AN EXPLORATION INTO SPARSE SIGNAL REPRESENTATION AND RECOVERY

INTERNAL PRESENTATION

NIMISHA T M
M110197EC

Introduction

- The classical sampling tries to protect the frequency spectrum of the signal being sampled.
- This implies that the sampled version has redundant information.
- Thus the question arises, "can we remove this redundancy while sampling itself?".
- This paved the way to COMPRESSED SENSING (CS)
- It's a new method to capture and represent compressible signals at the rate well below Nyquist rate.

Problem definition

- To find a structured measurement basis with improved recovery compared to random measurements.

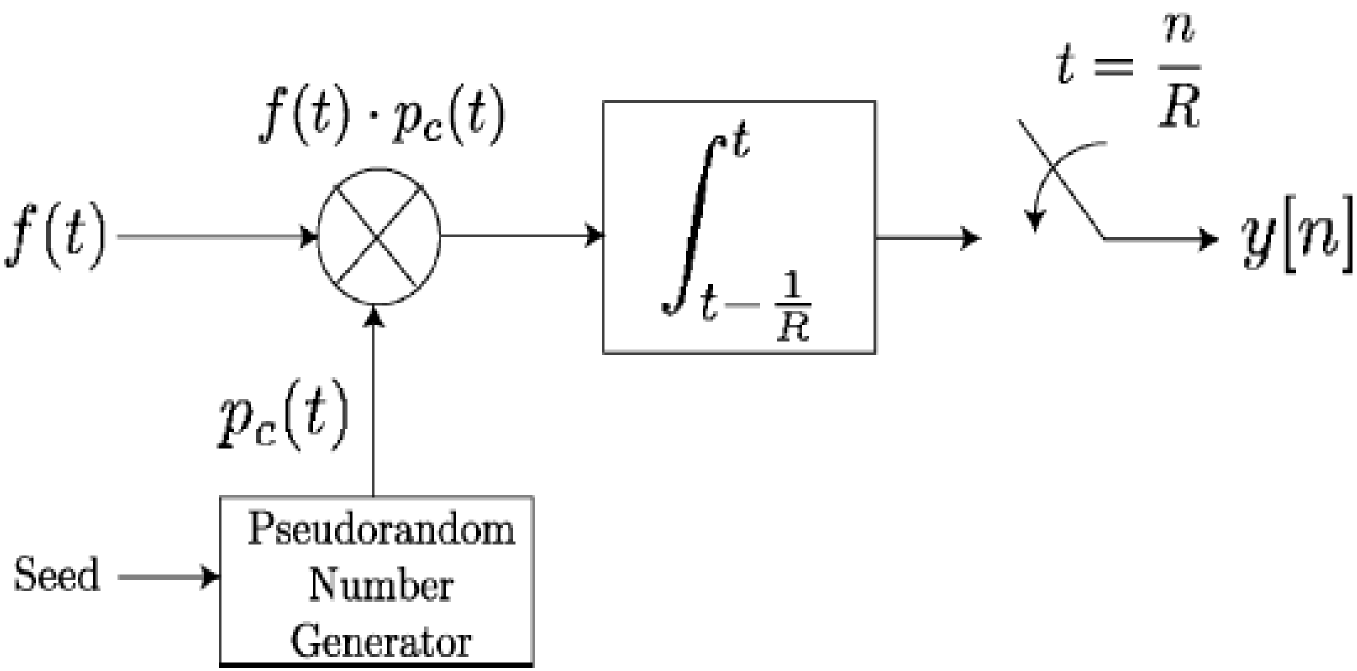
- To find a sparsifying basis for a class of signals non-adaptively and to obtain a measurement basis from it.

Work done in previous semester : Random Demodulator

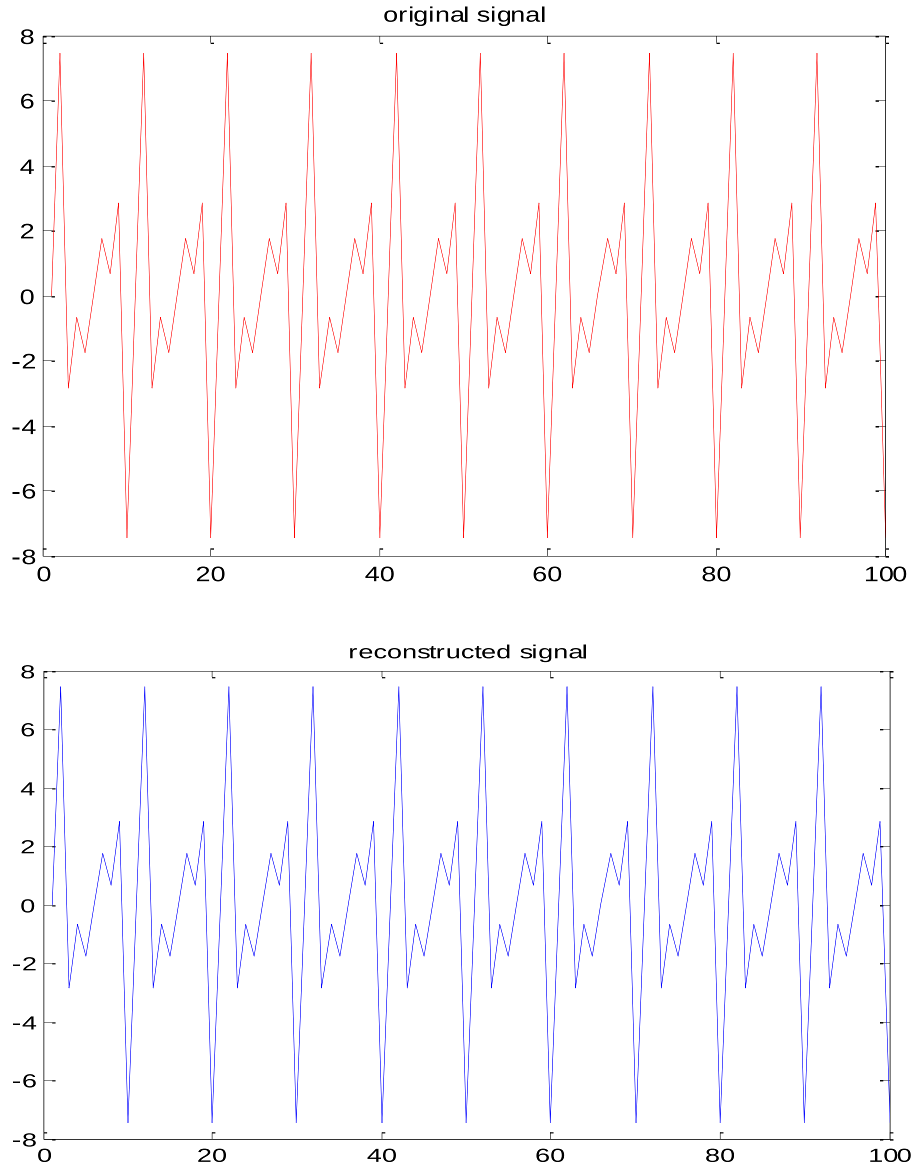
- Random Demodulator (RD) is used for Compressive Sensing of Analog signals.

- Recovery of the sparse set of coefficients that represent signal from sub-Nyquist samples.

