MAD 6607: Coding Theory

Edoardo Persichetti Test 1

Make sure you explain all your answers clearly and in detail. Answers such as "2" or "Yes" will be treated as incomplete and won't receive full credit.

Exercise 1 What is a linear code and how is it defined?

Exercise 2 Consider the code $\mathcal{C}$ given by the following generator matrix.

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

a) What are the code length and dimension?

b) Write down all the codewords of $\mathcal{C}$ and explain how you obtained them. How many are there and why?

c) What is the minimum distance of this code? How many errors can it correct?

d) Write G in systematic form.

e) Write a parity-check matrix H for $\mathcal{C}$.

f) Verify that H is a parity-check matrix. What property did you use?

Exercise 3 Consider again the code $\mathcal{C}$ and suppose to receive the word $y = 1010111$.

a) Calculate the syndrome of y.

b) Use syndrome decoding to decode y.

Exercise 4 Extend the code $\mathcal{C}$ by adding an overall parity check, and write the new parity-check matrix. What are the new code parameters?

Exercise 5 Give the definition of dual code. Then, find the dual of code $\mathcal{C}$ and list all of its codewords. Is this code self-dual or weakly self-dual (or neither)? Explain.

Exercise 6 Write a matrix defining the code $\mathcal{H}_2$ over $\mathbb{F}_3$. Then, find the dual of this code and list all of its codewords. Is this code self-dual or weakly self-dual (or neither)? Explain.

Hint: remember that parity-check matrices of Hamming codes have pairwise linearly independent columns.