flow-network
(standard form)
- graph $G(V, E)$ directed
- source s, sink t
- capacities c

→ Max-flow solver → maximum-flow f

Networks with antiparallel edges

Network
- graph $G(V, E)$ directed
- Source s, sink t
- G may have antiparallel edges
- capacities c

→

flow-network
- graph $G'(V', E')$ directed
- source s, sink t
- capacities c

→ Max-flow solver → max-flow for G' → max-flow for G

Networks with multiple sources, multiple sinks

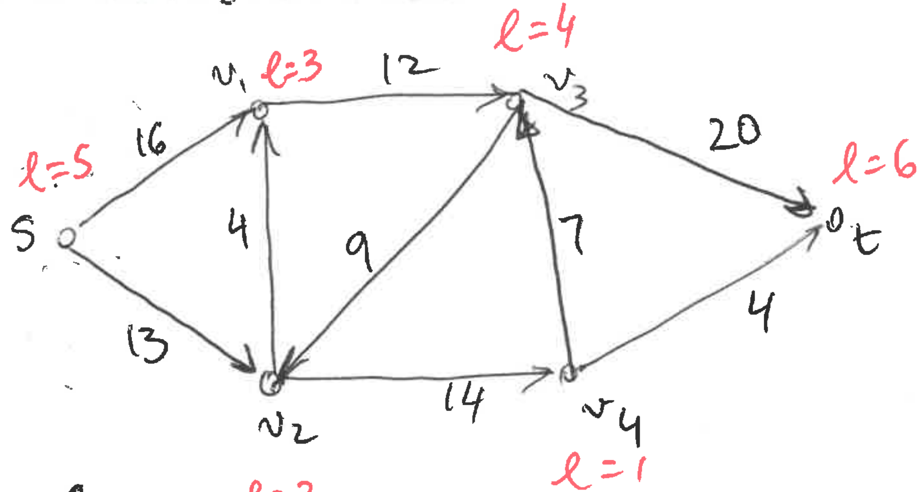
Network
- graph $G(V, E)$ directed
- multiple sources, multiple sinks
- capacities c

→

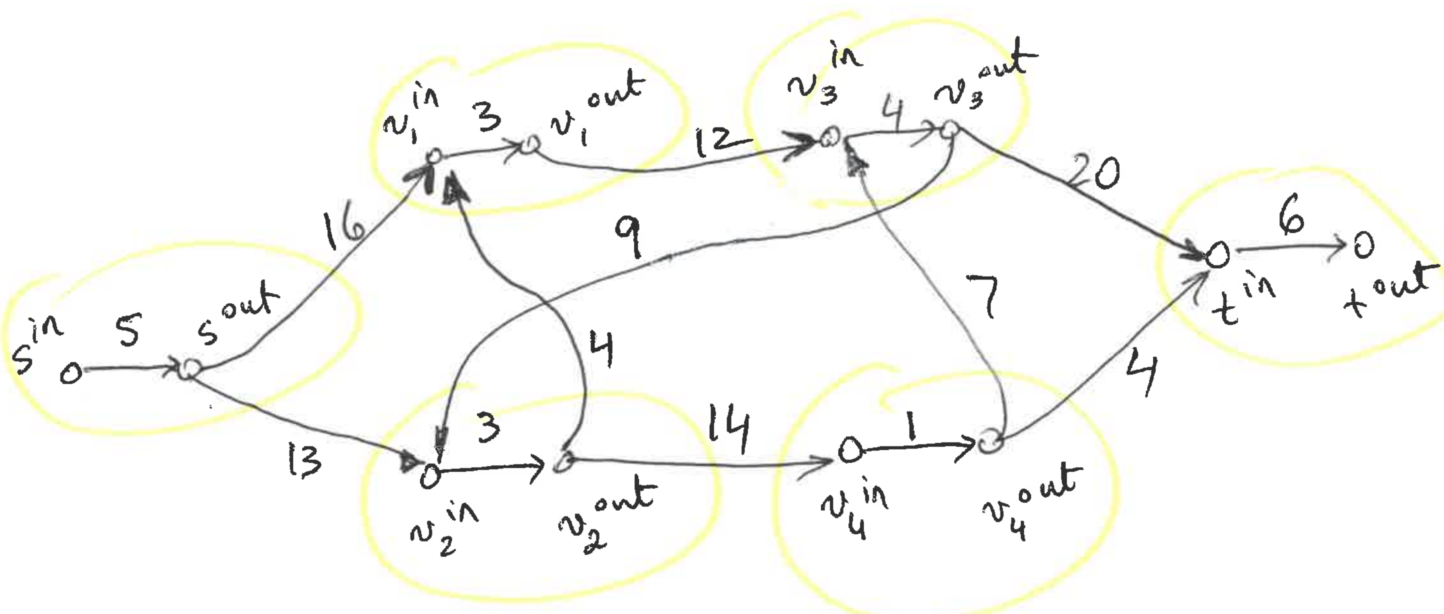
flow network
- graph $G'(V', E')$ directed
- source s, sink t
- Capacities c