RD

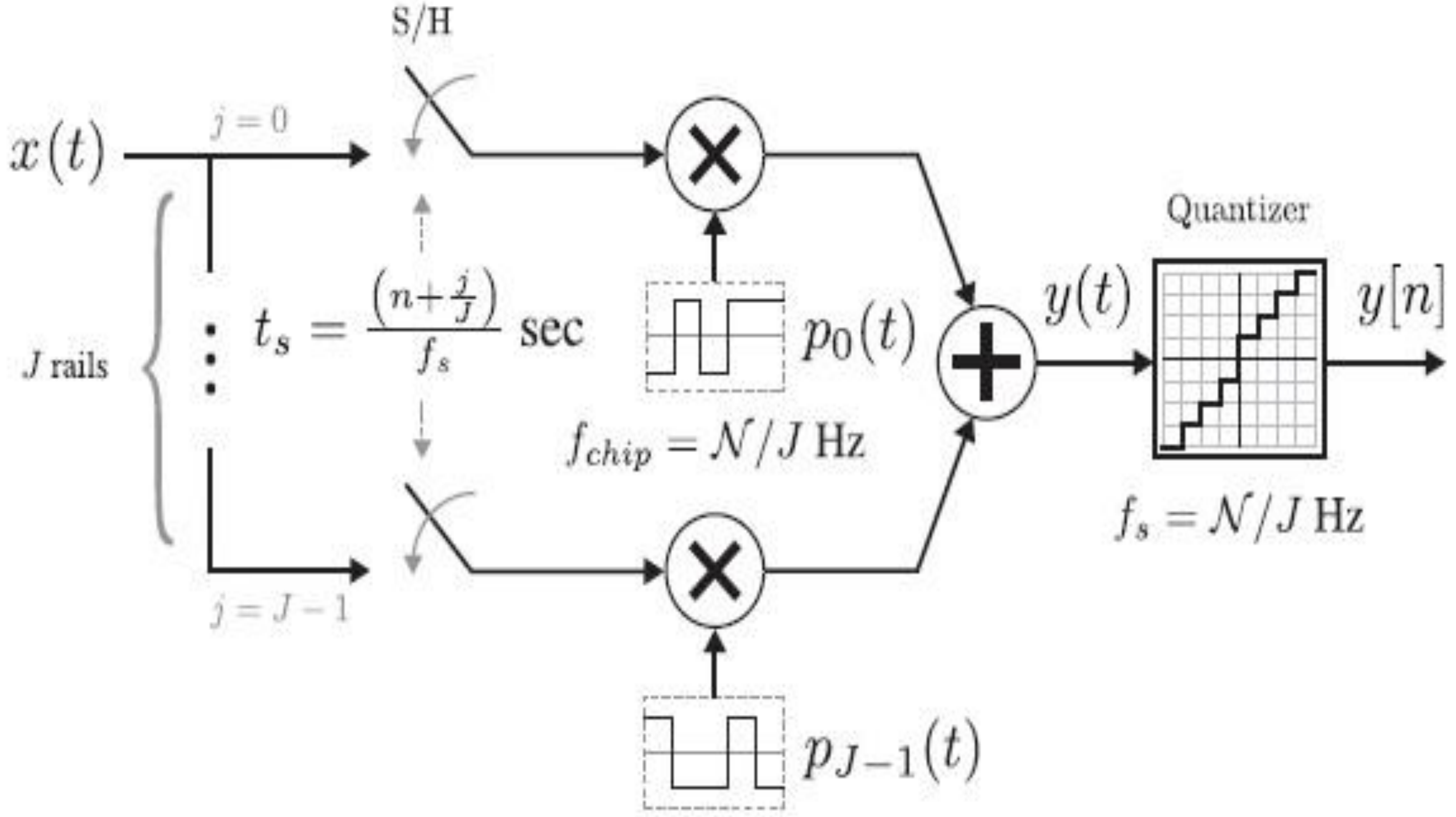

$f(t) \cdot p_c(t)$
$t = \frac{n}{R}$
$f(t)$
$\int_{t-\frac{1}{R}}^{t}$
$y[n]$
$p_c(t)$
Seed
Pseudorandom Number Generator

