# TX

## Chapitre I :

Energie : $E_x = \int_{\mathbb{R}} |x(t)|^2 \, dt$

Puissance : $P_x = \lim_{\theta \to +\infty} \frac{1}{2\theta} \int_{-\theta}^{\theta} |x(t)|^2 \, dt$

$x(t) = \sum X_n e^{j2\pi f_0 t}$

$X_n = \frac{1}{T_0} \int_{(T_0)} x(t) e^{-j2\pi f_0 t}$

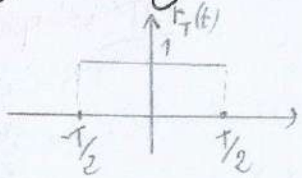
} pour signal périodique

*:

$X(f) = TF\{x(t)\} = \int_{\mathbb{R}} x(t) e^{-j2\pi f t} \, dt$

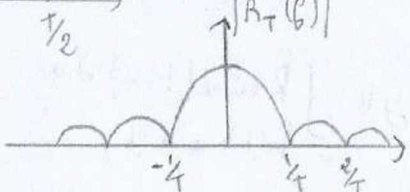
$x(t) = TF^{-1}\{X(f)\} = \int_{\mathbb{R}} X(f) e^{j2\pi f t} \, df$

Signal rectangulaire :

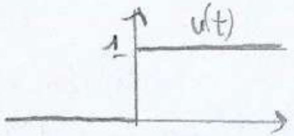



$R_T(f) = T \,\mathrm{sinc}(Tf)$

↳ $\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$




Fonction echelon :




$TF\{u(t)\} = \frac{1}{2}\delta(f) + \frac{1}{2\pi j f}$

Cosinus

$TF\{\cos(2\pi f_0 t + \varphi_0)\} = \frac{1}{2}\left(e^{j\varphi_0}\delta(f-f_0) + e^{-j\varphi_0}\delta(f+f_0)\right)$

Sinus

$TF\{\sin(2\pi f_0 t + \varphi_0)\} = \frac{1}{2j}\left(e^{j\varphi_0}\delta(f-f_0) - e^{-j\varphi_0}\delta(f+f_0)\right)$

fréquence pure $e^{i2\pi f_0 t}$ :

$TF\{e^{i2\pi f_0 t}\} = \delta(f-f_0)$

## Propriétés de TF :

$TF\{x(t-t_0)\} = e^{-j2\pi f t_0} X(f)$

$TF\{x(t) e^{j2\pi f_0 t}\} = X(f-f_0)$

$TF\{x(-t)\} = X(-f)$

$TF\{x^*(t)\} = X^*(-f)$

$TF\{x(at)\} = \frac{1}{|a|} X\left(\frac{f}{a}\right)$

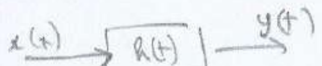
$TF\{x^{n}(t)\} = (2j\pi f)^n X(f)$

* si $X(f) = TF\{x(t)\}$ alors $TF\{X(t)\} = x(-f)$

Impulsion de Dirac

$\Delta(f) = TF\{\delta(t)\} = 1$

## Produit de convolution




$y(t) = \int h(u) x(t-u) \, du$
$= h(t) * x(t)$

Rq : $\delta(t) * x(t) = x(t) * \delta(t) = x(t)$

$\delta(x) \cdot f(x) = f(0) \cdot \delta(x)$

$\delta(x-x_0) f(x) = f(x_0) \cdot \delta(x-x_0)$

$\delta(x-x_0) * f(x) = f(x-x_0)$

$\int_{\mathbb{R}} \delta(x) \, dx = 1$

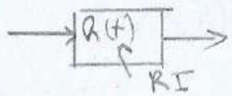
Propriétés

$TF\{x(t) * y(t)\} = X(f)\,Y(f)$

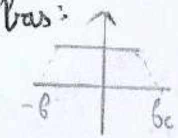
$TF\{x(t) \cdot y(t)\} = X(f) * Y(f)$

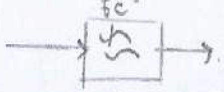
## Réponse fréquentielle des filtres:

