Disjoint-paths in a directed graph

Input
graph $G(V,E)$ directed
vertices s and t

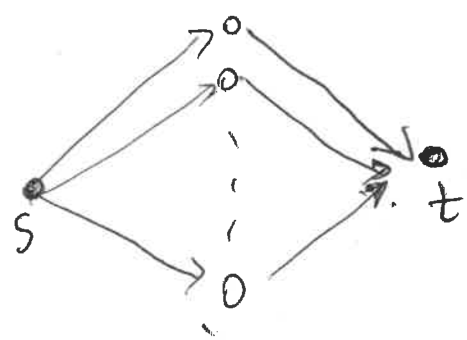
$\longrightarrow$

Flow Network
- graph $G(V,E)$ directed
- source s and sink t
- capacity 1 on all edges

$\longrightarrow$ Max-flow solver $\longrightarrow$ max-flow f $\longrightarrow$ starting from s, follow the flow (with value >0) to return disjoint paths

RT analysis

Ford-Fulkerson $\Rightarrow$ $RT = O(|f^*| \cdot E)$

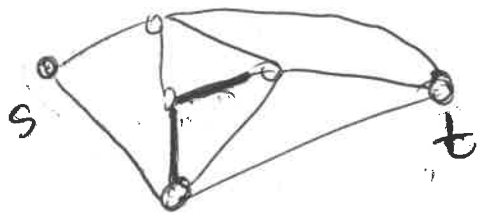


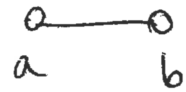
$|f^*| \leq |V| - 2$

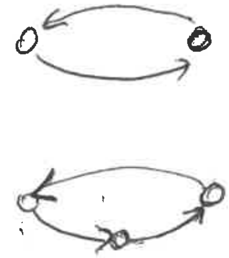
$|f^*| = O(V)$

$\boxed{RT = O(V \cdot E)}$

Disjoint paths in undirected graphs







Input

graph $G(V,E)$ undirected
vertices s and t

→

Flow Network

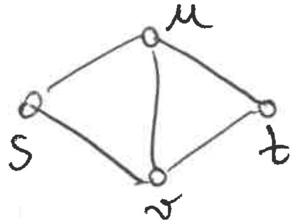
- build directed graph $G'(V',E')$
  - replace each edge (u,v) by two directed edges 

  - remove edges into s, and out of t
  - remove antiparallel edges
- source s, sink t
- capacity 1 on all edges

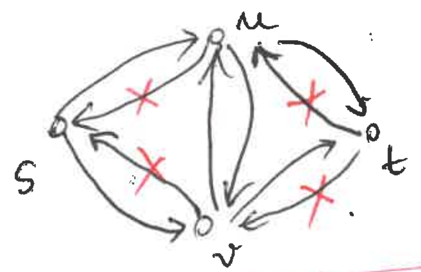
→ Max-flow Solver → Max-flow f → remove flow on antiparallel edges → starting from s, follow the flow (with value >0) to return disjoint paths

example

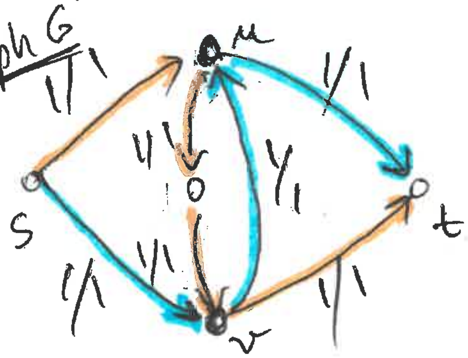
graph $G(V,E)$



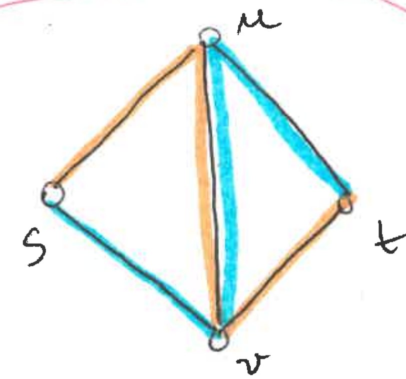
graph $G'(V',E')$



graph G'



- remove antiparallel edges
- add capacity 1



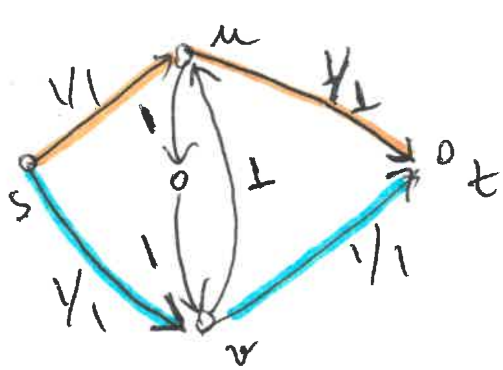
graph G

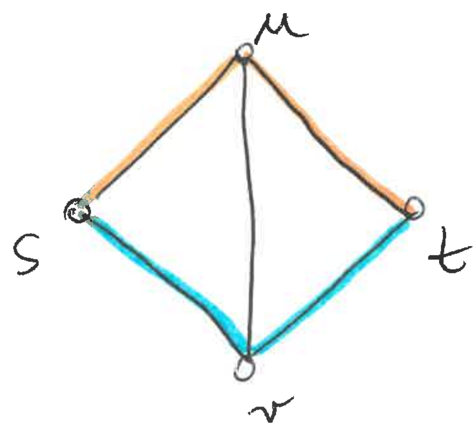
Not correct: overlapping edges in the resulting paths!

remove flow on antiparallel edges:

$\delta = \min\{f(u,v), f(v,u)\}$

$\delta = 1$





graph G

- edge disjoint paths in G

RT analysis

Ford - Fulkerson $\Rightarrow RT = O(|f^*| \cdot E')$

$|f^*| \leq |V| - 2 \Rightarrow |f^*| = O(V)$

$|E'| \leq 2 \cdot |E| + |E| = 3 \cdot |E| \Rightarrow |E'| = O(E)$

$\Rightarrow$

$\Rightarrow \boxed{RT = O(V \cdot E)}$