Midterm Exam

Started: Mar 3 at 4:36pm

Quiz Instructions

Question 1

5 pts

What is the Θ - notation for the expression below?

$$\left(1+\frac{1}{2}+\frac{1}{4}+\ldots+\left(\frac{1}{2}\right)^{n}\right)+n\left(\lg n\right)^{100}+n^{2}+1^{n}$$

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Question 2

5 pts

What is the running time for the pseudocode below? Express the running time using Θ - notation as a function of *n*.

```
for i =1 to nlgn
    for j = 1 to i
        x=x+1
    for k = 1 to n
        x=x+2
```

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Question 3

10 pts

Solve the following recurrence using the Master Theorem. State the case and the constant values used:

$$T(n) = T\left(\frac{n}{4}\right) + n^3$$

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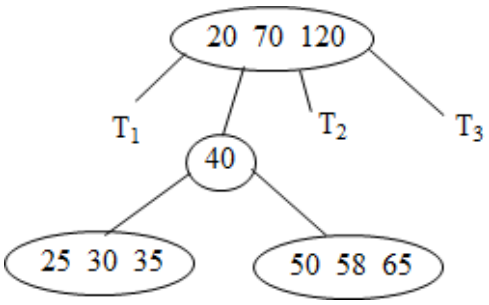
Question 4

10 pts

Consider the B-tree below, which has the minimum degree t = 2. Illustrate the operation B-Tree-Insert (T, 39).



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Question 5

10 pts

Select all the statements below which are TRUE:

- [ ] Backtracking is usually applied to optimization problems.
- [ ] The Traveling Salesman Problem can be solved using exhaustive search in time $\theta(n!)$.
- [ ]

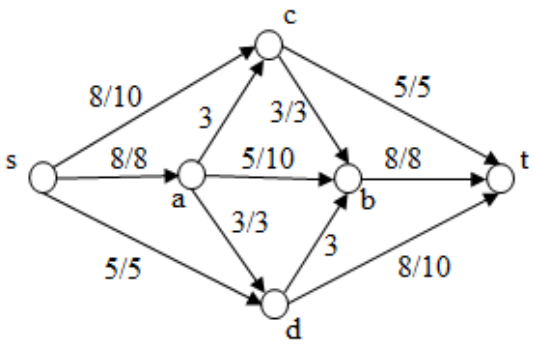
Let T be a B-tree with n keys and minimum degree t > 1. If the root x is not a leaf, then x must have between t and 2t children.

- [ ] The Hamiltonian Cycle problem can be solved in polynomial time using Brute Force.
- [ ] Let G(V, E) be a graph. The running time to check if a group of n vertices form an independent set is $\theta\left(n^2\right)$.
- [ ] Let T be a balanced BST, where all the leaves have the same depth. Then the running time for TREE-DELETE(T,z) is O(lgn).

Question 6

10 pts

Consider the flow network below:



a) (2 pts) What is the value of the flow |f| ?

b) (3 pts) Write the residual network G_f.

c) (2 pts) Can you find an augmenting path in G_f? If yes, write the path.

d) (3 pts) Is f a maximum-flow? If yes, then find a cut(S,T) such that |f| = c(S,T).

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Question 7

10 pts

Let A and B be two matrices of size n x n, where n is a power of 3. The objective is to compute the product AB. We design a Divide-and-Conquer algorithm for computing the product AB as follows:

- if n > 1, then divide A and B into nine n/3 x n/3 matrices:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \text{ and } B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

and compute the product as:

$$AB = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{pmatrix}$$

- The base case is when n = 1

a) (3 pts) What is the running time for each step: divide, conquer, combine ?

b) (5 pts) Write the recurrence for the Divide-and Conquer algorithm, an compute the running time by solving this recurrence.

c) (2 pts) Which algorithm is more efficient: this Divide-and-Conquer algorithm or the Brute Force solution?

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