Circulations w/ demands and lower bounds

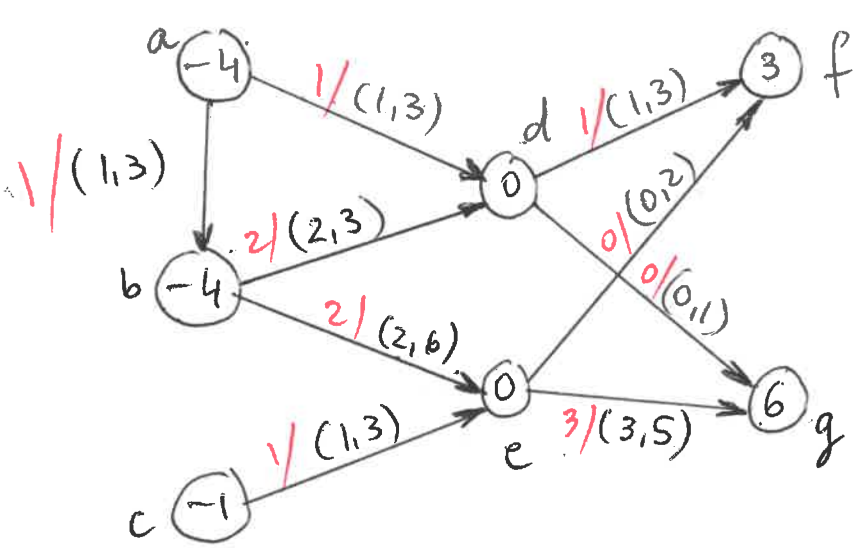
3.17.2017

$G \longrightarrow G' \longrightarrow G''$

circulation w/ demands and lower bounds | circulation w/ demands | max-flow problem

Example

Consider the graph $G(V,E)$ below, with the following constraints for demands, capacity, and lower bounds:

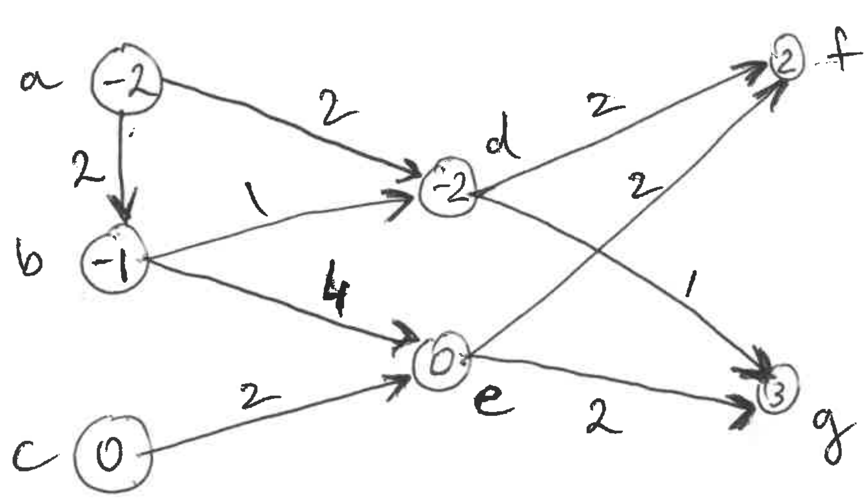


flow f_0

Find a feasible circulation.

Solution

graph G'



- start with a circulation f_0 s.t. $f_0 = \ell_e$
- then adjust the demands in the vertices:

$$\begin{cases} L_v = f_0^{in}(v) - f^{out}(v) \\ \text{new demand} = d_v - L_v \end{cases}$$