Result obtained-RD

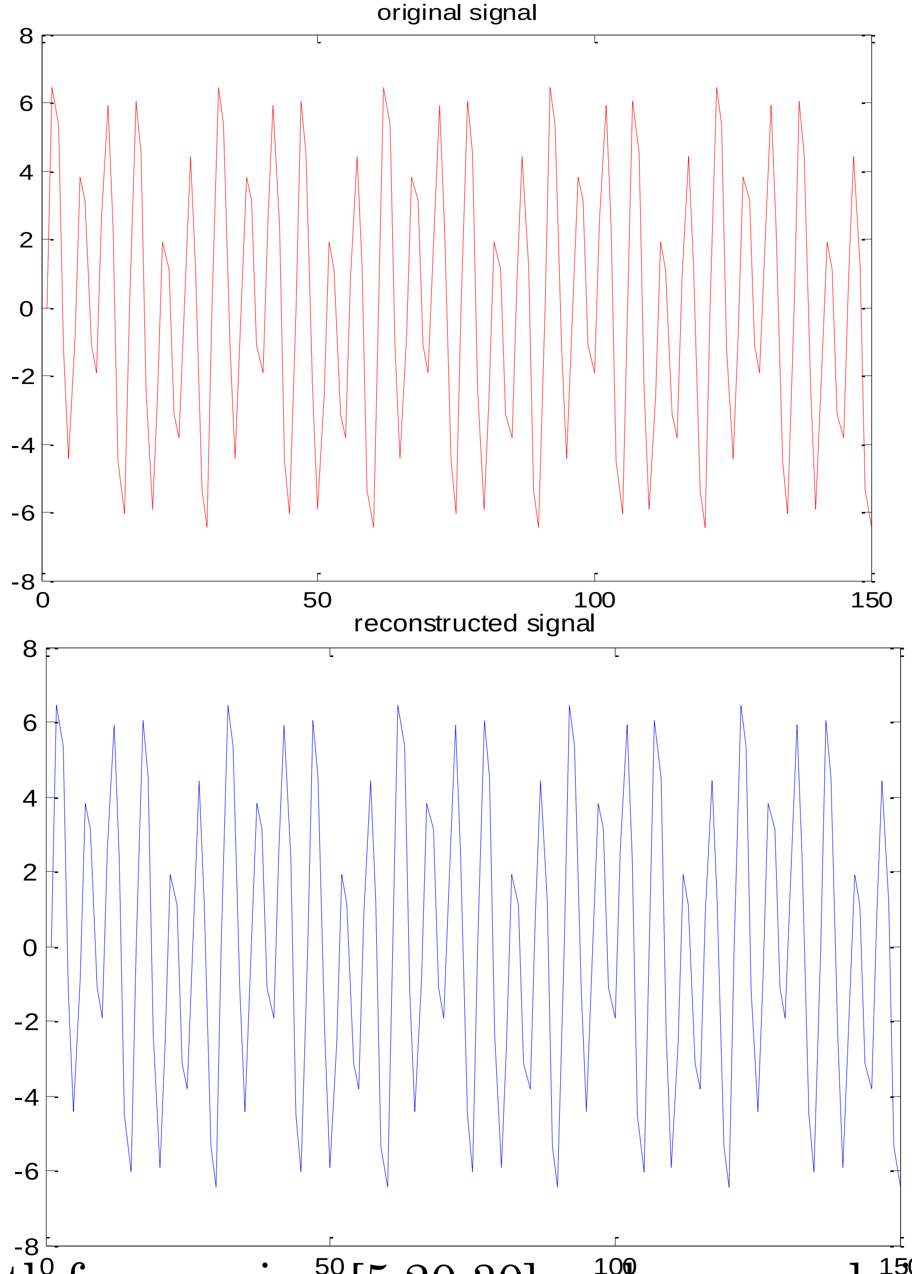


Input signal with frequency [10,20 ,30] Hz and recovered signal from M=17

POLY-PHASE RANDOM DEMODULATOR

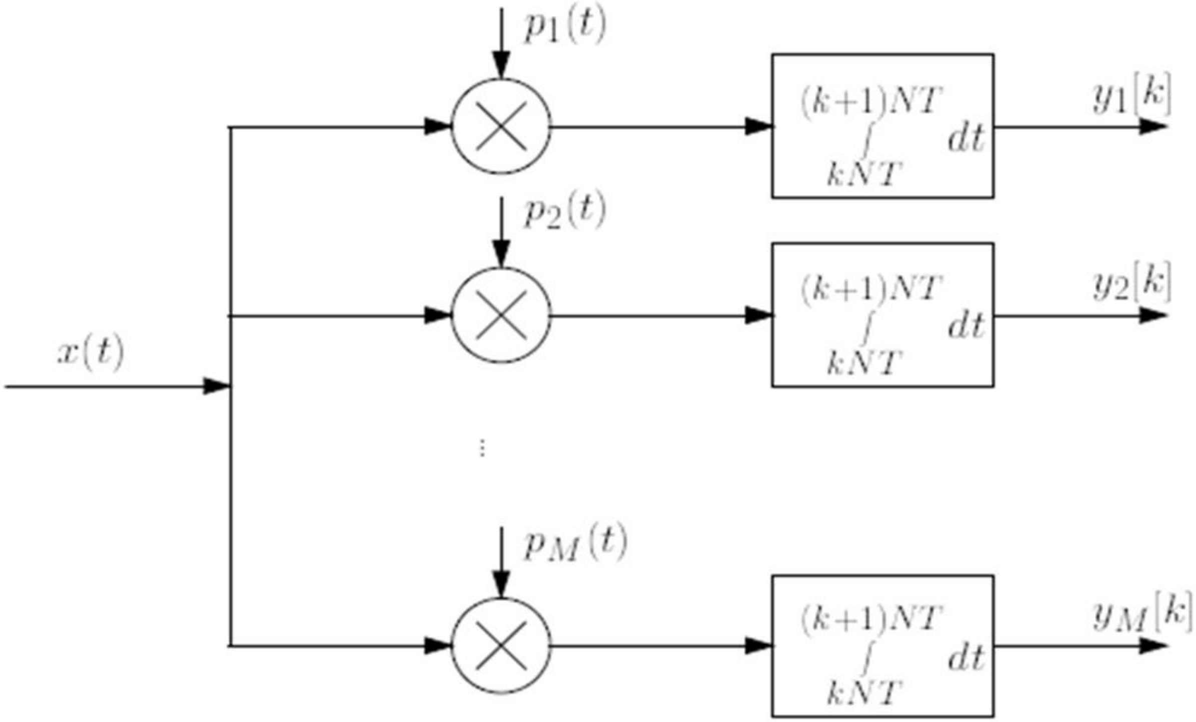


RESULT



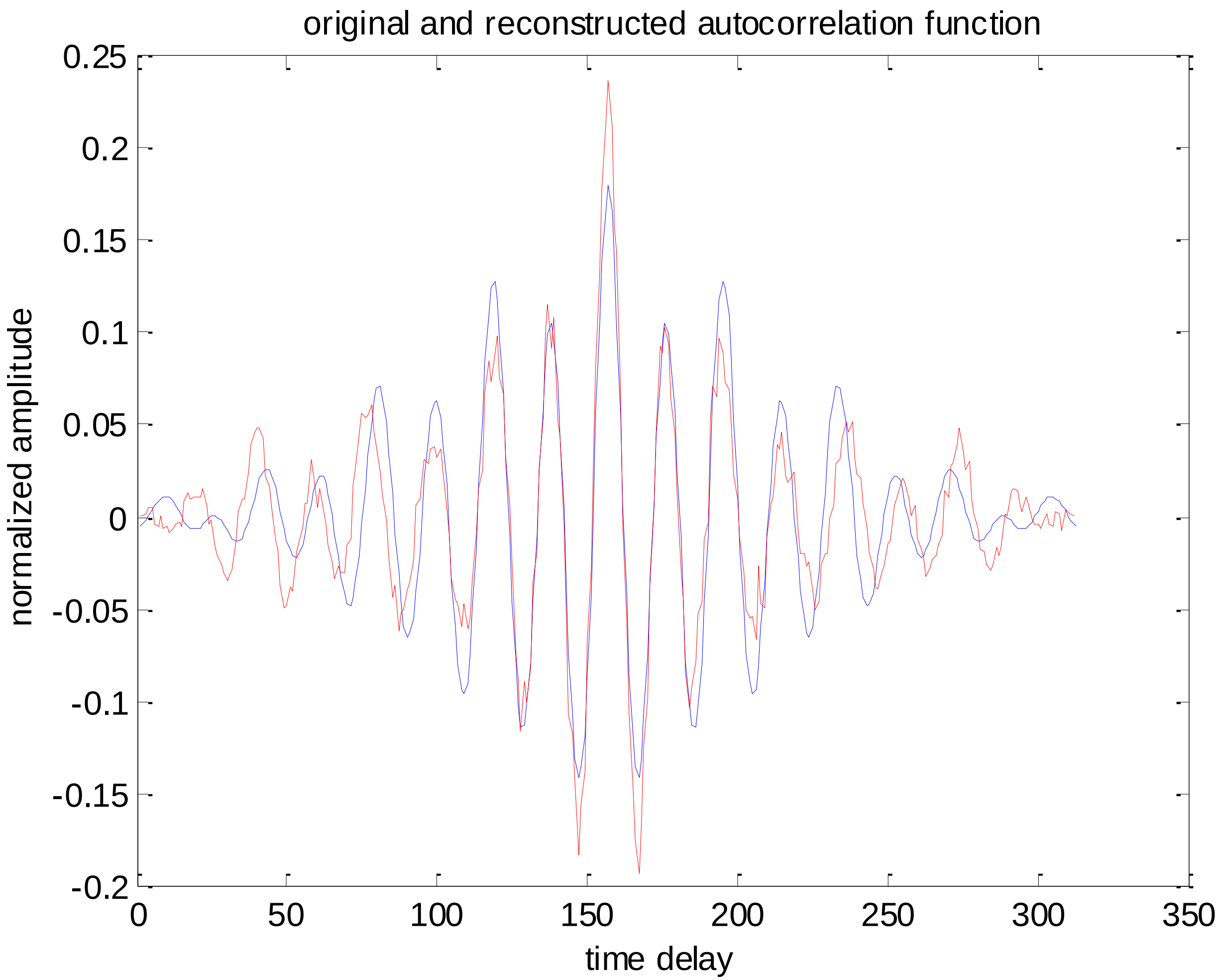
Input signal with frequencies=[5 20 30] and recovered signal from M=16 and number of rails =5

