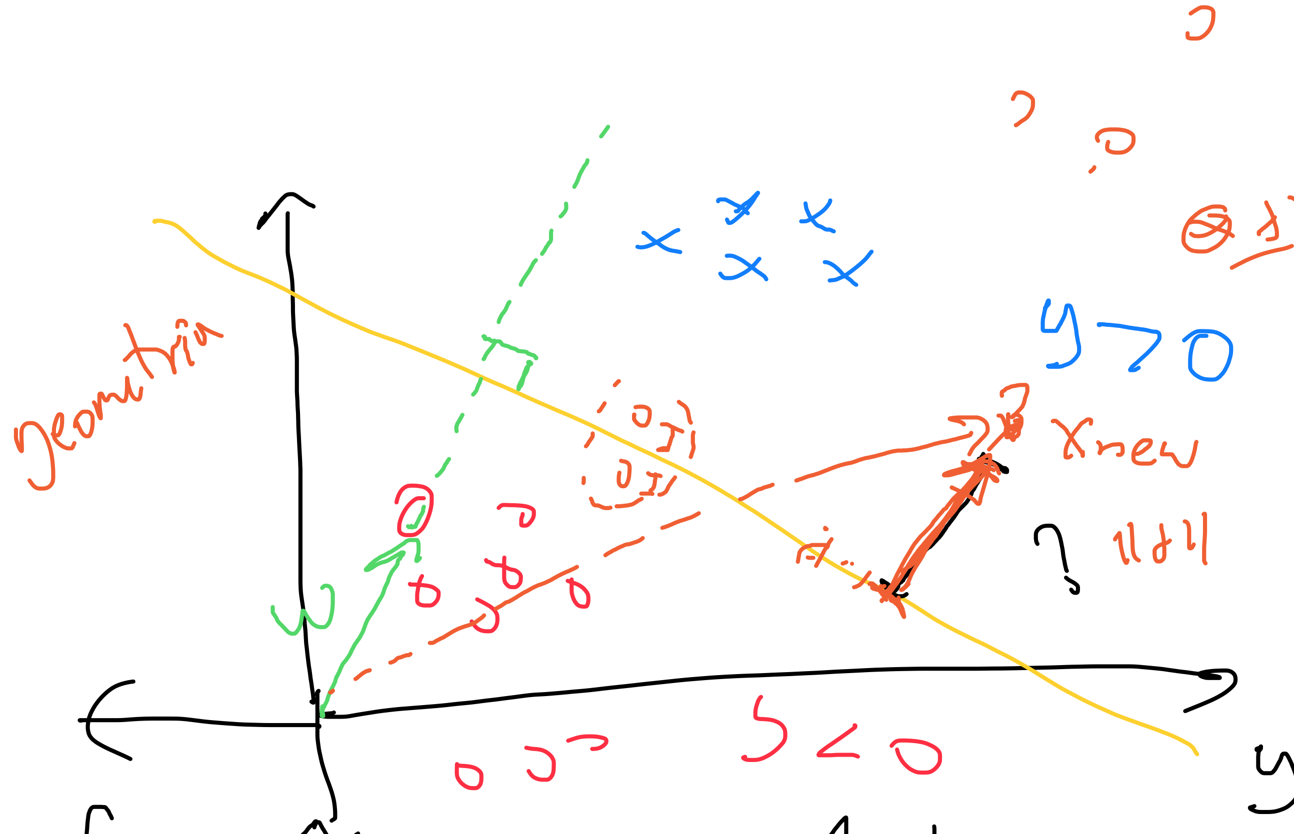
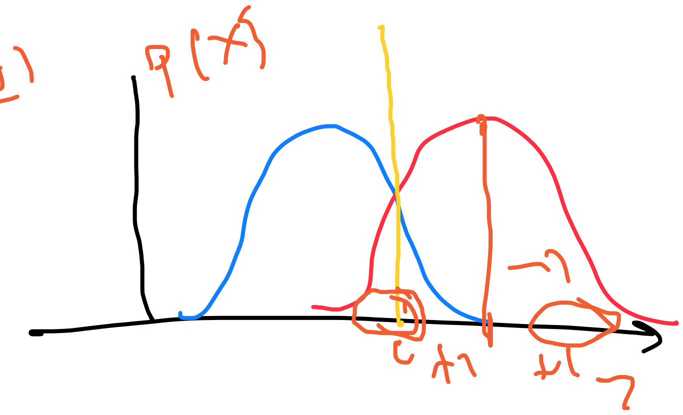


