⑤ Lucky Puck

Vancouver has	400	Edmonton	350
Winnipeg needs	250	Saskatoon	350

Shipping Costs

Source \ Destination	Winnipeg	Saskatoon
Vancouver	5	8
Edmonton	6	2

5.1 Let X_1 = helmets from Vancouver to Winnipeg
X_2 = " Vancouver to Saskatoon
X_3 = " Edmonton to Winnipeg
X_4 = " Edmonton to Saskatoon

5.2

$$X_1 + X_2 \leq 400$$
$$X_3 + X_4 \leq 350$$
$$X_1 + X_3 = 250 \quad (1)$$
$$X_2 + X_4 = 350 \quad (2)$$

Minimize $5X_1 + 8X_2 + 6X_3 + 2X_4 = L$ (3)
$X_1, X_2, X_3, X_4 \geq 0$ (3a)

Convert to MAXIMIZE by NEGATING (3)

$$-5X_1 - 8X_2 - 6X_3 - 2X_4 = L' \quad (4)$$

Let $X_1 = X_1' - X_1''$, $X_2 = X_2' - X_2''$, $X_3 = X_3' - X_3''$, $X_4 = X_4' - X_4''$
with $X_1', X_1'', X_2', X_2'', X_3', X_3'', X_4', X_4'' \geq 0$

4 becomes
$$-5(X_1' - X_1'') - 8(X_2' - X_2'') - 6(X_3' - X_3'') - 2(X_4' - X_4'') = L'' \quad (5)$$

Convert (1) & (2) to inequalities and they become

$$X_1' - X_1'' + X_3' - X_3'' \leq 250 \quad (6)$$
$$X_1' - X_1'' + X_3' - X_3'' \geq 250 \quad (7)$$
$$X_2' - X_2'' + X_4' - X_4'' \leq 350 \quad (8)$$
$$X_2' - X_2'' + X_4' - X_4'' \geq 350 \quad (9)$$

Negate 7, 9 and the system becomes

$$X_1' - X_1'' + X_2' - X_2'' \leq 400$$
$$X_3' - X_3'' + X_4' - X_4'' \leq 350$$

$$X_1' - X_1'' + X_3' - X_3'' \leq 250$$
$$X_1'' - X_1' + X_3'' - X_3' \leq -250 \quad (7)\ \text{negated}$$
$$X_2' - X_2'' + X_4' - X_4'' \leq 350$$
$$X_2'' - X_2' + X_4'' - X_4' \leq -350 \quad (9)\ \text{negated}$$

MAXIMIZE $-5X_1' + 5X_1'' - 8X_2' + 8X_2'' - 6X_3' + 6X_3'' - 2X_4' + 2X_4''$ (10)

Let $X_1 = X_1'$, $X_2 = X_1''$, $X_3 = X_2'$, $X_4 = X_2''$, $X_5 = X_3'$, $X_6 = X_3''$, $X_7 = X_4'$, $X_8 = X_4''$

The system becomes (LOTS of work for 7 points)

SUBJECT TO:

$$X_1 - X_2 + X_3 - X_4 \leq 400$$
$$X_5 - X_6 + X_7 - X_8 \leq 350$$
$$X_1 - X_2 + X_5 - X_6 \leq 250$$
$$X_2 - X_1 + X_6 - X_5 \leq -250$$
$$X_3 - X_4 + X_7 - X_8 \leq 350$$
$$X_4 - X_3 + X_8 - X_7 \leq -350$$

MAXIMIZE

$$-5X_1 + 5X_2 - 8X_3 + 8X_4 - 6X_5 + 6X_6 - 2X_7 + 2X_8$$

With $X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8 \geq 0$

Ex. 3

To Slack form?

$$Z = -5X_1 + 5X_2 - 8X_3 + 8X_4 - 6X_5 + 6X_6 - 2X_7 + 2X_8$$
$$X_9 = 400 - X_1 + X_2 - X_3 + X_4$$
$$X_{10} = 350 - X_5 + X_6 - X_7 + X_8$$
$$X_{11} = 250 - X_1 + X_2 - X_5 + X_6$$
$$X_{12} = -250 - X_2 + X_1 - X_6 + X_5$$
$$X_{13} = 350 - X_3 + X_4 - X_7 + X_8$$
$$X_{14} = -350 - X_4 + X_3 - X_6 + X_7$$

(5) Solution:

Solution is intuitively obvious. Looking at Shipping costs, Two lowest are Edmonton/Saskatoon, and Vancouver/Winnipeg.

Therefore ship 350 from Edmonton to Saskatoon and 250 from Vancouver to Winnipeg. Since these Lie on a vertex ($X_2=0, X_3=0$) this is IN the space.