PSBS sampler

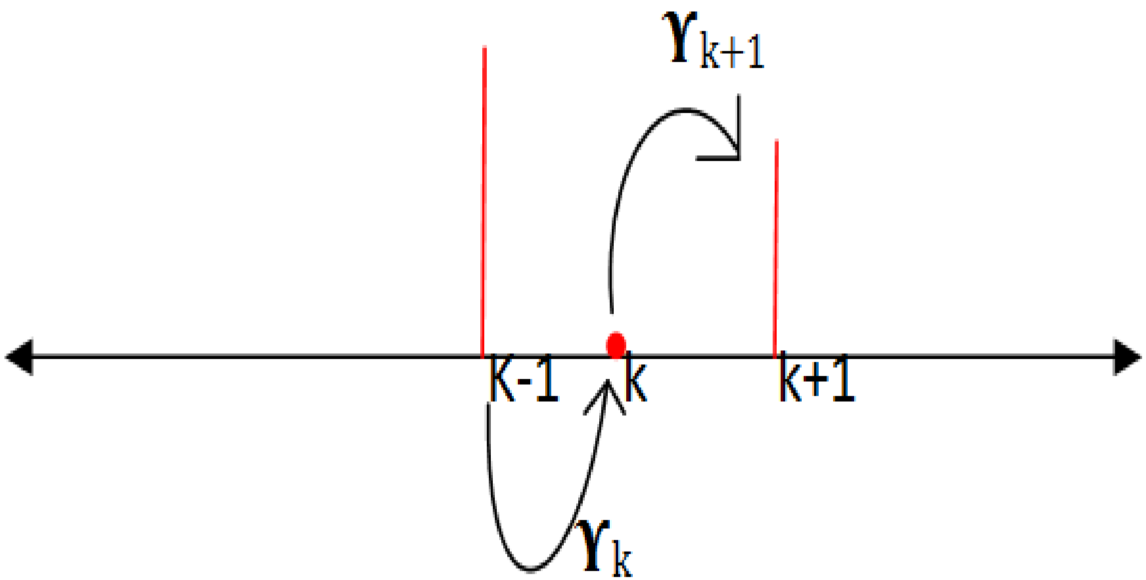


$$y_i[k] = \int_{kNT}^{(k+1)NT} p_i(t)x(t)dt$$

Result obtained-PSBS

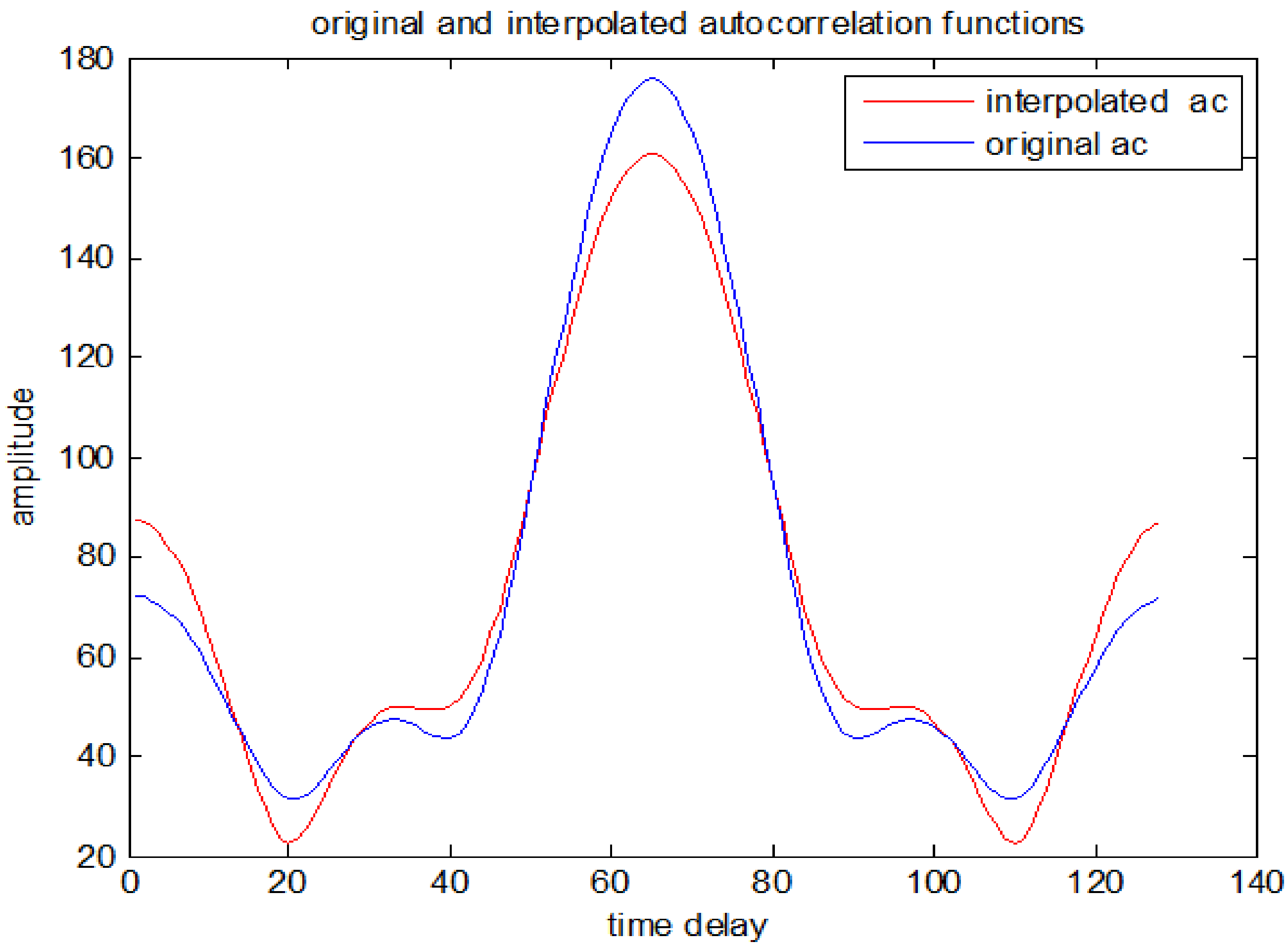


Single channel PSBS and autocorrelation interpolation



Knowing R(k-1) and R(k+1) we have to find R(k)

RESULT



original and interpolated autocorrelation functions
180
160
140
120
100
80
60
40
20
amplitude
0
20
40
60
80
100
120
140
time delay
interpolated ac
original ac

Different Recovery techniques and performance analysis

- $y = \Phi x = \Phi \psi s = As$

N X 1 where $x = \psi s$, s has only K non-zero values

$K < M << N$

M X 1 M X N

- Recovery techniques mainly classified into two

1) Convex optimization
2) Greedy algorithms

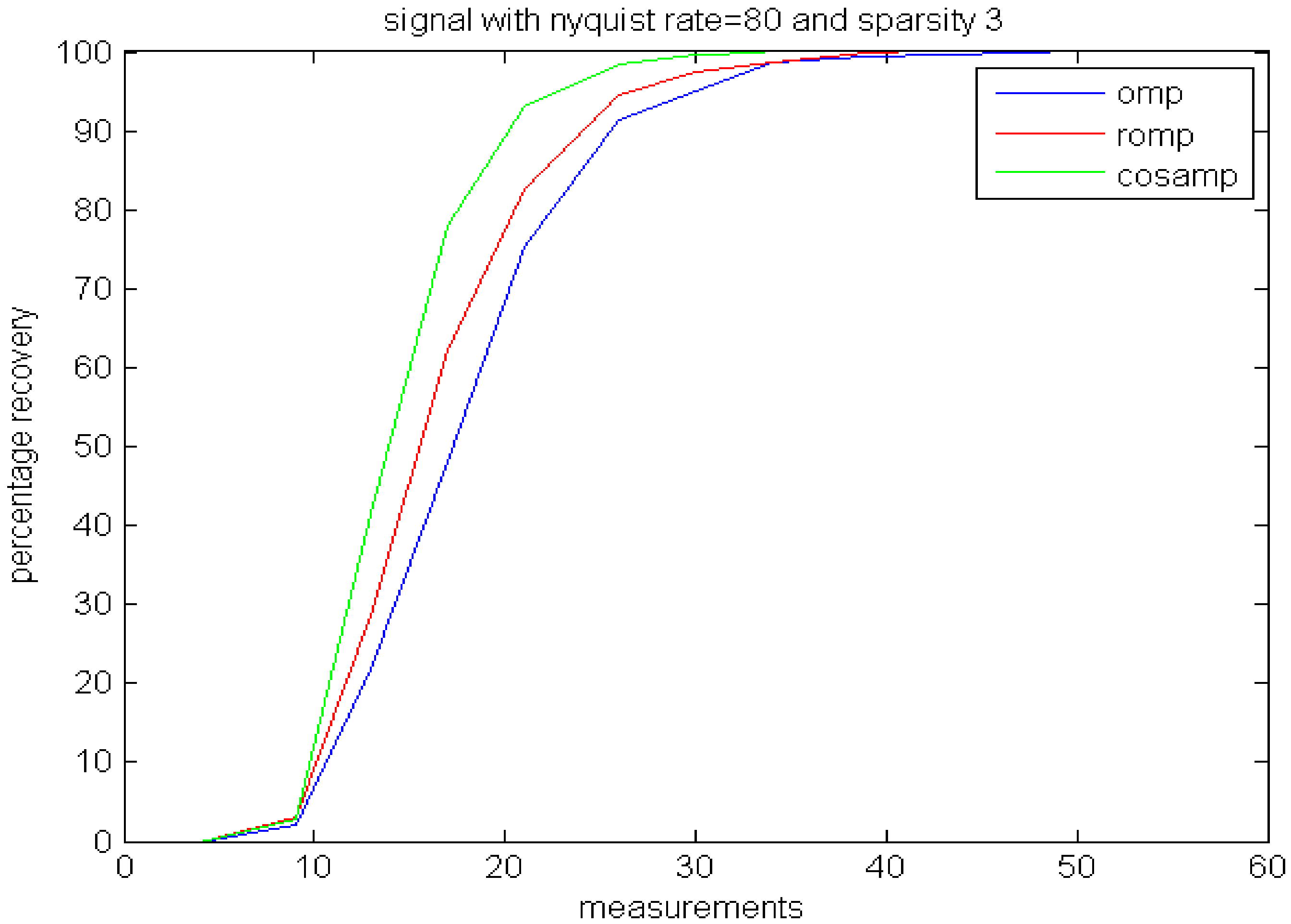
Performance Analysis

- Experimental setup: A sparse signal of length N=80 and sparsity K=3 was taken as input.
- Upper bound for mean square error :0.01

Percentage of successful recovery for various M values.

No: of measurements	M=13	M=26	M=39
OMP	22%	90%	98%
ROMP	26%	94%	99%
CoSaMP	41%	98%	100%

Recovery in errorless case



- **Inference:**

  1) CoSaMP has better performance compared to the other two in noiseless case.

Erroneous case:

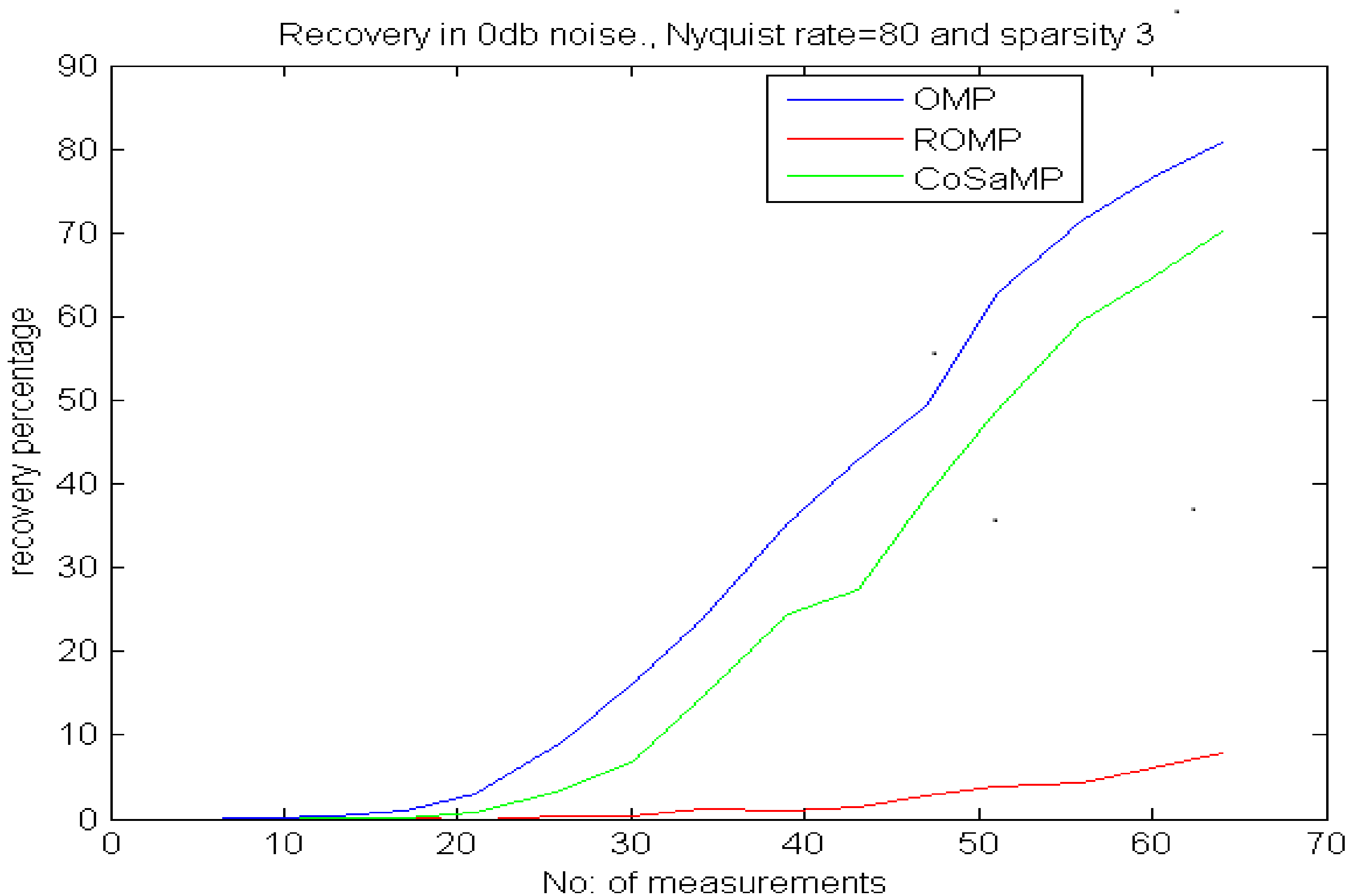
Case 1: Noise is added during transmission of measurements.

