Q5

(B)

Data ID	x_1	x_2	y
X^1	0	2	-1
X^2	2	0	-1
X^3	0	0	1
X^4	2	2	1

(i) Kernel Function

$K(X^i, X^j) = (1 + X^i . X^j)^2 ...(1)$

We know $X^i . X^j = [X_1^i, X_2^i] . [X_1^j, X_2^j] = X_1^i . X_1^j + X_2^i . X_2^j$

$(1 + X^i . X^j)^2$

$= 1 + (X^i . X^j)^2 + 2.(X^i . X^j)$

$= 1 + (X_1^i . X_1^j + X_2^i . X_2^j)^2 + 2.(X_1^i . X_1^j + X_2^i . X_2^j)$

$= 1 + (X_1^i . X_1^j)^2 + (X_2^i . X_2^j)^2 + 2.(X_1^i . X_1^j . X_2^i . X_2^j) + 2.(X_1^i . X_1^j) + 2.(X_2^i . X_2^j) ...(2)$

We know $X^i . X^j = \phi(X^i) . \phi(X^j) ...(3)$

From (1), (2) and (3),

$\phi(X^i) = 1 + (X_1^i)^2 + (X_2^i)^2 + \sqrt{2}.(X_1^i . X_2^i) + \sqrt{2}.(X_1^i) + \sqrt{2}.(X_2^i)$

$\phi(X^j) = 1 + (X_1^j)^2 + (X_2^j)^2 + \sqrt{2}.(X_1^j . X_2^j) + \sqrt{2}.(X_1^j) + \sqrt{2}.(X_2^j)$

$\phi(v) = < 1, v_1^2, v_2^2, \sqrt{2}v_1 v_2, \sqrt{2}v_1, \sqrt{2}v_2 >$

(ii) Transformed Space

Data ID	1	x_1^2	x_2^2	$\sqrt{2}x_1x_2$	$\sqrt{2}x_1$	$\sqrt{2}x_2$	y
X^1	1	0	4	0	0	$2\sqrt{2}$	-1
X^2	1	4	0	0	$2\sqrt{2}$	0	-1
X^3	1	0	0	0	0	0	1
X^4	1	4	4	$4\sqrt{2}$	$2\sqrt{2}$	$2\sqrt{2}$	1