8a) Show

$3n^3 - 7n + 500 = O(n^4)$

this implies $g(n) = n^4$

$0 \le 3n^3 - 7n + 500 \le c \cdot n^4$

$0 \le 3n^3 - 7n + 500$

using Descartes rule of signs we see that there is one negative zero. It can be estimated to be on the interval $(-6, -5)$. Therefore for values of $n > -5$ $f(n) > 0$

Show $3n^3 - 7n + 500 \le c \cdot n^4$ for some c & n

Let $c = 1$

$3n^3 - 7n + 500 \le n^4$

$0 \le n^4 - 3n^3 + 7n - 500$

It can be shown that the inequality is true for $n < -4.19$ $n \ge 5.61$

Since $n > -5$ we choose the MAXIMUM $n \ge 5.61$

$c = 1, n_0 = 6$

8b)

$3n^3 - 2n + 10 = \omega(n^2)$

Show

$0 \le c \cdot n^2 \le 3n^3 - 2n + 10$

$0 \le cn^2$ true for all $c > 0$

$cn^2 \le 3n^3 - 2n + 10$

$0 \le 3n^3 - cn^2 - 2n + 10$

$0 \le n^2(3n - c) + (10 - 2n)$

$n > c/3$

$10 - 2n$ may be ignored for values of $n > 0$ since n^2 grows faster than $10 - 2n$

$n_0 = \max(c/3, 0)$

Let $c = 6$ $n_0 \ge 2$

$n_0 \ge 2, c = 6$