COT 6405
ANLYSIS OF ALGORITHMS

Advanced Data Structure (B-Trees)

Computer & Electrical Engineering and Computer Science Dept.
Florida Atlantic University

Spring 2017

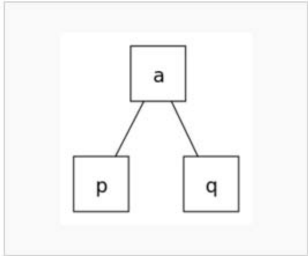
Elementary Data Structures

- Undergraduate Algorithms course:
  - elementary data structures: stacks, queues, linked lists, hash tables, priority queues, binary search trees (BST) (ref. CLRS)
- Objective: data structure where the *dictionary operations* (insert, delete, search) take efficient RT
  - More specifically O(log n)
- **Binary Search Trees (BST)** :
  - all dictionary operations take O(h), where h – height of the tree
  - Not balanced $\Rightarrow$ h = O(n)

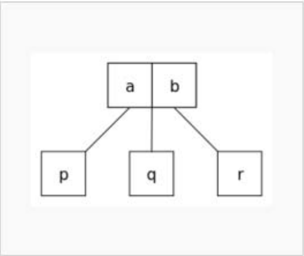
Balanced Search Trees, height = $\Theta(\log n)$

Two approaches:

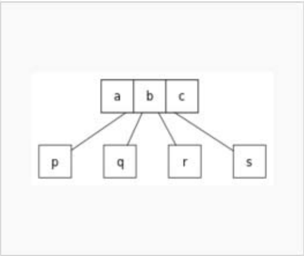
- Transform an unbalanced BST to a balanced one
  - AVL tree: difference between the height of the left & right subtrees of a node never exceeds 1
  - Red-black tree: for any node, the height of a subtree is at most twice as large as the other subtree
  - If insertion/deletion destroys balance $\Rightarrow$ use ***rotations*** to restore the balance
- Representation change: allow more than one element in a node of a search tree
  - Perfectly balanced
  - 2-3-4 trees, B-trees



2-node



3-node



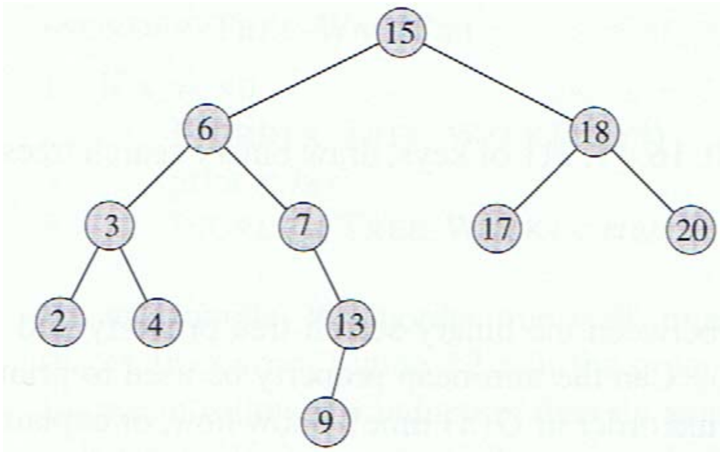
4-node

Balanced Search Trees

- Next:
  - Review BST (CLRS ch 12)
  - Study B-tree (CLRS ch 18)

Binary Search Trees (BST) - REVIEW

- tree T implementation:
  - T.root
  - each node is an object with fields:
    - key (and satellite data)
    - pointers: left, right, p
- the keys of a BST must satisfy the BST property: for any node x
  - if y is a node in x's left subtree then y.key $\leq$ x.key
  - if y is a node in x's right subtree then x.key $\leq$ y.key

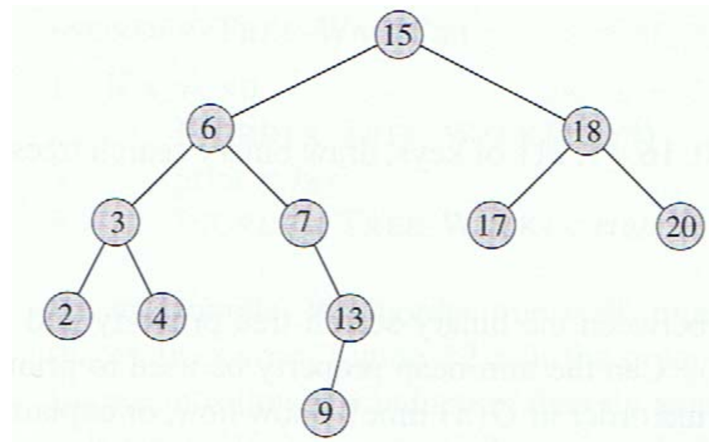


- what is the maximum height h ?