- the capacity of each edge is $c_e - \ell_e$

vertex a

$L_a = 0 - 2 = -2$

new demand $= -4 - (-2) = -2$

vertex b

$L_b = 1 - 4 = -3$

new demand $= -4 - (-3) = -1$

vertex c

$L_c = 0 - 1 = -1$

new demand $= -1 - (-1) = 0$

vertex d

$L_d = 3 - 1 = 2$

new demand $= 0 - 2 = -2$

vertex e

$L_e = 3 - 3 = 0$

new demand $= 0 - 0 = 0$

vertex f

$L_f = 1 - 0 = 1$

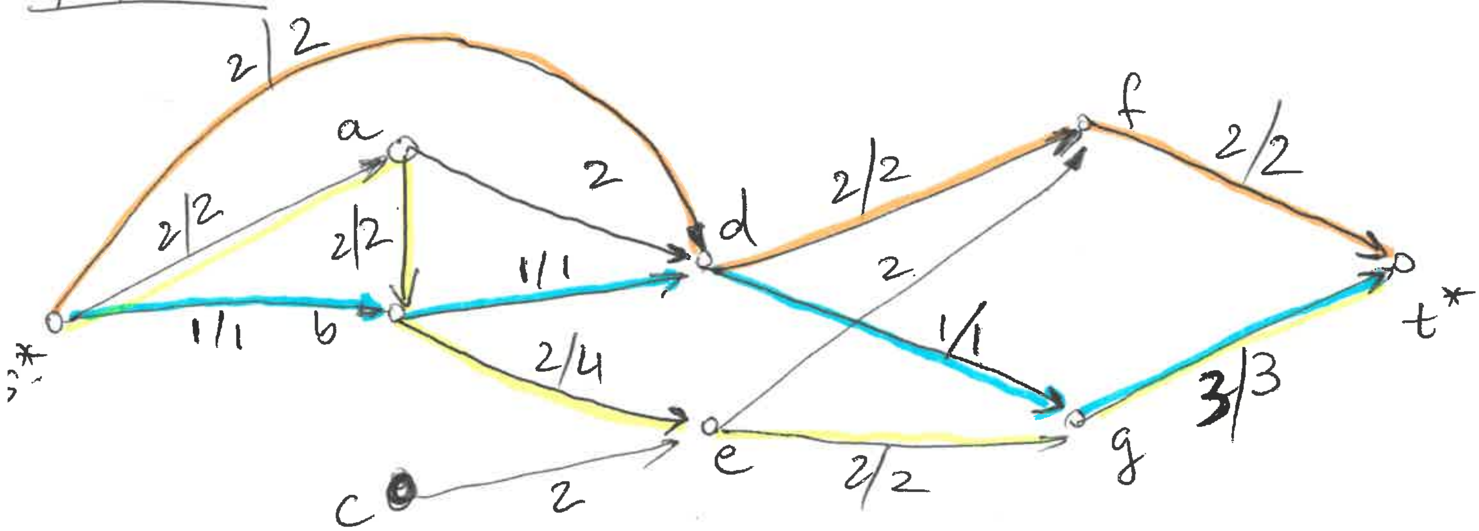
new demand $= 3 - 1 = 2$

vertex g

$L_g = 3 - 0 = 3$

new demand $= 6 - 3 = 3$

graph G''



find a max-flow for G''

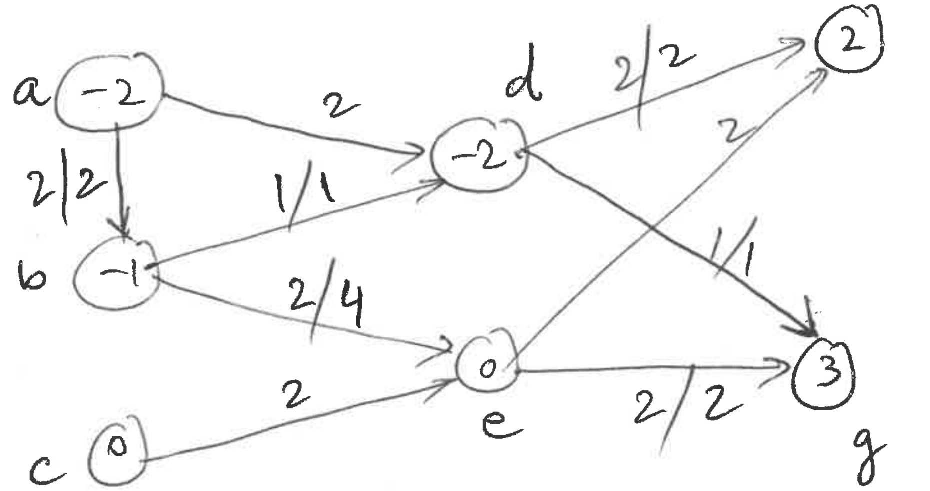
$p = \langle s^*, d, f, t^* \rangle \quad c_f(p) = 2$

