Problem 2 Hierarchical Clustering

Euclidean Distance Matrix:

| | А | В | С | D | E | F | G | Н | I | J |
|---|------|------|------|------|------|------|------|------|-------|-------|
| А | 0.00 | 7.00 | 4.12 | 5.39 | 3.16 | 3.61 | 6.40 | 6.08 | 8.06 | 10.63 |
| В | | 0.00 | 3.16 | 2.83 | 8.54 | 5.83 | 5.83 | 8.49 | 11.31 | 8.00 |
| С | | | 0.00 | 1.41 | 5.39 | 2.83 | 4.00 | 5.83 | 8.60 | 7.62 |
| D | | | | 0.00 | 6.08 | 3.16 | 3.16 | 5.66 | 8.49 | 6.32 |
| Е | | | | | 0.00 | 3.00 | 5.39 | 3.61 | 5.00 | 9.43 |
| F | | | | | | 0.00 | 2.83 | 3.16 | 5.83 | 7.07 |
| G | | | | | | | 0.00 | 3.16 | 5.83 | 4.24 |
| Н | | | | | | | | 0.00 | 2.83 | 6.32 |
| I | | | | | | | | | 0.00 | 8.00 |
| J | | | | | | | | | | 0.00 |

(a)

Single Link Hierarchical Clustering:

(Similarity of two clusters is based on the two most similar closest points in different clusters)

Step 1: {C} will merge with {D} at 1.41

| | А | В | C,D | Е | F | G | Н | I | J |
|-----|------|------|------|------|------|------|------|-------|-------|
| Α | 0.00 | 7.00 | 4.12 | 3.16 | 3.61 | 6.40 | 6.08 | 8.06 | 10.63 |
| В | | 0.00 | 2.83 | 8.54 | 5.83 | 5.83 | 8.49 | 11.31 | 8.00 |
| C,D | | | 0.00 | 5.39 | 2.83 | 3.16 | 5.66 | 8.49 | 6.32 |
| E | | | | 0.00 | 3.00 | 5.39 | 3.61 | 5.00 | 9.43 |
| F | | | | | 0.00 | 2.83 | 3.16 | 5.83 | 7.07 |
| G | | | | | | 0.00 | 3.16 | 5.83 | 4.24 |
| Н | | | | | | | 0.00 | 2.83 | 6.32 |
| I | | | | | | | | 0.00 | 8.00 |
| J | | | | | | | | | 0.00 |

Step 2: {C,D} will merge with {B} at 2.83

Here 2.83 distance is same for {C,D} and {B} & {H} and {I}, so I have chosen the first one.

| | А | C,D,B | Е | F | G | Н | I | J |
|-------|------|-------|------|------|------|------|------|-------|
| Α | 0.00 | 4.12 | 3.16 | 3.61 | 6.40 | 6.08 | 8.06 | 10.63 |
| C,D,B | | 0.00 | 5.39 | 2.83 | 3.16 | 5.66 | 8.49 | 6.32 |
| Е | | | 0.00 | 3.00 | 5.39 | 3.61 | 5.00 | 9.43 |
| F | | | | 0.00 | 2.83 | 3.16 | 5.83 | 7.07 |
| G | | | | | 0.00 | 3.16 | 5.83 | 4.24 |
| Н | | | | | | 0.00 | 2.83 | 6.32 |
| I | | | | | | | 0.00 | 8.00 |
| J | | | | | | | | 0.00 |

Step 3: {C,D,B} will merge with {F} at 2.83

| | А | C,D,B,F | Е | G | Н | 1 | J |
|---------|------|---------|------|------|------|------|-------|
| А | 0.00 | 3.61 | 3.16 | 6.40 | 6.08 | 8.06 | 10.63 |
| C,D,B,F | | 0.00 | 3.00 | 2.83 | 3.16 | 5.83 | 6.32 |
| Е | | | 0.00 | 5.39 | 3.61 | 5.00 | 9.43 |
| G | | | | 0.00 | 3.16 | 5.83 | 4.24 |
| Н | | | | | 0.00 | 2.83 | 6.32 |
| 1 | | | | | | 0.00 | 8.00 |
| J | | | | | | | 0.00 |

Step 4: {C,D,B,F} will merge with {G} 2.83

| | А | C,D,B,F,G | Е | Н | I | J |
|-----------|------|-----------|------|------|------|-------|
| А | 0.00 | 3.61 | 3.16 | 6.08 | 8.06 | 10.63 |
| C,D,B,F,G | | 0.00 | 3.00 | 3.16 | 5.83 | 4.24 |
| E | | | 0.00 | 3.61 | 5.00 | 9.43 |
| Н | | | | 0.00 | 2.83 | 6.32 |
| I | | | | | 0.00 | 8.00 |
| J | | | | | | 0.00 |