  max h = n-1, therefore h = O(n)

BST-walk: prints all the keys in the tree

- Inorder tree walk:
  - print x's left subtree
  - print node x's key
  - print x's right subtree

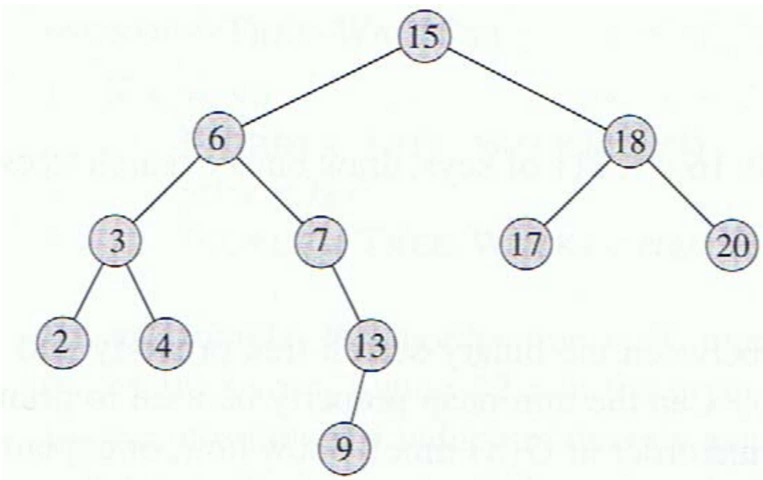



```
INORDER-TREE-WALK(x)
if x ≠ NIL
    INORDER-TREE-WALK(x.left)
    print x.key
    INORDER-TREE-WALK(x.right)
```

- Initial call: INORDER-TREE-WALK (T.root)
- RT = $\Theta(n)$
- example: 2, 3, 4, 6, 7, 9, 13, 15, 17, 18, 20
- **Property:** *prints the keys in sorted order*

BST-walk

- Preorder tree walk:
  - print node x's key
  - print x's left subtree
  - print x's right subtree



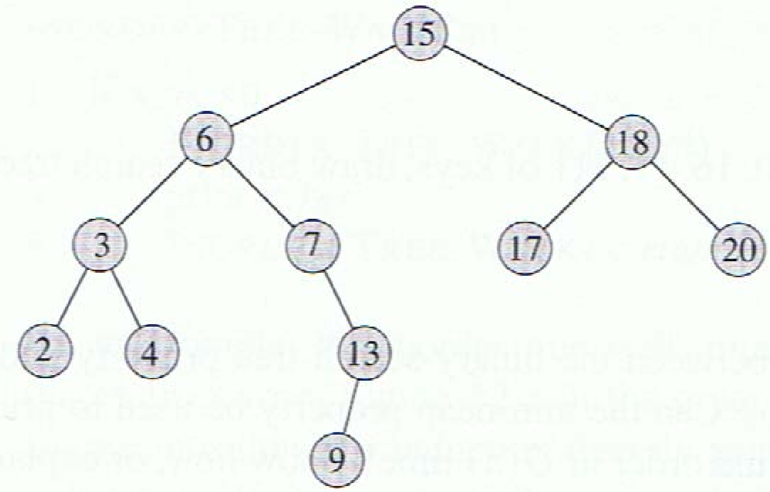
- Postorder tree walk:
  – print x's left subtree
  – print x's right subtree
  – print node x's key

Querying a BST

- operations: search, min, max, successor, predecessor
- all operations take RT = O(h)