→ Max-flow solver → max-flow for G' → max-flow for G

Suppose that in addition to edge capacities, a flow network has vertex capacities. That is, each vertex v has a limit l(v) on how much flow can pass through v. Show how to transform a flow network G = (V, E) with vertex capacities into an equivalent flow network G' = (V', E') without vertex capacities, such that a maximum flow in G' has the same value as a maximum flow in G. How many vertices and edges does G' have?



graph G



graph G'

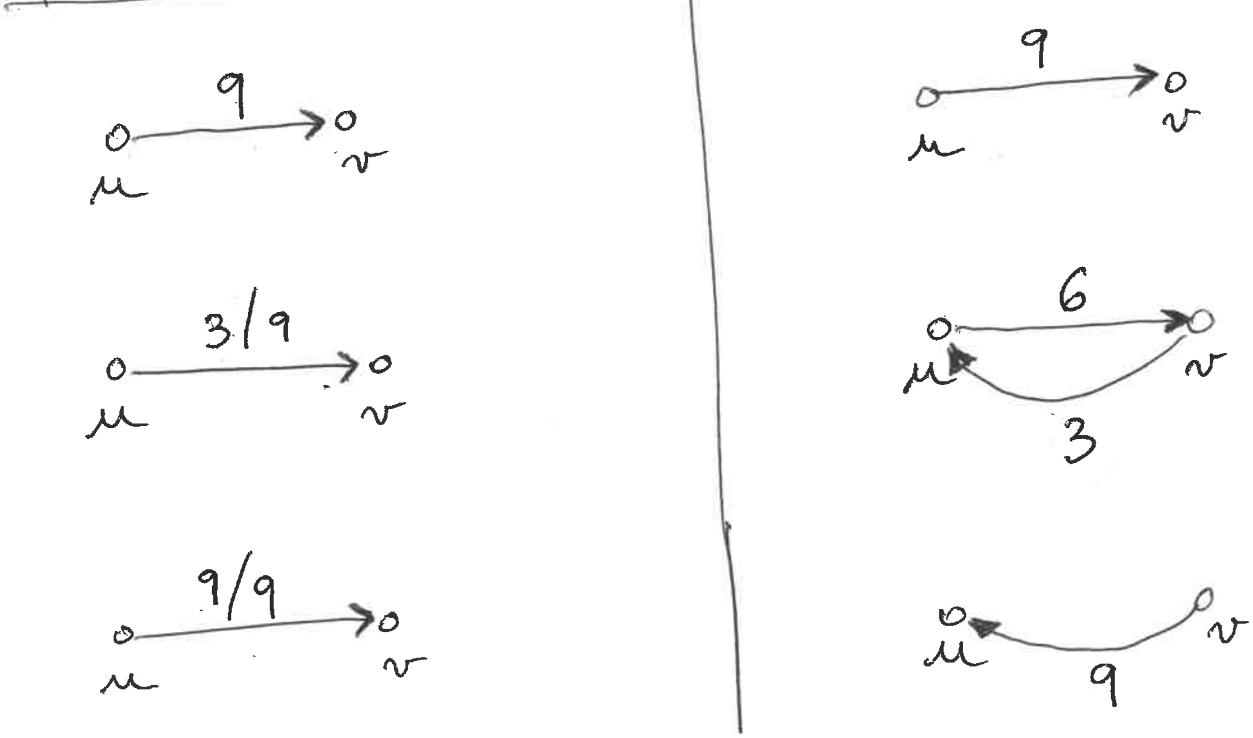
$$|V'| = 2 \cdot |V|$$

$$|E'| = |E| + |V|$$

residual capacity

$$c_f(u,v) = \begin{cases} c(u,v) - f(u,v) & \text{if } (u,v) \in E \\ f(v,u) & \text{if } (v,u) \in E \\ 0 & \text{otherwise} \end{cases}$$

