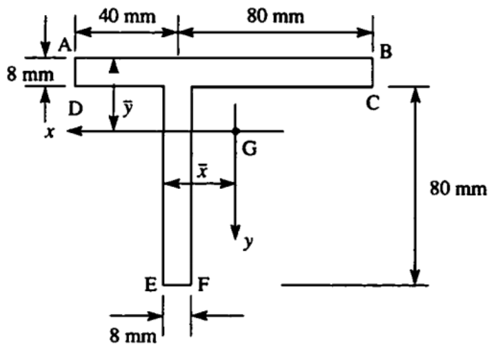
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur
AE333: AEROSPACE STRUCTURES I

Practice Set #2 **August 25, 2024**

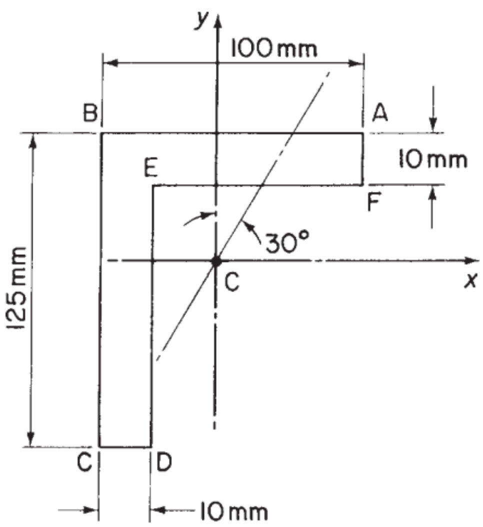
Unsymmetrical Bending

Questions

Problem 1: A beam having the cross-section shown in figure below is subjected to a hogging bending moment of 1500 Nm in a vertical plane. Calculate the maximum direct stress due to bending stating the point at which it acts.

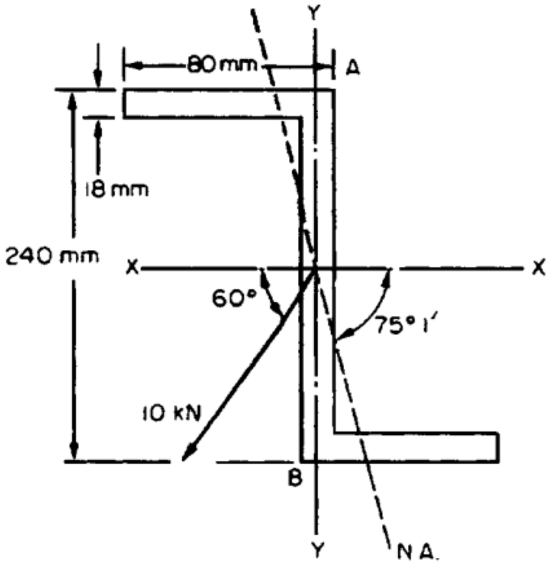


Problem 2: Figure given below shows the section of an angle purlin. A bending moment of 3000 Nm is applied to the purlin in a plane at an angle of 30° to the vertical y axis. If the sense of the bending moment is such that its components M_x and M_y both produce tension in the positive xy quadrant, calculate the maximum direct stress in the purlin stating clearly the point at which it acts.

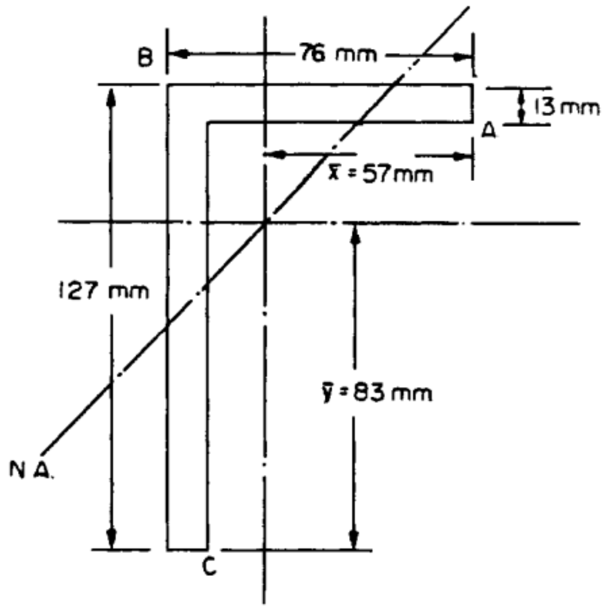


Problem 3: A horizontal cantilever 2 m long is constructed from the z-section shown in figure below. A load of 10 kN is applied to the end of the cantilever at an angle of 60° to the horizontal as shown. Assuming that no twisting moment is applied to the section,

1. Determine the stresses at points A and B. ($I_{xx} = 48.3$ x $10^{-6} m^4$, $I_{yy} = 4.4$ x $10^{-6} m^4$)



Problem 4: A cantilever of length 1.2 m and of the cross section shown in figure below carries a vertical load of 10 kN at its outer end, the line of action being parallel with the longer leg and arranged to pass through the shear centre of the section (i.e. there is no twisting of the section). Find the stress set up in the section at points A, B and C, given that the centroid is located as shown. Determine also the angle of inclination of the N.A.