SEARCH

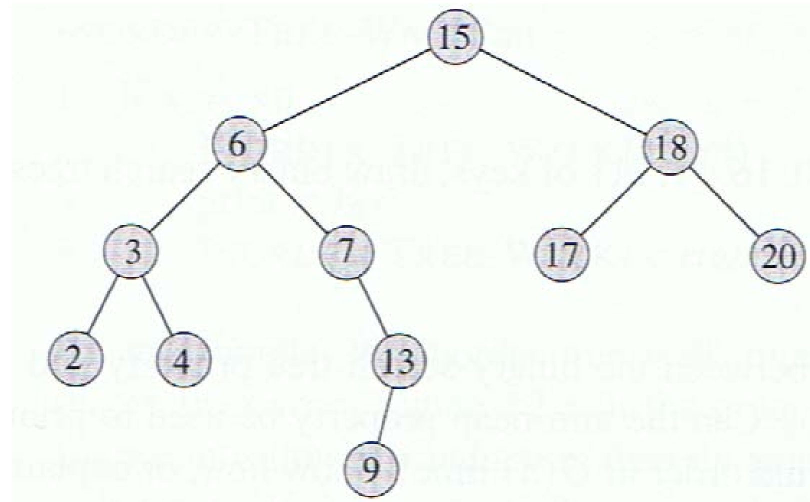
```
TREE-SEARCH(x, k)
if x == NIL or k == x.key
    return x
if k < x.key
    return TREE-SEARCH(x.left, k)
else return TREE-SEARCH(x.right, k)
```



- Initial call: TREE-SERACH (T.root, k)
- RT = O(h)

Minimum & Maximum

```
TREE-MINIMUM(x)
while x.left ≠ NIL
    x = x.left
return x
```



- Initial call: TREE-MINIMUM (T.root)
- RT = O(h)

```
TREE-MAXIMUM(x)
while x.right ≠ NIL
    x = x.right
return x
```

- Initial call: TREE-MAXIMUM (T.root)
- RT = O(h)

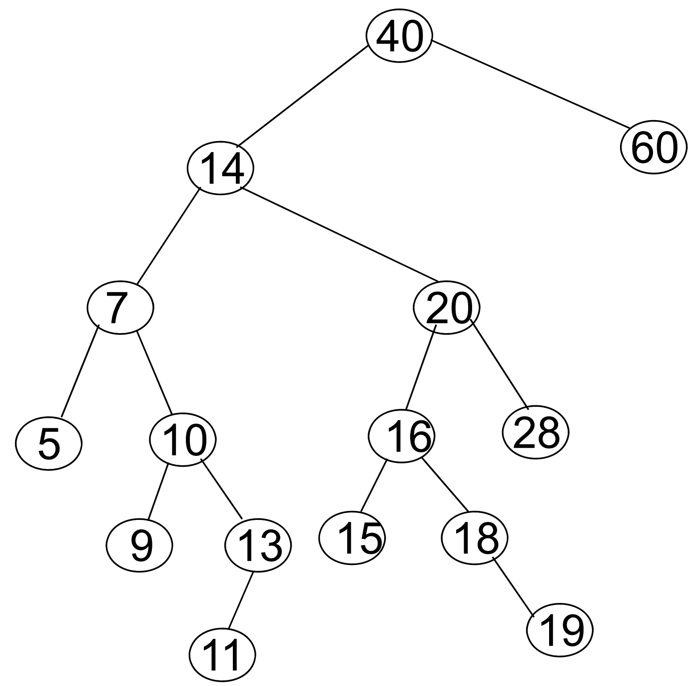
Successor

- Assuming the keys are distinct,

the successor of x is the node y

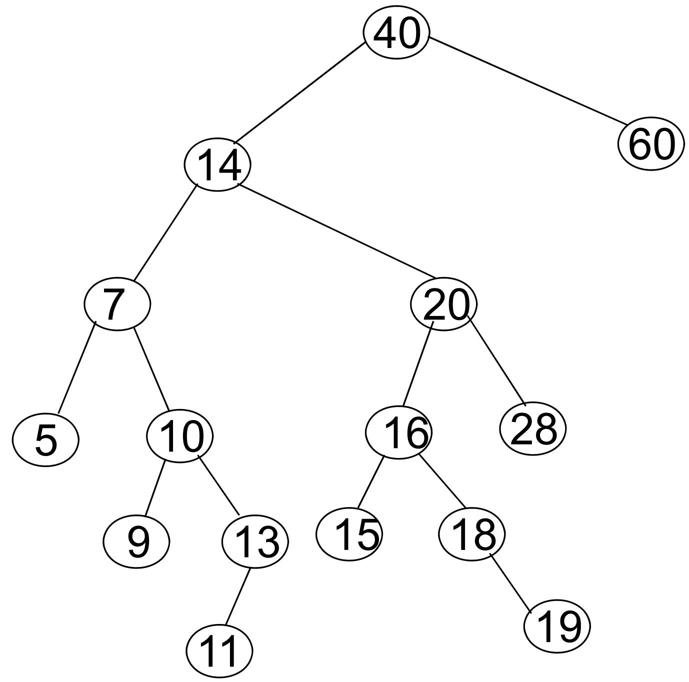
with the smallest key $\geq$ x.key

- Successor of x
  - if x.right $\neq$ NIL, then the successor is the TREE-MINIMUM(x.right)
  - if x.right = NIL, then the successor is the first ancestor larger than x



