Quantum Teleportation & Entanglement in Quantum Information Theory

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The source of the adopted Alice and Bob characters: https://www.freepik.com/premium-ai-image/3d-rendering-female-scientist-with-flask-white-background_73976178.htm

Introduction

I think we all share this feeling...

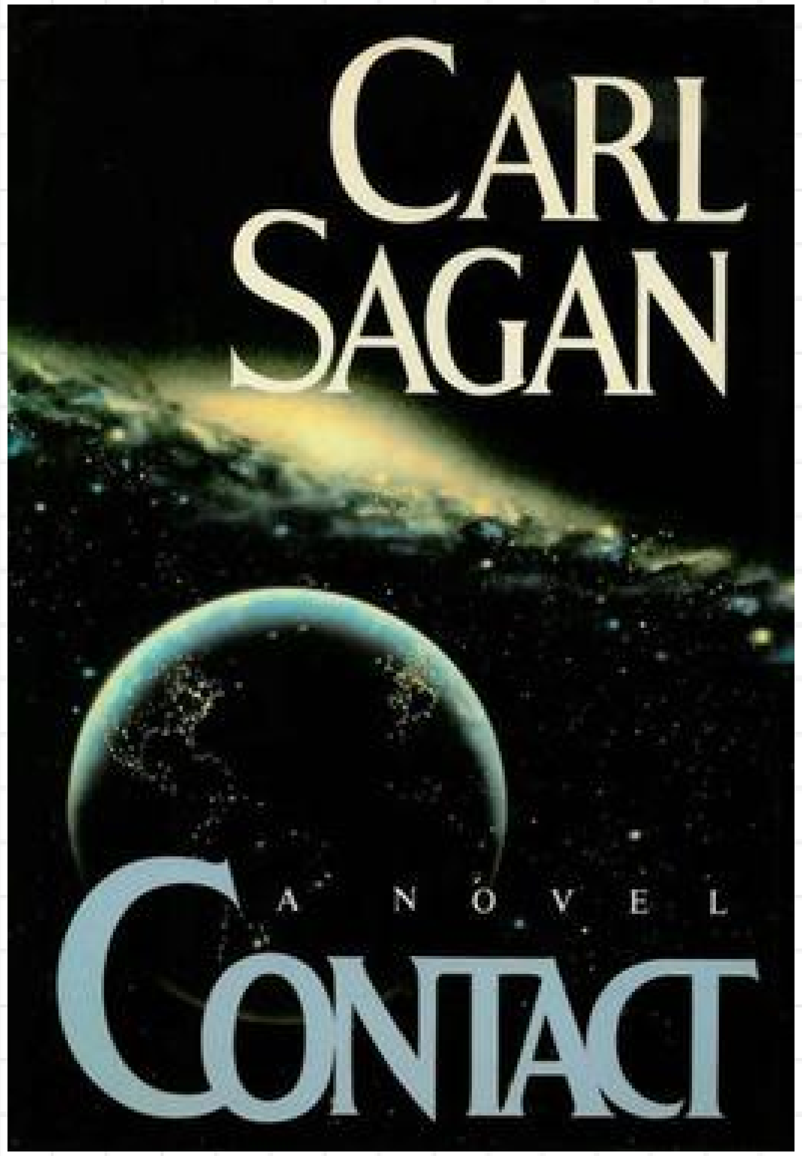
Outline

1. A Story of Obsession.
2. What is a Qubit?
3. How to Entangle Qubits?
4. How Does the Teleportation Work?
5. Applications.
6. Unlocked Gates.
7. Conclusion.

A story of Obsession

In the Sci-fi novel Contact, a physicist named Ele Arroway used a special type of a black hole (a wormhole) to teleport from one universe to another.

Surprisingly, this part of the novel caught the attention of many great physicists (Kip Thorne, Leonard Susskind, Juan Maldacena, Daniel Jafferis, etc.)



