

Question 1

$$A = \langle 2, 3, 4, 6, 7, 8, 9, 12 \rangle$$

Question 3

$$\Theta(n^2)$$

Question 6

$$\Theta(n^6)$$

Question 7

$$f_6, f_5, f_3, f_1, f_4, f_2$$

Question 8

- **O-notation**

$$c, n_0 = ? \quad 0 \leq 3n^3 - 7n + 500 \leq cn^4$$

$$3n^3 - 7n + 500 \geq 0$$

$$3n(n^2 - 7/3) + 500 \geq 0 \quad \text{true for } n \geq \sqrt{7/3} = 1.527$$

$$3n^3 - 7n + 500 \leq cn^4$$

$$cn^4 - 3n^3 + 7n - 500 \geq 0$$

$$\text{let } c = 3$$

$$3n^3(n-1) + 7(n - 500/7) \geq 0$$

$$\text{True for } n \geq 1 \text{ and } n \geq 500/7 = 71.428$$

$$\text{Solution: } c = 3, n_0 = 72$$

- **ω - notation**

$$n_0 = ? \quad 0 \leq cn^2 < 3n^3 - 2n + 10$$

$$3n^3 - 2n + 10 - cn^2 > 0$$

$$n^3 - cn^2 + 2n^3 - 2n + 10 > 0$$

$$n^2(n - c) + 2n(n^2 - 1) + 10 > 0$$

$$n \geq c \quad n \geq 1$$

$$n_0 = \max(c, 1)$$

Question 9

1. First Recurrence

$$T(n-1) = 2T(n-2) + 3$$

$$T(n) = 4T(n-2) + 2 \cdot 3 + 3$$

$$T(n-2) = 2T(n-3) + 3$$

$$T(n) = 2^3T(n-3) + 2^2 \cdot 3 + 3$$

$$T(n) = 2^4T(n-4) + 2^3 \cdot 3 + 2^2 \cdot 3 + 3$$

$$T(n) = 2^iT(n-i) + 2^{i-1} \cdot 3 + 2^{i-2} \cdot 3 + \dots + 2 \cdot 3 + 3$$

$$T(n) = 2^{n-1}T(1) + 2^{n-2} \cdot 3 + 2^{n-3} \cdot 3 + \dots + 2 \cdot 3 + 3$$

$$T(n) = 2^{n-1}4 + 3(2^{n-1} - 1)/(2 - 1) = 2^{n-1}4 + 3 \cdot 2^{n-1} - 3 = 7 \cdot 2^{n-1} - 3$$

$$T(n) = \Theta(2^n)$$

2. Second Recurrence

$$m = \lg n \rightarrow n = 2^m$$

$$T(2^m) = T(2^{3m/4}) + 5$$

$$S(m) = T(2^m)$$

$$S(m) = S(3m/4) + 5$$

$$5 = \Theta(1)$$

$$\text{Case 2 Master Thm} \rightarrow S(m) = \Theta(\lg m)$$

$$T(n) = \Theta(\lg \lg n)$$

Question 10

to be discussed in class