# Computer Assignment 3: Start up problem of Couette flow

Consider stationary viscous fluid bounded by two parallel plates extended to infinity, as shown in the Figure 1, so that no end effects are encountered. The


Figure 1: Flow between parallel plate.

spacing between the two plates is $h$. The lower plate is suddenly accelerated in the x-direction to velocity $U_0 = 40$ m/s. As a result, the fluid motion will be developed, which is governed by

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$$

where, $\nu$ is kinematic viscosity of the fluid.

1. Compute the velocity profile u=u(y,t) using both (i) Explicit Forward Time Central Space (FTCS) scheme and (ii) Crank-Nicolson scheme for the given initial and boundary conditions:

    - Initial Conditions:
      $u(y, 0) = U_0$ m/s for $y = 0$ and $u(y, 0) = 0$ m/s for $0 < y \leq h$
    - Boundary Conditions at $t \geq 0$:
      $u(y, t) = U_0$ m/s for $y = 0$ and $u(y, t) = 0$ m/s for $y = h$

2. Given h=0.04 m, $\nu = 0.000217\ m^2/s$.

3. Consider 61 grid points in the y-direction.

4. Find the time step for the explicit FTCS scheme from the relation

    $$\frac{\nu \Delta t}{\Delta x} \leq 0.5$$

    for stability and use it for your simulation.

5. Verify that the explicit FTCS is unstable for $\frac{\nu \Delta t}{\Delta x} > 0.5$.

6. Plot the velocity profiles (i.e. y versus u(y,t)) at time instant 0.15 sec, 0.8 sec and 1.5 sec using both explicit FTCS and Crank-Nicolson schemes. Compare them on the same plot. The $\Delta t$ and $\Delta y$ values should be kept same for both the schemes for fair comparison. Use different line type and colors in the plot for FTCS, Crank-Nicolson and exact solution. The exact solution is given as

$$u(y,t) = U_0 \left\{ \sum_{k=0}^{\infty} erfc[2k\eta_1 + \eta] - \sum_{k=0}^{\infty} erfc[2(k+1)\eta_1 - \eta] \right\}$$

Expanding it to a few terms (that is to be used by you in the program)

$$\begin{aligned} u(y,t) = U_0[&erfc(\eta) - erfc(2\eta_1 - \eta) + erfc(2\eta_1 + \eta) - erfc(4\eta_1 - \eta) \\ &+ erfc(4\eta_1 + \eta) - erfc(6\eta_1 - \eta) + \ldots] \end{aligned}$$

where

$$erfc(\eta) = 1 - erf(\eta)$$

$$erf(\eta) = \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-z^2} dz = \frac{2}{\pi} \left( \eta - \frac{\eta^3}{3} + \frac{\eta^5}{10} - \frac{\eta^7}{42} + \frac{\eta^9}{216} - \ldots \right)$$

$$\eta = \frac{y}{2\sqrt{\nu t}} \qquad \eta_1 = \frac{h}{2\sqrt{\nu t}}$$

7. Plot y versus ERROR for both explicit FTCS and Crank-Nicolson schemes on the same graph, when the error in the numerical solution is defined as

$$ERROR = \frac{\text{analytical value} - \text{computed value}}{\text{analytical value}}$$

Provide your comment based on the observation of this plot. The $\Delta t$ and $\Delta y$ values should be kept same for both the schemes for fair comparison.

General Instructions

- **Checklist for submission:**
  1. Flowchart of the C code written by you.
  2. The code (written in C only)[1] with proper inline documentation for each function. (Use meaningful variable names).
  3. A "README.txt" file which contains the proper description on how to run your code and get the plots.
  4. Brief report, in pdf format, where all the above mentioned results in terms of plots and comments are included. The plots submitted by you must be reproducible independently by the TA's from your code(s).

- **Instruction for submission:**
  - Rename your program file as your roll number (example: 204010006.c).
  - Rename the report as your roll number (example: 204010006.pdf)
  - Submit all the **4 documents** as stated in the Checklist above.

- **Notes:**
  - Marks will be given only if the program is working and showing correct result. No step marks will be given.
  - Assignment will not be evaluated if "instruction for submission" are not followed properly.

- **Copying program from each other or from any other source will lead to severe penalty.**

– E N D –

[1] You will be given zero mark for the entire assignment if any other computer language is used.