A story of Obsession

In one of my trials to analyze one of the articles written in Quanta Magazine (in 2020)...

EPR = ER?

THEORETICAL PHYSICS

Newfound Wormhole Allows Information to Escape Black Holes

By NATALIE WOLCHOVER | OCTOBER 23, 2017 |

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Physicists theorize that a new "traversable" kind of wormhole could resolve a baffling paradox and rescue information that falls into black holes.

Quantum Cloning?

Quantum Teleportation?

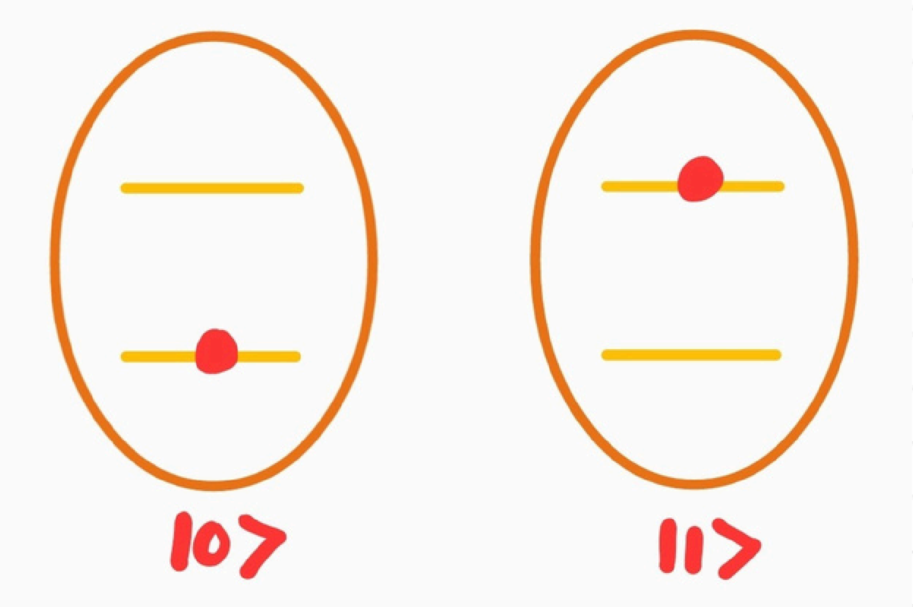
Qubits?

Quantum Entanglement?

What is a Qubit?

A Qubit (Quantum bit) is a two-level quantum system that defines the smallest unit of quantum information in Quantum information theory. It is analogues to the classical bit in classical computing (1).

two- level atom [1] with |0> & |1> basis for a qubit in ground state and 1st excited state respectively.

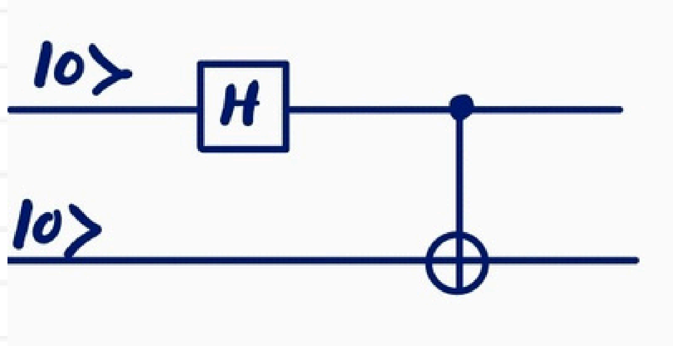


[1]Other examples of Qubits are photons, trapped ions, nuclear magnetic resonance, quantum dots, solid-state qubits, and superconducting qubits (2).

How to Entangle Qubits?

The qubits are said to be "Entangled" when the states of them are correlated [2].