Successor

```
TREE-SUCCESSOR(x)
if x.right ≠ NIL
    return TREE-MINIMUM(x.right)
y = x.p
while y ≠ NIL and x == y.right
    x = y
    y = y.p
return y
```

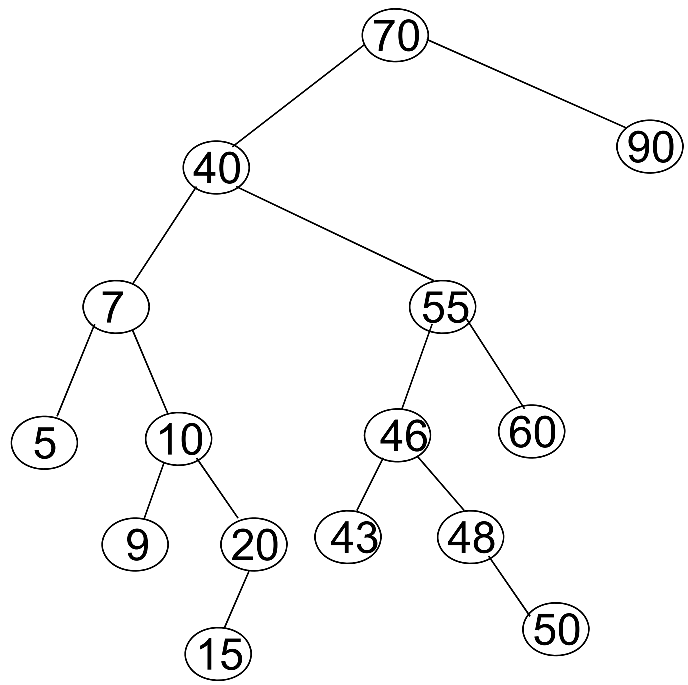


- RT = O(h)

Insert operation

- Assume z.key = some value, z.left = NIL, z.right = NIL

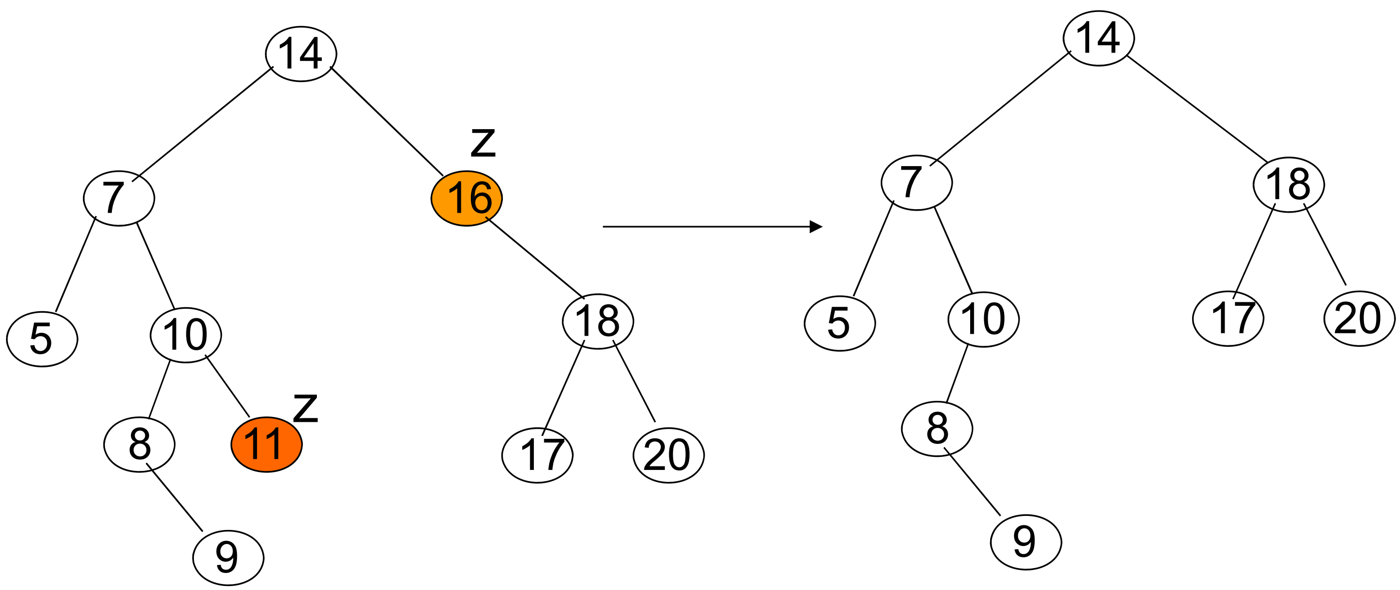
```
TREE-INSERT(T, z)
  y = NIL
  x = T.root
  while x ≠ NIL
      y = x
      if z.key < x.key
          x = x.left
      else x = x.right
  z.p = y
  if y == NIL
      T.root = z          // tree T was empty
  elseif z.key < y.key
      y.left = z
  else y.right = z
```