Solutions

Solution 1:

The position of the centroid, G, of the section may be found by taking moments of areas about some convenient point. Thus

$$(120 \times 8 + 80 \times 8)\bar{y} = 120 \times 8 \times 4 + 80 \times 8 \times 48$$

which gives $\bar{y} = 21{\cdot}6$ mm

and $(120 \times 8 + 80 \times 8)\bar{x} = 80 \times 8 \times 4 + 120 \times 8 \times 24$

giving $\bar{x} = 16$ mm

The second moments of area referred to axes Gxy are now calculated.

$$I_x = \frac{120 \times (8)^3}{12} + 120 \times 8 \times (17{\cdot}6)^2 + \frac{8 \times (80)^3}{12} + 80 \times 8 \times (26{\cdot}4)^2$$

$$= 1{\cdot}09 \times 10^6 \text{ mm}^4$$

$$I_y = \frac{8 \times (120)^3}{12} + 120 \times 8 \times (8)^2 + \frac{80 \times (8)^3}{12} + 80 \times 8 \times (12)^2$$

$$= 1{\cdot}31 \times 10^6 \text{ mm}^4$$

$$I_{xy} = 120 \times 8 \times (-8) \times (-17{\cdot}6) + 80 \times 8 \times (+12) \times (+26{\cdot}4)$$

$$= 0{\cdot}34 \times 10^6 \text{ mm}^4$$

Since $M_x = -1500$ N m and $M_y = 0$ we have, from Eqs. (9.31)

$$\bar{M}_x = -1630 \text{ N m} \quad \text{and} \quad \bar{M}_y = +505 \text{ N m}$$

Substituting these values and the appropriate second moments of area in Eq. (9.30), we obtain

$$\sigma_z = -1{\cdot}5y + 0{\cdot}39x \qquad \text{(i)}$$

Inspection of Eq. (i) shows that σ_z is a maximum at F where $x = 8$ mm, $y = 66{\cdot}4$ mm. Hence

$$\sigma_{z,\max} = -96 \text{ N/mm}^2 \quad \text{(compressive)}$$

Solution 2:

$$M_x = 3000 \times 10^3 \cos 30° = 2.6 \times 10^6 \text{ N mm}$$

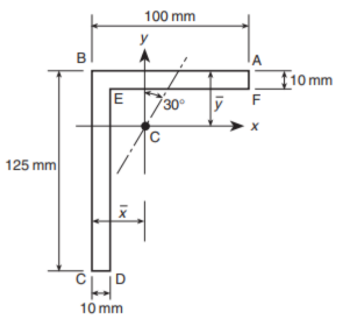
$$M_y = 3000 \times 10^3 \sin 30° = 1.5 \times 10^6 \text{ N mm}$$

initially, the position of and taking moments of
the centroid of area, C, must be found. Referring to Fig
area about the edge BC

$$(100 \times 10 + 115 \times 10)\bar{x} = 100 \times 10 \times 50 + 115 \times 10 \times 5$$

i.e.

$$\bar{x} = 25.9 \text{ mm}$$



Now taking moments of area about AB

$$(100 \times 10 + 115 \times 10)\bar{y} = 100 \times 10 \times 5 + 115 \times 10 \times 67.5$$

from which

$$\bar{y} = 38.4 \text{ mm}$$

The second moments of area are then

$$I_{xx} = \frac{100 \times 10^3}{12} + 100 \times 10 \times 33.4^2 + \frac{10 \times 115^3}{12} + 10 \times 115 \times 29.1^2$$

$$= 3.37 \times 10^6 \text{ mm}^4$$

$$I_{yy} = \frac{10 \times 100^3}{12} + 10 \times 100 \times 24.1^2 + \frac{115 \times 10^3}{12} + 115 \times 10 \times 20.9^2$$

$$= 1.93 \times 10^6 \text{ mm}^4$$

$$I_{xy} = 100 \times 10 \times 33.4 \times 24.1 + 115 \times 10(-20.9)(-29.1)$$

$$= 1.50 \times 10^6 \text{ mm}^4$$

Substituting for M_x, M_y, I_{xx}, I_{yy} and I_{xy}

$$\sigma_z = 0.27x + 0.65y \qquad \text{(i)}$$

Since the coefficients of x and y in Eq. (i) have the same sign the maximum value of direct stress will occur in either the first or third quadrants. Then

$$\sigma_{z(A)} = 0.27 \times 74.1 + 0.65 \times 38.4 = 45.0 \text{ N/mm}^2 \quad \text{(tension)}$$

$$\sigma_{z(C)} = 0.27 \times (-25.9) + 0.65 \times (-86.6) = -63.3 \text{ N/mm}^2 \quad \text{(compression)}$$

The maximum direct stress therefore occurs at C and is 63.3 N/mm^2 compression.