$p = \langle s^*, a, b, e, g, t^* \rangle \quad c_f(p) = 2$

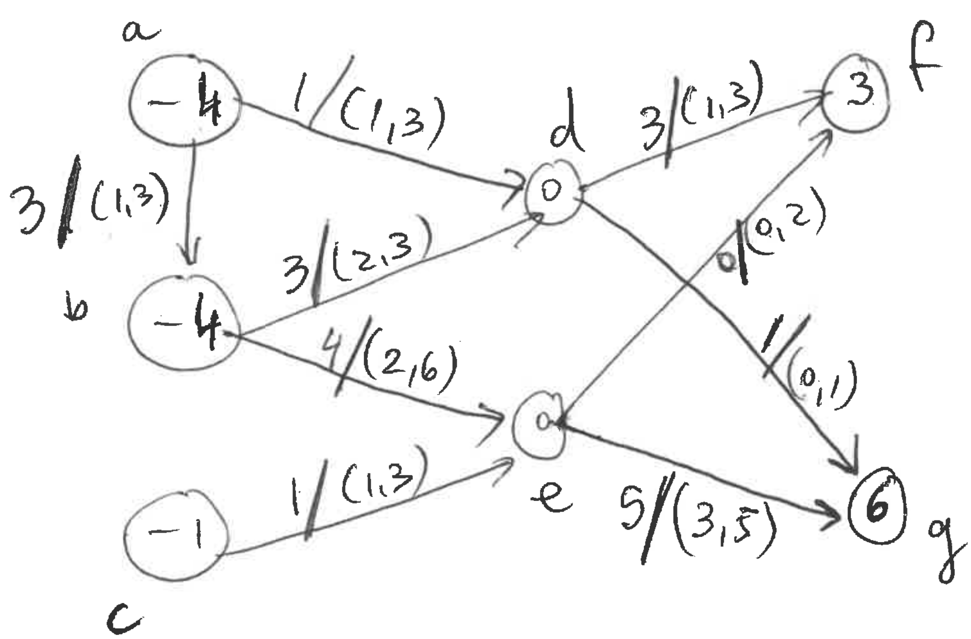
$p = \langle s^*, b, d, g, t^* \rangle \quad c_f(p) = 1$

$|f| = 5 = D \Rightarrow G'$ has a feasible circulation

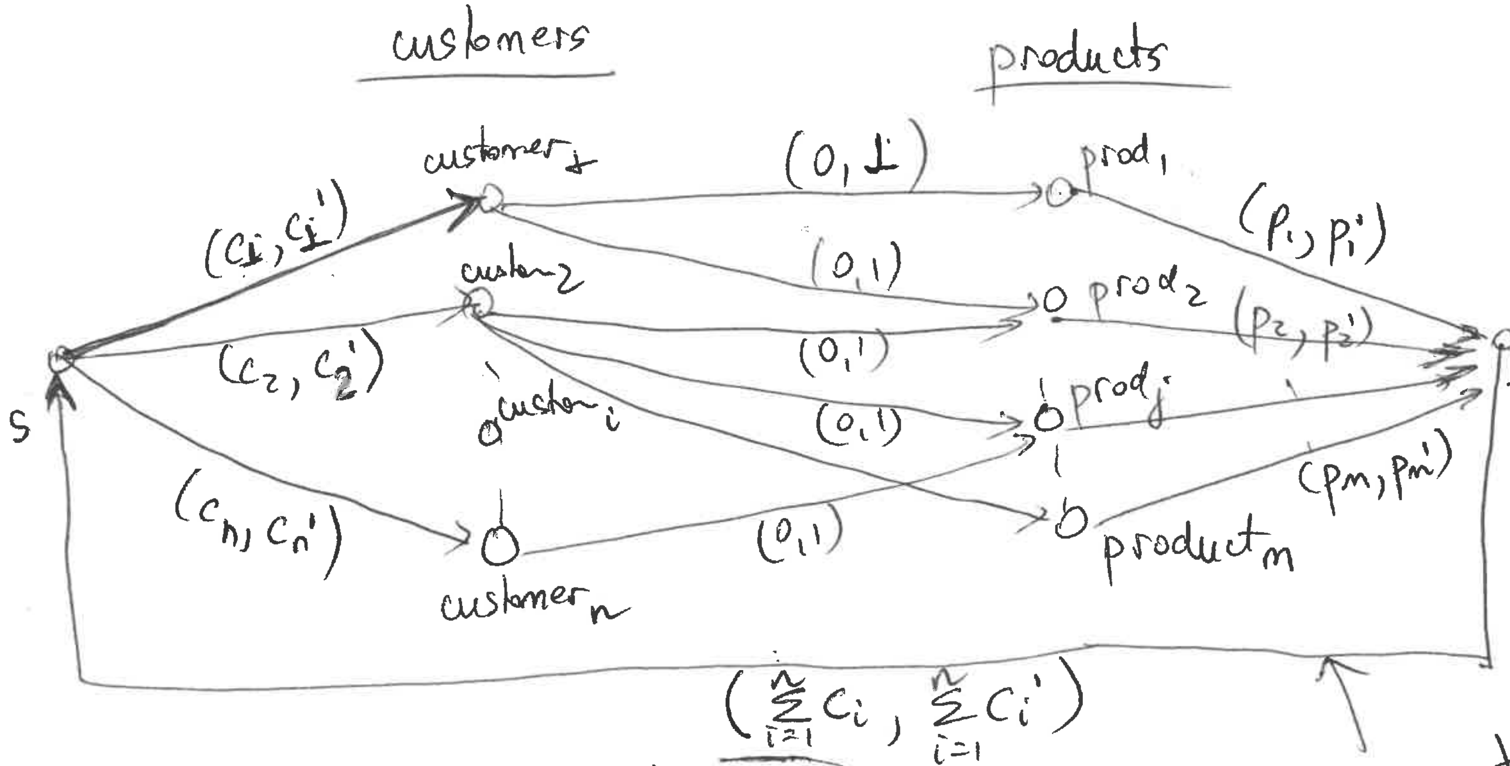
Circulation for G'