$H(f) = TF\{h(t)\}$

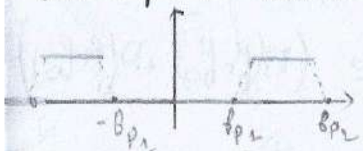


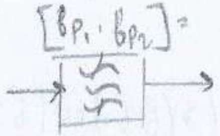

Filtre passe-bas:



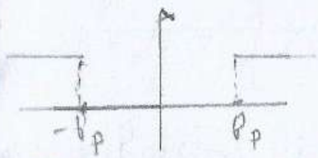



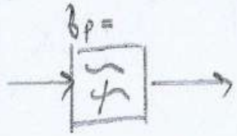

Filtre passe-bande




Filtre passe-haut



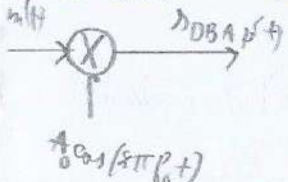



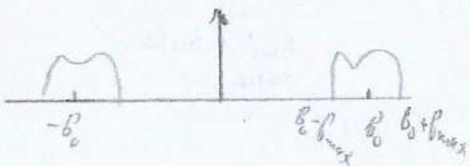

# Chapitre II:

## ① DBSP:

$s_{DBSP}(t) = A_0 \cos(2\pi f_0 t) \cdot m(t)$ $\quad (f_0 \ggg f_{max})$

$S_{DBSP}(f) = \frac{A_0}{2}\left(M(f-f_0) + M(f+f_0)\right)$



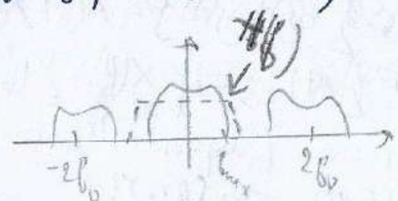




Demodulation

$r(t) = A_1 \cos(2\pi f_0 t)\, s_{DBSP}(t) = A_0 A_1 m(t) \cos^2(2\pi f_0 t)$

$= \frac{A_0 A_1}{2} m(t)\left(1 + \cos(4\pi f_0 t)\right)$

$R(f) = \frac{A_0 A_2}{2} M(f) + \frac{A_0 A_1}{4}\left(M(f - 2f_0) + M(f + 2f_0)\right)$

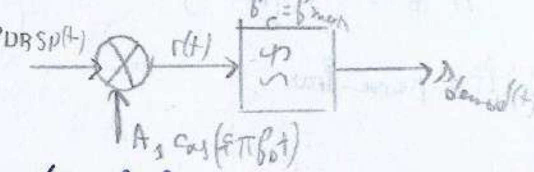



$s_{demod}(t) = r(t) * h(t)$

$S_{demod}(f) = R(f) \cdot H(f)$

$= \frac{A_0 A_1}{2} M(f)$

$s_{demod}(t) = \frac{A_0 A_1}{2} m(t)$

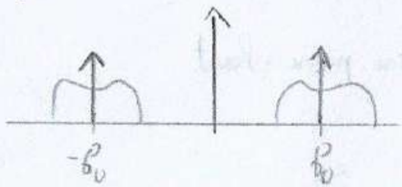



## ② DBAP:

$s_{DBAP}(t) = A_0\left(1 + k_a m(t)\right) \cos(2\pi f_0 t)$

(*) $|k_a m(t)| < 1$ *

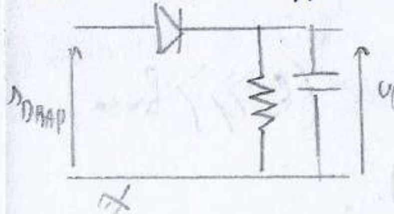
$S_{DBA}(f) = \frac{A_0}{2}\left(\delta(f-f_0) + \delta(f+f_0)\right) + \frac{k A_0}{2}\left(M(f-f_0) + M(f+f_0)\right)$

