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**Assignment 5**

**Exercise 9.1:**

P(X = -1) = 0.2
P(X = 2 ) = 0.5
P(X = 6 ) = 0.3

Expected Value = $(-1 * 0.2) + (2 * 0.5) + (6 * 0.3) = 2.6$

Mean = (-1 + 2 + 6) / 3 = $2\frac{1}{3}$

Variance = $\dfrac{(-1-\frac{1}{3})^2+(2-\frac{1}{3})^2+(6-\frac{1}{3})^2}{3} = \dfrac{110}{9}$

Standard Deviation = $\sqrt{Variace} = \sqrt{\frac{110}{9}} = 3.496$

**Exercise 9.2:**

(a) Marginal Distributions
$P(X) = [(0.12 + 0.08 + 0.10), (0.20 + 0.04 + 0.25), (0.08 + 0.10 + 0.03)]$
$P(Y) = [(0.12 + 0.20 + 0.08), (0.08 + 0.04 + 0.10), (0.10 + 0.25 + 0.03)]$

$$P(X) = [0.3, 0.49, 0.21]$$
$$P(Y) = [0.4, 0.22, 0.38]$$

| | -1 | 1 | 2 | $P(X)$ |
|---|---|---|---|---|
| 0 | 0.12 | 0.08 | 0.10 | 0.3 |
| 1 | 0.20 | 0.04 | 0.25 | 0.49 |
| 3 | 0.08 | 0.10 | 0.03 | 0.21 |
| $P(Y)$ | 0.4 | 0.22 | 0.38 | |

(b) $X$ and $Y$ Independent? **Solution: No**

Lets label our original joint distribution as $F$. If we are dealing with something that is independent, we should get $F(X,Y) = P(X) * P(Y)$ for all $(x, y)$ in range, or each cell in the table. This is because $P(A, B) = P(A) * P(B)$ with independent events.

$$F(X,Y)$$

| | -1 | 1 | 2 | $P(X)$ |
|---|---|---|---|---|
| 0 | 0.12 | 0.08 | 0.10 | 0.3 |
| 1 | 0.20 | 0.04 | 0.25 | 0.49 |
| 3 | 0.08 | 0.10 | 0.03 | 0.21 |
| $P(Y)$ | 0.4 | 0.22 | 0.38 | |

$$P(X) * P(Y)$$

| | -1 | 1 | 2 |
|---|---|---|---|
| 0 | 0.12 | 0.07 | 0.11 |
| 1 | 0.20 | 0.11 | 0.19 |
| 3 | 0.08 | 0.05 | 0.08 |

As we can see, when we multiply the margins, we do not get the same the same as $F(X,Y)$, so because $F(X,Y) \neq P(X) * P(Y)$, we know that they are not Independent.

(c) Exp(X) and Exp(Y)

$$\text{Exp(X)} = (0 * 0.3) + (1 * 0.49) + (3 * 0.21) = 1.12$$

$$\text{Exp(Y)} = (-1 * 0.4) + (1 * 0.22) + (2 * 0.38) = 0.58$$

(d) Distribution of $X + Y$.

We must first find all the possible values for $X + Y$. $X$ can be $[0, 1, 3]$. $Y$ can be $[-1, 1, 2]$ This means that the possible values for $X + Y$ are $[-1, 1, 2, 0, 3, 4, 5]$, thus

$P(-1) = 0.12$
$P(0) = 0.20$
$P(1) = 0.08$
$P(2) = 0.10 + 0.04 + 0.08 = 0.22$
$P(3) = 0.25$
$P(4) = 0.10$
$P(5) = 0.03$

(e) $P(X|Y = 2)$ and $P(Y|X = 1)$

i. $P(X|Y = 2)$

We know that

$$P(X|Y = 2) = \frac{P(X, Y = 2)}{P(Y = 2)}$$

Now lets compute

$$P(Y = 2) = 0.38$$

**This means that**

$$P(X = 0|Y = 2) = \frac{0.1}{0.38} = 0.26315 = 26.32\%$$
$$P(X = 1|Y = 2) = \frac{0.25}{0.38} = 0.65789 = 65.79\%$$
$$P(X = 3|Y = 2) = \frac{0.03}{0.38} = 0.07894 = 7.895\%$$

ii. $P(Y|X = 1)$

We know that

$$P(Y|X = 1) = \frac{P(X = 1, Y)}{P(X = 1)}$$

Now lets compute

$$P(X = 1) = 0.49$$

**This means that**

$$P(Y = -1|X = 1) = \frac{0.20}{0.49} = 0.40816 = 40.82\%$$

$$P(Y = 1|X = 1) = \frac{0.04}{0.49} = 0.08163 = 8.163\%$$

$$P(Y = 2|X = 1) = \frac{0.25}{0.49} = 0.5102 = 51.02\%$$

**Exercise 9.3:**

Let X be a random variable with values 0, 1, 3, and let Y be a random variable with values -1, 1, 2. Suppose that P(X=1) = 0.4, P(X=2) = 0.4, and P(X=3) = 0.1, with the following values of $P(Y|X)$

$P(Y|X)$

| | -1 | 1 | 2 |
|---|---|---|---|
| 0 | 0.5 | 0.3 | 0.2 |
| 1 | 0.2 | 0.7 | 0.1 |
| 3 | 0.4 | 0.1 | 0.5 |

(a) Joint Distribution of $X, Y$

We are given all $P(X)$ and also all $P(Y|X)$. We must now solve for $P(X, Y)$ for the joint distribution. We know that $P(X) * P(Y|X) = P(X, Y)$, so we just need to multiply in the correct locations to have the correct values. This results in

$P(X, Y)$

| | -1 | 1 | 2 |
|---|---|---|---|
| 0 | 0.25 | 0.15 | 0.1 |
| 1 | 0.08 | 0.28 | 0.04 |
| 3 | 0.04 | 0.01 | 0.05 |

(b) Distribution of $Y$

$$P(Y = -1) = 0.25 + 0.08 + 0.04 = 0.37 = 37\%$$
$$P(Y = 1) = 0.15 + 0.28 + 0.01 = 0.44 = 44\%$$
$$P(Y = 2) = 0.1 + 0.04 + 0.05 = 0.19 = 19\%$$

(c) Corresponding table for $P(X|Y)$

$$P(X, Y) = P(X|Y) * P(Y)$$
$$\frac{P(X, Y)}{P(Y)} = P(X|Y)$$

$P(X, Y)$

| | -1 | 1 | 2 |
|---|---|---|---|
| 0 | 0.6757 | 0.341 | 0.5263 |
| 1 | 0.216 | 0.636 | 0.2105 |
| 3 | 0.108 | 0.0227 | 0.2623 |

(d) Distribution of $X + Y$

(e) $\mathrm{Exp}(X)$, $\mathrm{Exp}(Y)$, $\mathrm{Exp}(X + Y)$.
Theorem 9.8 from page 260. (Proof on page 261). says that $\mathrm{Exp}(X+Y) = \mathrm{Exp}(X) + \mathrm{Exp}(Y)$