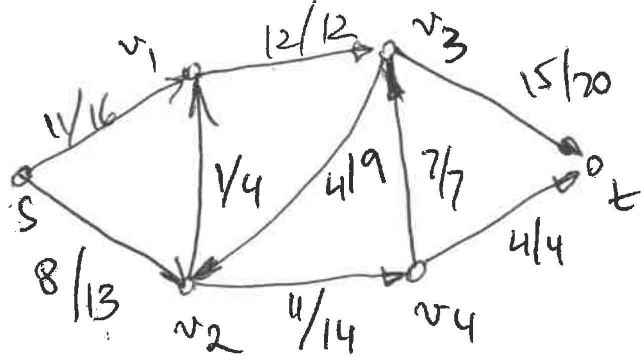
G flow network	G_f residual network



residual network $G_f(V, E_f)$

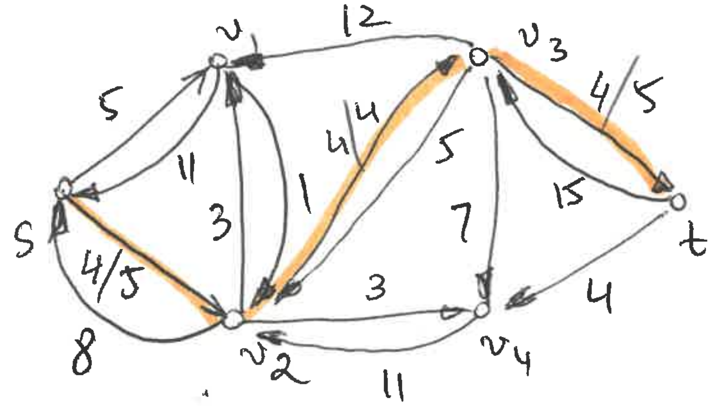
$$|E_f| \leq 2 \cdot |E|$$

G
(flow-network)



$|f| = 19$

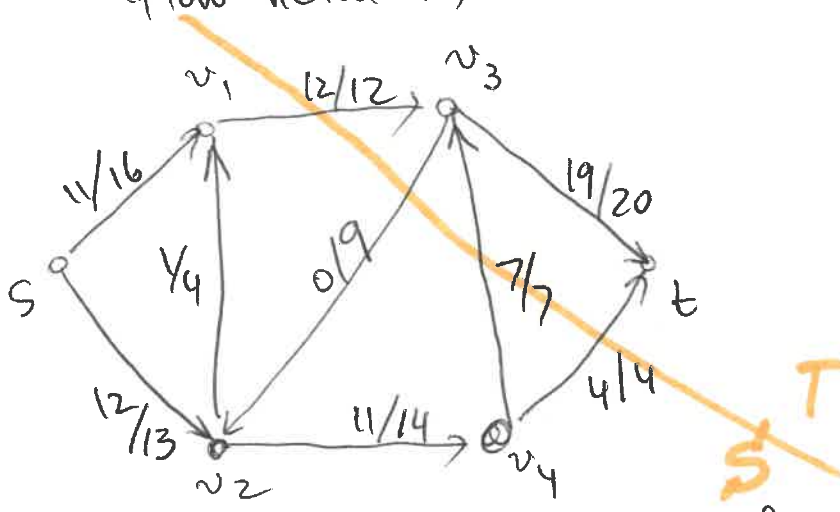
G_f
(residual network)



