac{3m}{4}}) + 5$ — constant function!

Let $S(m) = T(2^m)$

Let $S(m) = T(2^m) = S(\frac{3}{4}m) \quad a=1,\ b=\frac{4}{3}$

$S(m) = S(\frac{3}{4}m) + 2^{\lg 5} \qquad a=1,\ b=4/3,\ c=\lg 5$

Since $\log_{\frac{4}{3}} 1 = 0 \qquad 0 < \lg 5$

Use Case 1 of Master Theorem

$$T(m) = \Theta(m^{\lg 5})$$
$$= \Theta(m^{2.32})$$
$$= \Theta((\lg n)^{2.32})$$