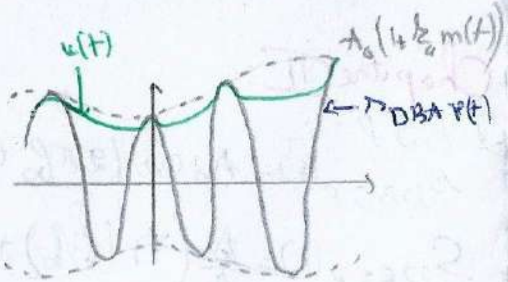



Circuit ??

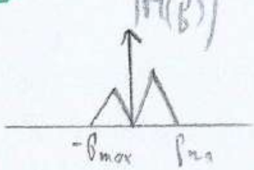
## Demodulation:

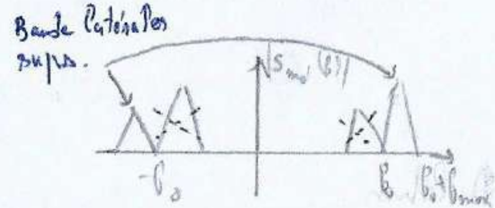
Détection d'enveloppe




## BLU




$S_{BLU}(f) = A_0 M(f-f_0)\, u(f-f_0) + A_0 M(f+f_0)\, u(-f-f_0)$

$$S_{BLU}(f) = A_0\left(G_1(f-f_0) + G_2(f+f_0)\right)$$

$$\text{tq } \begin{cases} G_1(f) = M(f)\,u(f) \\ G_2(f) = M(f)\,u(-f) \end{cases}$$

$$s_{BLU}(t) = A_0\left(g_1(t)e^{j2\pi f_0 t} + g_2(t)e^{-j2\pi f_0 t}\right)$$

$$\text{tq } \begin{cases} g_1(t) = m(t) * TF^{-1}\{u(f)\} \\ g_2(t) = m(t) * TF^{-1}\{u(-f)\} \end{cases}$$

$$\text{or } TF\left\{\frac{1}{2}\delta(t) + \frac{1}{2\pi j t}\right\} = U(-f)$$

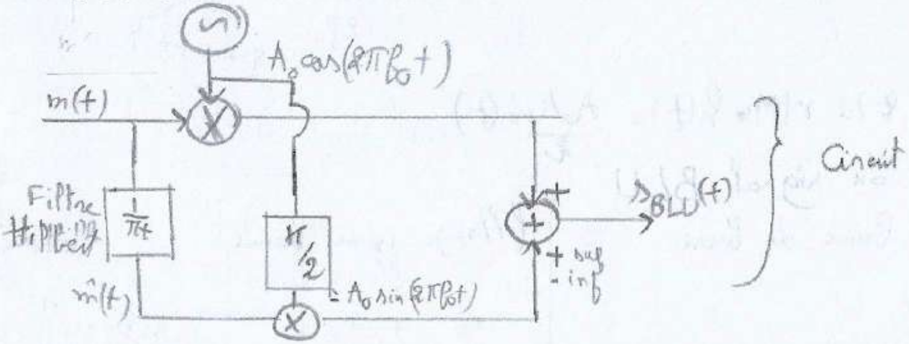
$$\Rightarrow s_{BLU}(t) = A_0\left(m(t) * \left(\frac{1}{2}\delta(t) - \frac{1}{2\pi j t}\right)e^{j2\pi f_0 t} + m(t) * \left(\frac{1}{2}\delta(t) + \frac{1}{2\pi j t}\right)e^{-j2\pi f_0 t}\right)$$

