

# Q5

## (B)

Data ID	$x_1$	$x_2$	$y$
$X^1$	0	2	-1
$X^2$	2	0	-1
$X^3$	0	0	1
$X^4$	2	2	1

### (i) Kernel Function

$$K(X^i, X^j) = (1 + X^i \cdot X^j)^2 \dots (1)$$

$$\text{We know } X^i \cdot X^j = [X_1^i, X_2^i] \cdot [X_1^j, X_2^j] = X_1^i \cdot X_1^j + X_2^i \cdot X_2^j$$

$$(1 + X^i \cdot X^j)^2$$

$$= 1 + (X^i \cdot X^j)^2 + 2 \cdot (X^i \cdot X^j)$$

$$= 1 + (X_1^i \cdot X_1^j + X_2^i \cdot X_2^j)^2 + 2 \cdot (X_1^i \cdot X_1^j + X_2^i \cdot X_2^j)$$

$$= 1 + (X_1^i \cdot X_1^j)^2 + (X_2^i \cdot X_2^j)^2 + 2 \cdot (X_1^i \cdot X_1^j \cdot X_2^i \cdot X_2^j) + 2 \cdot (X_1^i \cdot X_1^j) + 2 \cdot (X_2^i \cdot X_2^j) \dots (2)$$

$$\text{We know } X^i \cdot X^j = \phi(X^i) \cdot \phi(X^j) \dots (3)$$

From (1), (2) and (3),

$$\phi(X^i) = 1 + (X_1^i)^2 + (X_2^i)^2 + \sqrt{2} \cdot (X_1^i \cdot X_2^i) + \sqrt{2} \cdot (X_1^i) + \sqrt{2} \cdot (X_2^i)$$

$$\phi(X^j) = 1 + (X_1^j)^2 + (X_2^j)^2 + \sqrt{2} \cdot (X_1^j \cdot X_2^j) + \sqrt{2} \cdot (X_1^j) + \sqrt{2} \cdot (X_2^j)$$

$$\phi(v) = \langle 1, v_1^2, v_2^2, \sqrt{2}v_1v_2, \sqrt{2}v_1, \sqrt{2}v_2 \rangle$$

### (ii) Transformed Space

Data ID	1	$x_1^2$	$x_2^2$	$\sqrt{2}x_1x_2$	$\sqrt{2}x_1$	$\sqrt{2}x_2$	$y$
$X^1$	1	0	4	0	0	$2\sqrt{2}$	-1
$X^2$	1	4	0	0	$2\sqrt{2}$	0	-1
$X^3$	1	0	0	0	0	0	1
$X^4$	1	4	4	$4\sqrt{2}$	$2\sqrt{2}$	$2\sqrt{2}$	1