$y = 0$ frontera



$w \perp f(x) = 0$

$\hat{y} = 0$

$$\left\{ x_n \in \mathbb{R}^2,\ y_n \in \{-1, +1\} \right\}_{n=1}^{N}$$

o x

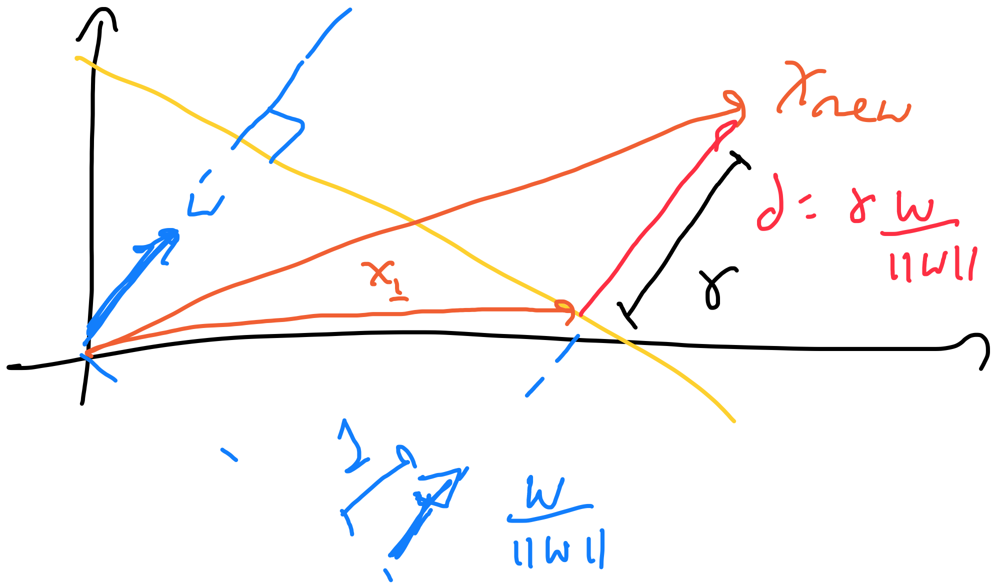
$$f(x_n) = \hat{y}_n \in \{-1, +1\}$$

Modelo lineal: $\hat{y}_n = f(x_n) = w^T x_n + b;\ w \in \mathbb{R}^2$

$$f(x_n) = 0;\ \forall x_n,\ \hat{y}_n = 0$$

$$y_{new} = \operatorname{sgn}\Big(\underbrace{w^T x_{new} + b}_{\in \mathbb{R}}\Big) = \begin{cases} +1 & w^T x_{new} + b \geq 0 \\ -1 & w^T x_{new} + b < 0 \end{cases}$$