Step 5: {H} will merge with {I} 2.83

| | А | C,D,B,F,G | Е | H,I | J |
|-----------|------|-----------|------|------|-------|
| А | 0.00 | 3.61 | 3.16 | 6.08 | 10.63 |
| C,D,B,F,G | | 0.00 | 3.00 | 3.16 | 4.24 |
| E | | | 0.00 | 3.61 | 9.43 |
| H,I | | | | 0.00 | 6.32 |
| J | | | | | 0.00 |

Step 5: {C,D,B,F,G} will merge with {E} 3.00

| | А | C,D,B,F,G,E | H,I | J |
|-------------|------|-------------|------|-------|
| Α | 0.00 | 3.16 | 6.08 | 10.63 |
| C,D,B,F,G,E | | 0.00 | 3.16 | 4.24 |
| H,I | | | 0.00 | 6.32 |
| J | | | | 0.00 |

Step 6: {C,D,B,F,G,E} will merge with {H,I} at height 3.16

| | А | C,D,B,F,G,E,H,I | J |
|-----------------|------|-----------------|-------|
| A | 0.00 | 3.16 | 10.63 |
| C,D,B,F,G,E,H,I | | 0.00 | 4.24 |
| J | | | 0.00 |

Step 7: {C,D,B,F,G,E,H,I} will merge with {A} at height 3.16

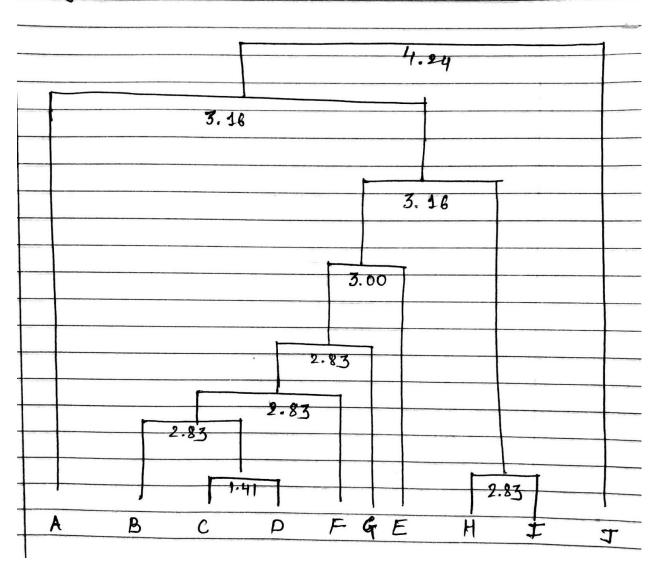
| | C,D,B,F,G,E,H,I,A | J |
|-------------------|-------------------|------|
| C,D,B,F,G,E,H,I,A | 0.00 | 4.24 |
| J | | 0.00 |

Step 8: {C,D,B,F,G,H,I,A} will merge with {J} at height 4.24

Dendrogram:

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Single Link



Complete Link Hierarchical Clustering:

(Similarity of two clusters is based on the two **least** similar (most distant) points in different clusters)

Step 1: {C} will merge with {D} at 1.41

| | А | В | C,D | E | F | G | Н | I | J |
|-----|------|------|------|------|------|------|------|-------|-------|
| Α | 0.00 | 7.00 | 5.39 | 3.16 | 3.61 | 6.40 | 6.08 | 8.06 | 10.63 |
| В | | 0.00 | 3.16 | 8.54 | 5.83 | 5.83 | 8.49 | 11.31 | 8.00 |
| C,D | | | 0.00 | 6.08 | 3.16 | 4.00 | 5.83 | 8.60 | 7.62 |
| E | | | | 0.00 | 3.00 | 5.39 | 3.61 | 5.00 | 9.43 |
| F | | | | | 0.00 | 2.83 | 3.16 | 5.83 | 7.07 |
| G | | | | | | 0.00 | 3.16 | 5.83 | 4.24 |
| Н | | | | | | | 0.00 | 2.83 | 6.32 |
| I | | | | | | | | 0.00 | 8.00 |
| J | | | | | | | | | 0.00 |

Step 2: {G} will merge with {F} at 2.83

| | А | В | C,D | E | G,F | Н | I | J |
|-----|------|------|------|------|------|------|-------|-------|
| Α | 0.00 | 7.00 | 5.39 | 3.16 | 6.40 | 6.08 | 8.06 | 10.63 |
| В | | 0.00 | 3.16 | 8.54 | 5.83 | 8.49 | 11.31 | 8.00 |
| C,D | | | 0.00 | 6.08 | 4.00 | 5.83 | 8.60 | 7.62 |
| Е | | | | 0.00 | 5.39 | 3.61 | 5.00 | 9.43 |
| G,F | | | | | 0.00 | 3.16 | 5.83 | 7.07 |
| Н | | | | | | 0.00 | 2.83 | 6.32 |
| I | | | | | | | 0.00 | 8.00 |
| J | | | | | | | | 0.00 |

