$\boxed{T(n) = \Theta(n \cdot \lg^2 n)}$

1.20.2017

Correctness

- show that $T(n) = O(n \cdot \lg^2 n)$ using induction

$T(n) \le c\, n \lg^2 n$ for some const $c > 0$

Inductive step

assume that $T(k) \le c k \lg^2 k$ for all $k < n$

show that $T(n) \le c n \lg^2 n$

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + n \lg n \le \cancel{2} c \frac{n}{\cancel{2}} \lg^2 \frac{n}{2} + n \lg n \overset{?}{\le} c n \lg^2 n$$

$$c \lg^2 \frac{n}{2} + \lg n \le c \cdot \lg^2 n$$

$$c(\lg n - \underbrace{\lg 2}_{=1})^2 + \lg n \le c \cdot \lg^2 n$$

$$\cancel{c \lg^2 n} - 2c \lg n + c + \lg n \le \cancel{c \cdot \lg^2 n}$$

$$c \le 2c \lg n - \lg n$$

$$c \le (2c - 1) \lg n$$

$$\begin{cases} \text{let } c = 1 \\ 1 \le \lg n \Rightarrow n \ge 2 \end{cases}$$

- show that $T(n) = \Omega(n \lg^2 n)$

$T(n) \ge d\, n \lg^2 n$ for some const $d > 0$

Inductive step

assume that $T(k) \geq d\,k\,\lg^2 k$ for all $k < n$

show that $T(n) \geq d\,n\,\lg^2 n$

$$T(n) = 2\,T\left(\tfrac{n}{2}\right) + n\lg n \geq \underbrace{\cancel{2}\cdot d\,\frac{n}{\cancel{2}}\lg^2\frac{n}{2} + n\lg n \overset{?}{\geq} d\,n\lg^2 n}$$

$$d\lg^2\frac{n}{2} + \lg n \geq d\lg^2 n$$

(same computation)

$$d \geq (2d-1)\lg n$$

true for $d = \frac{1}{2}$

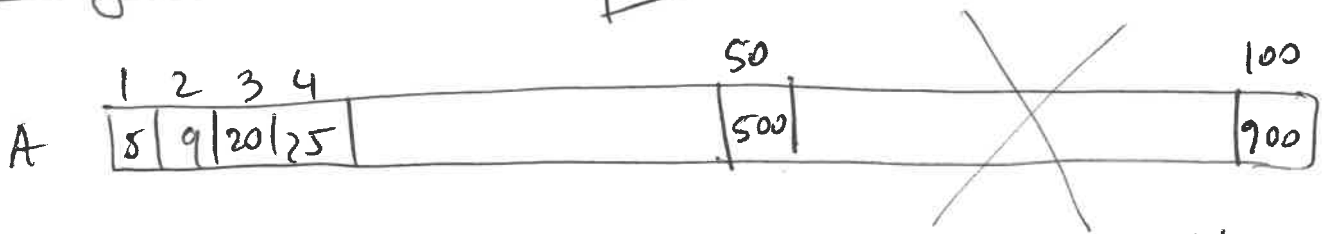
Conclusion:

$T(n) = \Theta(n\lg^2 n)$ is the correct solution

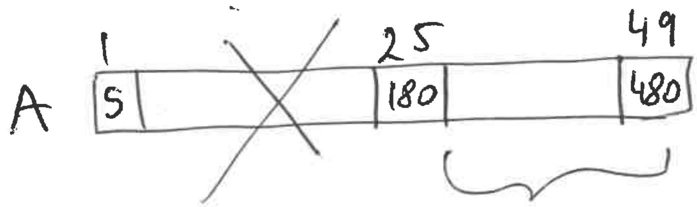
Sublinear Time

Binary Search

$P = 250$



$\left\lfloor \frac{1+100}{2} \right\rfloor = 50$



$\left\lfloor \frac{1+49}{2} \right\rfloor = 25$

$\boxed{RT = O(\lg n)}$

Linear time $O(n)$

- merge two sorted arrays into a sorted array

$A = \langle 2, 4, 6, 20 \rangle$ n elms

$B = \langle 5, 10, 14, 18 \rangle$ n elms

$C = \langle 2, 4, 5, 6, 10, 14, 18, 20 \rangle$

at most $2n$ iterations $\Rightarrow RT = O(n)$

Independent set of size K

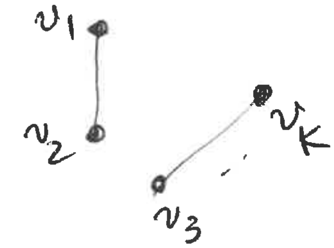
- How many sets of size K does the graph have?