$$\tilde{y} = y + e$$

Case 2: Measurement itself is noisy

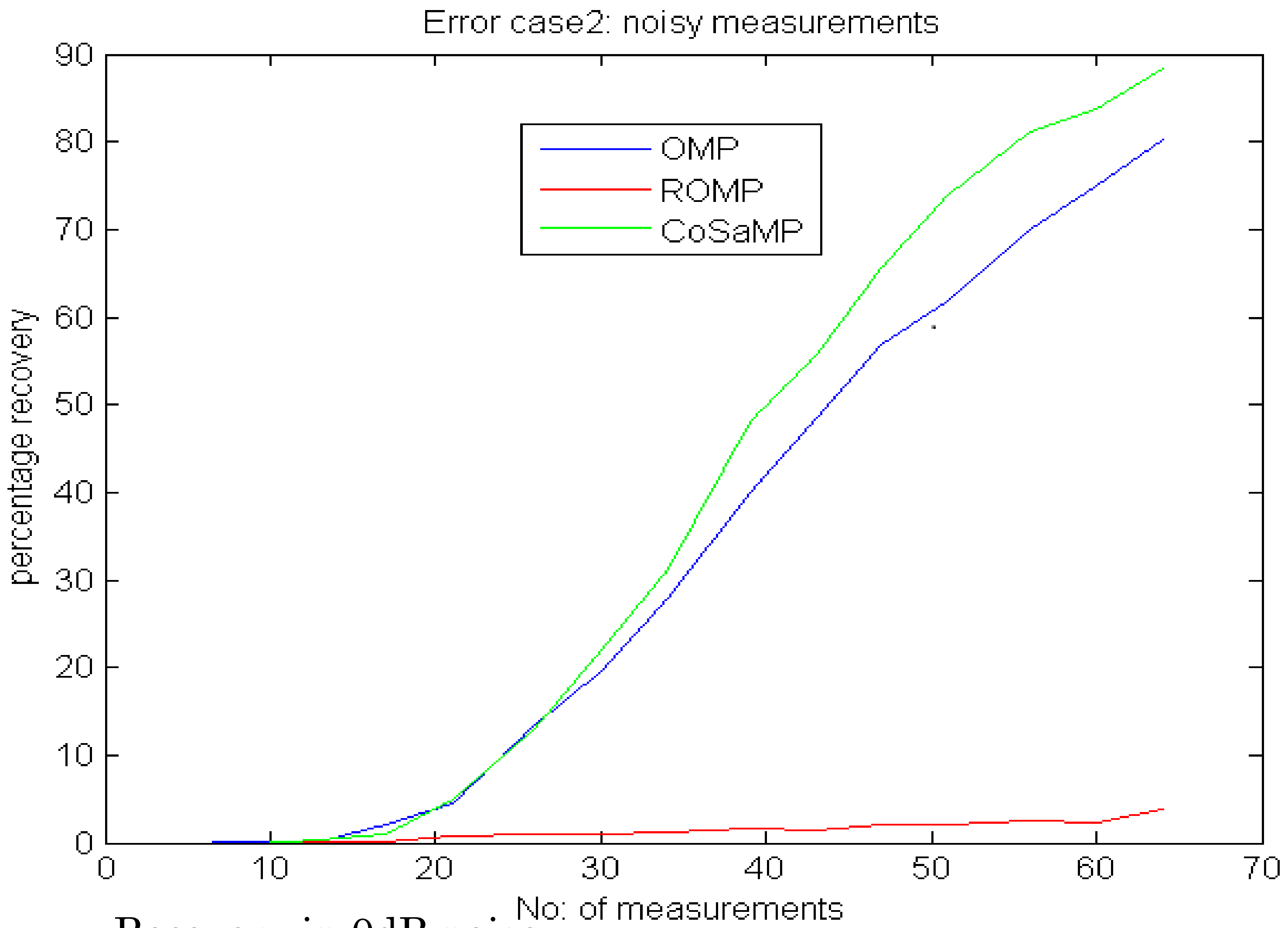
$$\tilde{y} = \Phi(x + x_i)$$

Recovery in erroneous case-1



Recovery in 0dB noise

Recovery in erroneous case-2



Recovery in 0dB noise

Interference cancellation

- Assumptions: signal and interference are orthogonal and interference is sparse with sparsity K_{i}

$$\tilde{y} = \Phi(x + x_i)$$

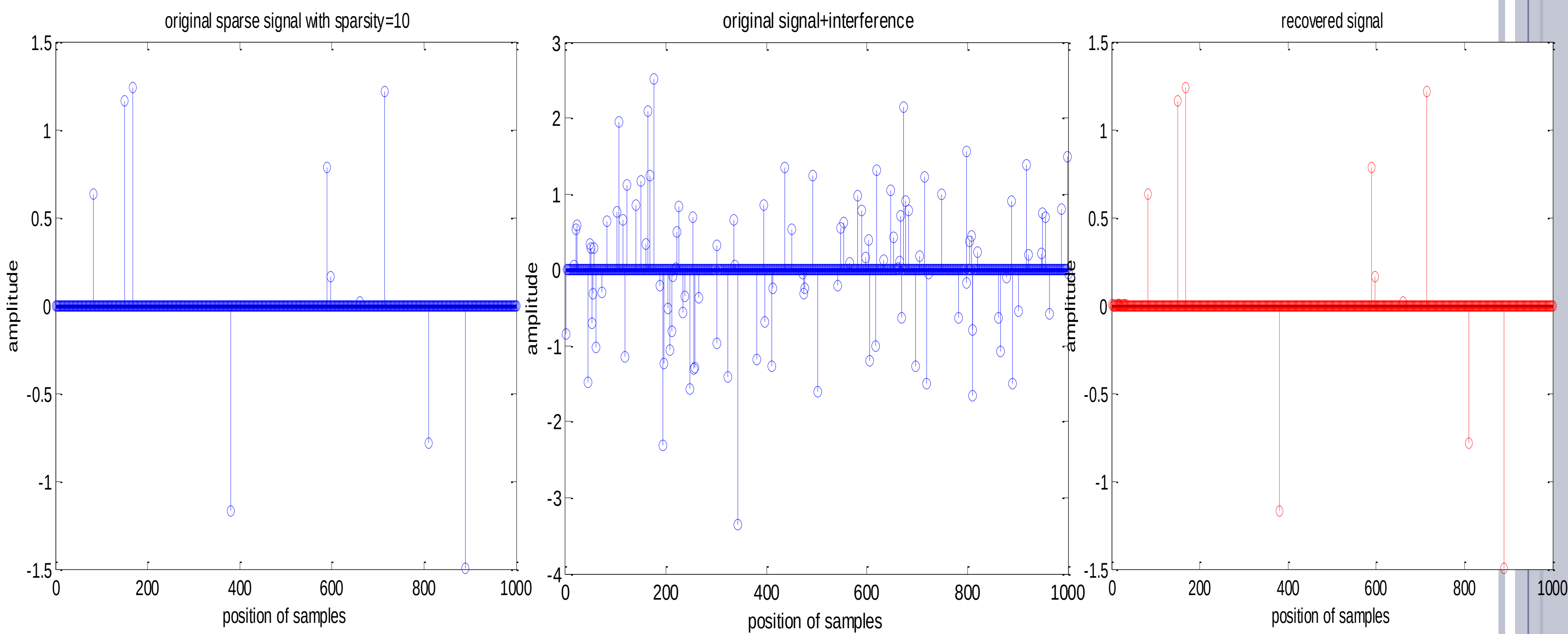
- Solution: Find a projection operator (P) whose null space is the range space of (Φ_{j}), where Φ_{j} is columns of Φ corresponding to the position of non-zero values in x_i
- Thus

$$P = I - \Phi_j \Phi_j^*$$

$$P\tilde{y} = P\Phi(x + x_i)$$

$$P\tilde{y} = P\Phi x$$

Result



Signal length N=1000 and No: of measurements M=150

Observation

- Interference cancellation leads to decrease in information power.
- Information power before removing noise : 0.3551 units
- Information power after removing noise : 0.2537 units
- Noise removal followed by recovery shows better result.

