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**Assignment 5**

**Exercise 9.1:**

$$P(X = -1) = 0.2$$

$$P(X = 2) = 0.5$$

$$P(X = 6) = 0.3$$

$$\text{Expected Value} = (-1 * 0.2) + (2 * 0.5) + (6 * 0.3) = 2.6$$

$$\text{Mean} = (-1 + 2 + 6) / 3 = 2\frac{1}{3}$$

$$\text{Variance} = \frac{(-1 - \frac{1}{3})^2 + (2 - \frac{1}{3})^2 + (6 - \frac{1}{3})^2}{3} = \frac{110}{9}$$

$$\text{Standard Deviation} = \sqrt{\text{Variance}} = \sqrt{\frac{110}{9}} = 3.496$$

**Exercise 9.2:**

(a) Marginal Distributions

$$P(X) = [(0.12 + 0.08 + 0.10), (0.20 + 0.04 + 0.25), (0.08 + 0.10 + 0.03)]$$

$$P(Y) = [(0.12 + 0.20 + 0.08), (0.08 + 0.04 + 0.10), (0.10 + 0.25 + 0.03)]$$

$$P(X) = [0.3, 0.49, 0.21]$$

$$P(Y) = [0.4, 0.22, 0.38]$$

|        | -1   | 1    | 2    | $P(X)$ |
|--------|------|------|------|--------|
| 0      | 0.12 | 0.08 | 0.10 | 0.3    |
| 1      | 0.20 | 0.04 | 0.25 | 0.49   |
| 3      | 0.08 | 0.10 | 0.03 | 0.21   |
| $P(Y)$ | 0.4  | 0.22 | 0.38 |        |

(b)  $X$  and  $Y$  Independent? **Solution: No**

Lets label our original joint distribution as  $F$ . If we are dealing with something that is independent, we should get  $F(X, Y) = P(X) * P(Y)$  for all  $(x, y)$  in range, or each cell in the table. This is because  $P(A, B) = P(A) * P(B)$  with independent events.

| $F(X, Y)$ |      |      |      |        |
|-----------|------|------|------|--------|
|           | -1   | 1    | 2    | $P(X)$ |
| 0         | 0.12 | 0.08 | 0.10 | 0.3    |
| 1         | 0.20 | 0.04 | 0.25 | 0.49   |
| 3         | 0.08 | 0.10 | 0.03 | 0.21   |
| $P(Y)$    | 0.4  | 0.22 | 0.38 |        |

| $P(X) * P(Y)$ |      |      |      |  |
|---------------|------|------|------|--|
|               | -1   | 1    | 2    |  |
| 0             | 0.12 | 0.07 | 0.11 |  |
| 1             | 0.20 | 0.11 | 0.19 |  |
| 3             | 0.08 | 0.05 | 0.08 |  |

As we can see, when we multiply the margins, we do not get the same the same as  $F(X, Y)$ , so because  $F(X, Y) \neq P(X) * P(Y)$ , we know that they are not Independent.

(c)  $\text{Exp}(X)$  and  $\text{Exp}(Y)$

$$\text{Exp}(X) = (0 * 0.3) + (1 * 0.49) + (3 * 0.21) = 1.12$$

$$\text{Exp}(Y) = (-1 * 0.4) + (1 * 0.22) + (2 * 0.38) = 0.58$$

(d) Distribution of  $X + Y$ .

We must first find all the possible values for  $X + Y$ .  $X$  can be  $[0, 1, 3]$ .  $Y$  can be  $[-1, 1, 2]$   
This means that the possible values for  $X + Y$  are  $[-1, 1, 2, 0, 3, 4, 5]$ , thus

$$P(-1) = 0.12$$

$$P(0) = 0.20$$

$$P(1) = 0.08$$

$$P(2) = 0.10 + 0.04 + 0.08 = 0.22$$

$$P(3) = 0.25$$

$$P(4) = 0.10$$

$$P(5) = 0.03$$

(e)  $P(X|Y = 2)$  and  $P(Y|X = 1)$

i.  $P(X|Y = 2)$

We know that

$$P(X|Y = 2) = \frac{P(X, Y = 2)}{P(Y = 2)}$$

Now lets compute

$$P(Y = 2) = 0.38$$

**This means that**

$$P(X = 0|Y = 2) = \frac{0.1}{0.38} = 0.26315 = 26.32\%$$

$$P(X = 1|Y = 2) = \frac{0.25}{0.38} = 0.65789 = 65.79\%$$

$$P(X = 3|Y = 2) = \frac{0.03}{0.38} = 0.07894 = 7.895\%$$

ii.  $P(Y|X = 1)$

We know that

$$P(Y|X = 1) = \frac{P(X = 1, Y)}{P(X = 1)}$$

Now lets compute

$$P(X = 1) = 0.49$$

**This means that**

$$P(Y = -1|X = 1) = \frac{0.20}{0.49} = 0.40816 = 40.82\%$$

$$P(Y = 1|X = 1) = \frac{0.04}{0.49} = 0.08163 = 8.163\%$$

$$P(Y = 2|X = 1) = \frac{0.25}{0.49} = 0.5102 = 51.02\%$$

**Exercise 9.3:**