CNOT gate and the generation of Bell's states:



$$|00\rangle \xrightarrow{H} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\,|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle\,|0\rangle + |1\rangle\,|0\rangle) = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

$$\xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|0\rangle\,|0\rangle + |1\rangle\,|1\rangle)$$

Hadamard Gate: A matrix that superpose a deterministic qubit state.
CNOT Gate: A matrix that flips the state of the target qubit if the control qubit is in state |1>.

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[2]You can go through all the details of Alice and Bob thought experiment on quantum entanglement in (3), (4), & (5). We will briefly go over this experiment in explaining Quantum Teleportation, but you might want to read about the dilemma of EPR and Einstein Vs. Bohr debate. Enjoy!

How to Entangle Qubits?

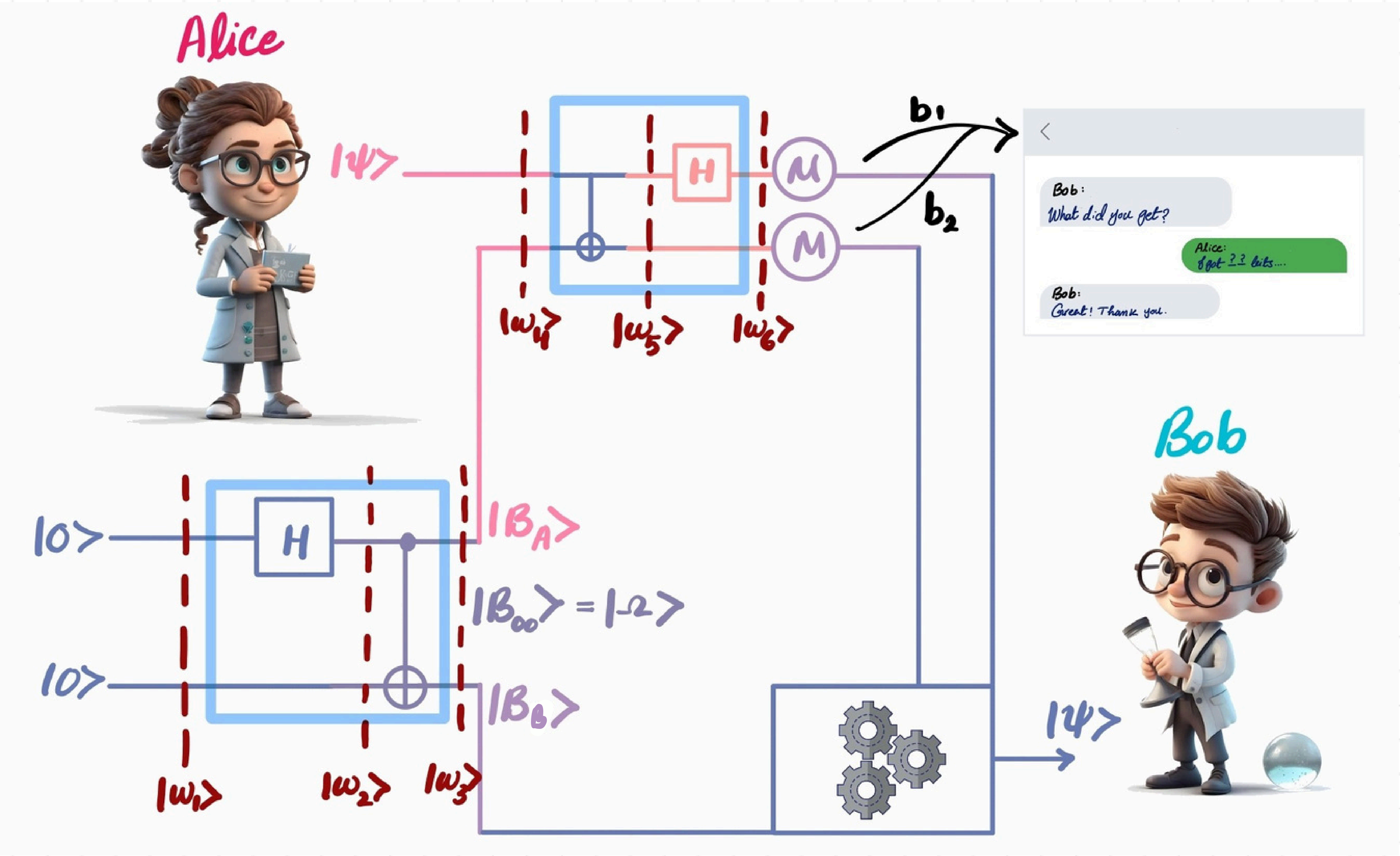
The unfactorizable states generated by the previous circuit are called