Work done in this semester: Problem Identified

- Measurements are obtained by projecting the signal to a random sequence with Gaussian distribution.
- Redundancy of a source X with a given power is given as

  Redundancy (r)= 1-H(X)/H(G)
- QUESTION: Is it possible to reduce the number of measurements, retaining the same redundancy achievable using Gaussian measurements, by choosing a measurement matrix with statistics same as that of source?

EXPERIMENTATION SETUP AND RESULT

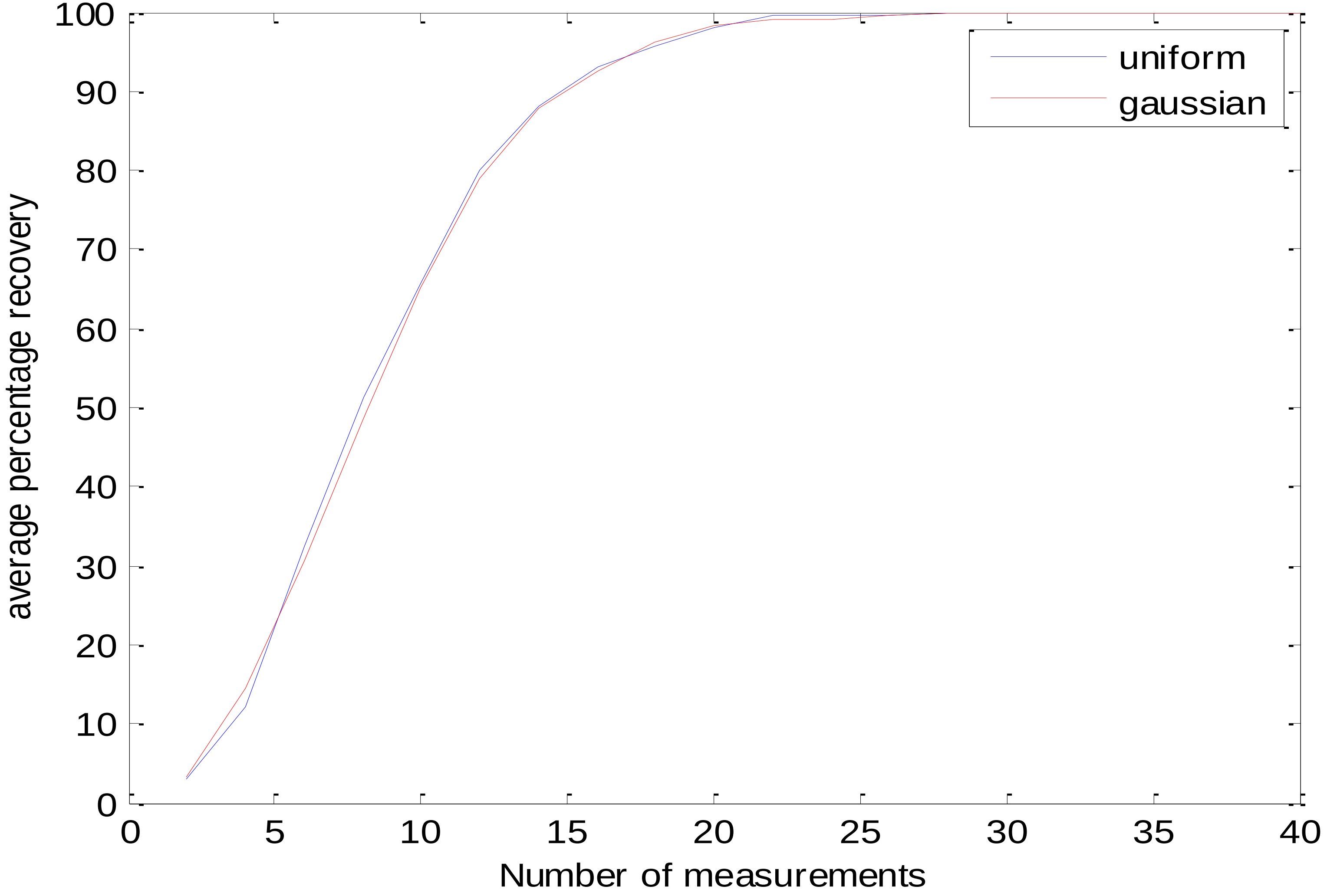
- A sinusoidal data of single frequency sampled at its Nyquist rate was taken as input (X).
- Two measurement matrices were taken

  - one with Gaussian distribution

  - other with Uniform distribution
- Redundancy for a particular number of measurement (M) was observed and found that $r_2 > r_1$;

  where $r_2 = 1\text{-}H(X)/H(U)$ and $r_1 = 1\text{-}H(X)/H(G)$.
- Thus number of measurements could be reduced but recovery was poor in that case.

Experimentation setup and result



Optimization of measurement matrix

- How to select the measurement matrix?
  - RIP and Incoherence
- Random sequence with certain pdf satisfies these criteria.
- But is this the best?

MEASUREMENT MATRIX BY ALTERING THE EIGENVALUES

- Does the M rows of Φ matrix capture equal information?
- Give a spherical structure to the random matrix.
- ALGORITHM

Choose a random Φ .

Represent Φ using Singular Value Decomposition (SVD) as:

$\Phi = U \Sigma V^T$

Now, replace the eigenvalues in Σ with its mean to get Σ'

New measurement matrix

$\Phi\_new = U \Sigma' V^T$

Measurement matrix From sparsifying matrix

- Why do we go for independent design of measurement matrix?
- Aim: To find a measurement matrix from the sparsifying matrix such that $A = \Phi \psi$ has a similar structure to that of ψ.
- Uses the concept of Multi-Dimensional Scaling (MDS).

MDS

- Projecting higher dimensional data to lower dimensional space such that pair wise distance is preserved.
- Suppose $X=\{x_1, x_2, \ldots x_N\}$ be a matrix with each $x_i \in R^N$ and D be the matrix of pair-wise distance.
- Now let $Y=\{y_1, y_2, .. y_N\}$ be the projected lower dimensional data with each $y_i \in R^M$ and $\dot{D}$ be the new matrix of pair-wise distance.
- We want $\dot{D}$ to be nearly equal to D.
- Thus, the solution is found out by Singular Value Decomposition of the matrix X.
- The solution turns out as

  $Y=U^T X$

  where U is the matrix of eigenvectors corresponding to the largest M eigenvalues of the matrix X.

Measurement matrix From sparsifying matrix

- Using the concept of MDS

  $A = U^T \psi$;where U the matrix of eigenvectors corresponding to the largest M eigenvalues of ψ
- Thus $\Phi = U^T$
- Since U is a rotation matrix it will be incoherent with ψ