RT = O(h)

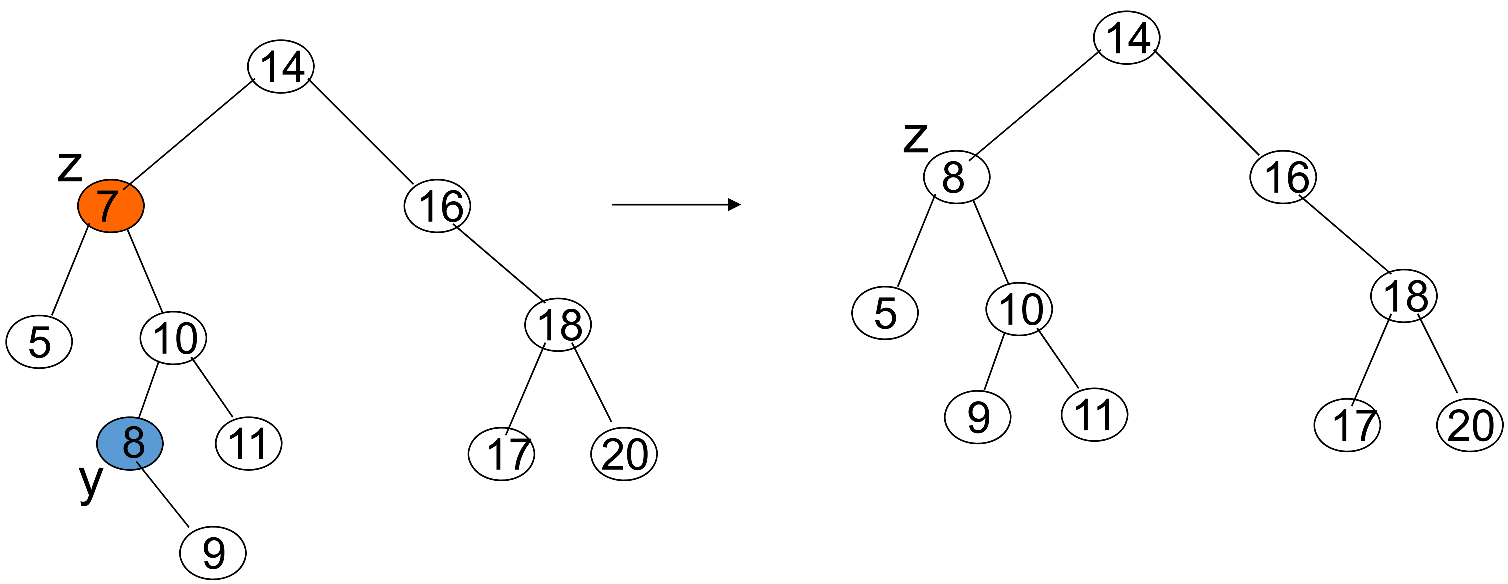
Delete operation

- z has no children
- z has one child



Delete operation

- z has two children



TRANSPLANT operation

- replace the subtree rooted at node u with the subtree rooted at node v

```
TRANSPLANT(T, u, v)
if u.p == NIL
    T.root = v
elseif u == u.p.left
    u.p.left = v
else u.p.right = v
if v ≠ NIL
    v.p = u.p
```

```
TREE-DELETE(T, z)
if z.left == NIL
    TRANSPLANT(T, z, z.right)
elseif z.right == NIL
    TRANSPLANT(T, z, z.left)
else y = TREE-MINIMUM(z.right)
    if y.p ≠ z
        TRANSPLANT(T, y, y.right)
        y.right = z.right
        y.right.p = y
    TRANSPLANT(T, z, y)
    y.left = z.left
    y.left.p = y
```

RT = O(h)