Circulation for G , computed as $f' + f_0$



Survey Design problem



flow on this edge = how many questions were asked in the survey

customer i : (c_i, c_i')

product j : (p_j, p_j')

given

- all demands $= 0$

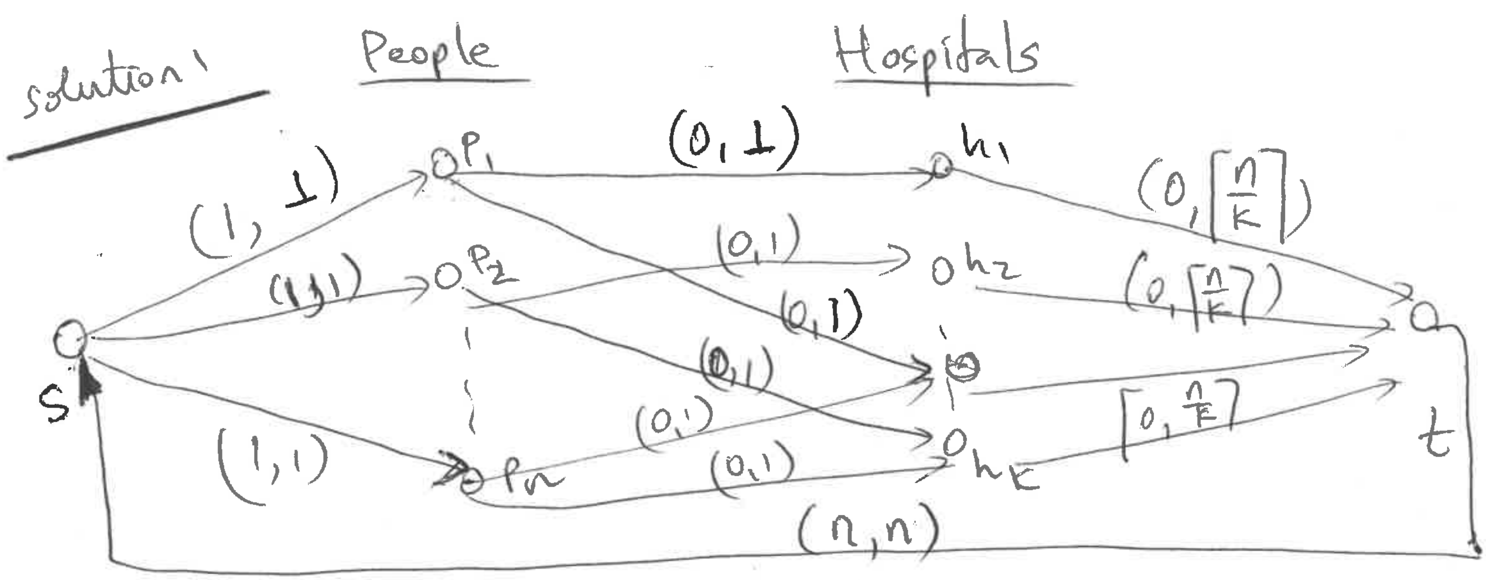
Network flow issues come up in dealing with natural disasters and other crises, since major unexpected events often require the movement and evacuation of large numbers of people in a short amount of time.

