COT 6405
ANLYSIS OF ALGORITHMS

Approximation Algorithms

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Outline

- Performance ratios
- The vertex-cover (VC) problem
- The traveling-salesman problem (TSP)

Reference: CLRS ch 35

Motivation

- Many problems of practical significance are NP-complete
- 3 ways to get around NP-completeness
  - if input sizes are small, use brute force (superpolynomial)
  - isolate important special cases that can be solved polynomial time
  - design near-optimal solutions (approximation algorithms)

Approximation ratio

- optimization problems: maximization or minimization
- let C – solution returned by an algorithm and C* – optimal solution
- an algorithm has an ***approximation ratio*** $\rho(n)$ if for any input size n, the cost C produced by the algorithm is within a factor of $\rho(n)$ of the cost C* of an optimal solution
- such an algorithm is called **ρ(n) – approximation** algorithm

Approximation ratio

- $\rho(n)$-approximation algorithm:

$$\max\left(\frac{C}{C^*}, \frac{C^*}{C}\right) \le \rho(n)$$

- Maximization problem:

  $0 \le C \le C^*$

$$\boxed{\frac{C^*}{C} \le \rho(n)}$$

- Minimization problem:

  $0 \le C^* \le C$

$$\boxed{\frac{C}{C^*} \le \rho(n)}$$

More on the approximation algorithms

- approximation ratio ρ is always ≥ 1
    - an 1 – approximation algorithm is actually the *optimal* solution
- **Polynomial Time Approximation Scheme (PTAS)**: for any input size *n*, and a fixed $\varepsilon > 0$, the scheme is a $(1+\varepsilon)$ – approximation algorithm, running in time polynomial on *n*
    - e.g. RT = $O(n^{2/\varepsilon})$
- **Fully PTAS**: if the RT is polynomial in *n* and $1/\varepsilon$
    - e.g. RT = $O((1/\varepsilon)^2 n^3)$

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