$$= A_0\left(m(t)\cos(2\pi f_0 t) - \hat{m}(t)\sin(2\pi f_0 t)\right)$$

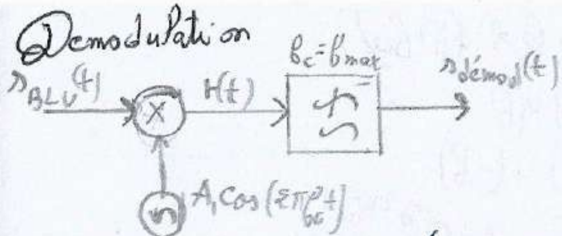
$$\text{tq : } \hat{m}(t) = TH\{m(t)\} = \frac{1}{\pi t} * m(t)$$

$$\Rightarrow s_{BLU}(t) = A_0\left(\cos(2\pi f_0 t)\,m(t) \underset{Sup}{\overset{Inf}{\pm}} \sin(2\pi f_0 t)\,\hat{m}(t)\right)$$

Modulateur BLU




Démodulation




$$r(t) = A_1\cos(2\pi f_0 t)\,A_0\left(m(t)\cos(2\pi f_0 t) - \hat{m}(t)\sin(2\pi f_0 t)\right)$$

$$= \frac{A_0A_1}{2}\left(\left(1+\cos(4\pi f_0 t)\right)m(t)\right) - \frac{A_0A_1}{2}\hat{m}(t)\sin(4\pi f_0 t)$$

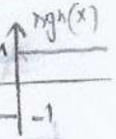
$$= \frac{A_0A_1}{2}m(t) + \frac{A_0A_1}{2}m(t)\cos(4\pi f_0 t) - \frac{A_0A_1}{2}\hat{m}(t)\sin(4\pi f_0 t)$$

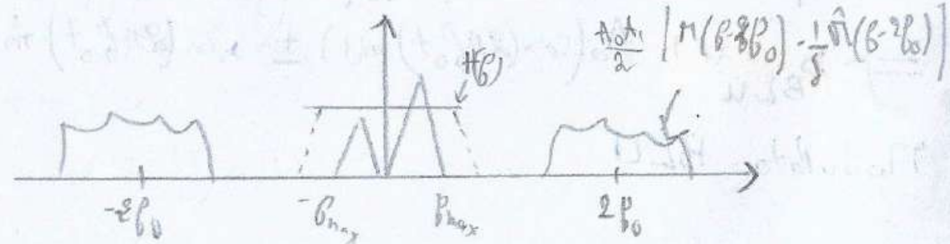
$$R(f) = \frac{A_0A_1}{2}M(f) + \frac{A_0A_1}{4}M(f-2f_0) + \frac{A_0A_1}{4}M(f+2f_0)$$

$$- \frac{A_0A_1}{4j}\hat{M}(f-2f_0) + \frac{A_0A_1}{4j}(f+2f_0)$$

$$\text{tq : } \hat{M}(f) = TF\{\hat{m}(t)\} = TF\left\{\frac{1}{\pi t}\right\}TF\{m(t)\}$$

$$= -j\,sgn(f)\,M(f)$$



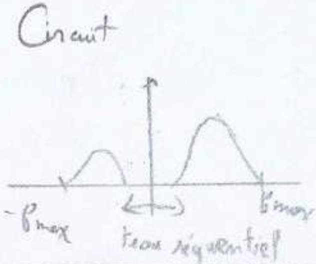




$$s_{démod}(t) = r(t) * h(t) = \frac{A_0A_1}{2}m(t)$$

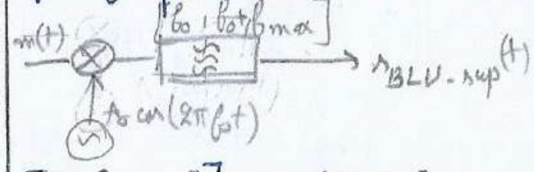
Génération du signal BLU

Filtrage en Bande de Base

Circuit




Filtrage passe Bande




$[f_0 - f_{max}\,;\,f_0]$ pour BLU-inf

## Chapitre III :

$\Psi(t)$ : angle instantanée

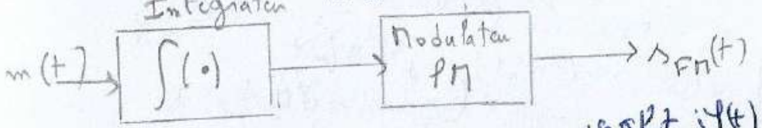
$$s_{ang}(t) = A_0 \cos\big(2\pi f_0 t + \underbrace{\varphi(m(t))}_{\text{phase instantanée}}\big)$$