Solution 3:

(a) For this section I_{xy} for the web is zero since its centroid lies on both axes and hence h and k are both zero. The contributions to I_{xy} of the other two portions will be negative since in both cases either h or k is negative.

$$\therefore \quad I_{xy} = -2(80 \times 18)(40-9)(120-9)10^{-12}$$
$$= -9.91 \times 10^{-6}\ \text{m}^4$$

Now, at the built-in end,

$$M_x = +10\,000 \sin 60^\circ \times 2 = +17\,320 \ \text{Nm}$$
$$M_y = -10\,000 \cos 60^\circ \times 2 = -10\,000 \ \text{Nm}$$

Substituting in eqns. (1.20) and (1.21),

$$17\,320 = PI_{xy} + QI_{xx} = (-9.91P + 48.3Q)10^{-6}$$
$$-10\,000 = -PI_{yy} - QI_{xy} = (-4.4P + 9.91Q)10^{-6}$$
$$\therefore \quad 1.732 \times 10^{10} = -9.91P + 48.3Q \qquad (1)$$
$$-1 \times 10^{10} = -4.4P + 9.91Q \qquad (2)$$

$(1) \times \dfrac{4.4}{9.91}$,

$$0.769 \times 10^{10} = -4.4P + 21.45Q \qquad (3)$$

(3) − (2),

$$1.769 \times 10^{10} = 11.54Q$$
$$\therefore \quad Q = 1533 \times 10^6$$

and substituting in (2) gives

$$P = 5725 \times 10^6$$

The inclination of the N.A. relative to the X axis is then given by

$$\tan \alpha_{\text{N.A.}} = -\frac{P}{Q} = -\frac{5725}{1533} = -3.735$$
$$\alpha_{\text{N.A.}} = \mathbf{-75^\circ 1'}$$

This has been added to Fig. 1.15 and indicates that the points A and B are on either side of the N.A. and equidistant from it. Stresses at A and B are therefore of equal magnitude but opposite sign.

Now

$$\sigma = Px + Qy$$
$$\therefore \quad \text{stress at A} = 5725 \times 10^6 \times 9 \times 10^{-3} + 1533 \times 10^6 \times 120 \times 10^{-3}$$
$$= \mathbf{235 \ MN/m^2 \ (tensile)}$$

Similarly,

$$\text{stress at } B = \mathbf{235 \ MN/m^2 \ (compressive)}$$

Solution 4:

The product second moment of area of the section is given by eqn. (1.3).

$$I_{xy} = \Sigma Ahk$$
$$= \{76 \times 13(\tfrac{1}{2} \times 76 - 19)(44 - \tfrac{1}{2} \times 13)$$
$$+ 114 \times 13[-(83 - \tfrac{1}{2} \times 114)][-(19 - \tfrac{1}{2} \times 13)]\}10^{-12}$$
$$= (0.704 + 0.482)10^{-6} = 1.186 \times 10^{-6}\ \text{m}^4$$

From eqn. (1.20) $\quad M_x = PI_{xy} + QI_{xx} = 10\,000 \times 1.2 = 12\,000$

i.e. $$1.186P + 4Q = 12\,000 \times 10^6 \qquad (1)$$

Since the load is vertical there will be no moment about the Y axis and eqn. (1.21) gives

$$M_y = -PI_{yy} - QI_{xy} = 0$$
$$\therefore \quad -1.08P - 1.186Q = 0$$
$$\therefore \quad \frac{P}{Q} = -\frac{1.186}{1.08} = -1.098$$

But the angle of inclination of the N.A. is given by eqn. (1.22) as

$$\tan \alpha_{\text{N.A.}} = -\frac{P}{Q} = 1.098$$

i.e. $$\alpha_{\text{N.A.}} = \mathbf{47^\circ 41'}$$

Substituting $P = -1.098Q$ in eqn. (1),

$$1.186(-1.098Q) + 4Q = 12\,000 \times 10^6$$
$$\therefore \quad Q = \frac{12\,000 \times 10^6}{2.69} = 4460 \times 10^6$$
$$\therefore \quad P = -4897 \times 10^6$$

If the N.A. is drawn as shown in Fig. 1.14 at an angle of $47^\circ 41'$ to the XX axis through the centroid of the section, then this is the axis about which bending takes place. The points of maximum stress are then obtained by inspection as the points which are the maximum perpendicular distance from the N.A.

Thus B is the point of maximum tensile stress and C the point of maximum compressive stress.

Now from eqn (1.19) the stress at any point is given by

$$\sigma = Px + Qy$$
$$\therefore \quad \text{stress at } A = -4897 \times 10^6(57 \times 10^{-3}) + 4460 \times 10^6(31 \times 10^{-3})$$
$$= \mathbf{-141 \ MN/m^2 \ (compressive)}$$
$$\text{stress at } B = -4897 \times 10^6(-19 \times 10^{-3}) + 4460 \times 10^6(44 \times 10^{-3})$$
$$= \mathbf{289 \ MN/m^2 \ (tensile)}$$
$$\text{stress at } C = -4897 \times 10^6(-6 \times 10^{-3}) + 4460 \times 10^6(-83 \times 10^{-3})$$
$$= \mathbf{-341 \ MN/m \ (compressive)}$$