"entangled states"

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \quad |\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle),$$
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), \quad |\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle).$$

Which form maximally entangled states "Bell's states" (equally weighted states)

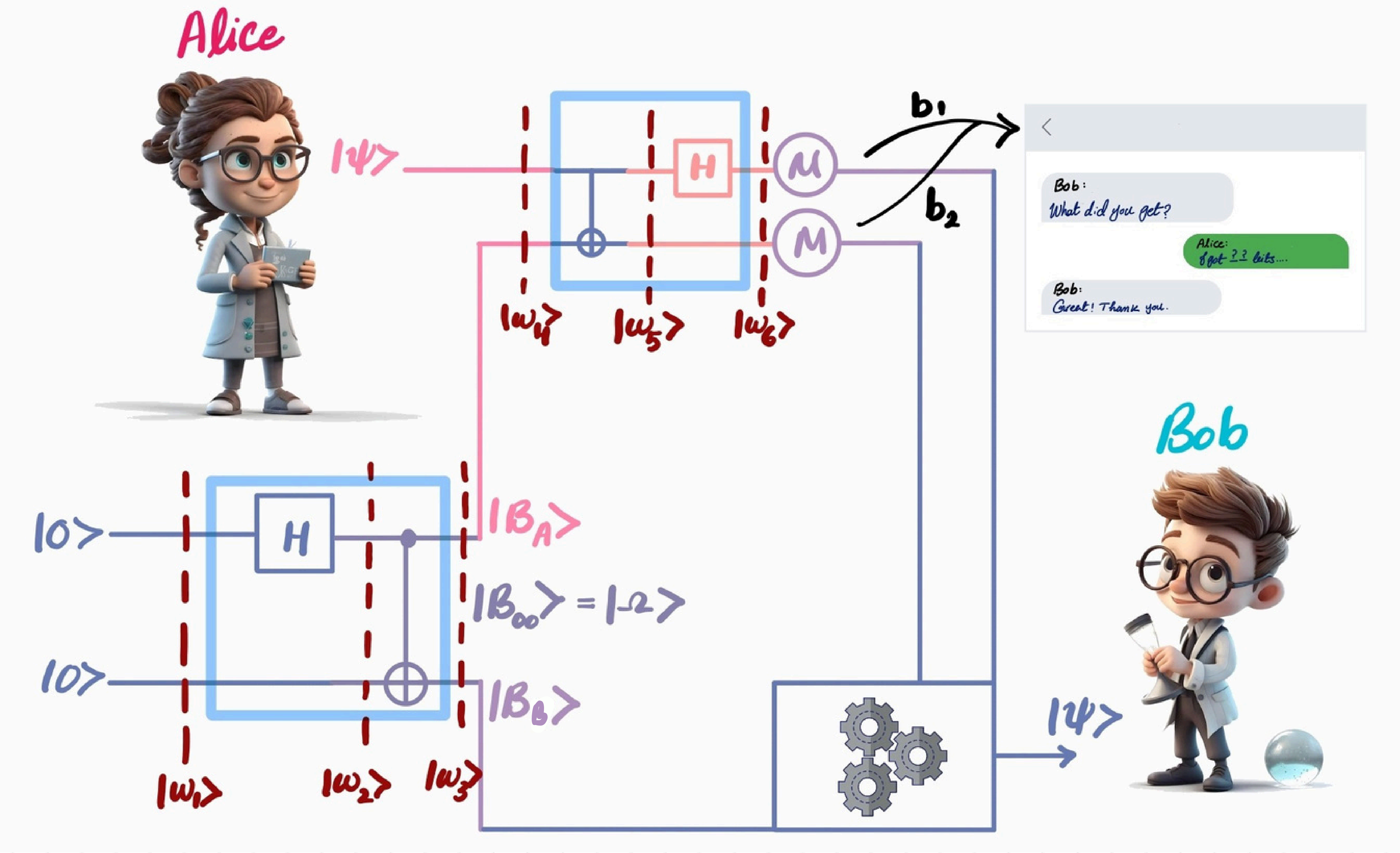
Teleportation of Quantum Information



Defining the protocol (6):

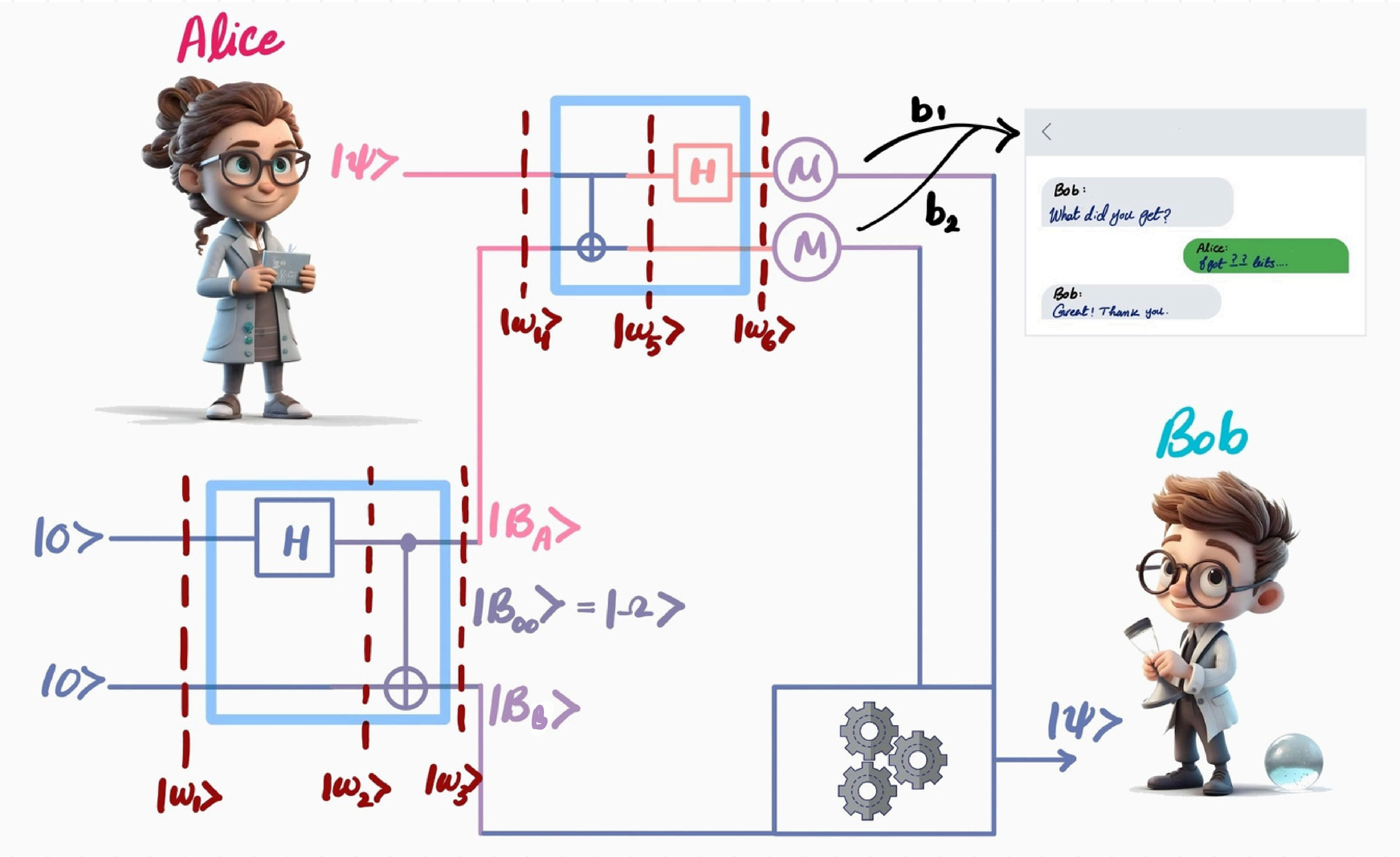
- No Cloning (recreated information but not a copy of the original).
- Classical transmitting channel (no propagation faster than C).

