Solutions of Sec 4.1

1. Determine whether each of the following variables x is discrete or continuous:
   a) Let x represent the number of times you do laundry this month. Discrete
   b) Let x represent your annual salary given to the nearest cent. Discrete
   c) Let x represent your height at age 10. Continuous
   d) Let x represent number of math classes that you have taken in your life. Discrete

2. Determine if each of the following tables represents a probability distribution:

(a)

X	-5	6	9
$P(x)$	0.5	0.25	0.25

Yes, this is a probability distribution, since all of the probabilities are between 0 and 1, and they add to 1.

(b)

X	1	2	3	4
$P(x)$	0.4	0.4	0.4	0.2

This is not a probability distribution, since the probabilities add to 1.4, not 1.

(c)

X	1	2	3	4
$P(x)$	0.4	0.4	0.4	-0.2

This is not a probability distribution, since P(4) = – 0.2, and probabilities cannot be negative.

3. Make a probability distribution from the following frequency distribution represent the number of fish caught in a 6-hour period:

Number of fish caught	0	1	2	3	4
Frequency	88	72	30	8	2

To find the probabilities, we need to divide the frequencies by the total number of observations, 200, so we get:

Number of fish caught	0	1	2	3	4
Probability	0.44	0.36	0.15	0.04	0.01

4. Calculate the expected value, variance, and standard deviation for each of the following probability distributions:

(a)

X	-5	6	9
$P(x)$	0.5	0.25	0.25

First, we need to calculate the expected value (as it is used in the rest of the calculations), so

$$\mu = -5 \cdot 0.5 + 6 \cdot 0.25 + 9 \cdot 0.25 = 1.25.$$

Then the variance is

$$\sigma^2 = (-5 - 1.25)^2 \cdot 0.5 + (6 - 1.25)^2 \cdot 0.25 + (9 - 1.25)^2 \cdot 0.25 = 40.1875,$$

so the standard deviation is $\sigma = \sqrt{40.1875} = 6.33936$.

(b)

Number of fish caught	0	1	2	3	4
Probability	0.44	0.36	0.15	0.04	0.01

The expected value is

$$\mu = 0 \cdot 0.44 + 1 \cdot 0.36 + 2 \cdot 0.15 + 3 \cdot 0.04 + 4 \cdot 0.01 = 0.82.$$

The variance is

$$\begin{aligned}\sigma^2 &= (0 - 0.82)^2 \cdot 0.44 + (1 - 0.82)^2 \cdot 0.36 + (2 - 0.82)^2 \cdot 0.15 + (3 - 0.82)^2 \cdot 0.04 \\ &\quad + (4 - 0.82)^2 \cdot 0.01 \\ &= 0.8076\end{aligned}$$

and the standard deviation is $\sigma = \sqrt{0.8076} = 0.8987$.

Solutions of Sec 4.2

1. When flipping a weighted coin (with the probability of heads being 0.6), what is the probability that it will come up heads exactly 5 times when it is flipped 10 times?

This describes a binomial distribution with $n = 10$, $p = 0.6$, and $q = 0.4$.

We want to find

$$P(5) = \frac{10!}{5!5!}\, 0.6^5 \cdot 0.4^5 = 0.2007.$$

2. When randomly guessing on a multiple choice test with 8 questions, where each question has 4 options, what is the probability that you will get at least 7 questions correct? What is the expected number of questions a student will get correct without studying for the exam? What is the standard deviation?

First note that $n = 8$, $p = 0.25$, and $q = 0.75$. We want to find

$$P(7 \text{ or } 8) = P(7) + P(8) = \frac{8!}{1!7!}\, 0.25^7 \cdot 0.75^1 + \frac{8!}{0!8!}\, 0.25^8 \cdot 0.75^0$$
$$= 0.000381.$$

(Recall that $0! = 1$ and $0.75^0 = 1$.)

The average number of questions a student will get correct is

$\mu = 8 \cdot 0.25 = 2$, and the standard deviation is

$$\sigma = \sqrt{8 \cdot 0.25 \cdot 0.75} = 1.22474.$$