<u>Restricted Isometry Property</u>

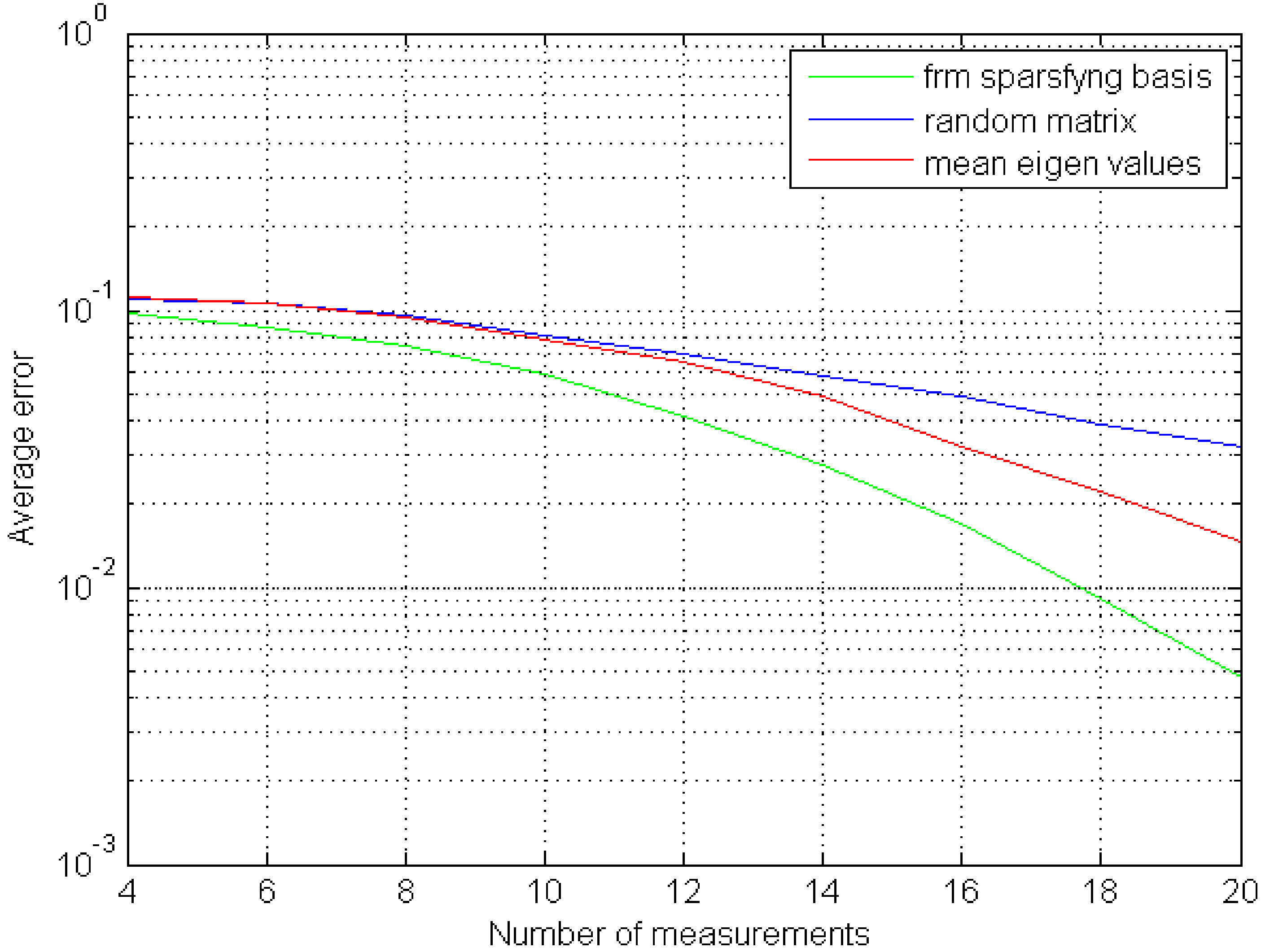
$$y = \Phi x = \Phi \psi s$$

$$= U^T \psi s = U^T U \Sigma V^T s$$

$$= \Sigma V^T s \text{ ; for } M=N$$

$$\sigma_{\min}^2 \|s\|^2 < \|y\|^2 < \sigma_{\max}^2 \|s\|^2$$

Result



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Best basis for the class of speech signals

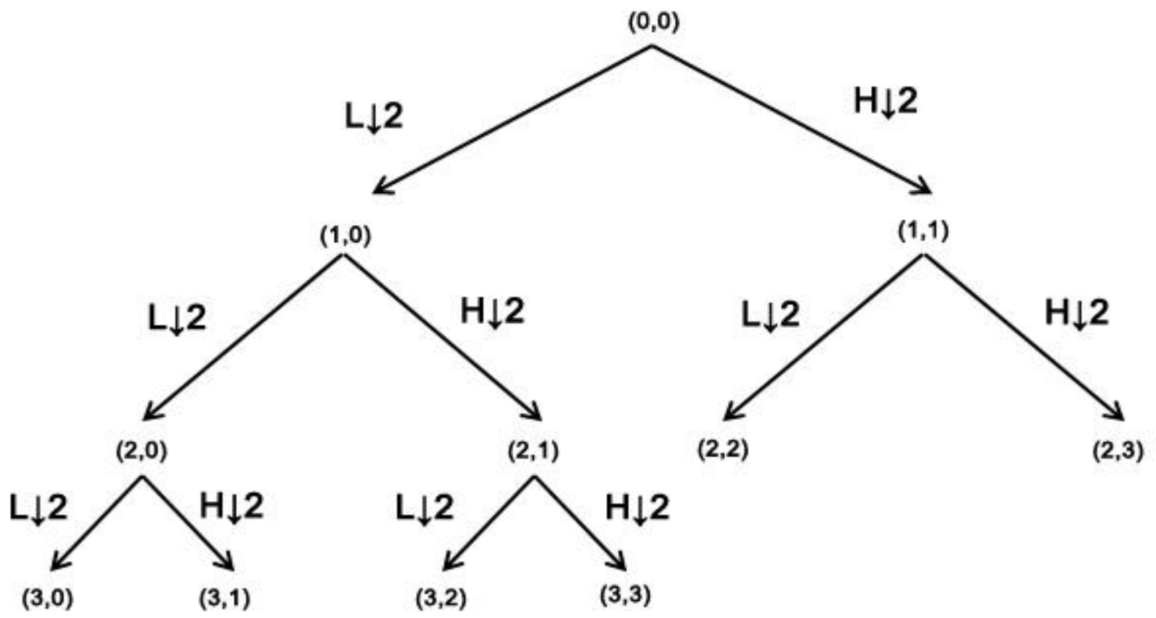
- Scheme based on minimization of Shannon entropy
- Dimension **d= exp (H(X)).** Reduction of H(X) leads to reduction of d.
- For a normalized signal, energy of each sample is considered as probability of that component.

Entropy based best basis algorithm

1. Normalize the signal
2. Do the decomposition that reduces the entropy
3. If H is the entropy of the parent node and H1 and H2 are the entropy of the children nodes obtained by decomposing the parent, we choose the decomposition only if $H > H1+H2$

BEST BASIS FOR THE CLASS OF SPEECH SIGNALS CONTD…

- The following tree structure was obtained with six terminal nodes



- Impulse response at each of the terminal nodes forms the basis corresponding to that node.

CS OF SPEECH SIGNALS

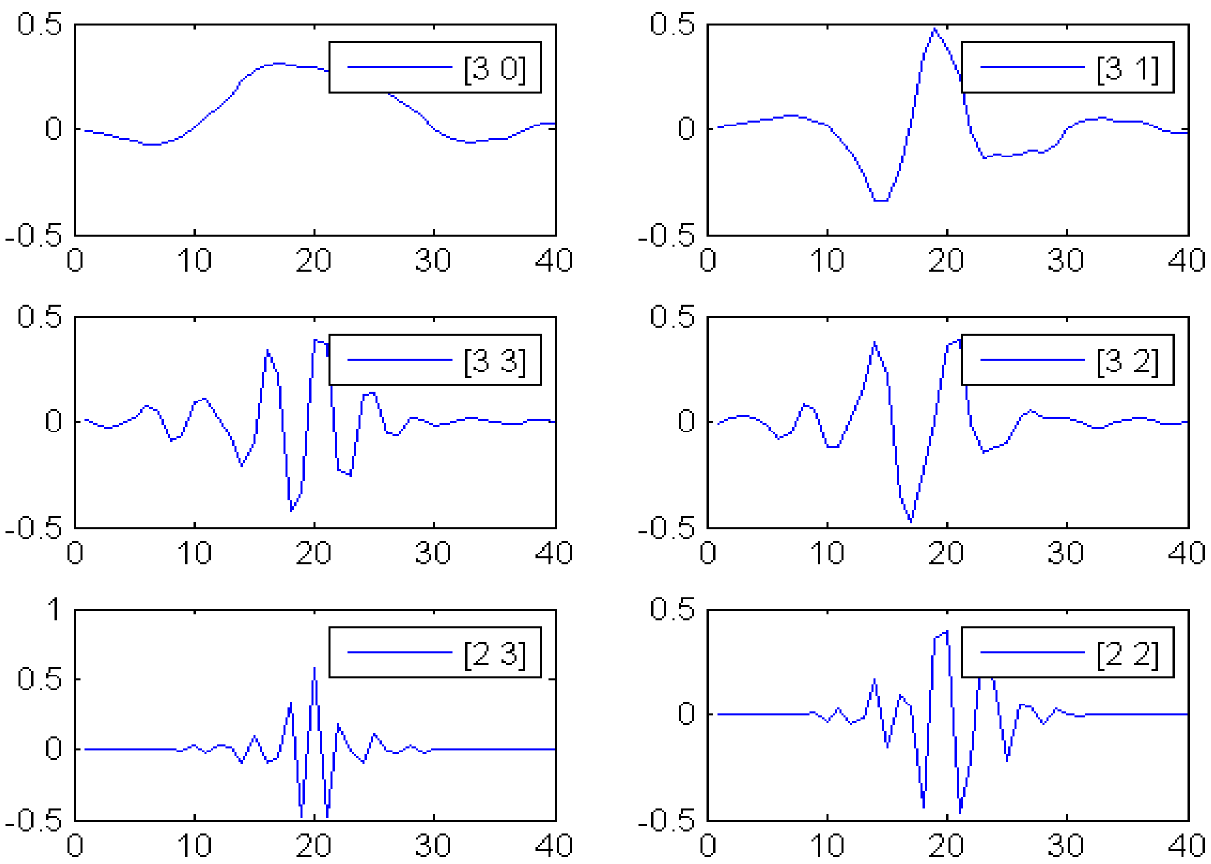
- First step is to find the sparsifying basis using the entropy based best basis algorithm.
- Next we have to derive a measurement matrix from the sparsifying matrix using MDS based technique.
- Recovery is done using OMP.