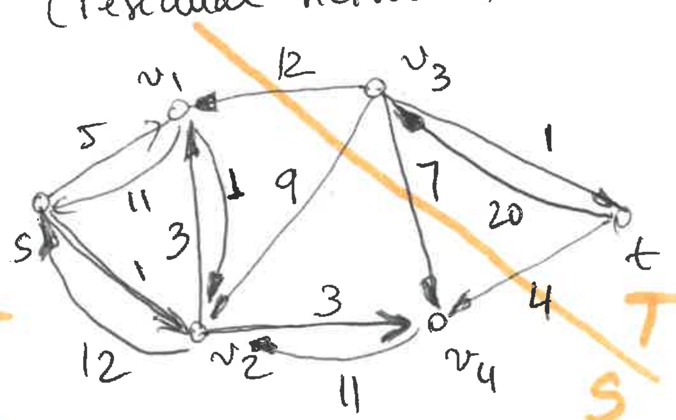
augmenting path $p = \langle s, v_2, v_3, t \rangle$
residual capacity $c_f(p) = \min\{c_f(s, v_2), (v_2, v_3), (v_3, t)\} = \min\{5, 4, 5\} = 4$

$|f_p| = 4$

G
(flow-network)



G_f
(residual network)



flow f was augmented by f_p

$|f \uparrow f_p| = |f| + |f_p| = 19 + 4 = 23$

the net flow is now

$|f| = 23$

$c(S, T) = 23$

since $|f| = c(S, T)$
$\Downarrow$
flow is max!

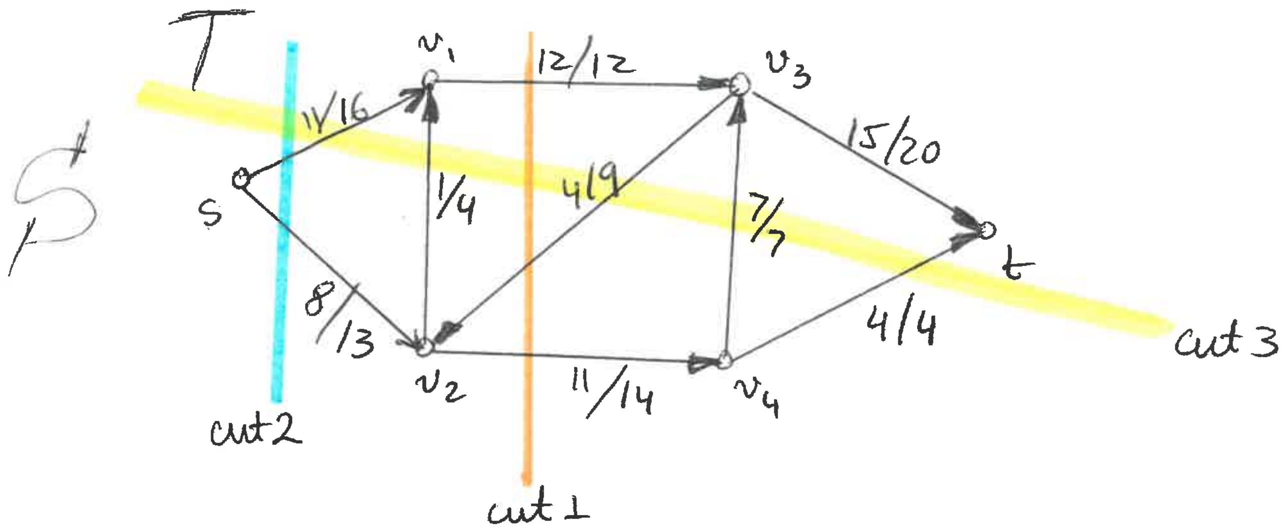
- cannot find an augmenting path
$\Rightarrow$ we reached the max-flow!

cut(S, T)

$S = \{s, v_1, v_2, v_4\}$

$T = \{t, v_3\}$

Cuts of flow networks



cut 1	cut 2	cut 3
$f(S,T) = 19$	$f(S,T) = 19$	$f(S,T) = 19$
$c(S,T) = 26$	$c(S,T) = 29$	$c(S,T) = 31$

$S = \{s, v_1, v_2\}$

$T = \{t, v_3, v_4\}$

For a given flow f, the net flow across any cut is the same and it equals the flow value $|f|$