Step 4: {H} will merge with {I} at 2.83

| | А | В | C,D | E | G,F | H,I | J |
|-----|------|------|------|------|------|-------|-------|
| А | 0.00 | 7.00 | 5.39 | 3.16 | 6.40 | 8.06 | 10.63 |
| В | | 0.00 | 3.16 | 8.54 | 5.83 | 11.31 | 8.00 |
| C,D | | | 0.00 | 6.08 | 4.00 | 8.60 | 7.62 |
| Е | | | | 0.00 | 5.39 | 5.00 | 9.43 |
| G,F | | | | | 0.00 | 5.83 | 7.07 |
| H,I | | | | | | 0.00 | 8.00 |
| J | | | | | | | 0.00 |

Step 5: {A} will merge with {E} at 3.16

| | A,E | В | C,D | G,F | H,I | J |
|-----|------|------|------|------|-------|-------|
| A,E | 0.00 | 8.54 | 5.39 | 6.40 | 8.06 | 10.63 |
| В | | 0.00 | 3.16 | 5.83 | 11.31 | 8.00 |
| C,D | | | 0.00 | 4.00 | 8.60 | 7.62 |
| G,F | | | | 0.00 | 5.83 | 7.07 |
| H,I | | | | | 0.00 | 8.00 |
| J | | | | | | 0.00 |

Step 6: {B} will merge with {C,D} at 3.16

| | A,E | B,C,D | G,F | H,I | J |
|-------|------|-------|------|-------|-------|
| A,E | 0.00 | 8.54 | 6.40 | 8.06 | 10.63 |
| B,C,D | | 0.00 | 5.83 | 11.31 | 8.00 |
| G,F | | | 0.00 | 5.83 | 7.07 |
| H,I | | | | 0.00 | 8.00 |
| J | | | | | 0.00 |

Step 7: {B,C,D} will merge with {G,F} at 5.83

| | A,E | B,C,D,F,G | H,I | J |
|-----------|------|-----------|-------|-------|
| A,E | 0.00 | 8.54 | 8.06 | 10.63 |
| B,C,D,F,G | | 0.00 | 11.31 | 8.00 |
| H,I | | | 0.00 | 8.00 |
| J | | | | 0.00 |

Step 7: {B,C,D,F,G} will merge with {J} at 8.00

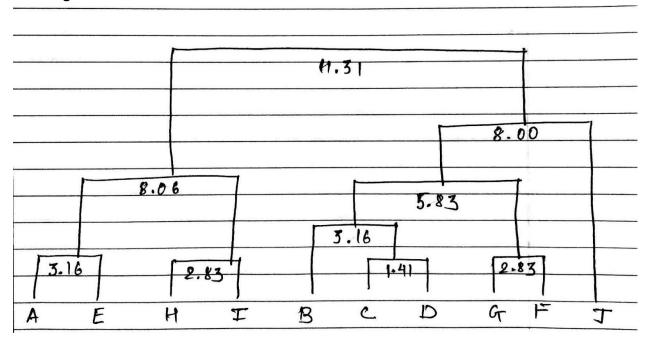
| | A,E | B,C,D,F,G,J | H,I |
|-------------|------|-------------|-------|
| A,E | 0.00 | 10.63 | 8.06 |
| B,C,D,F,G,J | | 0.00 | 11.31 |
| H,I | | | 0.00 |

Step 8: {A,E} will merge with {H,I} at 8.06

| | A,E,H,I | B,C,D,F,G,J |
|-------------|---------|-------------|
| A,E,H,I | 0.00 | 11.31 |
| B,C,D,F,G,J | | 0.00 |

Step 9: {A,E,H,I} will merge with {B,C,D,F,G,J} at 11.31

Dendrogram:



(b) If we assume there are three clusters, which of the single and complete link hierarchical clustering will give better resulted clusters?

Single Link Hierarchical:

3 Clusters: {B,C,D,E,G,H,I}, {J} and {A}

Single Link SSE: 107.875 ((Calculated in Q2.pynb)

Complete Link Hierarchical:

3 Clusters: {B,C,D,G,F}, {J} and {A,E,H,I}

SSE: 72.0 (Calculated in Q2.pynb)

As we can visualize from dendrogram and cluster formation, Single link tends to provide elongated clusters whereas complete link short and circular clusters.

Conclusion: If there are three clusters, based on Sum of Squared Error calculated and dendrogram, Complete link gives better result.

(c) Compare your resulted clusters from 2(b) with the resulted clusters using K-means in Question 1 by calculating their corresponding Sum of Squared Error (SSE). Based on their SSE results, which resulted clusters, 1(b) or 2(b), are better?

From Q1, SSE for K-Means Clustering is: 60.8333333333333 (Calculated in Q1.pynb)

Comparing results with Q1(b) and Q2(b),

K-Means provide better results, as it provides least Sum of Squared Errors.