Fréquence instantannée de $s_{ang}(t) \stackrel{\Delta}{=} F_i(t) = \frac{1}{2\pi}\frac{d\Psi(t)}{dt}$

$$\varphi(t) = 2\pi \underbrace{k_{FM}}_{\text{cst de modulation}} \int_0^t m(u)\,du$$

$$F_i(t) = f_0 + k_{FM}\, m(t)$$

$$\Delta F = |F_i(t) - f_0|_{max} = k_{FM}\, |m(t)|_{max} \quad \text{(deviation max)}$$




$$s_{FM}(t) = A_0 \cos(2\pi f_0 t + \varphi(t)) = \frac{A_0}{2}\left(e^{j2\pi f_0 t} \cdot e^{j\varphi(t)} + e^{-j2\pi f_0 t}\, e^{-j\varphi(t)}\right)$$

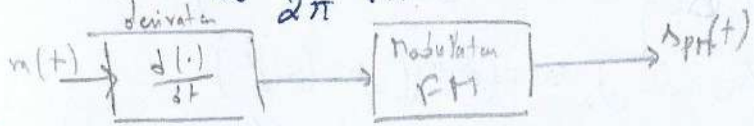
enveloppe complexe de $s_{FM}(t)$ $e^{j\varphi(t)} = x(t)$

$$S_{FM}(f) = \frac{A_0}{2}\left(X(f-f_0) + X^*(-f-f_0)\right)$$

**Rq** : Signal modulé selon PM

$$\varphi(t) = k_{PM}\, m(t)$$

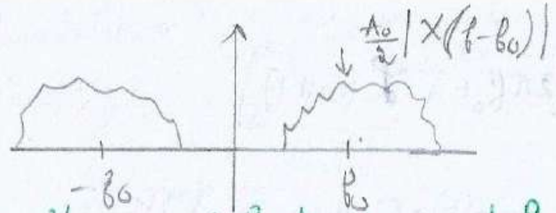
$$F_i(t) = f_0 + \frac{1}{2\pi} k_{PM}\, m'(t)$$




Fin Rq.

hyp $|X(f)| \xrightarrow[|f| \to +\infty]{} 0$ $\quad \left(E < +\infty \Rightarrow \int_{\mathbb{R}} |x(t)|^2 dt = \int_{\mathbb{R}} |X(f)|^2 df < +\infty\right)$

Relation de Parseval




### Cas d'un modulant sinusoïdal :

$$m(t) = A_0 \cos(2\pi f_m t)$$

$$s_{FM}(t) = A_0 \cos\left(2\pi f_0 t + \beta \sin(2\pi f_m t)\right) \qquad \left(\beta = \frac{k_{FM} A_m}{f_m},\ \Delta F = k_{FM} A_m\right)$$

$$= A_0\, \mathrm{Re}\left\{ e^{j2\pi f_0 t} \cdot \underbrace{e^{j\beta \sin(2\pi f_m t)}}_{g(t)} \right\}$$

$$g(t) = e^{j\beta \sin(2\pi f_m t)} = \sum G_n e^{2j\pi n f_m t}$$

où $G_n = \frac{1}{T_m}\int_0^{T_m} e^{j\beta \sin(2\pi f_m t)} e^{-j2\pi n f_m t} dt$