  n = no. of vertices

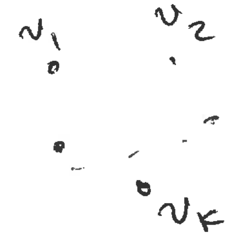
let $K=4$

$$\binom{n}{4} = \frac{n!}{(n-4)!\,4!} = \frac{n(n-1)(n-2)(n-3)}{4!} = \Theta(n^4)$$

$$\binom{n}{K} = \Theta(n^K)$$



- Check if a group of size K is independent or not



$$\binom{K}{2} = \frac{K!}{(K-2)!\cdot 2!} = \Theta(K^2)$$

K const $\Rightarrow \Theta(1)$

total RT $= \Theta(n^K \cdot \underset{\substack{\uparrow \\ K \text{ is const}}}{K^2}) = \Theta(n^K)$

Finding an independent set of max-size.

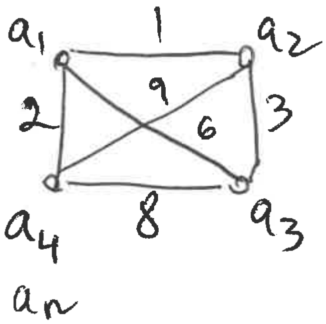
2^n subsets

$v_1 \; v_2 \; v_3 \; - \; v_5 \; - \; v_n$

$\boxed{0 \; 1 \; 1 \; 0 \; 1 \; 0 \; - \; - \; 1}$

2^n binary strings of length n

Traveling Salesman Problem



let a_1 = starting city

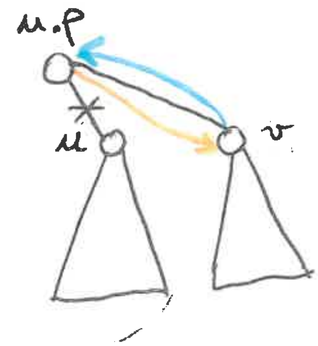
$$\left.\begin{array}{l} a_1 a_2 a_3 a_4 \\ a_1 a_2 a_4 a_3 \\ - \; - \; - \; - \end{array}\right\} (n-1)! \text{ tours}$$

$\Theta(n)$ to compute the cost of a tour

$\boxed{\text{total RT} = \Theta(n!)}$

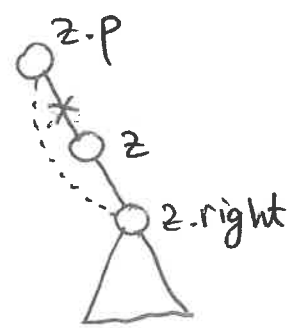
BST-Delete operation

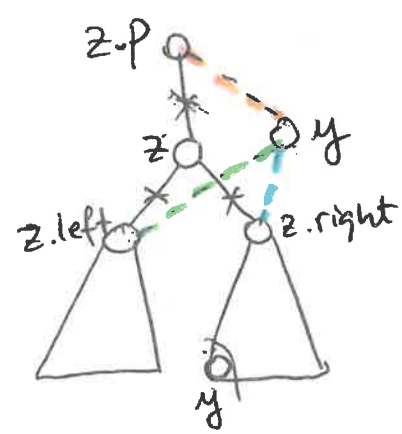
```
TRANSPLANT(T, u, v)
if u.p == NIL
    T.root = v
elseif u == u.p.left
    u.p.left = v
else u.p.right = v
if v ≠ NIL
    v.p = u.p
```

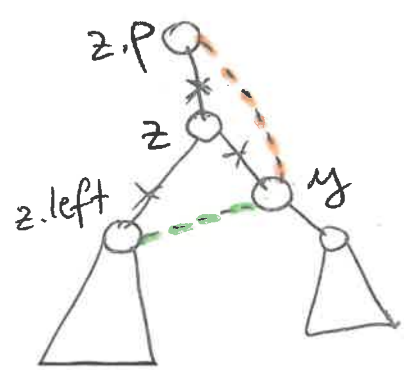


$\boxed{RT = \Theta(1)}$

```
TREE-DELETE(T, z)
if z.left == NIL
    TRANSPLANT(T, z, z.right)
elseif z.right == NIL
    TRANSPLANT(T, z, z.left)
else y = TREE-MINIMUM(z.right)
    if y.p ≠ z
        TRANSPLANT(T, y, y.right)
        y.right = z.right
        y.right.p = y
    TRANSPLANT(T, z, y)
    y.left = z.left
    y.left.p = y
```







$\boxed{RT = O(h)}$ – because of the TREE-MINIMUM call