Teleportation of Quantum Information



- 2 qubits of state |0> are introduced.
- Entangled qubits |BA> and |BB> with Alice and Bob respectively.
- A secret message encoded in state |ψ> is operated together with |BA> by CNOT.

Teleportation of Quantum Information



- A measurement done by Alice will collapse the state of the qubits to one of 4 possible states.
- Alice can send her results to Bob by e.g. text message.
- Bob can perform some operations on |BB> to rebuild the secret quantum state |ψ>.

Just a Sense of Beauty

$$|\omega_1\rangle = |0\rangle\,|0\rangle \tag{1}$$

$$|\omega_2\rangle = H\,|0\rangle\,|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\,|0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) \tag{2}$$

$$|\omega_3\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = |B_{00}\rangle \tag{3}$$

$$|\omega_4\rangle = |\psi\rangle\,|B_{00}\rangle\,, |\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

$$|\omega_4\rangle = \frac{1}{\sqrt{2}}(\alpha\,|0\rangle + \beta\,|1\rangle)(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(\alpha\,|000\rangle + \alpha\,|011\rangle + \beta\,|100\rangle + \beta\,|111\rangle) \tag{4}$$

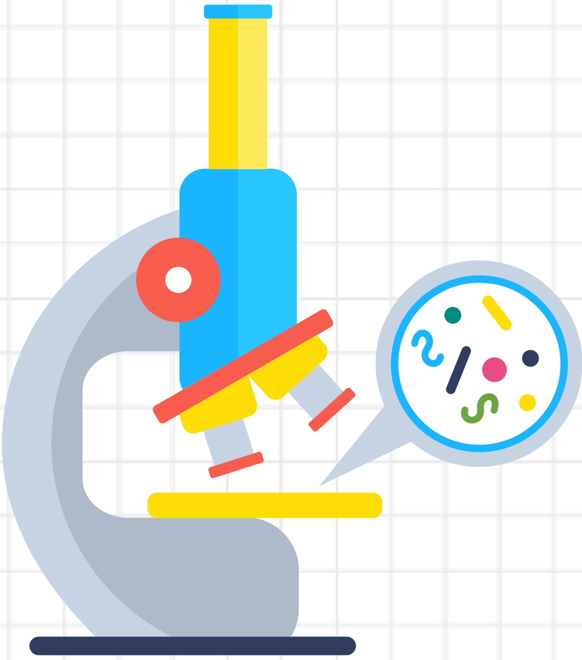
$$|\omega_5\rangle = \frac{1}{\sqrt{2}}(\alpha\,|000\rangle + \alpha\,|011\rangle + \beta\,|110\rangle + \beta\,|101\rangle) \tag{5}$$

$$|\omega_6\rangle = \frac{1}{2}(|00\rangle\,(\alpha\,|0\rangle + \beta\,|1\rangle) + |10\rangle\,(\alpha\,|0\rangle - \beta\,|1\rangle) + |11\rangle\,(\alpha\,|1\rangle - \beta\,|0\rangle) + |01\rangle\,(\alpha\,|1\rangle + \beta\,|0\rangle)) \tag{6}$$