$$= \frac{1}{2\pi}\int_0^{2\pi} e^{j\beta \sin(v)} e^{-jnv} dv \quad \text{en posant } v = 2\pi f_m t$$

$$= J_n(\beta) = \frac{1}{2\pi}\int_0^{2\pi} e^{j(\beta \sin(v) - nv)} dv$$

$$\begin{cases} J_n^*(\alpha) = J_n(\alpha) \\ J_{-n}(\alpha) = (-1)^n J_n(\alpha) \\ \lim\limits_{|n| \to +\infty} J_n(\alpha) = 0 \end{cases}$$

$$\Rightarrow s_{FM}(t) = A_0\, \mathrm{Re}\left\{ e^{j2\pi f_0 t} \sum_n J_n(\beta) e^{j2\pi n f_m t} \right\}$$

$$= A_0 \sum J_n \cos\left(2\pi(f_0 + n f_m)t\right)$$

$$\Rightarrow S_{FM}(f) = \frac{A_0}{2} \sum J_n(\beta)\left(\delta(f - f_0 - n f_m) + \delta(f + f_0 + n f_m)\right)$$

Bande de Carson : $B = 2(\beta + 1) f_m = 2(\Delta F + f_m)$

## Chapitre IV :

Soit $x(t)$ un signal aléatoire

$x(t,\omega)$ une réalisation

$F_{x(t)}(\alpha) = P\{x(t) \leq \alpha\} \Leftrightarrow f_{x(t)}(\alpha) = \dfrac{d F_{x(t)}(\alpha)}{d\alpha}$

↳ Fonction de répartition ↳ densité de probabilité.

- $E[x(t)] = m_x(t)$ : moyenne statistique
- $E[x^2(t)] : P_x(t)$ : puissance moyenne instantanée
- $\sigma_x^2(t) = E[x^2(t)] - m_x^2(t)$ : variance

$= E[(x(t) - m_x(t))^2]$

Rq : pour un signal complexe : $P_x(t) = E[|x(t)|^2]$

$\sigma_x^2(t) = E[|x(t)|^2] - |m_x^2(t)|$

$E(X) = \int_{\mathbb{R}} \alpha f_X(\alpha) d\alpha$

$E(X^2) = \int_{\mathbb{R}} \alpha^2 f_X(\alpha) d\alpha$

$E[g(X)] = \int_{\mathbb{R}} g(\alpha) f_X(\alpha) d\alpha$

S.A stationnaires au sens Large (SSL)

C1 - $E[x(t)] = m_x$ (ne dépend pas de $t$)

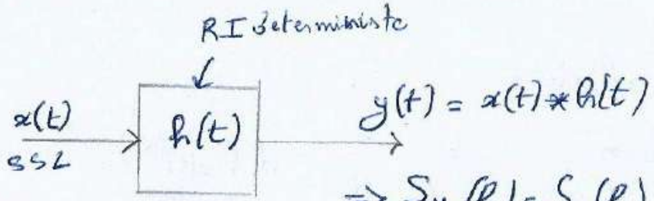
C2 - $R_x(t, t-\tau) = E[x(t) x^*(t-\tau)] = R(\tau)$ (ne dépend que de $\tau$)

↳ fct d'autocorrélation

$\Rightarrow S_x(f) = TF\{R_x(\tau)\} = \int_{\mathbb{R}} R_x(\tau) e^{-j2\pi f \tau} d\tau$

↳ densité spectrale de puissance (dsp) ou spectre de puissance.