Regresión



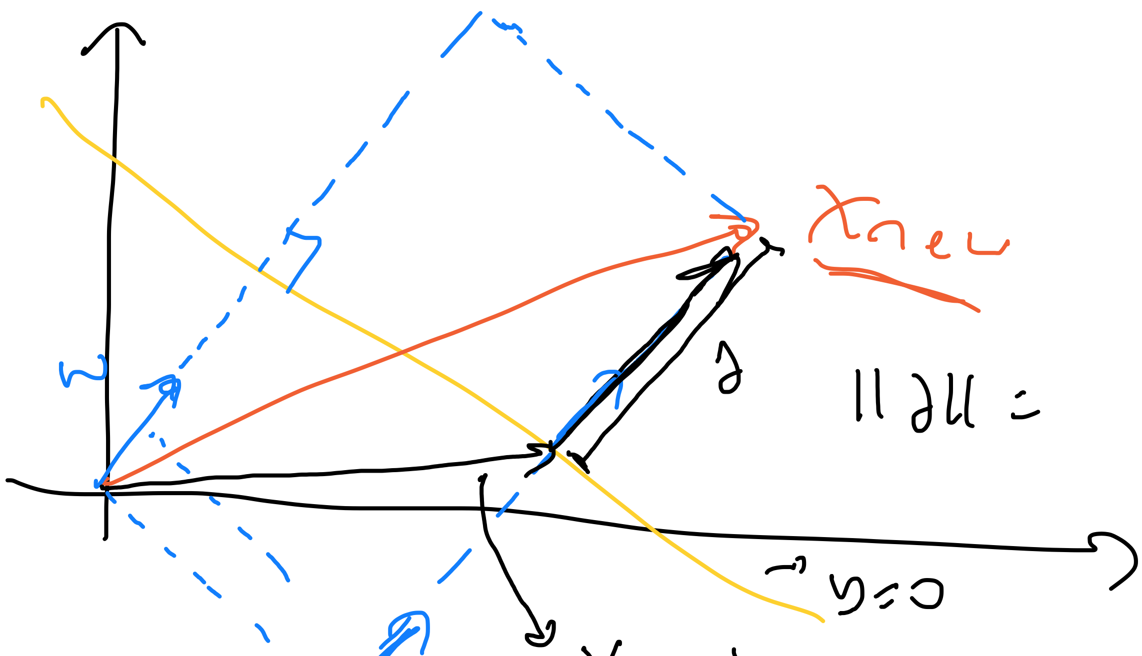
$; \gamma = \|d\|$

$$x_{new} = x_1 + \gamma \frac{w}{\|w\|} \; ; \; \gamma = \|d\| \; ; \; f(x_{new}) = f(x_1 + d) = w^T x_1 + b + w^T d$$

$$f(x_{new}) = \underbrace{\cancel{f(x_1)}}_{0} + w^T d = w^T \gamma \frac{w}{\|w\|} = \gamma \frac{w^T w}{\|w\|} = \gamma \frac{\|w\|^{2}}{\cancel{\|w\|}}$$

$$\boxed{\gamma = \frac{f(x_{new})}{\|w\|}}$$

$\rightarrow$ distancia de x_{new} a $f(x) = 0$. frontera.

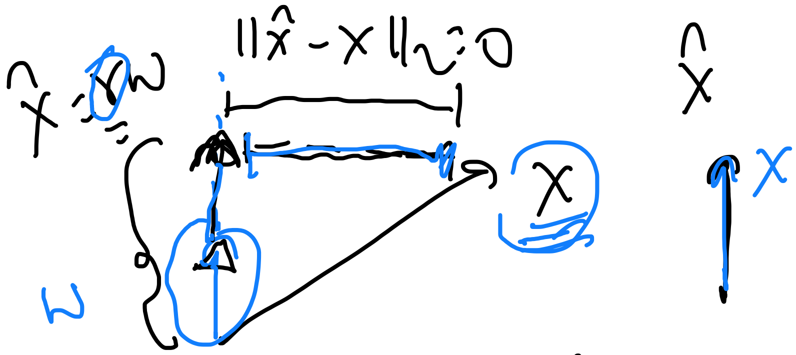


$x_\perp + d = x_{new}$

$f(x) = w^T x + b$

$$f(x_\perp) = w^T x_\perp + b = 0$$

$$f(x_{new}) = w^T (x_\perp + d) + b = w^T x_\perp + w^T d + b$$
$$= \underbrace{w^T x_\perp + b}_{0} + w^T d$$
$$= 0 + w^T d$$

