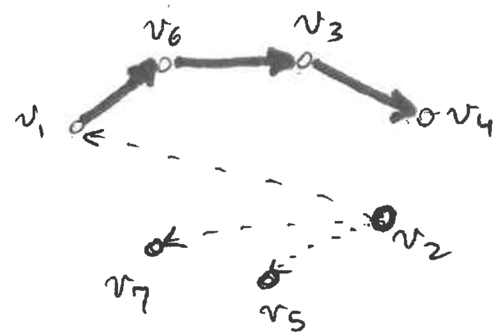
2.15.2017

# TSP problem (Traveling Salesman Person)

- How do we compute the **bound**?

- assume the partial solution

$\underline{v_1}$ $\underline{v_6}$ $\underline{v_3}$ $\underline{v_4}$ _ _ _




$v_1 : w(v_1, v_6)$

$v_2 : \min\{w(v_2, v_5), w(v_2, v_7), w(v_2, v_1)\}$

$v_3 : w(v_3, v_4)$

$v_4 : \min\{w(v_4, v_2), w(v_4, v_5), w(v_4, v_7)\}$

$v_5 : \min\{w(v_5, v_2), w(v_5, v_7), w(v_5, v_1)\}$

$v_6 : w(v_6, v_3)$

$v_7 : \min\{w(v_7, v_1), w(v_7, v_2), w(v_7, v_5)\}$

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bound $= \Sigma$

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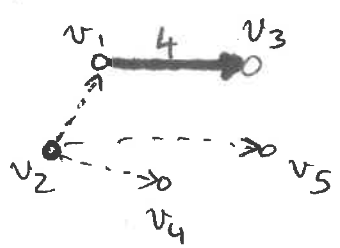
example:




| | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ |
|---|---|---|---|---|---|
| $v_1$ | 0 | 14 | 4 | 10 | 20 |
| $v_2$ | 14 | 0 | 7 | 8 | 7 |
| $v_3$ | 4 | 5 | 0 | 7 | 16 |
| $v_4$ | 11 | 7 | 9 | 0 | 2 |
| $v_5$ | 18 | 7 | 17 | 4 | 0 |

$v_1 : w(v_1, v_3) = 4$

$v_2 : \min\{w(v_2, v_1), w(v_2, v_4), w(v_2, v_5)\} = \min\{14, 8, 7\} = 7$

$v_3 : \min\{w(v_3, v_2), w(v_3, v_4), w(v_3, v_5)\} = \min\{5, 7, 16\} = 5$

$v_4 : \min\{w(v_4, v_1), w(v_4, v_2), w(v_4, v_5)\} = \min\{11, 7, 2\} = 2$

$v_5 : \min\{w(v_5, v_1), w(v_5, v_2), w(v_5, v_4)\} = \min\{18, 7, 4\} = 4$

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bound $= 4 + 7 + 5 + 2 + 4 = 22$

# Divide-and-conquer

$$T(n) = \text{Divide}(n) + a \cdot T\left(\frac{n}{b}\right) + \text{Combine}(n)$$

$a \geq 1$

$b > 1$

$$\boxed{T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n)}$$

→ solve with Master Thm

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- $\boxed{T(n) = \Theta(n \lg n)}$

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + \Theta(n)$$

Divide-and-conquer

- alg. recurses on 2 subproblems of size $\frac{n}{2}$
- divide + combine take $O(n)$

$$n = n^{\log_2 2} = n$$

⇒ case 2 Master Thm

$$\boxed{T(n) = \Theta(n \lg n)}$$

- $\boxed{T(n) = \Theta(n)}$

$$T(n) = T\left(\frac{n}{2}\right) + \Theta(n)$$

Divide-and-conquer alg:

- recurses on 1 subproblem of size $\frac{n}{2}$
- divide + combine take $O(n)$

Master Thm

$cn$ vs $n^{\log_2 1} = n^0 = 1$

$cn = \Omega(n^{\varepsilon})$ for $\varepsilon = 1$

regularity condition checks

$\Rightarrow$ case 3 Master Thm $\boxed{T(n) = \Theta(n)}$

- $\boxed{T(n) = \Theta(\lg n)}$

$$T(n) = T\left(\frac{n}{2}\right) + \Theta(1)$$

Divide-and-conquer:

- alg recurses on 1 subproblem of size $\frac{n}{2}$
- divide + combine is $\Theta(1)$

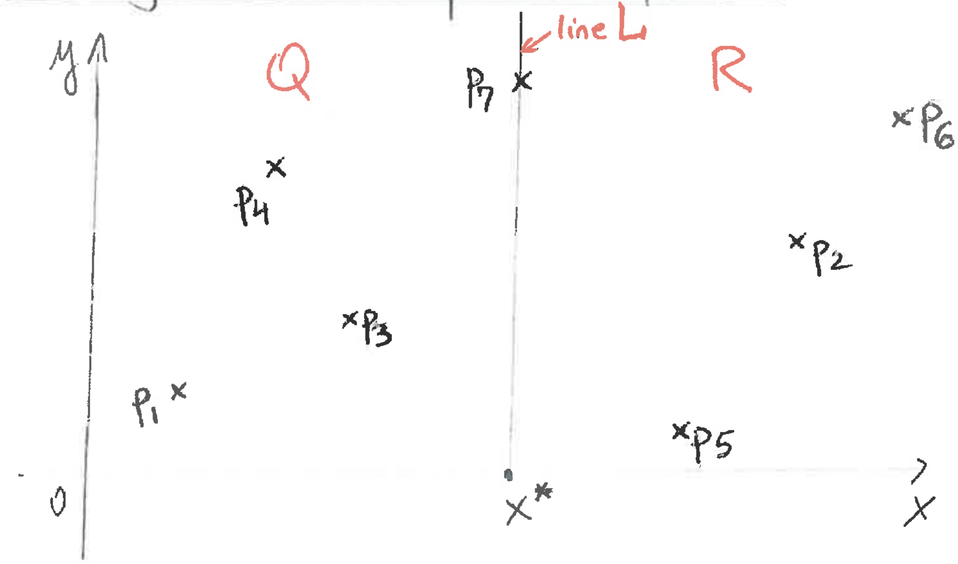
Master Thm

$c = n^{\log_2 1} = n^0 = 1$

$c = \Theta(1) \Longrightarrow$ case 2 Master Thm

$\boxed{T(n) = \Theta(\lg n)}$

# Finding the closest pair of points




$P = \langle p_1, p_2, p_3, p_4, p_5, p_6, p_7 \rangle$ $\Rightarrow n = 7$ points

$P_x = \langle \overbrace{p_1, p_4, p_3, p_7}^{Q}, \overbrace{p_5, p_2, p_6}^{R} \rangle$ → sort points by x-coordinate

$P_y = \langle p_5, p_1, p_3, p_2, p_4, p_6, p_7 \rangle$ → sort points by y-coordinate

→ take $O(n \cdot \lg n)$ and it is performed <u>only once</u>!

$\lceil \frac{n}{2} \rceil + \lfloor \frac{n}{2} \rfloor = n$

number of points in Q

number of points in R

• can use a flag array to remember which points are in Q, R

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 1 | 1 | 0 |

$Q_x = \langle p_1, p_4, p_3, p_7 \rangle$

$R_x = \langle p_5, p_2, p_6 \rangle$

$Q_y = \langle p_1, p_3, p_4, p_7 \rangle$

$R_y = \langle p_5, p_2, p_6 \rangle$

$O(n)$

← <u>Divide step</u>