$P_x = R_x(0) = \int_{\mathbb{R}} S_x(f) df$




$\Rightarrow S_y(f) = S_x(f) \cdot |H(f)|^2$

## Chapitre V :

$D_b = \dfrac{1}{T_b}$

$D_s = \dfrac{D_b}{\log_2(M)}$

$e(t) = \sum_k a_k h(t - kT_b)$

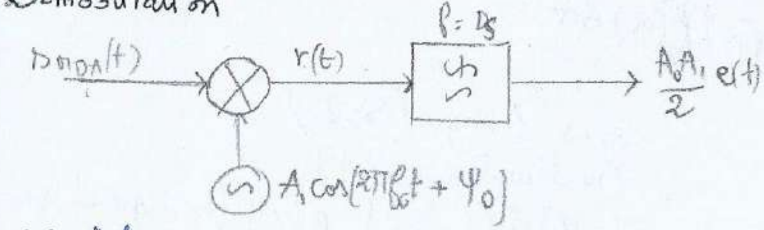
$S_e(f) = S_a(f) \cdot |H(f)|^2$

$\gamma_a'(k) = \dfrac{E\{(a_n - m_a)(a_{n-k} - m_a)^*\}}{\sigma_a}$

1) Modulation par déplacement d'Amplitude (MDA)

$s_{MDA}(t) = A_0 e(t) \cos(2\pi f_0 t + \varphi_0)$

Démodulation




$r(t) = \dfrac{A_0 A_1}{2} (1 + \cos(4\pi f_0 t + 2\varphi_0)) e(t)$

$= \underbrace{\dfrac{A_0 A_1}{2} e(t)}_{\text{en Bande de Base}} + \underbrace{\dfrac{A_0 A_1}{2} \cos(4\pi f_0 t + 2\varphi_0) e(t)}_{HF}$

2) Modulation d'amplitude sur 2 porteuses en quadrature de phase (MAQ)

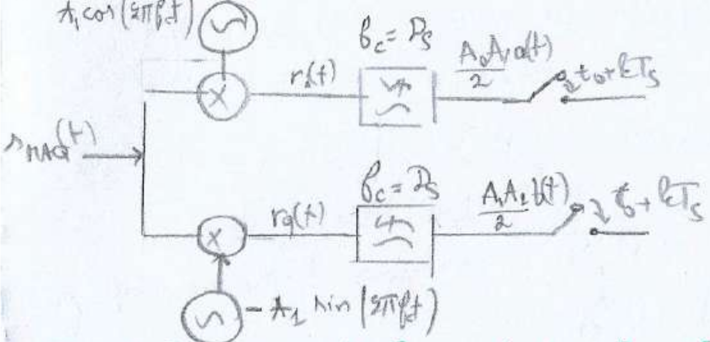
$s_{MAQ}(t) = A_0 a(t)\cos(2\pi f_0 t) - A_0 b(t)\sin(2\pi f_0 t)$

$a_k, b_k \in \{\pm d, \pm 3d, \dots \pm(M-1)d\}$

Couple de symboles $(a_k, \underbrace{b_k}_{n\ \text{bits}})$ de longueur $2n$ et qui peut prendre $M^2$ valeurs possibles. On parle de MAQ à $M^2$ états

$(MAQ\text{-}M^2)$

Démodulation




3) Modulation par déplacement de phase (MDP) ou PSK : phase shift Keying:

$$s_{MDP}(t) = A_0\cos(2\pi f_0 t + \phi(t))$$

avec $\phi(t) = \sum_k \phi_k h(t-kT_s)$

$\phi_k \in \{\theta_0 + (2m+1)\frac{\pi}{M} ; 0 \le m \le M-1\}$

* sur chaque intervalle $[kT_s, (k+1)T_s[$, $s_{MDP}(t) = A_0\cos(2\pi f_0 t + \phi_k)$

↳ $s_{MDP}(t) = A_0 \sum_k \cos(2\pi f_0 t + \phi_k)\, h(t-kT_s)$

tq $\phi_k = \sum_n \phi_n h(t-nT_s)$

$$\Rightarrow s_{MDP}(t) = A_0\Big(\overbrace{\sum_k \cos(\phi_k)\, h(t-kT_s)}^{a(t)}\Big)\cos(2\pi f_0 t)$$
$$- A_0\Big(\underbrace{\sum_k \sin(\phi_k)\, h(t-kT_s)}_{b(t)}\Big)\sin(2\pi f_0 t)$$