Análisis de componentes principales. $z_n = x_n w$; $\hat{x}_n = z_n w^T$.

$$\min_w \mathcal{E}_x\{\|x_n - \hat{x}_n\|_2^2\} = \mathcal{E}_x\{\langle x_n - z_n w^T, x_n - z_n w^T \rangle\}$$

$$= \mathcal{E}_x\{x_n x_n^T - 2 x_n (z_n w^T)^T + z_n w^T (z_n w^T)^T\}; \quad x_n \in \mathbb{R}^{1\times P}$$

$$w \in \mathbb{R}^{P\times 1}$$

$$= \mathcal{E}_x\{x_n x_n^T - 2 x_n w w^T x_n^T + x_n w \underbrace{w^T w}_{1} w^T x_n^T\}; \quad w^T w = 1$$

$$= \mathcal{E}_x\{x_n x_n^T - 2 x_n w w^T x_n^T + x_n w w^T x_n^T\}$$

$$= \mathcal{E}\{x_n x_n^T - x_n w w^T x_n^T\}$$

$$\min_w \mathcal{E}_x\{\|x_n - \hat{x}_n\|_2^2\} = \min_w -\mathcal{E}_x\{x_n w w^T x_n^T\}$$

s.t. $w^T w = 1$

$z_n = x_n w$.

$$\min_w -\mathcal{E}_x\{z_n z_n^T\} = -\mathcal{E}_x\{z_n^T z_n\} \qquad z_n \in \mathbb{R}$$

$$= -\mathcal{E}_x\{w^T x_n^T x_n w\} = -w^T \mathcal{E}\{x_n^T x_n\} w; \quad \mathcal{E}_x\{x_n^T x_n\} = R_{xx} \in \mathbb{R}^{P\times P}$$

$$\min_w -w^T \Sigma_x w \quad \text{s.t. } w^T w = 1, \; z_n = x_n w. \qquad = \max_w w^T \Sigma_x w \quad \text{s.t. } w^T w = 1$$

$$L(w, \lambda) = w^T \Sigma_x w - \lambda (w^T w - 1); \quad \frac{\partial L}{\partial w} = 2\Sigma_x w - 2\lambda w^T w = 0$$

$$\boxed{\Sigma_x w = \lambda w} \rightarrow \text{eig}$$

$$\boxed{\begin{aligned} &w^T \Sigma_x w = \lambda w^T w = \lambda \\ &\mathcal{E}\{w^T x_n^T x_n w\} = \mathcal{E}\{z_n^T z_n\} = \sigma_z^2 = \lambda \end{aligned}}$$