Consider the following scenario. Due to large-scale flooding in a region, paramedics have identified a set of n injured people distributed across the region who need to be rushed to hospitals. There are k hospitals in the region, and each of the n people needs to be brought to a hospital that is within a half-hour's driving time of their current location (so different people will have different options for hospitals, depending on where they are right now).

At the same time, one doesn't want to overload any one of the hospitals by sending too many patients its way. The paramedics are in touch by cell phone, and they want to collectively work out whether they can choose a hospital for each of the injured people in such a way that the load on the hospitals is *balanced*: Each hospital receives at most $\lceil n/k \rceil$ people.

Give a polynomial-time algorithm that takes the given information about the people's locations and determines whether this is possible.

solution:



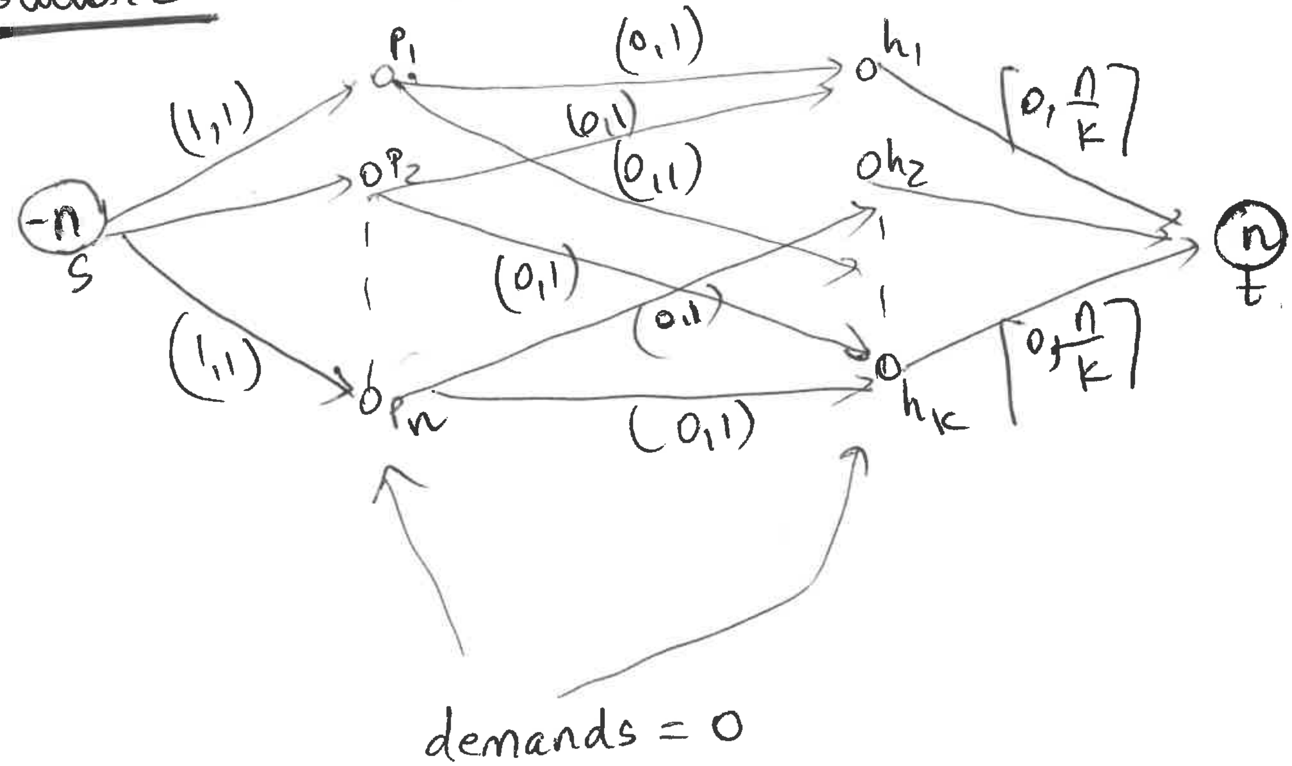
$n=9$
$k=3$

$n=11$
$k=3$

$\lceil \frac{n}{k} \rceil = \lceil \frac{11}{3} \rceil = 4$ 3, 4, 4

- all demands = 0

Solution 2



RT analysis

Ford-Fulkerson $RT = O(|f^*| \cdot E)$

$|f^*| = n$

$|E| \leq n \cdot k + n + k + 1 \Rightarrow |E| = O(n \cdot k)$

$\Rightarrow \boxed{RT = O(n^2 \cdot k)}$ polynomial RT