M	Probability	Resultant state	Operation
00	1/4	$\alpha\,|0\rangle + \beta\,|1\rangle$	I
10	1/4	$\alpha\,|0\rangle - \beta\,|1\rangle$	Z $|B_b\rangle$
01	1/4	$\alpha\,|1\rangle + \beta\,|0\rangle$	X $|B_b\rangle$
11	1/4	$\alpha\,|1\rangle - \beta\,|0\rangle$	Z(X $|B_b\rangle$)

- Calculations showing the result of each step in the circuit diagram.
- In |w6>, you can see the 4 fore mentioned equally weighted states(7).
- The set of gates used to reconstruct |ψ> are shown in the table

Applications



An entanglement-enhanced microscope (8)

With such a microscope, the visibility of the image showed an improvement due to the usage of entangled photon pairs compared to the typical source of light.

Safe data transmission

In information security, the tools of quantum mechanics serves as an excellent solution for data transmission.

Applications

Super-dense coding (9)

With quantum computers, an entangled cubit can carry two classical bits. Hence, Quantum computers can work twice the speed of the classical computers.

Note: Quantum computation is not a technological progress. It just gives unique tools that do not make sense for classical computers!

Applications

Photosynthesis (10)

A study on the photosynthesis of plants showed that although light harvesting molecules are not strongly coupled through mechanical means, the transfer of solar energy in light harvesting complexes is instantaneous!

This would **ONLY** be possible via quantum entanglement!

Unlocked Gates!

Quantum Super-Resolution

Monogamy of Entanglement

Standard Model Physics

General Relativity!

Beyond the Standard Model Physics

Conclusion

1. A qubit is the quantum analogue for the classical bit.
2. Entangled states are the states that we cannot attribute separable states for individual qubits.
3. Teleportation is possible for quantum states under a defined protocol.
4. Quantum Information science is a relatively new science, that is still under development, but it has many promising features implemented in useful applications.

Before I came here I was confused about this subject. Having listened to your lecture I am still confused. But on a higher level.

Enrico Fermi

References

1. A bit on the bit. Georgescu, Iulia. 2016, Nature Physics, Vol. 12, pp. 888-888.
2. Introduction to Superconducting Qubits. Radhi, Kumail. s.l. : University of Bahrain, College of Science, Department of Physics, 2023.
3. Duarte, F J. Fundamentals of Quantum Entanglement. London : IOP Publishing, 2022.
4. Introduction to Quantum Entanglement. Guo, Yuying. Shenzhen : AIP Publishing, 2019.
5. Entanglement verification and steering when Alice and Bob cannot be trusted. Eric G. Cavalcanti, Michael J. W. Hall, Howard M. Wiseman. A 87, s.l. : Americal Physical Society, 2013.
6. An Introduction to Quantum Teleportation. Adam Miranowicz, Kiyoshi Tamaki. s.l. : Math. Sciences (Suri-Kagaku), 2003, Vol. 473.
7. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Charles H. Bennett, Gilles Brassard, Claude Crépeau, Richard Jozsa, Asher Peres, and William K. Wootters. s.l. : American Physical Society, 1993, Vol. 70.

References

8. An Entanglement-Enhanced Microscope. Takafumi Ono, Ryo Okamoto, & Shigeki Takeuchi. s.l. : Nature Communications, 2013, Vol. 4.
9. Quantum Entanglement and Its Application in Quantum. Zou, Nanxi. s.l. : Journal of Physics: Conference Series, 2021, Vol. 1827.
10. K. Birgitta Whaley, Mohan Sarovar, Akihito Ishizaki. Quantum entanglement phenomena in photosynthetic light harvesting complexes. Procedia Chemistry. 3, 2011.