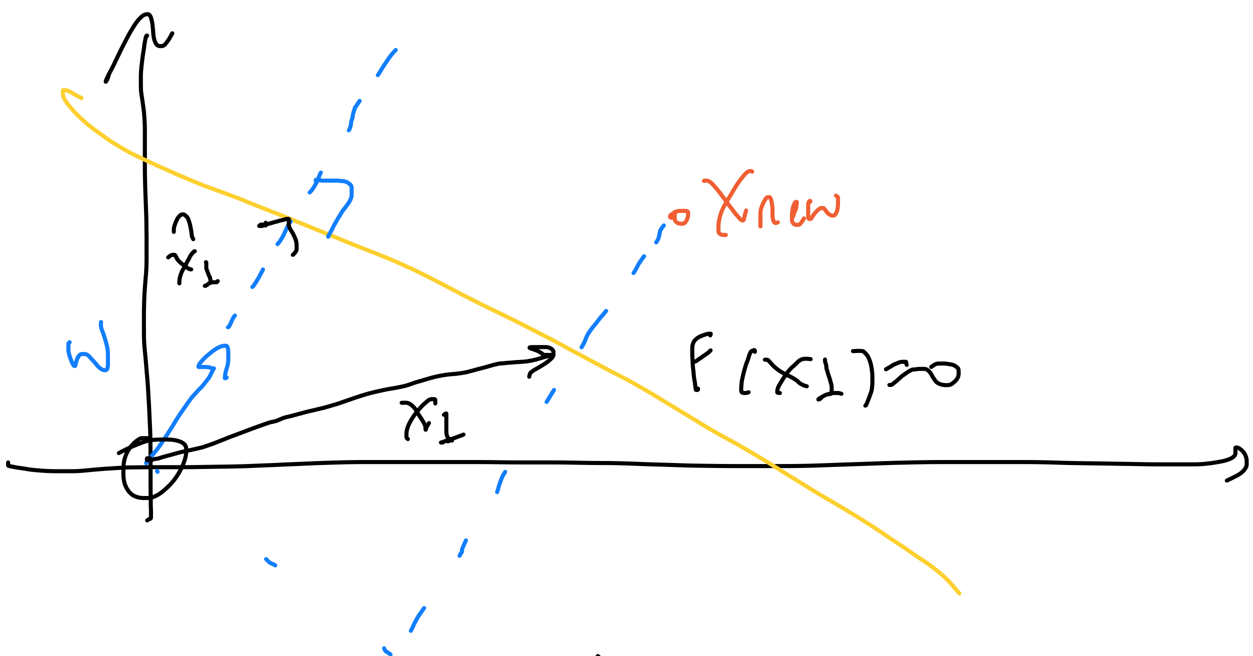


$$\min_\gamma \|\hat{x} - x\|_2, \quad \frac{\partial \|\hat{x} - x\|_2}{\partial \gamma} = \frac{\partial}{\partial \gamma}\left(\hat{x}^T \hat{x} - 2\hat{x}^T x + x^T x\right)$$

$$= \frac{\partial}{\partial \gamma}\left\{(\gamma w)^T \gamma w - 2\gamma w^T x + x^T x\right\} = 2\gamma w^T w - 2 w^T x = 0$$

$$\gamma = \frac{w^T x}{w^T w} = \frac{\langle w, x\rangle}{\|w\|_2^2} \in \mathbb{R}$$

$$\|\hat{x}\| = \|\gamma w\| = (\gamma^2 w^T w)^{1/2}$$
$$= \gamma (\|w\|^2)^{1/2} = \gamma \|w\| = \frac{w^T x}{\|w\|^2} \|w\| = \boxed{\frac{w^T x}{\|w\|} = \|\hat{x}\|}$$



$\hat{x}_\perp = r w$; $\quad r = \dfrac{\langle x_\perp, w\rangle}{\|w\|^2} = \dfrac{\langle w, x_\perp\rangle}{\|w\|^2}$

$\|\hat{x}_\perp\| = \|r w\| = (r^2 w^T w)^{1/2} = r\,(\|w\|^2)^{1/2} = r\|w\|$

$\|\hat{x}_\perp\| = \text{dist}(f(x_\perp), 0) = r\|w\| = \dfrac{w^T x_\perp}{\|w\|^{\cancel{2}}}\cancel{\|w\|} = \dfrac{w^T x_\perp}{\|w\|}$

$f(x_\perp) = w^T x_\perp + b = 0$; $\quad w^T x_\perp = -b$; $\quad \|\hat{x}_\perp\| = \dfrac{-b}{\|w\|}$