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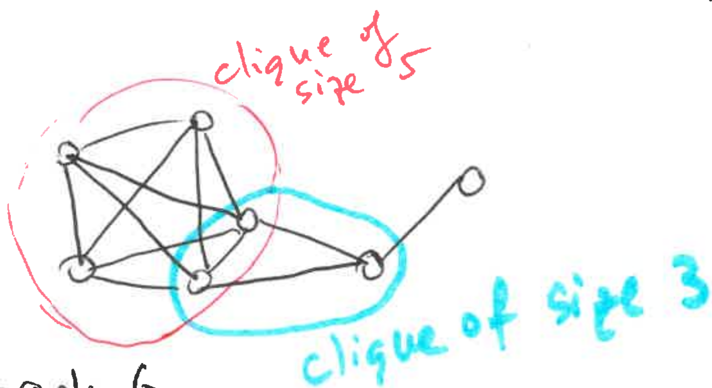
The clique problem

Let $G(V,E)$ be an undirected graph

A clique is a subset of vertices $V' \subseteq V$ s.t. for any two vertices $u,v \in V'$, $(u,v) \in E$.



graph G

Problem definitions

- optimization problem : Given a graph $G(V,E)$ undirected, find a clique of maximum size.
- decision problem : Given a graph $G(V,E)$ undirected and a value k, does G have a clique of size k?

CLIQUE problem

Brute-force let $n = |V|$

- take all groups of k vertices $\binom{n}{k} = \Theta(n^k)$
- check if a group is a clique $\binom{k}{2} = \Theta(k^2)$

total RT $= O(n^k \cdot k^2)$

if $k = \Theta(n)$ $\Rightarrow$ then total RT $= O(n^n \cdot n^2)$

superpolynomial

Thm CLIQUE is NP-complete.

- CLIQUE $\in$ NP

certificate = $V' = \{v_1, v_2, \ldots, v_k\}$ - vertices in the clique

verification alg. (polynomial)

adj-lists graph representation

- check that the vertices are distinct $\Rightarrow O(n)$
- for each vertices $v_i, v_j \in V'$, check if $(v_i, v_j) \in E$ $\Rightarrow O(n^3)$

$O(n^3)$

- CLIQUE is NP-hard

$$3\text{-CNF-SAT} \leq_p \text{CLIQUE}$$

Reduction algorithm - takes an instance of 3-CNF-SAT and reduces it (in polynomial time) to an instance of the CLIQUE problem.

let ϕ be a 3-CNF formula

exp: $\phi = (x_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_2 \vee x_3)$

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge \ldots \wedge C_k$$

clause C_r has 3 literals $C_r = l_1^r \vee l_2^r \vee l_3^r$

Construct a graph G:

- for each clause $C_r = l_1^r \vee l_2^r \vee l_3^r$ add 3 vertices v_1^r, v_2^r, v_3^r to G
- add an edge (v_i^r, v_j^s) if $r \neq s$ and if the corresponding literals are consistent (cannot be x_1 and $\bar{x}_1$ for example)