Experimentation setup

- An audio signal sampled at 8000Hz was taken as input. A block of 40 samples were taken at a time.

- The impulse response at each of the terminal nodes were found by passing a delta at the (0,0) node and reconstructing the coefficients obtained at each terminal node.

IMPULSE RESPONSE AT EACH NODE



RESULTS

- The representation basis (ψ) was formed using the six impulse responses and its even translates thus forming a 40 x 240 matrix, and a 40 x 40 matrix.

No: of measurements (M)	% of compression	SNR in dB using SVD (40X40)	SNR in dB using SVD (40X240)	SNR in dB using random matrix
20	50	7.7556	5.4642	-0.0515
24	40	11.4210	11.6564	0.9151
30	25	15.2802	13.3712	3.9993
32	20	17.9644	13.9388	4.3979
36	10	23.4415	16.4167	8.8104

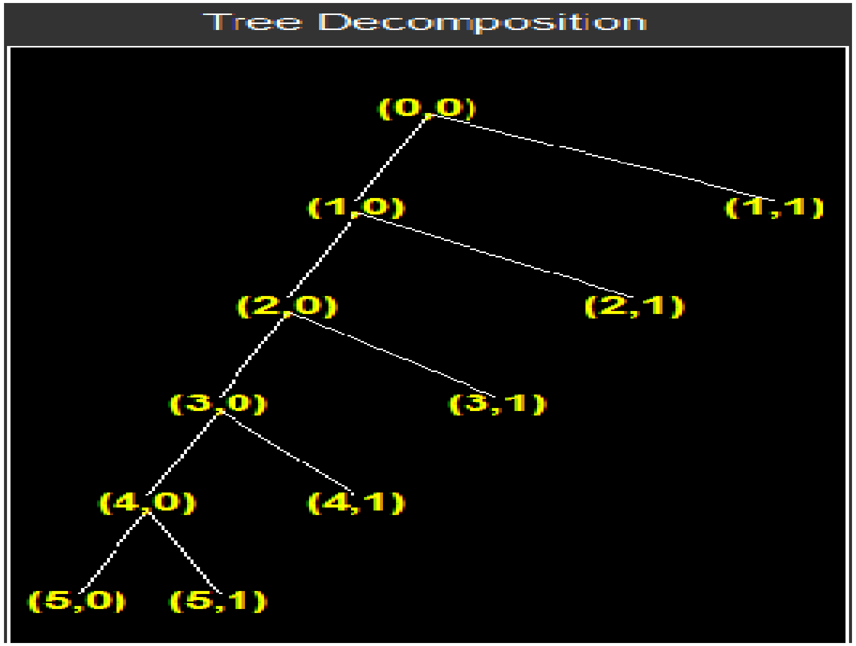
AUDIO RESULT

- Original signal
- Reconstructed signal with a compression of 40% using SVD based technique
- Reconstructed signal with a compression of 40% using random projection

CS OF TIME LIMITED SIGNAL (PULSE)

- Nyquist rate is infinity.
- Sampling it at any bounded rate leads to sub-Nyquist sampling.

Best tree for time limited signals

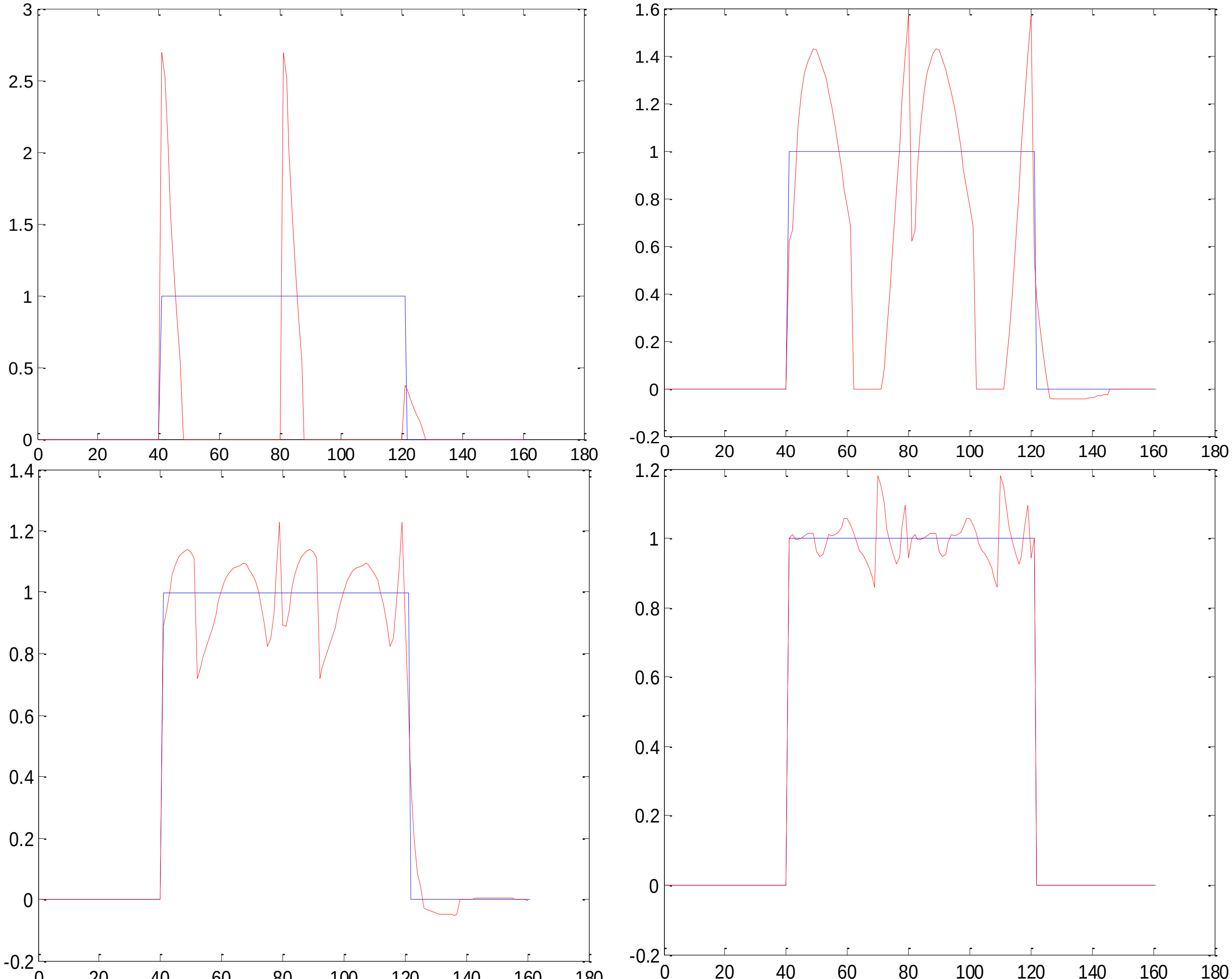


RESULT

- Recovery in SNR for an infinite band signal from measurements taken at 80% of sampling rate.

Time interval in seconds	Sampling period (T) in seconds	SNR in dB
[-10 10]	1/4	17
[-5 5]	1/6	16.5
[-1 1]	1/20	16.88
[-0.5 0.5]	1/30	17.8

Result showing recovery at different measurements



Critical Evaluation and conclusion

- The first phase comprised of implementing RD and PSBS thereby understanding the concept of CS in analog domain.
- Extending the work of PSBS to single channel opens a new way of thinking, for interpolation of under-sampled data to get the full information regarding the power spectrum of the signal.
- A possible solution for this problem can be found by studying Reproducing Kernel Hilbert Space.
- In the second phase different recovery algorithms and its performance were studied.

CRITICAL EVALUATION AND CONCLUSION CONTD..

- In the third phase, methods to optimize the measurement matrix were explored.
- Entropy based best basis for the class of signal was found but its optimality is not guaranteed.
- Methods to find an optimum sparse representation can be explored.
- A structured measurement basis showed better recovery compared to other methods and such a matrix satisfies Restricted Isometry Property.

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THANK YOU